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A NOTE ON THE SMITH PREDICTOR

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A note on the Smith predictor.

Abstract

The problem of parametrization of the Smith predictor is adressed. It is shown that the controller can be conveniently parametrized in terms of the desired closed-loop system dynamics. Servo properties and disturbance rejection ability of the control system are also investigated. The robustness of the closed-loop system is analysed, and the result indicates how a robust control design can be achieved.

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Contents

1. INTRODUCTION

Many technical and industrial processes have time delays. Common examples on processes where time delays appear naturally, are systems with product transports and material flows. For these systems the feature is also referred to as transport delay. Physically, a time delay in a system can be visualized as the time it takes before a change in the control signal is observed in the output measurement.

Processes with time delays are non-minimum-phase and infinite dimensional systems. Control engineers noticed early that these systems are quite difficult to control with conventional PID regulators.

The invention of the Smith predictor, or controller, by Otto Smith in 1925, was a real break-through for control theory. This regulator has the property that it is able to control time delay systems. The tuning of a Smith controller is as simple as the tuning of a conventional regulator. A limitation of the regulator is that it is restricted to stable processes. It has another difficulty associated with it as well - a model of the process time delay has to be included in the controller. Time delays are difficult to implement using analog techniques, and consequently the Smith controller is almost exclusively found in computer based control systems.

The Smith predictor has been subject to much analysis. Among recent publications we will mention a few. In Åström (1977) it is shown that the controller gives a large amount of phase advance, a feature which is necessary in the control of systems with time delays. The sensitivity of control systems incorporating Smith predictors, is examined in Ioannides et al (1979). Different aspects of implementation and internal properties are presented in Bengtsson and Egardt (1978).

The present report is concerned with the parametrization of the controller. It is shown that the Smith predictor can be parametrized conveniently in terms of the desired closed-loop system dynamics. This alternative representation facilitates tuning and design. A robustness result for the controller, based on the general results in Mannerfelt (1981), is also presented. It is shown that in reasonable designs, the closed-loop system bandwidth can not be much higher than that of the open-loop system.

The paper is organized as follows: In Section 2 the Smith predictor is presented in its usual form. Section 3 is devoted to the derivation of the alternative

parametrization. Some properties of the control system, e.g.
robustness, are discussed in Section 4. The conclusions are
found in Section 5, and Section 6 contains the references.

2. THE SMITH PREDICTOR

In this section we will briefly discuss the Smith predictor in its originally proposed form. As was previously mentioned, conventional control of processes with time delays is not a simple matter. Such a control system configuration is shown in Fig. 2.1. If the controller is of the usual P, PI or PID types, the system performance is often of very limited quality. The regulator should remember the history of old control actions in order to improve performance. This feature is included in the Smith predictor.

The configuration of a control system using a Smith predictor is shown in Fig. 2.2. The process to be controlled is assumed to be linear, time-invariant and stable, and it has the transfer function $G_P(s)$. The Smith controller is shown inside the dotted rectangle. It consists of several parts: a model of the process dynamics $G_M(s)$ and a model of the process time delay $\exp(-sT_M)$. There is also a block marked $G_R(s)$, which represents a regulator designed to control the plant model without time delay, i.e. $G_M(s)$. The regulator $G_R(s)$ should thus be such that the corresponding closed-loop model system
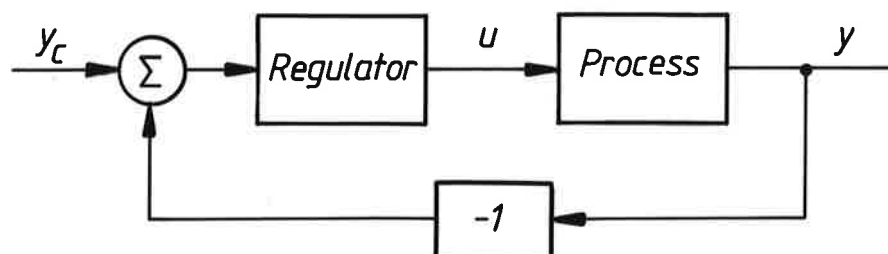

