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DOMINANT POLE DESIGN

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ABSTRACT

This paper describes a new method for designing simple feedback systems. The idea is to position only those closed loop poles which have a dominant influence on the closed loop response. Techniques for approximate determination of the dominant poles are derived, as well as design methods based on these formulas. Apart from applications to design, the results may also be used to assess the complexity of the compensators needed to satisfy different specifications. The method works for systems where the closed loop dynamics can be approximated by dominant poles.

1. INTRODUCTION

This paper was motivated by work on automatic tuning of simple regulators. See Åström (1981, 1982), Hägglund (1981) and Åström and Hägglund (1983, 1984a, 1984b) which indicated the need for improved design methods for simple regulators of the PID type. The classical Ziegler-Nichols tuning rules have the advantage of being very simple to use since they are based on knowledge of one point on the Nyquist curve of the system only. See Ziegler and Nichols (1943). The Ziegler-Nichols design method does, however, give poor control of the damping of the closed loop system. Related methods based on amplitude and phase margins discussed in Åström and Hägglund (1984a) also have the same difficulty. See Hägglund and Åström (1984).

A natural extension of the Ziegler-Nichols method is to try to find techniques which are based on knowledge of several points on the Nyquist curve of the open loop system. In Hägglund and Åström (1984), a new method was proposed which uses two points on the Nyquist curve. It may be regarded as a special case of pole-placement where it is only attempted to position the dominant closed loop poles. This is in contrast to normal pole placement methods where all closed loop poles are positioned. The design was derived using conformal mapping arguments. In this paper, a more general derivation of the dominant pole design method is presented. It contains the method of Hägglund and Åström (1984) as a special case.

The paper is organized as follows. The notion of dominant poles is reviewed in Section 2. Approximate methods for determining the dominant poles are given in Section 3. The key result is a very simple method for determining poles from the Nyquist curve of the loop transfer function. The formula developed in Section 3 is used to derive design methods for PI, PD and PID-regulators in Section 4. The specifications given are primarily related to the frequency and the damping of the dominant poles. A few examples of the application of the design method are also given. In Section 5, the design method is used to control several models of processes which are common in process control. The main results of the paper are summarized in Section 6, and references are given in Section 7.

2. DOMINANT POLES

Consider a closed loop system obtained by negative feedback around a linear system with the transfer function G(s). See Fig. 1. The transfer function of the closed loop system from the command signal to the output is given by

$$G_c(s) = \frac{G(s)}{1+G(s)} \tag{2.1}$$

Many properties of the closed loop system can be deduced from the poles and the zeros of $G_c(s)$. The zeros of $G_c(s)$ are the same as the zeros of G(s) i.e. the zeros of the plant and the regulator. The closed loop poles are the roots of the equation

$$1 + G(s) = 0 \tag{2.2}$$

The pole-zero configurations of closed loop systems may vary considerably. Many simple feedback loops will, however, have a configuration of the type shown in Fig. 2 where the principal characteristics of the response is given by a complex pair of poles p_1, p_2 called the dominant poles. The response is also somewhat influenced by real poles and zeros, p_3 and z_1 respectively. The steady state properties are influenced by the dipole p_4, z_2. Poles and zeros whose real parts are much smaller than the real part of the dominant poles have little influence on the transient response. Classical control was very much concerned with closed loop systems having the pole-zero configuration shown in Fig. 2. See Mulligan (1949), Truxal (1955), Elgerd and Stephens (1959), Horowitz (1963).

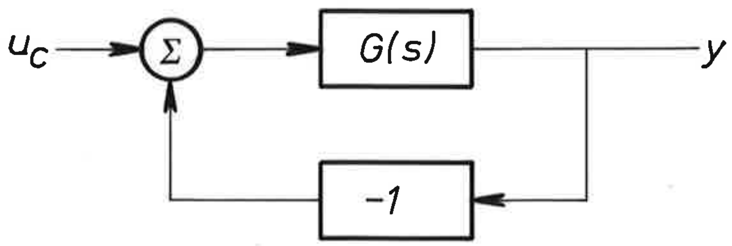

Fig. 1 Block diagram of a simple feedback system.

