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A NEW METHOD FOR DESIGN OF PID REGULATORS

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Abstract. The idea of conformal mapping is used to develop methods for analysis and design of PID control. It is first shown that the notions of amplitude margin and phase margin are too simplistic for design. The reason is that they are based on knowledge of one point on the Nyquist curve only. Simple methods for approximative determination of the dominant closed loop poles based on knowledge of two points on the Nyquist curve are first given. A design method based on knowledge of two points on the Nyquist curve is then presented, as well as techniques for determining these points automatically.

Contents

1. INTRODUCTION

The majority of the regulators used in industry are of the PI(D) type. A large industrial plant may have hundreds of them. Instrument engineers and plant personnel are used to select, install and operate such regulators. Many different methods have been proposed for tuning PI(D) regulators. The Ziegler-Nichols (1943) method is one of the more popular schemes. Many regulators are, however, poorly tuned. The purpose of this paper is to propose improved design methods for PI(D) control, which are based on knowledge of two points on the open loop Nyquist curve in the neighbourhood of the cross-over frequency. A powerful method for determining suitable points is also presented. The method is an extension of the technique presented in Åström and Hägglund (1983), which is based on determination of one point only of the Nyquist curve.

The paper is organized as follows. The notions of phase and amplitude margins are analysed in Section 2. It is shown that design specifications can not be expressed in terms of these margins only. Examples are given of systems with the same margins which have widely different transient responses. A technique for estimating the dominant closed loop poles from the open loop Nyquist curve is then presented. The estimate is based on the knowledge of a portion of the Nyquist curve close to the critical point $s = -1$. It is derived using a conformal mapping argument. Two points on the Nyquist curve are needed to estimate the dominant poles. It is demonstrated that the procedure gives the dominant closed loop poles with a reasonable accuracy. A design technique for positioning the dominant poles is presented in Section 3. Starting from the knowledge of two points on the Nyquist curve, a PID regulator which gives a closed loop system with dominant poles having specified relative damping and frequency is obtained. Methods for automatic determination of relevant points on the Nyquist curve are finally discussed in Section 4. A powerful scheme for automatic tuning of simple PID regulators is obtained by combining the estimation technique with the design methods discussed in Section 3. Computer programs related to the paper are enclosed in the Appendix.

2. ANALYSIS

This section is introduced by two examples, which show that the notions of amplitude margin and phase margin are too simple design criteria. The following analysis gives a general explanation, and indicates how to derive better criteria.

Example 1. Nyquist curves for three different systems are given in Figure 1, together with their corresponding closed loop step responses. The open loop transfer functions are

$$G_1(s) = \frac{1.4(1+0.5s)e^{-0.4s}}{s^2}$$

$$G_2(s) = \frac{0.072e^{-5s}}{s(1+5s)^2}$$

$$G_3(s) = \frac{1.65e^{-12s}}{1+20s}$$

All the systems have the amplitude margin $A_m = 2$. It is clear from the figure that there is a considerable variation in damping between the three systems. The values of the resonance peaks of the systems are $M_p = 17$, $M_p = 1.8$, and $M_p = 1.2$ respectively. The closed loop responses are obtained with unit feedback. □

Example 2. Nyquist curves for three different systems are shown in Figure 2, together with their corresponding closed loop step responses. The open loop transfer functions are

$$G_4(s) = \frac{1.25e^{-15s}}{(1+5s)^2}$$

$$G_5(s) = \frac{0.1}{s(1+5s)^2}$$

$$G_6(s) = \frac{1.4(1+1.2s)e^{-0.2s}}{s^2}$$

All the systems have the phases margin $\varphi_m = 45°$. The damping varies considerably between the three systems. The values of the resonance peaks of the systems are $M_p = 7$, $M_p = 1.4$, and $M_p = 1.5$ respectively. □

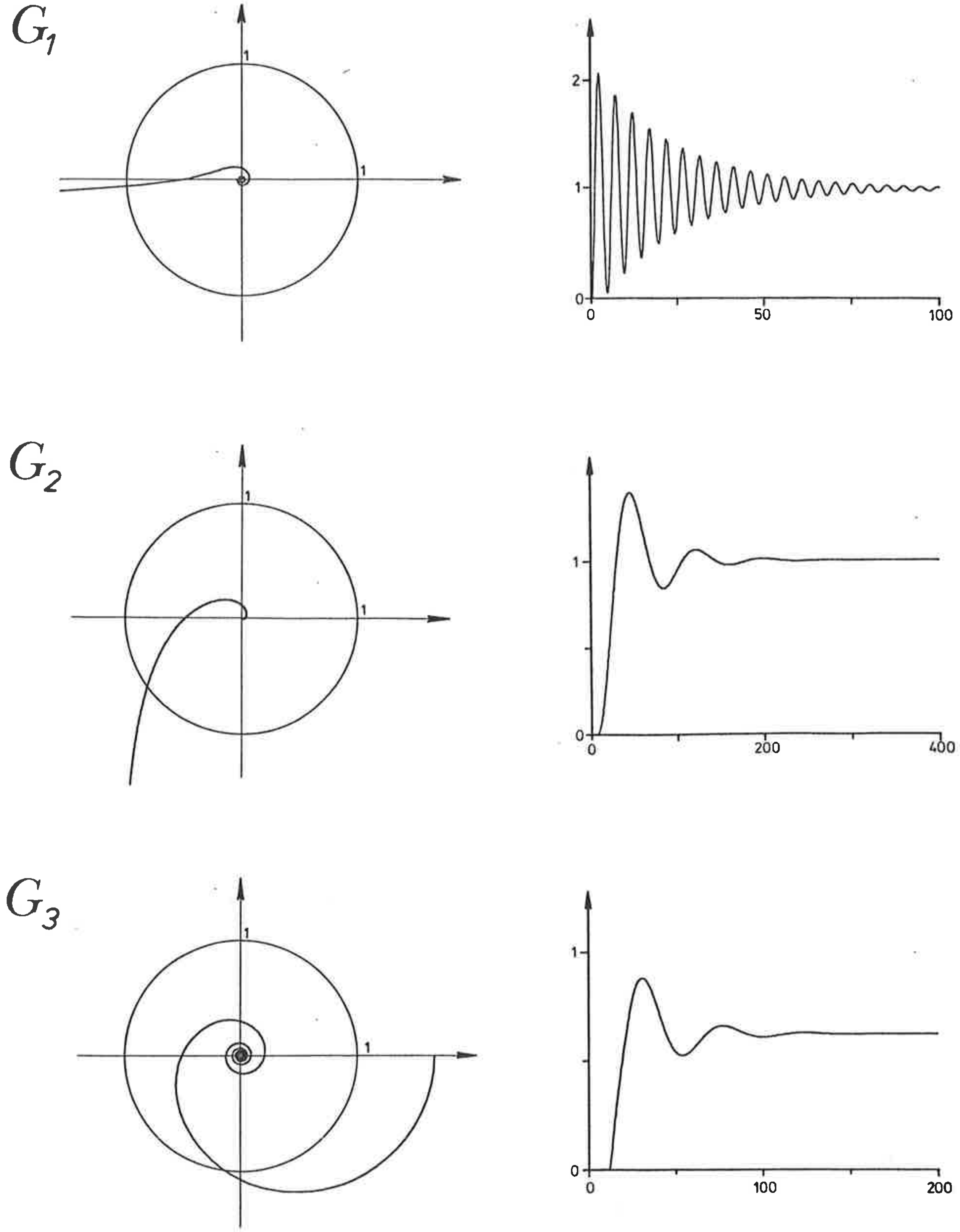

Figure 1. The Nyquist curves of the three systems with equal amplitude margin in Example 1, and their corresponding closed loop step responses.