Fig. 2.1 - A conventional control system.

$$H_M(s) = \frac{G_R(s)G_M(s)}{1 + G_R(s)G_M(s)} \qquad (2.1)$$

has desirable properties, such as stability, acceptable bandwidth and sufficient damping etc. Often $G_R(s)$ is chosen as a PI or PID controller. The tuning of $G_R(s)$ can then be made according to standard rules, wellknown to process operators and control engineers. The originally difficult control problem is thus transferred to a simple standard case by using the Smith predictor.

It can be shown that the Smith controller cancels all the poles of the open-loop system. In some control problems it is not suitable to use controllers with this property, and consequently the Smith predictor is not always the best choice. More about this and other theoretical results concerning the Smith predictor can be found in Åström (1982), and in the references mentioned in the introduction.

In the following section, we will rearrange the configuration of the Smith predictor as it is shown in Fig. 2.2. This leads to an alternative parametrization that has some appealing properties.
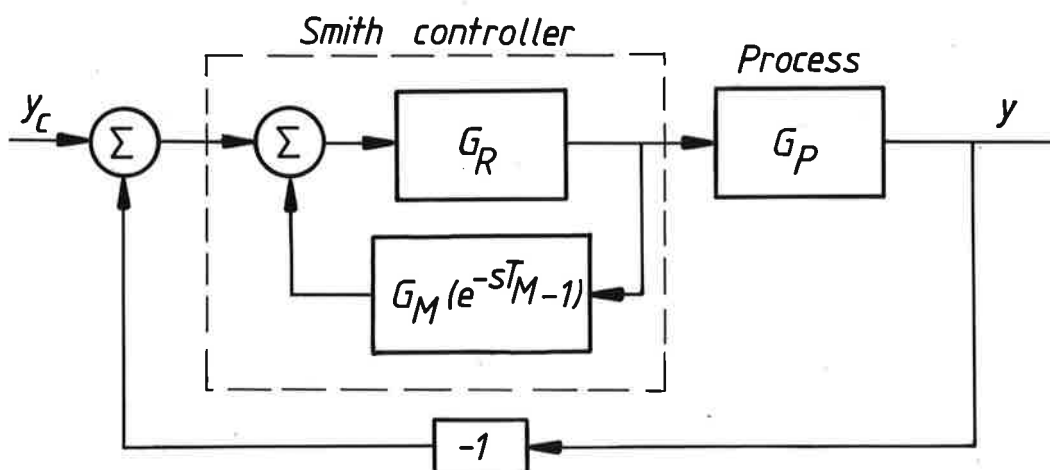

Fig. 2.2 - A control system using a Smith controller.

3. AN ALTERNATIVE PARAMETRIZATION

An alternative representation of the Smith controller will be derived. Similar ideas have previously been used e.g. in the area of Model Reference Control, and in the design of Self-tuning Poleplacement regulators, see Åström et al (1978). The discussion is restricted to the discrete time case, since the sampled version of the controller is far more common and important than the one in continuous time.

Let the sampled process dynamics have the pulse transfer operator

$$G_P(q^{-1}) \qquad (3.1)$$

where q^{-1} is the backward shift operator that represents a time delay of one sample interval h. A discrete time model of the process is given by

$$G_M(q^{-1})q^{-k} \qquad (3.2)$$

where $G_M(\infty) \neq 0$, i.e. $G_M(q^{-1})$ contains a direct feed-through term. The factor q^{-k} is the pulse transfer operator of the time delay model, and consequently $T_M = k \cdot h$.

A block diagram of the discrete time control system is analogous to the one shown in Fig. 2.2. The regulator G_R is designed to control the process model without time delay G_M.

To derive the alternative regulator parametrization, introduce the desired closed-loop model system

$$H_M = \frac{G_R G_M}{1 + G_R G_M} \qquad (3.3)$$

The transfer function $H_M(z^{-1})$ should have, in addition to suitable dynamics, a unity steady state gain, i.e. $H_M(1) = 1$.