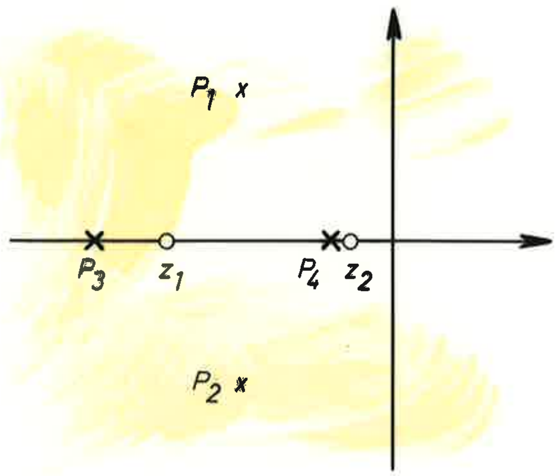

Fig. 2 Pole-zero configuration of a simple feedback system.

Even if many closed loop systems have a pole-zero configuration similar to the one shown in Fig. 2, there are, however, exceptions. Systems with mechanical resonances, which may have poles and zeros close to the imaginary axis, are generic examples of systems which do not fit the pole-zero pattern of Fig. 2.

3. DETERMINATION OF DOMINANT POLES

A simple method for estimating the dominant poles from knowledge of the Nyquist curve of the open loop system will now be described. Consider the loop transfer function G as a map from the s-plane to the G-plane. See Fig. 3. The map of the imaginary axis in the s-plane is the Nyquist curve which is indicated by the full line in Fig. 3b. The closed loop poles are given by the characteristic equation

$$G(s) + 1 = 0$$

Therefore, the map of a straight line through the dominant poles in the s-plane is a curve which goes through the critical point C = -1 in the G-plane. This curve is dashed in Fig. 3b. Since the map is conformal, the straight line A'C' is mapped into the curve AC which intersects the Nyquist curve orthogonally. The triangle A'B'C' is also mapped conformally to ABC. If ABC can be approximated by a triangle the following condition holds

$$\frac{G(i\omega_2) - G(i\omega_1)}{i\omega_2 - i\omega_1} = \frac{1+G(i\omega_2)}{\sigma} \tag{3.1}$$

This equation can be used to determine the dominant poles approximatively. The

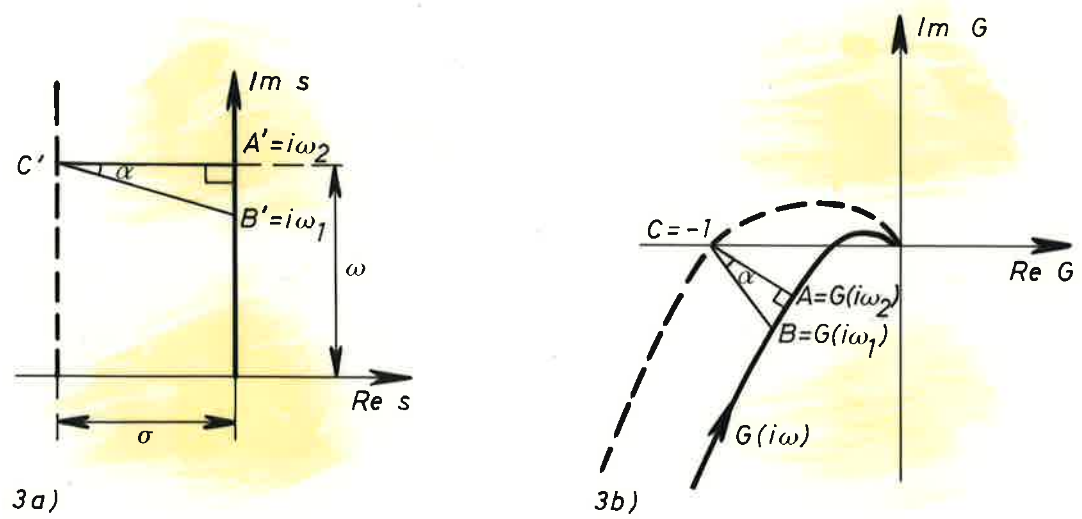

Fig. 3 Representation of the transfer function G as a map of C to C.

procedure can be expressed as follows. Determine a point A on the Nyquist curve such that the normal at A goes through the critical point C. The frequency ω_2 at A is then the argument such that $G(i\omega_2) = A$. To determine σ consider a neighbouring point ω_1 and compute σ from (3.1). The approximation will be good if the graph ABC is close to a triangle.