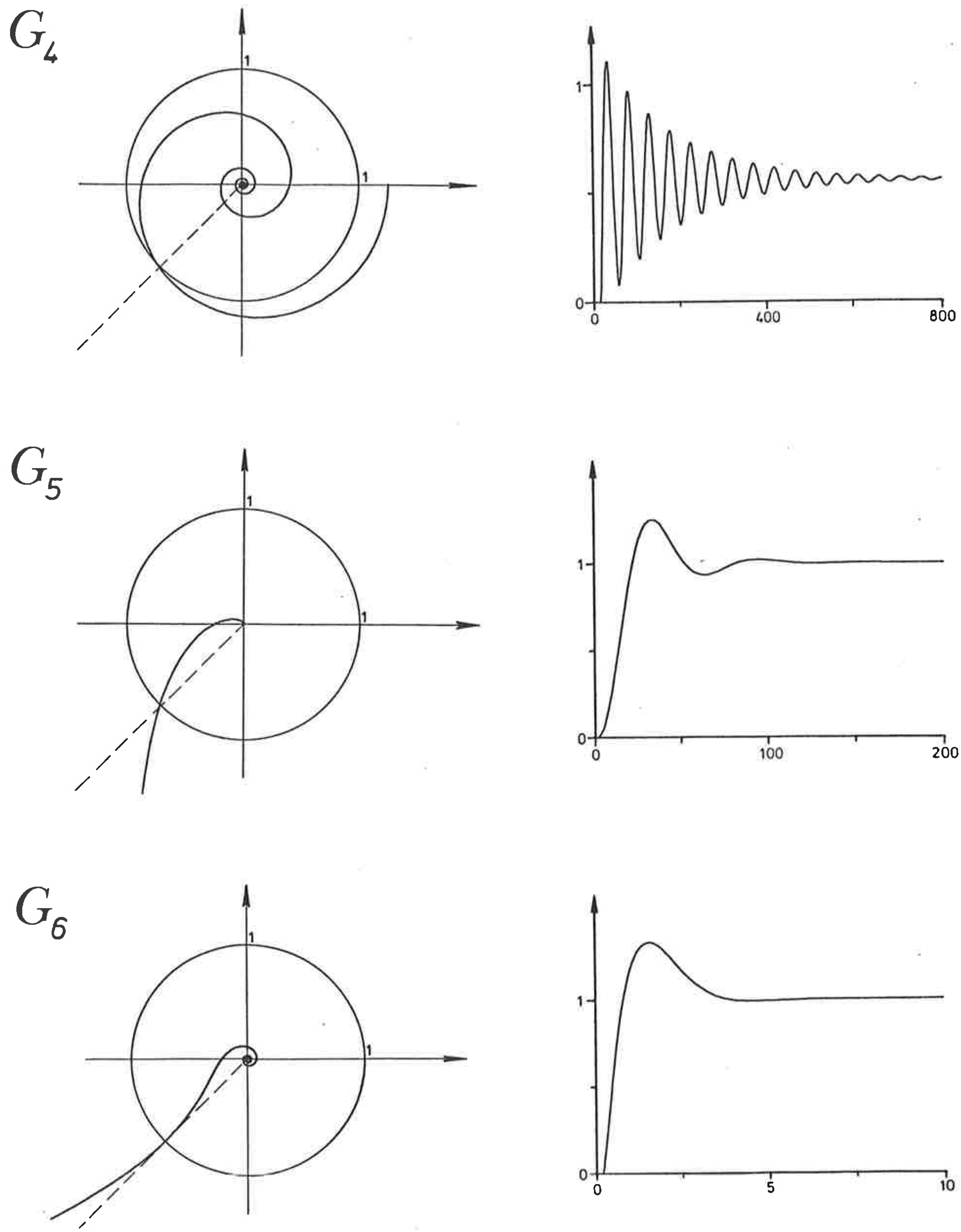

Figure 2. The Nyquist curves of the three systems with equal phase margin in Example 2, and their corresponding closed loop step responses.

The examples show clearly that the notions of amplitude margin and phase margin are too simple design criteria. The reason for this will now be demonstrated by analysis.

Analysis

The transfer function G(s) may be viewed as a conformal map from the complex s-plane to the complex G-plane. The Nyquist curve is the image in the G-plane of the positive imaginary axis in the s-plane. To judge the properties of the closed loop system, it is useful to know the poles of the closed loop system. They are the zeros of the characteristic equation

$$G(s) + 1 = 0 \tag{2.1}$$

i.e. the points in the s-plane which are mapped to G = -1.

To discuss the inverse map G^{-1}, it is useful to consider a manifold of several sheets in the s-plane. This is necessary because there are frequently several points in the s-plane which maps into G = -1. Physically this corresponds to the common situation when the closed loop system has many poles. For simple systems the response is, however, often dominated by a pair of complex poles which are called the dominant poles. A simple approximate method to determine the dominant poles from the Nyquist curve of a system will now be given.

Consider the map of a line parallel to the imaginary axis in the s-plane which goes through G = -1. See Figure 3. A point A′ on this line is obviously a root to the characteristic equation (2.1). Now consider the map A′B′C′ of the triangle ABC in the s-plane. This map is in general not a triangle because its sides will not be straight lines. If the points are sufficiently close together, it will however be close to a triangle. Under this assumption it is easy to determine the point A from knowledge of B′ and C′. Assume e.g. that B′ and C′ are chosen symmetrically at either side of the normal to the Nyquist curve through G = -1. Let the angle α be small and let B′ and C′ correspond to ω_1 and ω_2. The following approximate formulas are then obtained for the dominant poles.