It is now straightforward to rearrange the control system according to Fig. 3.1. In this configuration the control signal is computed as

$$u = H_M [G_M^{-1}(y_c - y) + q^{-k}u] \qquad (3.4)$$

The regulator can thus be parametrized only in terms of the process model and the desired closed-loop system H_M.

When using the usual Smith controller configuration, shown in Fig. 2.2, the structure of the regulator G_R has to be specified a priori. The parameters of G_R must be either tuned to properly control G_M, or computed to give the resulting closed-loop system H_M suitable properties. In most applications G_R is chosen as a PI or PID controller, which in fact is an unnecessary limitation.

The alternative representation proposed in this paper, is parametrized in terms of the desired closed-loop system. This gives a process operator new possibilities to directly on line specify, or alter, the closed-loop system performance. There is no longer any need to bother about the structure of the "internal" regulator G_R. For example, it is possible to specify a 2:nd order closed-loop model H_M as in the Bandwidth Design method, see Aström (1979). Such a model contains only two parameters, bandwidth and damping. The corresponding regulator G_R may have many more parameters if
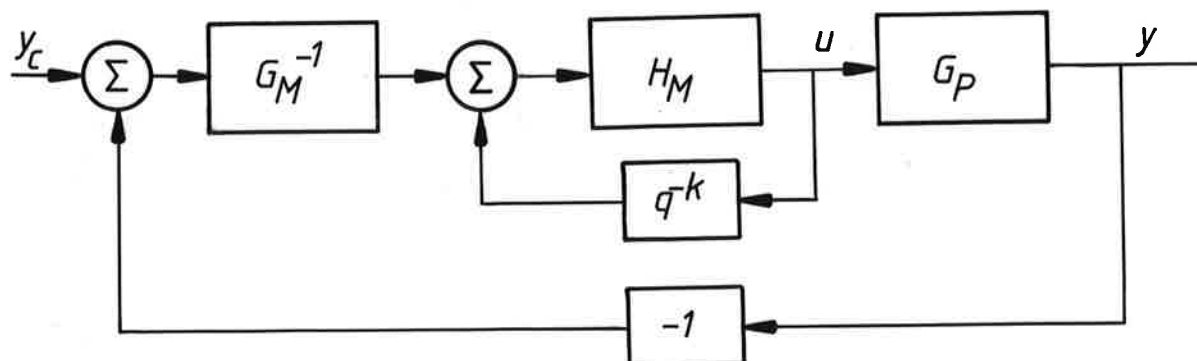

Fig. 3.1 - An alternative representation of the system.

the model G_M is of high order.

It is intuitively appealing to see that the regulator (3.4)
can be interpreted in terms of model following control. The
control signal is the sum of a feed-forward signal from the
command input, and a feed-back signal from the output error
between the model and the process:

$$u = G_M^{-1} H_M y_c + G_M^{-1} H_M [G_M q^{-k} u - y] \qquad (3.5)$$

The regulator (3.4) uses the inverse process model G_M^{-1}.
This transfer function is unstable if the process model G_M

is non-minimum phase, i.e. if it has zeros outside the unit
circle. In order to avoid this problem, it is necessary to
include the non-minimum phase zeros of G_M in H_M. The

combined transfer function $G_M^{-1} H_M$, relevant in the regulator
realization (3.4) is then stable, and the problem is solved.

A short summary of the previous results: the Smith predictor
can be parametrized as (3.4), where the non-minimum phase
zeros of G_M are also restricted to be zeros of H_M. The

corresponding factors must be cancelled <u>before</u> the
controller is implemented in order to maintain stability.
This is a feature that is in common for all control system
designs, and nothing special for the Smith controller. The
proposed parametrization is well suited for modern process
control, and facilitate regulator tuning for the process
operator. In the next section, some properties of the
control system will be analysed.