An analytic derivation

To provide further insight, the equation (3.1) will now be derived analytically. For this purpose consider the equation (2.2). A Taylor series expansion around $s = i\omega$ gives

$$0 = 1 + G(-\sigma+i\omega) = 1 + G(i\omega) - \sigma G'(i\omega) + \ldots$$

Neglecting terms of second and higher order in σ we find

$$1 + G(i\omega) - \sigma G'(i\omega) = 0 \tag{3.2}$$

This equation is equivalent to (3.1) as $\omega_1 \rightarrow \omega_2 = \omega$. Notice that ω must be chosen so that the normal to the Nyquist curve at ω goes through the critical point. Otherwise σ in (3.2) will not be real. This analytic derivation shows that the formula (3.1) will give good results for small σ, i.e. when the dominant poles are close to the imaginary axis. The approximation (3.2) will not hold if the function $G(s)$ has singularities inside the circle with center in $i\omega$ and radius ω. This means that σ must be smaller than ω.

Examples

A few examples which illustrate the formula (3.2) for approximative determination of the dominant poles will now be given.

EXAMPLE 3.1

Consider a system with the loop transfer function

$$G(s) = \frac{k}{s(s+1)}$$

Hence

$$G'(s) = -\frac{k}{s^2(s+1)} - \frac{k}{s(s+1)^2}$$

Equation (3.2) becomes

$$1 + \frac{k}{s(s+1)} + \frac{\sigma k}{s^2(s+1)} + \frac{\sigma k}{s(s+1)^2} = 0$$

Hence

$$s^2(s+1)^2 + ks(s+1) + \sigma k(s+1) + \sigma ks = 0$$

or

$$s^4 + 2s^3 + (1+k)s^2 + k(2\sigma+1)s + \sigma k = 0$$

Introducing $s = i\omega$ we get

$$\begin{cases} \omega^4 - (1+k)\omega^2 + \sigma k = 0 \\ -2\omega^2 + k(2\sigma+1) = 0 \end{cases}$$

These equations have the solution

$$\sigma = \frac{1}{2}\sqrt{1 + \frac{2}{k}}$$

$$\omega = \sqrt{\frac{k}{2} + \frac{1}{2}\sqrt{k^2 + 2k}}$$

The relative damping is

$$\zeta = \sqrt{\frac{2+k}{2+k+2k^2+2k\sqrt{k^2+2k}}}$$

The following numerical values are obtained for k = 1.

σ	$= 0.866$	(0.500)
ω	$= 1.17$	(0.866)
ζ	$= 0.59$	(0.500)

The correct values are given in parentheses. □

EXAMPLE 3.2

Consider a system with the loop transfer function

$$G(s) = \frac{k}{s(s+1)^2}$$

Hence

$$G'(s) = -\frac{k}{s^2(s+1)^2} - \frac{2k}{s(s+1)^3}$$

Equation (3.2) becomes

$$s^5 + 3s^4 + 3s^3 + (1+k)s^2 + k(1+3\sigma)s + \sigma k = 0$$

Introducing $s = i\omega$ gives

$$\begin{cases} 3\omega^4 - (1+k)\omega^2 + \sigma k = 0 \\ \omega^4 - 3\omega^2 + k(1+3\sigma) = 0 \end{cases}$$

These equations have the solution

$$\sigma = \frac{(8-k)\sqrt{32k+9k^2} - 24k - 3k^2}{128k}$$

$$\omega = \sqrt{\frac{3k + \sqrt{32k + 9k^2}}{16}}$$

For k = 1 the solution becomes

$\sigma = 0.14$ (0.122)