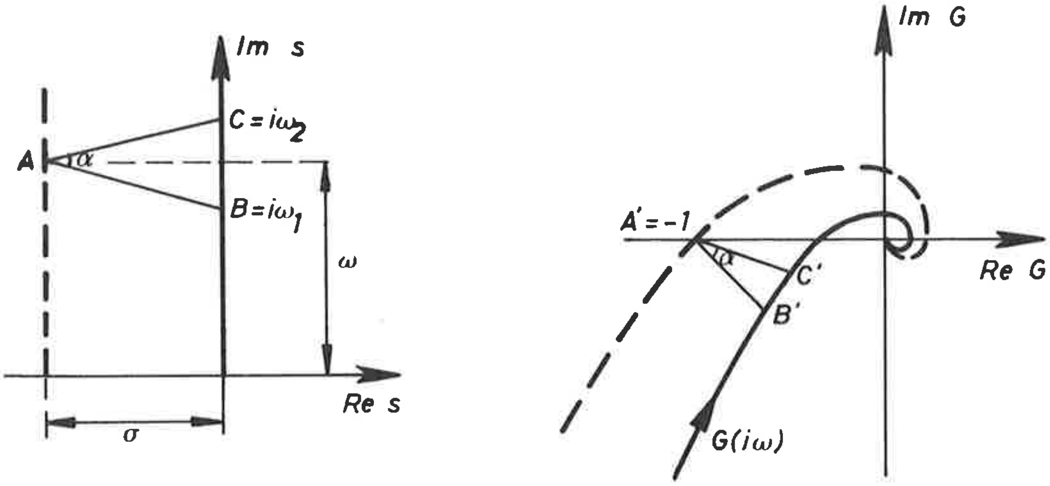

Figure 3. The conformal mapping from the s-plane to the G-plane.

$$\omega = \frac{1}{2}(\omega_1 + \omega_2) \tag{2.2}$$

$$\sigma = \frac{1}{\alpha}(\omega_2 - \omega_1) \tag{2.3}$$

The relative damping is given by

$$\zeta = \frac{\sigma}{\sqrt{\sigma^2 + \omega^2}} \tag{2.4}$$

It is clear from these formulas, that the damping of the dominant poles is determined not only by the shape of the Nyquist curve, but also by how rapidly ω changes along the Nyquist curve.

To get a feel for the accuracy of the approximate formulas, they will be applied to the systems in Example 1 and Example 2. For the Nyquist curves in Example 1 we get

$$G_1: \begin{cases} \sigma = 0.044 & (0.043) \\ \omega = 1.30 & (1.29) \\ \zeta = 0.034 & (0.033) \end{cases}$$

$$G_2: \begin{cases} \sigma = 0.027 & (0.024) \\ \omega = 0.088 & (0.082) \\ \zeta = 0.29 & (0.28) \end{cases}$$

$$G_3: \begin{cases} \sigma = 0.051 & (0.042) \\ \omega = 0.143 & (0.136) \\ \zeta = 0.34 & (0.30) \end{cases}$$

where the true values are given in parenthesis. The corresponding results for the systems in Example 2 are

$$G_4: \begin{cases} \sigma = 0.0076 & (0.0060) \\ \omega = 0.135 & (0.131) \\ \zeta = 0.056 & (0.046) \end{cases}$$

$$G_5: \begin{cases} \sigma = 0.057 & (0.044) \\ \omega = 0.12 & (0.104) \\ \zeta = 0.43 & (0.39) \end{cases}$$

$$G_6: \begin{cases} \sigma = 2.86 & (1.06) \\ \omega = 3.0 & (1.06) \\ \zeta = 0.69 & (0.71) \end{cases}$$

The approximate formulas thus give a fairly accurate estimate of the dominant poles, except in case G_6. The reason is that the Nyquist curve of G_6 has a shape such that the normal is very close to $G = -1$ for a wide range of ω-values. This causes the large error in the σ and ω estimates. Note that the value of the relative damping nevertheless is quite good even in this case. It thus appears, that the approximate formulas are useful in order to assess the damping of the dominant poles from the open loop Nyquist curves.

Amplitude and phase margins

It is clear from the preceding analysis that amplitude and phase margins alone do not suffice to give good estimates of the damping of the dominant poles. More insight into this is obtained by approximating the Nyquist curve locally by straight lines as shown in Figure 4. Assuming that A'B'C' can be approximated by a triangle, the following approximate expressions are obtained for the dominating poles.

$$\omega = \omega_1 + (\omega_2 - \omega_1)\frac{A_m(A_m + 1)\sin^2(\varphi_m/2)}{(A_m - 1)^2 + 4A_m\sin^2(\varphi_m/2)} \tag{2.5}$$

$$\sigma = (\omega_2 - \omega_1)\frac{A_m(A_m - 1)\sin(\varphi_m)}{(A_m - 1)^2 + 4A_m\sin^2(\varphi_m/2)} \tag{2.6}$$

where ω_1 and ω_2 are the frequencies where the Nyquist curve intersects the unit circle and the negative real axis respectively. From these equations it

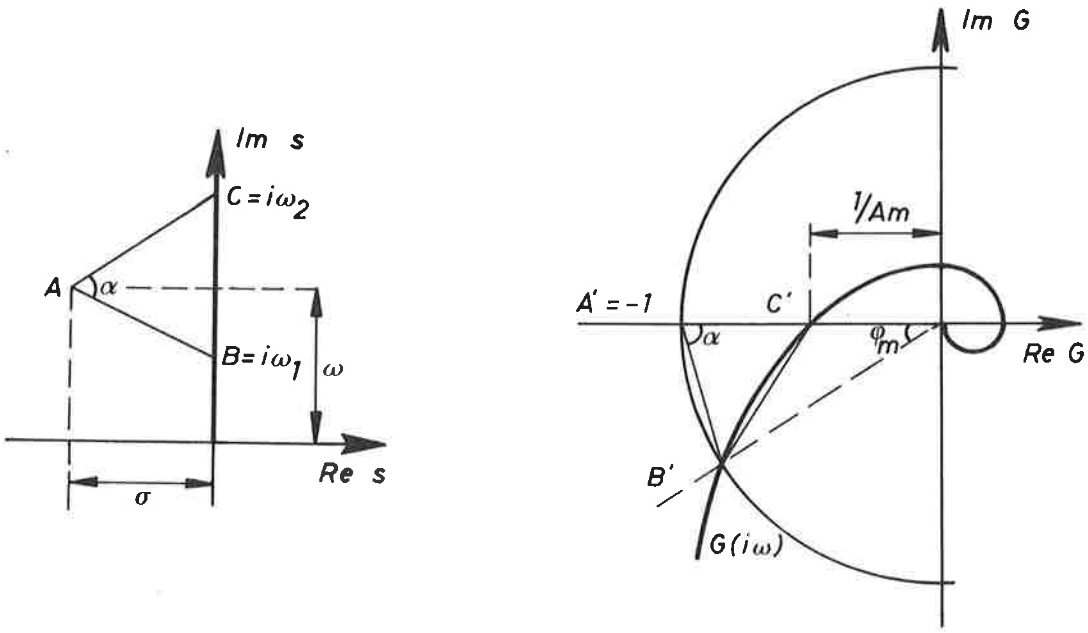

Figure 4. A map of the s-plane in the G-plane.

follows that knowledge of either the amplitude margin or the phase margin is not enough to determine σ and ω. The quantity $\omega_2 - \omega_1$, which tells how rapidly ω changes along the Nyquist curve, is also needed.

The above analysis shows that knowledge of at least two points on the Nyquist curve is needed to determine the dominant pose of simple feedback loops. It also explains qualitatively why the Ziegler-Nichols design or designs based on phase or amplitude margins alone do not suffice.

3. DESIGN

In the previous section it was shown that the dominant closed loop poles can bee determined approximately from two points on the Nyquist curve of the open loop transfer function. By introducing a controller in the loop, the dominant poles may be moved to new desired positions. The corresponding design problem may then be expressed in terms of the frequency ω and the relative damping ζ of the dominant poles.