4. PROPERTIES

In this section we will investigate some properties of a
control system, using a Smith predictor. The results of
course are independent of the parametrization of the
controller. However, they can conveniently be expressed in
terms of the process model without time delay G_M, and the

desired closed-loop transfer function H_M.

Servo_properties

The Smith predictor is used mainly when solving two
particular problems: to give the control system a specified
servo performance, and to eliminate steady state and slow
disturbances. To investigate the servo properties, we form
the input-output relation of the closed-loop system. The
open-loop relation

$$y = G_P u \tag{4.1}$$

and the control law

$$u = H_M [G_M^{-1}(y_c - y) + q^{-k} u] \tag{4.2}$$

together give

$$y = \frac{G_P H_M G_M^{-1}}{1 + G_P H_M G_M^{-1} - H_M q^{-k}} y_c \tag{4.3}$$

If the process model is correct, i.e. $G_P = G_M q^{-k}$, this
relation becomes the ideal

$$y = H_M q^{-k} y_c \tag{4.4}$$

This can not be expected in general, since there are always

modelling errors in G_M. It is however interesting to see that the closed-loop transfer function in (4.3) always has unity steady state gain:

$$H(1) = \frac{G_P(1)H_M(1)G_M(1)^{-1}}{1 + G_P(1)H_M(1)G_M(1)^{-1} - H_M(1)} =$$

$$= \frac{G_P(1)G_M(1)^{-1}}{G_P(1)G_M(1)^{-1}} = 1 \qquad (4.5)$$

where it has been used that $H_M(1) = 1$. This indicates that the servo properties are acceptable at least for low frequencies. The reason why (4.5) is true irrespectively of G_P, is that the Smith controller has integral action.

Disturbance rejection

The other task of the regulator is to eliminate constant and slowly drifting disturbances. When analyzing this property, it is convenient to utilize the linearity of the control system, and with no loss of generality we regard the command input y_c as being zero. Assume that the process is described by the relation

$$y = G_P u + v \qquad (4.6)$$

where v is a disturbance. The control signal is given by equation (4.2). The output of the control system is then given by

$$y = \frac{1 - H_M q^{-k}}{1 + G_P H_M G_M^{-1} - H_M q^{-k}} v \qquad (4.7)$$

Because of the integral action of the controller it follows that constant disturbances are totally eliminated in the

output. At higher frequencies, however, the relation (4.7) shows that disturbances may in fact be amplified by the regulator. Suppose for example that the process model is correct, i.e. $G_M q^{-k} = G_P$. The relation (4.7) in this case becomes

$$y = [1 - H_M q^{-k}] \, v \tag{4.8}$$

For some frequency ω, the magnitude of the corresponding transfer function will be $1 + |H_M|$ which obviously is larger than unity.

Another measure of the output due to the disturbance is

$$\|y\|_\infty \leq \frac{1 + \|H_M\|_1}{1 - \|G_P H_M G_M^{-1} - H_M q^{-k}\|_1} \|v\|_\infty \tag{4.9}$$

Here $\|\cdot\|$ is a notation for signal and operator norm. A necessary condition for acceptable disturbance rejection is that the norm

$$\|G_P H_M G_M^{-1} - H_M q^{-k}\|_1 \tag{4.10}$$

is small. This is implied if either the modelling error

$$\|G_P - G_M q^{-k}\|_1 \tag{4.11}$$

is small, or if the norm of the ratio between the closed-loop and open-loop model transfer functions

$$\|H_M G_M^{-1}\|_1 \tag{4.12}$$

is small. We thus conclude, that if the disturbances at the process output have considerable contents at high frequencies, it may be important either to have a good process model or to reduce the gain of the desired closed-loop system.

It follows from equation (4.2) that the poles of the regulator are the zeros of the transfer function

$$G_M H_M^{-1} [1 - z^{-k} H_M]$$

(4.13)

The regulators ability to suppress slow disturbances, is mainly determined by its low frequency gain. It is possible to impose restrictions on H_M, such that the transfer function

$$1 - z^{-k} H_M$$

(4.14)

has several zeros at $z = 1$. In this way the regulator may be designed to have a suitable number of integrators.