$\omega = 0.77$ (0.745)

$\zeta = 0.18$ (0.16) □

Difference approximations

Equation (3.1), which may be considered as a difference approximation of (3.2), is more convenient to use than (3.2) when the Nyquist curve is determined experimentally. The equation (3.1) can be written as

$$\sigma = \frac{i(\omega_2-\omega_1)\left[1+G(i\omega_2)\right]}{G(i\omega_2) - G(i\omega_1)} \tag{3.3}$$

Notice that the complex numbers $1+G(i\omega_2)$ and $G(i\omega_2) - G(i\omega_1)$ are orthogonal if ω_1 and ω_2 are properly chosen. The frequency ω can then also be estimated as $\omega = \omega_2$. Two points on the Nyquist curve are obviously needed to use this formula. More accurate equations can be derived if more points are known. With three equidistant points $\omega - h$, ω and $\omega + h$ the following equation for σ is obtained by approximating the derivatives in (3.2) by differences

$$1 + G(i\omega) + \frac{i\sigma}{2h}\left[G[i(\omega+h)] - G[i(\omega-h)]\right] - \\ - \frac{\sigma^2}{2h^2}\left[G[i(\omega+h)] - 2G[i\omega] + G[i(\omega-h)]\right] = 0 \tag{3.4}$$

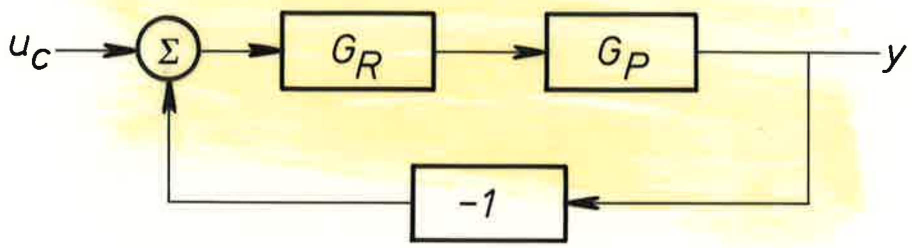

Fig. 4 Block diagram of a closed loop system.

A control design problem will now be presented. Consider the closed loop system shown in Fig. 4 with a plant and a compensator. The compensator G_R should be determined so that the closed loop system has desired properties. In a single control loop the specifications include

Transient behaviour
Rejection of load disturbances
Rejection of measurement noise

Assume that the closed loop system can be characterized by the dominant poles, $s = -\sigma \pm i\omega$. The transient behaviour is then largely governed by σ and ω. It is also influenced by the zeros of the process and the regulator to some extent.

Load disturbances may be reduced by integral action. To make sure that rejection of low frequency disturbances does not take too long time, it is required that the closed loop pole introduced via integral lies sufficiently close to the origin.

The effect of measurement noise is governed by the dominant poles and the high frequency gain of the regulator.

Since the dominant poles are characterized by two parameters, a regulator of the PI or PD type which has two adjustable parameters is sufficient to give desired dominant poles, provided that the desired bandwidth is not too high. A PID-regulator which has an additional parameter adds extra flexibility with respect to rejection of load disturbances.

PID-regulator which has an additional parameter adds extra flexibility with respect to rejection of load disturbances.

With these specifications it is straightforward to obtain an analytic formula for the design. Equation (3.2) gives the following condition

$$\left[1+G_p(i\omega)G_R(i\omega)\right] - \sigma\left[G_p'(i\omega)G_R(i\omega)+G_p(i\omega)G_R'(i\omega)\right] = 0 \tag{4.1}$$

This is an equation in complex variables. It thus gives two real equations which can be used to determine the parameters of a PI or a PD regulator. Since a PID-regulator has three parameters, an auxiliary condition is needed in this case. Such a condition can be to specify a given relation between the integral time T_i and the derivative time T_d, i.e.