To perform the design, it is assumed that the values of the open loop transfer function at two neighbouring frequencies, ω_1 and ω_2, are known, i.e.

$$\begin{aligned} G_o(i\omega_1) &= a_1 + ib_1 \\ G_o(i\omega_2) &= a_2 + ib_2 \end{aligned} \tag{3.1}$$

It is also assumed that the frequencies ω_1 and ω_2 are close to the cross-over frequency. The design is not restricted to any particular regulator structure. Almost any regulator with at least two adjustable parameters may be used. A PID regulator of the form

$$G_R(s) = K\left(1 + \frac{1}{sT_i} + sT_d\right) \tag{3.2}$$

is chosen as an illustration. Furthermore, it is assumed that there is a given relation between the integration time T_i and the derivative time T_d.

$$T_d = \alpha T_i$$

Hence

$$G_R(s) = K\left(1 + \frac{1}{sT} + s\alpha T\right) \tag{3.3}$$

This regulator has two adjustable parameters. The gain K moves the Nyquist curve radially from the origin. The time constant T twists the curve.

The design problem is then to determine a regulator so that the transfer function of the compensated system has desired values at the two frequencies, i.e.

$$G(i\omega_1) = G_o(i\omega_1)\, G_R(i\omega_1) = c_1 + id_1$$
$$G(i\omega_2) = G_o(i\omega_2)\, G_R(i\omega_2) = c_2 + id_2 \tag{3.4}$$

In the sequel, it is assumed that the desired frequency ω of the dominant poles is equal to ω_2. To obtain dominant poles with frequency ω_2, the normal of the Nyquist curve at ω_2 should go through -1. See Figure 5. The following relation is then obtained from the conformal mapping argument introduced in the previous section.

$$\frac{G(i\omega_2) - G(i\omega_1)}{i\omega_2 - i\omega_1} = \frac{G(i\omega_2) + 1}{\sigma} \tag{3.5}$$

Hence

$$\sigma = \frac{G(\omega_2) + 1}{G(i\omega_2) - G(i\omega_1)}\, i(\omega_2 - \omega_1) \tag{3.6}$$

Equation (2.4) gives

$$\sigma = \frac{\zeta\omega_2}{\sqrt{1-\zeta^2}} \tag{3.7}$$

Equations (3.6) and (3.7) now give

$$\frac{G(i\omega_2) - G(i\omega_1)}{G(i\omega_2) + 1} = \frac{\sqrt{1 - \zeta^2}}{\zeta} \cdot \frac{i(\omega_2 - \omega_1)}{\omega_2} \triangleq i\kappa \tag{3.8}$$

It follows from Equation (3.4) that

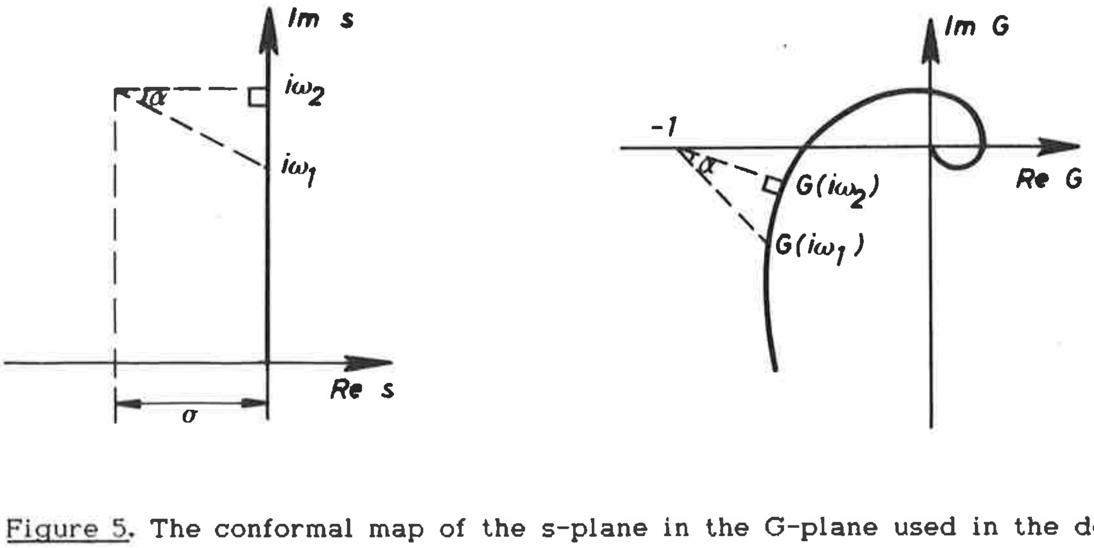

Figure 5. The conformal map of the s-plane in the G-plane used in the design method.

$$\frac{c_2 - c_1 + i(d_2 - d_1)}{c_2 + 1 + id_2} = i\varkappa \tag{3.9}$$

Equation (3.9) then gives

$$\begin{cases} c_2 - c_1 + \varkappa d_2 = 0 & (3.10) \\ d_2 - d_1 - \varkappa(c_2 + 1) = 0 & (3.11) \end{cases}$$

These conditions determine the parameters K and T of the PID regulator (3.3). Equation (3.10) gives a second order equation for T, from which T is solved. The gain K is then obtained from Equation (3.11).

The new design method will be illustrated by some examples. It will also be compared with the Ziegler-Nichols design. First, the introduction of the command signal in the loop will be discussed.

The PID controller introduces zeros in the loop transfer function. These zeros are influenced by the manner in which the command signal is introduced in the system. It is common practice not to introduce the command signal in the derivative action. Such a PID-regulator can be described by

$$u = K\left[e_p + \frac{1}{T_i}\int^t e(s)ds + T_d\frac{de_d}{dt}\right] \tag{3.12}$$

where

$$e_p = e = r - y$$

and

$$e_d = - y$$

This controller is used for the Ziegler-Nichols design below. The regulator has a zero at

$$s = -\frac{1}{T_i}$$

This zero may cause an excessive overshoot if it is too close to the real part of the dominant poles. The zero can be adjusted by the following modification of the controller.

$$\begin{cases} e = r - y \\ e_p = \beta r - y \qquad 0 < \beta \le 1 \\ e_d = - y \end{cases} \tag{3.13}$$

This means that the proportional part only acts on a fraction β of the reference signal. The regulator (3.12) with e_p, e_d and e defined by (3.13) introduces a zero at

$$s = - \frac{1}{\beta T_i}$$

This zero can be positioned properly by selecting β. Since the dominant poles are specified in the new design, it is possible to ensure that the zero not is too close to the real part of the dominant poles. In the following examples, the value of β is chosen as

$$\beta = \frac{1}{3\sigma T_i} \tag{3.14}$$

Example 3. Consider the linear system

$$G(s) = \frac{1}{(1+s)(1+0.2s)(1+0.05s)(1+0.01s)}$$

Two points on the Nyquist curve which are used for the design are given by

$$\begin{cases} G(8\cdot i) = - 0.0593 - i\cdot 0.0135 \\ G(10\cdot i) = - 0.0396 \end{cases}$$

Using these two values of $G(i\omega)$, the design method presented above can be applied. The following set of PID parameters is obtained for $\alpha = 0.25$, $\omega_2 = 10$ and $\zeta = 0.4$,

$$K = 14.17 \qquad T_i = 0.407 \qquad T_d = 0.1018 \qquad \beta = 0.17$$

These parameters can be compared with the following values obtained by the Ziegler-Nichols design.

$$K = 15.15 \qquad T_i = 0.314 \qquad T_d = 0.0785$$

Step and a load disturbance responses of the closed loop systems are given in Figure 6. It is well-known that the Ziegler-Nichols method gives a system with poor damping in situations like this. This is clearly seen in the figure.