Robustness

We saw previously, that in order to get acceptable system performance with high frequency disturbances, either the modelling error or the gain of the closed-loop system had to be small. These items are also important for the stability of the closed-loop system. We analyse the robustness properties of the control system with the general robustness results in Mannerfelt (1981).

Since both the process G_P and the process model $G_M q^{-k}$ are assumed to be stable, straightforward application of the robustness results to the configuration shown in Fig. 3.1, give that the closed-loop system is stable if the inequality

$$|G_P H_M G_M^{-1} - H_M z^{-k}| < 1$$

(4.15)

is satisfied on the unit circle $|z| = 1$. The inequality can also be expressed as

$$|G_P - G_M z^{-k}| < |G_M/H_M|$$

(4.16)

When both G_M and H_M are of the same magnitude it is thus possible to tolerate moderate modelling errors. At higher frequencies, where the modelling errors inevitably become

large, it is necessary not to demand too much of the desired closed-loop system H_M. Specifically, at the frequency interval where the process model $G_M q^{-k}$ is uncertain, the gain of H_M should be decreased considerably in order to guarantee that the robustness inequality is satisfied.

This result agrees well with the results we derived concerning disturbance rejection. The only thing that differs is that different norms are involved.

5. CONCLUSIONS

The problem of parametrization of the Smith predictor is adressed. An alternative representation of the controller, in terms of the desired closed-loop system, has been derived. The proposed regulator is simple to tune, especially if the desired model H_M has few parameters.

We have also briefly examined some properties of a control system using a Smith controller. Both the servo properties and the disturbance rejection are acceptable for low frequencies. At higher frequencies it may be necessary to have a good process model in order to get good control. The robustness properties of the control system have also been analysed. It was shown that at low and medium high frequencies, where usually the open-loop and desired closed-loop system transfer functions are of the same magnitude, the model accuracy is not critical. For higher frequencies where modelling errors become pronounced, it is necessary to decrease the gain of the closed-loop system in order to get robustness.

The Smith predictor is a simple and easily implemented controller, designed to solve a difficult control problem. It has several disadvantages, mainly a limited capacity to eliminate disturbances. There are alternative choices of more complex regulators with superior performance to solve the same problem, such as pole-placement regulators and observer models, see Åström et al (1978). Nevertheless, the simplicity of the Smith predictor, still makes it an economically interesting and attractive solution to the problem of time delay system control.

6. REFERENCES

Aström, K.J.(1977): Frequency domain properties of Otto Smith regulators. Int. J. Control 26, 307-314.

Aström, K.J., Westerberg, B.and Wittenmark, B.(1978): Self-Tuning Controllers Based on Pole-Placement Design.
Lund Inst. of Technology, Lund, Sweden.
CODEN: LUTFD2/(TFRT-3148)/1-052/(1978).

Aström, K.J.(1979): Simple Self-tuners I.
Lund Inst. of Technology, Lund, Sweden.
CODEN: LUTFD2/(TFRT-7184)/1-063/(1979).

Aström, K.J.(1982): Reglerteknik - en elementär introduktion. Kapitel 8. Dödtidskompensering. Lund Inst. of Technology, Lund, Sweden. (In Swedish). To be published.

Bengtsson, G. and Egardt, B.(1978): Internal properties and insensitive implementations of the Otto Smith regulator.
Lund Inst. of Technology, Lund, Sweden.
CODEN: LUTFD2/(TFRT-7133)/1-014/(1978).

Ioannides, A.C.,Rogers, G.C. and Latham, V.(1979): Stability Limits of a Smith Controller in Simple Systems Containing a Time Delay. Int. J. Control 26, 557-563.

Mannerfelt, C.F.(1981): Robust Control Design for Simplified Models. Lund Inst. of Technology, Lund, Sweden.
CODEN: LUTFD2/(TFRT-1026)/1-163/(1981).