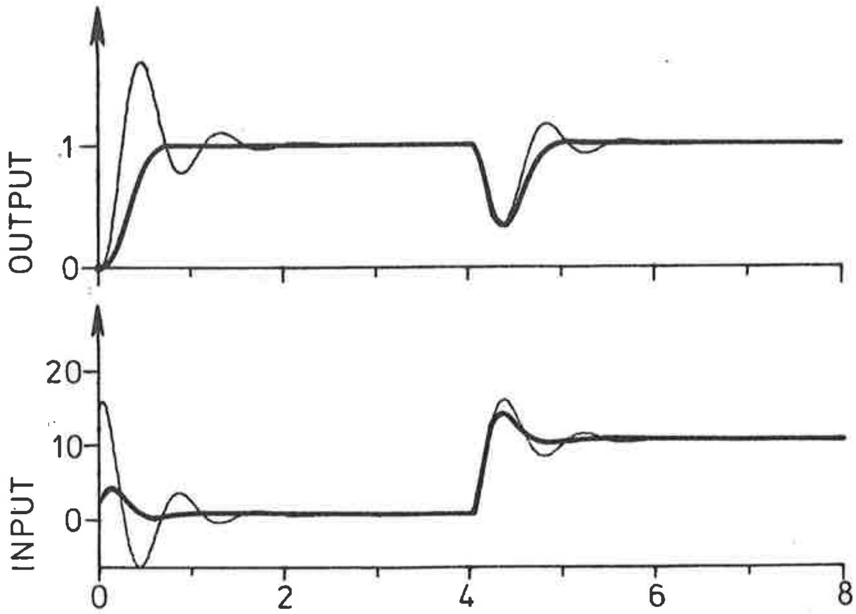

Figure 6. Step and load disturbance responses of the closed loop systems in Example 3 obtained with the new design (thick line) and the Ziegler-Nichols design (thin line).

Also notice that the responses of the systems are quite different even if the parameters are relatively close. □

Example 4. The new design method has also been applied to the six systems in Example 1 and Example 2. The controller parameters are given in Table 1. The step and load disturbance responses are given in Figures 7 and 8. The corresponding results for the Ziegler-Nichols design are also given in the graphs and in the table.

The differences between the controller parameters obtained by the two methods can roughly be characterized as follows.

G_1 The same gain, T_i and T_d about 2.5 times larger in the new design.
G_2 The gain 10% higher in the new design, T_i and T_d the same.
G_3 The same gain, T_i and T_d 20% smaller in the new design.
G_4 The gain 15% and T_i and T_d 30% smaller in the new design.
G_5 The gain 10% lower, T_i and T_d 30% larger in the new design.
G_6 The gain 25% higher in the new design, T_i and T_d the same. □

TABLE 1: The PID parameters obtained in Example 4.

	Ziegler-Nichols design			New design			
	K	T_i	T_d	K	T_i	T_d	β
G_1	1.13	1.65	0.41	1.12	4.11	1.03	0.090
G_2	1.21	28.3	7.08	1.33	28.3	7.07	0.22
G_3	1.20	20.0	5.00	1.18	15.7	3.93	0.28
G_4	0.69	23.8	5.95	0.58	16.9	4.23	0.31
G_5	2.40	15.7	3.93	2.25	20.5	5.12	0.17
G_6	2.6	0.43	0.11	3.24	0.41	0.103	0.23

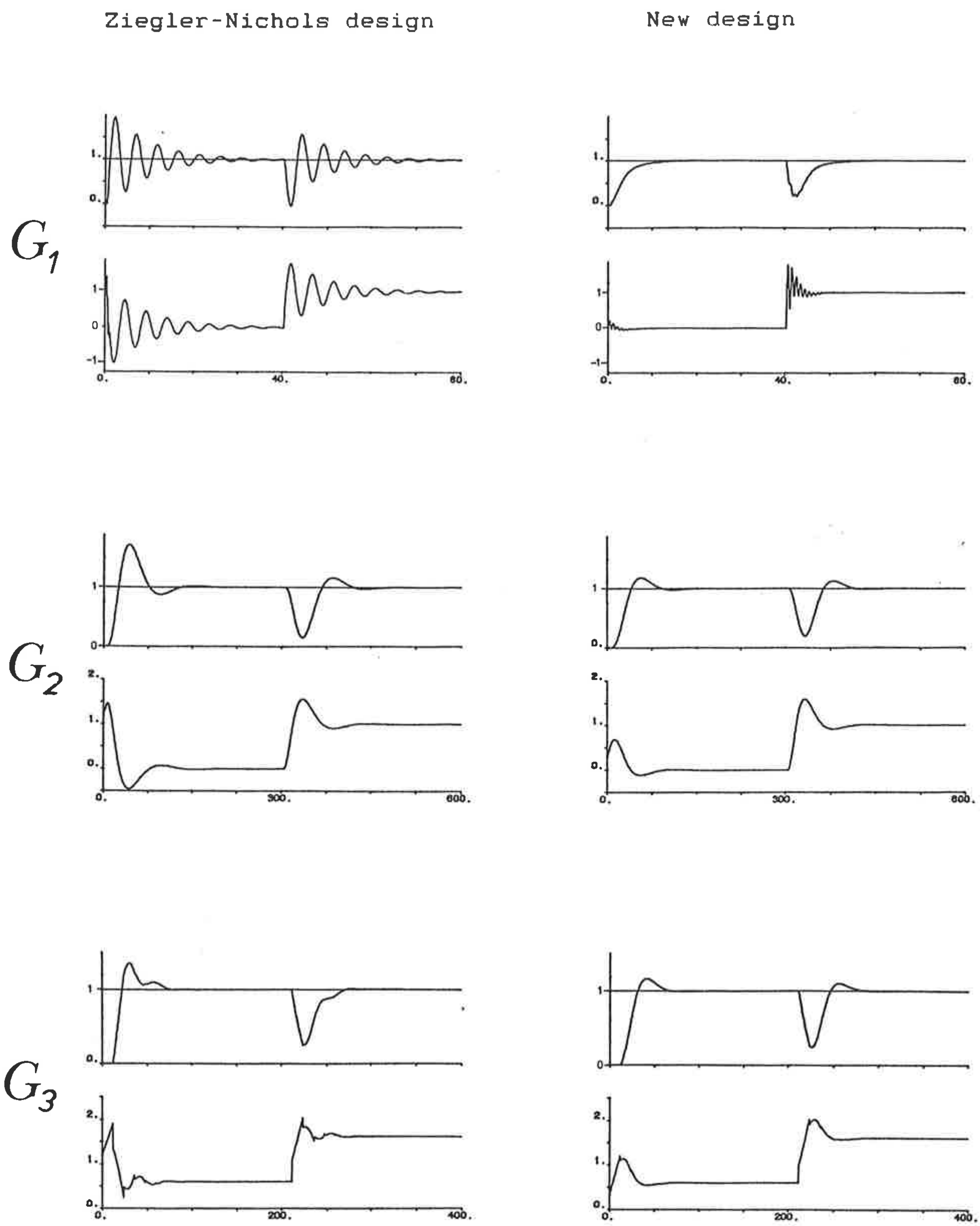

Figure 7. Step and load disturbance responses of the systems G_1, G_2 and G_3 controlled by the Ziegler-Nichols design and the new design method. The graphs show the output signal above the control signal.

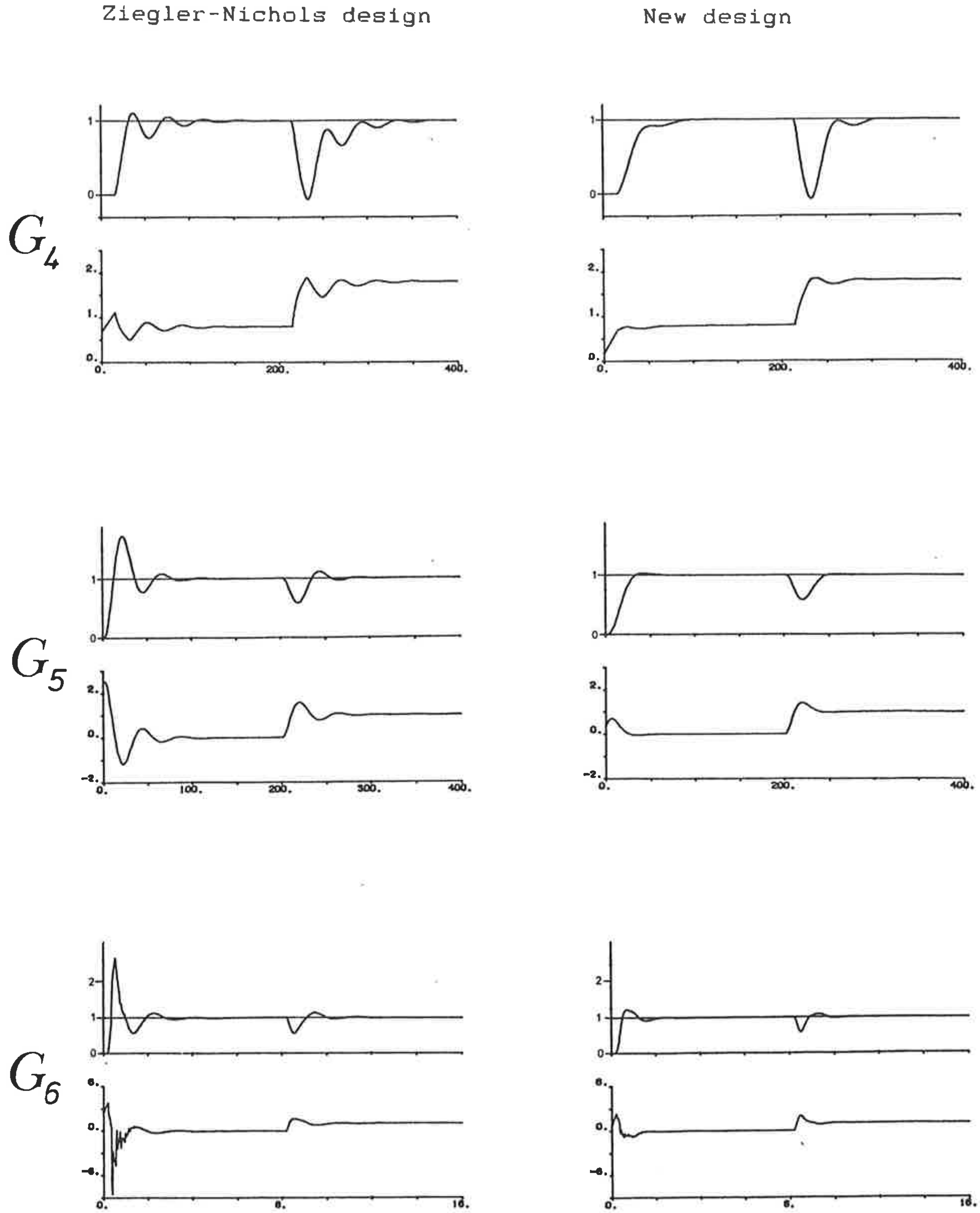

Figure 8. Step and load disturbance responses of the systems G_4, G_5 and G_6 controlled by the Ziegler-Nichols design and the new design method. The graphs show the output signal above the control signal.

4. DETERMINATION OF POINTS ON THE NYQUIST CURVE

To apply the design technique discussed in Section 3, it is necessary to know two points on the Nyquist curve which are close to the cross-over frequency. Any point on the Nyquist curve can easily be determined by frequency response analysis. Since the appropriate frequencies are not known apriori, an exhaustive search is needed to find suitable frequencies. The method introduced in Åström and Hägglund (1983) gives a convenient method for determining the point on the Nyquist curve which intersects the negative real axis. The method is based on the observation that a system with a phase lag of at least 180° may oscillate with period t_c under relay control. To determine the critical gain and the critical period, the system is provided with relay feedback as is shown in Figure 9. The error e is then a periodic signal with the period t_c. If d is the relay amplitude, it follows from a Fourier series expansion that the first harmonic of the relay output has the amplitude $4d/\pi$. If the process output is a, we get approximately

$$G(i\frac{2\pi}{t_c}) = \frac{\pi a}{4d} \tag{4.1}$$

This result also follows from the describing function approximation. The describing function N(a) for a relay is given by

$$N(a) = \frac{4d}{\pi a} \tag{4.2}$$

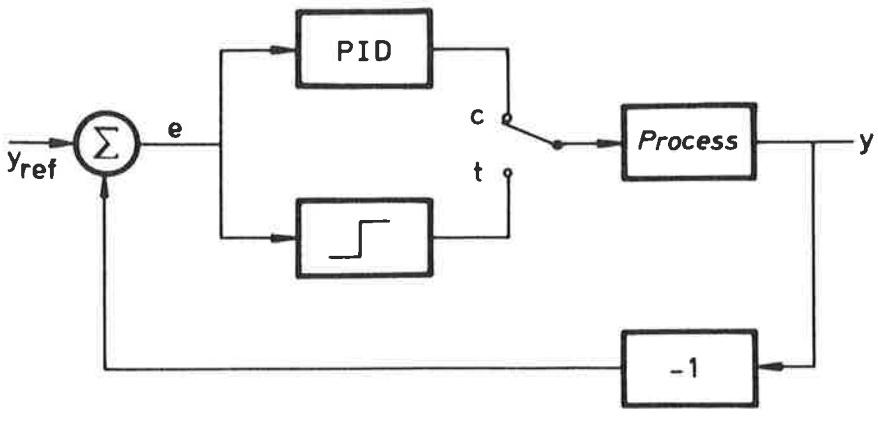

Figure 9. Block diagram of the auto-tuner. The system operates as a relay controller in the tuning mode (t) and as an ordinary PID regulator in the control mode (c).

There are advantages in having a relay with hysteresis. The negative reciprocal of the describing function of such a relay is

$$-\frac{1}{N(a)} = -\frac{\pi}{4d}\sqrt{a^2 - \varepsilon^2} - i\,\frac{\pi\varepsilon}{4d} \tag{4.3}$$

where d is the relay amplitude and ε is the hysteresis width. This function can be regarded as a straight line parallel to the real axis, in the complex plane. See Figure 10. By choosing the relation between ε and d it is therefore possible to determine a point on the Nyquist curve with a specified imaginary part. Several points on the Nyquist curve are easily obtained by repeating the experiment with different relations between ε and d.

Two experiments with relay feedback having different ratios ε/d thus give the information about the process which is needed in order to apply the design method given in Section 3. It is easy to control the amplitude of the limit cycle by a proper choice of the relay amplitude. Notice also that the estimation method will automatically generate the appropriate input signals.

Determination of amplitude and period

Methods for automatic determination of the frequency and the amplitude of the oscillation will be given to complete the description of the estimation method. The period of an oscillation can easily be determined by measuring the times between zero-crossings. The amplitude may be determined by measuring the peak-to-peak values of the output. These estimation methods

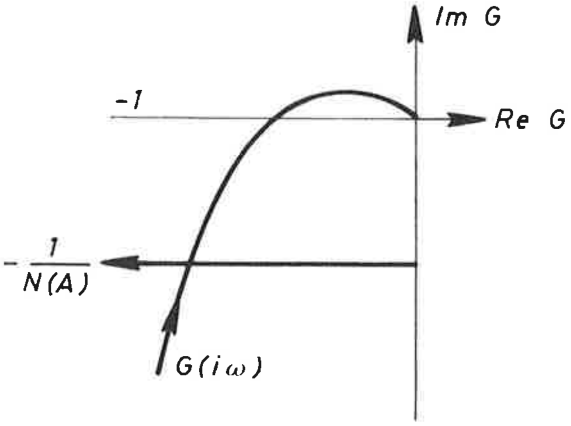

Figure 10. The negative reciprocal of the describing function N(a), and the Nyquist curve of G(s).

are easy to implement because they are based on counting and comparisons only. Since the describing function analysis is based on the first harmonic of the oscillation, the simple estimation techniques require that the first harmonic dominates. If this is not the case, it may be necessary to filter the signal before measuring. More elaborate estimation schemes like least squares estimation and extended Kalman filtering may also be used to determine the amplitude and the frequency of the limit cycle oscillation. Simulations and experiments on industrial processes have indicated that little is gained in practice by using more sophisticated methods for determining the amplitude and the period.

5. CONCLUSIONS

Tuning of simple regulators have not received much attention in the past 20 years. This paper hopefully indicates that is is possible to obtain simple design techniques that will improve upon the traditional practices. The possibility to combine these design methods with the techniques for automatic determination of certain points on the Nyquist curve offers interesting possibilities to arrive at robust automatic tuning of simple regulators. The availability of microprocessors makes it possible to implement the tuning techniques in a simple fashion.

6. ACKNOWLEDGEMENTS

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APPENDIX

This appendix contains some of the computer programs used in the design calculations and simulations presented in the report.

```
program NYQDES(input,output);

{This program computes the PID-parameters from the information
of the location of two points on the Nyqvist curve
Author: Tore Hagglund 1984-04-12}

const namelength=63;
      lo=1; hi=10;
var outfile:text;
    outfilename:packed array[1..namelength] of char;
    outlen, i, j:integer;
    a1, a2, b1, b2, c1, c2, d1, d2: real;
    w1, w2, alfa, z, kappa, n, a, b, c, d, e: real;
    K, Ti, Td, T, beta, s: real;

procedure filename(name: packed array[lo..hi:integer] of char;
                   var length:integer);extern;

procedure list(K, Ti, Td, beta, c1, d1, c2, d2: real);
begin
  writeln;
  write('K  = '); writeln(K);
  write('Ti = '); writeln(Ti);
  write('Td = '); writeln(Td);
  write('beta = '); writeln(beta);
  writeln(outfile);
  writeln(outfile, 'Regulator parameters:');
  write(outfile,'K  = '); writeln(outfile,K);
  write(outfile,'Ti = '); writeln(outfile,Ti);
  write(outfile,'Td = '); writeln(outfile,Td);
  write(outfile,'beta = '); writeln(outfile,beta);
  write('c1= '); write(c1); write('d1= '); writeln(d1);
  write('c2= '); write(c2); write('d2= '); writeln(d2);
  writeln(outfile);
  writeln(outfile, 'Location of compensated Nyqvist curve:');
  write(outfile,'c1= '); write(outfile,c1);
  write(outfile,'d1= '); writeln(outfile,d1);
  write(outfile,'c2= '); write(outfile,c2);
  write(outfile,'d2= '); writeln(outfile,d2);
end;

begin
  write('Output file: '); readln(outfilename);
  filename(outfilename, outlen);
  open(outfile,'$$$FIL',new);
  rewrite(outfile);
  writeln('Enter parameters of the open loop Nyqvist curve');
  writeln('Notation: G(iw) = a + ib');
  writeln;
  write('w1= '); readln(w1);
  write('a1= '); readln(a1);
  write('b1= '); readln(b1);
  write('w2= '); readln(w2);
  write('a2= '); readln(a2);
  write('b2= '); readln(b2);
```

```
  writeln;
  writeln(outfile, 'The parameters of the open loop Nyqvist curve');
  writeln(outfile, 'Notation: G(iw) = a + ib');
  writeln(outfile);
  write(outfile,'w1= '); write(outfile,w1);
  write(outfile,'a1= '); write(outfile,a1);
  write(outfile,'b1= '); writeln(outfile,b1);
  write(outfile,'w2= '); write(outfile,w2);
  write(outfile,'a2= '); write(outfile,a2);
  write(outfile,'b2= '); writeln(outfile,b2);
  writeln(outfile);
  while true do
  begin
    write('Specify relative damping. z = '); readln(z);
    write('Specify alfa in Td = alfa*Ti : alfa = '); readln(alfa);
    write(outfile, 'Relative damping. z= '); writeln(outfile,z);
    write(outfile, 'alfa = '); writeln(outfile,alfa);
    s:=z*w2/(1-z*z);
    kappa:=(w2-w1)/s;
    if alfa=0 then
    begin
      T:=(b1*w2-b2*w1-kappa*a2*w1)/w1/w2/(a2-a1-kappa*b2);
      K:=kappa/(-kappa*a2+b2-b1-(a2+kappa*b2)/(w2*T)+a1/(w1*T));
      Ti:=T;
      Td:=alfa*T;
      beta:=1/(3*s*Ti);
      c1:=K*(a1-b1*(alfa*w1*T-1/(w1*T)));
      c2:=K*(a2-b2*(alfa*w2*T-1/(w2*T)));
      d1:=K*(b1+a1*(alfa*w1*T-1/(w1*T)));
      d2:=K*(b2+a2*(alfa*w2*T-1/(w2*T)));
      list(K, Ti, Td, beta, c1, d1, c2, d2);
    end
    else begin
      for j:= 1 to 2 do
      begin
        n:=b1*w1-b2*w2+kappa*a2*w2;
        a:=(a2-a1+kappa*b2)/(alfa*n);
        b:=(b2*w1-b1*w2-kappa*a2*w1)/(alfa*w1*w2*n);
        T:=-a/2+2*(j-1.5)*sqrt(sqr(a/2)-b);
        c:=(-kappa*a2+b2-b1);
        d:=(a2+kappa*b2)*(w2*alfa*T-1/(w2*T));
        e:=-a1*(w1*alfa*T-1/(w1*T));
        K:=kappa/(c+d+e);
        Ti:=T;
        Td:=alfa*T;
        beta:=1/(3*s*Ti);
        c1:=K*(a1-b1*(alfa*w1*T-1/(w1*T)));
        c2:=K*(a2-b2*(alfa*w2*T-1/(w2*T)));
        d1:=K*(b1+a1*(alfa*w1*T-1/(w1*T)));
        d2:=K*(b2+a2*(alfa*w2*T-1/(w2*T)));
        list(K, Ti, Td, beta, c1, d1, c2, d2);
      end;
    end;
  end;
  close(outfile);
end.
```

```
CONTINUOUS SYSTEM SAM1

INPUT u
OUTPUT y dy
STATE x1 x2
DER dx1 dx2

dx1=x2+0.7*u
dx2=1.4*u

y=x1
dy=dx1

END
```

- -

```
CONTINUOUS SYSTEM PID

INPUT e de ep
OUTPUT u
STATE i
DER di

di=e/Ti
u=K*(ep+i+Td*de)

K:1.12
Ti:4.11
Td:1.03

END
```

- -

```
CONNECTING SYSTEM CONAM1

TIME t

td1[DELAY]=t-0.4
u1[DELAY]=u[PID]+v
e[PID]=yref-y[SAM1]
de[PID]=-dy[SAM1]
ep[PID]=beta*yref-y[SAM1]
u[SAM1]=y1[DELAY]
v=if t>t1 and t<t2 then -1 else 0

t1:40
t2:400
yref:1
beta:0.090

END
```

```
CONTINUOUS SYSTEM SAM2

INPUT u
OUTPUT y dy
STATE x1 x2 x3
DER dx1 dx2 dx3

dx1=-0.4*x1+x2
dx2=-0.04*x1+x3
dx3=0.00288*u

y=x1
dy=dx1

END
```

- -

```
CONTINUOUS SYSTEM PID

INPUT e de ep
OUTPUT u
STATE i
DER di

di=e/Ti
u=K*(ep+i+Td*de)

K:1.33
Ti:28.3
Td:7.07

END
```

- -

```
CONNECTING SYSTEM CONAM2

TIME t

td1[DELAY]=t-5
u1[DELAY]=u[PID]+v
e[PID]=yref-y[SAM2]
de[PID]=-dy[SAM2]
ep[PID]=beta*yref-y[SAM2]
u[SAM2]=y1[DELAY]
v=if t>t1 and t<t2 then -1 else 0

t1:300
t2:1000
yref:1
beta:0.22

END
```

```
CONTINUOUS SYSTEM SAM3

INPUT u
OUTPUT y dy
STATE x1
DER dx1

dx1=-0.05*x1+0.0825*u

y=x1
dy=dx1

END
```

- -

```
CONTINUOUS SYSTEM PID

INPUT e de ep
OUTPUT u
STATE i
DER di

di=e/Ti
u=K*(ep+i+Td*de)

K:1.18
Ti:15.7
Td:3.93

END
```

- -

```
CONNECTING SYSTEM CONAM3

TIME t

td1[DELAY]=t-12
u1[DELAY]=u[PID]+v
e[PID]=yref-y[SAM3]
de[PID]=-dy[SAM3]
ep[PID]=beta*yref-y[SAM3]
u[SAM3]=y1[DELAY]
v=if t>t1 and t<t2 then -1 else 0

t1:200
t2:400
yref:1
beta:0.28

END
```

```
CONTINUOUS SYSTEM SFM1

INPUT u
OUTPUT y dy
STATE x1 x2
DER dx1 dx2

dx1=-0.4*x1+x2
dx2=-0.04*x1+0.05*u

y=x1
dy=dx1

END
```

- -

```
CONTINUOUS SYSTEM PID

INPUT e de ep
OUTPUT u
STATE i
DER di

di=e/Ti
u=K*(ep+i+Td*de)

K:0.58
Ti:16.9
Td:4.23

END
```

- -

```
CONNECTING SYSTEM CONFM1

TIME t

td1[DELAY]=t-15
u1[DELAY]=u[PID]+v
e[PID]=yref-y[SFM1]
de[PID]=-dy[SFM1]
ep[PID]=beta*yref-y[SFM1]
u[SFM1]=y1[DELAY]
v=if t>t1 and t<t2 then -1 else 0

t1:200
t2:600
yref:1
beta:0.31

END
```

```
CONTINUOUS SYSTEM SFM2

INPUT u
OUTPUT y dy
STATE x1 x2 x3
DER dx1 dx2 dx3

dx1=-0.4*x1+x2
dx2=-0.04*x1+x3
dx3=0.004*u

y=x1
dy=dx1

END
```

- -

```
CONTINUOUS SYSTEM PID

INPUT e de ep
OUTPUT u
STATE i
DER di

di=e/Ti
u=K*(ep+i+Td*de)

K:2.25
Ti:20.5
Td:5.12

END
```

- -

```
CONNECTING SYSTEM CONFM2

TIME t

e[PID]=yref-y[SFM2]
de[PID]=-dy[SFM2]
ep[PID]=beta*yref-y[SFM2]
u[SFM2]=u[PID]+v
v=if t>t1 and t<t2 then -1 else 0

t1:200
t2:600
yref:1
beta:0.17

END
```

```
CONTINUOUS SYSTEM SFM3

INPUT u
OUTPUT y dy
STATE x1 x2
DER dx1 dx2

dx1=x2+1.68*u
dx2=1.4*u

y=x1
dy=dx1

END
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
CONTINUOUS SYSTEM PID

INPUT e de ep
OUTPUT u
STATE i
DER di

di=e/Ti
u=K*(ep+i+Td*de)

K:3.24
Ti:0.41
Td:0.103

END
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
CONNECTING SYSTEM CONFM3

TIME t

td1[DELAY]=t-0.2
u1[DELAY]=u[PID]+v
e[PID]=yref-y[SFM3]
de[PID]=-dy[SFM3]
ep[PID]=beta*yref-y[SFM3]
u[SFM3]=y1[DELAY]
v=if t>t1 and t<t2 then -1 else 0

t1:8
t2:20
yref:1
beta:0.23

END
```