$$T_d = \alpha T_i \tag{4.2}$$

A regulator also introduces zeros in the loop transfer function. These zeros are influenced by the manner in which the command signal is introduced in the system. It is common practice not to introduce the command signal in the derivative action. Such a PID-regulator can be described by

$$u = K\left[e_p + \frac{1}{T_i}\int_0^t e(s)ds + T_d\frac{de_d}{dt}\right] \tag{4.3}$$

where

$$e_p = e = r - y$$

and

$$e_d = -y$$

The regulator introduces a zero at

$$s = -\frac{1}{T_i}$$

This zero may cause an excessive overshoot if it is too close to the real part of the dominant poles. To avoid this the regulator may be modified by choosing

$$\begin{cases} e = r - y \\ e_p = \beta r - y & 0 < \beta \le 1 \\ e_d = -y \end{cases} \qquad (4.4)$$

This means that the proportional part only acts on a fraction β of the reference signal. The regulator (4.3) with e_p, e_d and e defined by (4.4) introduces a zero at

$$s = -\frac{1}{\beta T_i}$$

This zero can be positioned properly by selecting β. A reasonable choice is

$$\beta = \frac{1}{3\sigma T_i} \qquad (4.5)$$

since this choice will place the zero at -3σ, which is far away from the real part of the dominant poles.

Summarizing we find that the design procedure can be described as follows. Determine the parameters of the regulator such that (4.1), (4.2) and (4.3) hold. The design procedure is illustrated by several examples in the next section. This section ends with some examples where only PI or PD regulators are used. In these cases, the controller is uniquely determined by Equation (4.1).

EXAMPLE 4.1 - PD control of a double integrator

With PD-control of a double integrator the loop transfer function becomes

$$G(s) = \frac{k+k_d s}{s^2} = \frac{k}{s^2} + \frac{k_d}{s}$$

where k is the proportional gain and k_d is the derivative gain. The design equation (4.1) becomes

$$1 + \frac{k}{s^2} + \frac{k_d}{s} + \sigma\left[\frac{2k}{s^3} + \frac{k_d}{s^2}\right] = 0$$

Hence

$$s^3 + k_d s^2 + (k+\sigma k_d)s + 2\sigma k = 0$$

Introducing $s = i\omega$ this gives

$$i\omega(-\omega^2+k+\sigma k_d) - k_d\omega^2 + 2\sigma k = 0$$

Hence

$$k = \frac{\omega^4}{2\sigma^2+\omega^2} \approx \left(\omega^2+\sigma^2\right)\left[1 - \frac{3\sigma^2}{\omega^2}\right]$$

$$k_d = \frac{2\sigma\omega^2}{2\sigma^2+\omega^2} \approx 2\sigma\left[1 - \frac{2\sigma^2}{\omega^2}\right]$$

where the approximations yield if σ/ω is small. With PD-control of a double integrator it is possible to obtain an arbitrary pole placement. The exact gains which give the poles $-\sigma \pm i\omega$ are $k = \omega^2+\sigma^2$ and $k_d = 2\sigma$. □

The gains in Example 4.1 give a closed loop system with a frequency

$$\omega_1 = \sqrt{k-k_d^2/4} = \omega\,\frac{\sqrt{\omega^4+\sigma^2\omega^2}}{\omega^2+2\sigma^2} \approx \omega\left[1 - \frac{3}{2}\frac{\sigma^2}{\omega^2}\right]$$

and a relative damping

$$\zeta_1 = \frac{k_d}{2\sqrt{k}} = \frac{\sigma}{\sqrt{\omega^2+\sigma^2}} \cdot \sqrt{\frac{\omega^2+\sigma^2}{\omega^2+2\sigma^2}} \approx \zeta\left[1 - \frac{\sigma^2}{2\omega^2}\right]$$

The equations indicate the error due to the approximations used. Notice that the error in the relative damping is smaller than the error in the frequency. With $\sigma/\omega = 0.5$ the error in the relative damping is 9 % while the error in the frequency is about 25 %. The errors may be reduced by using more terms in the Taylor series expansion used to derive the design equation. Straightforward calculations give the following series expansion for the design equation

$$1 + k\left[\frac{1}{s^2} + \frac{2\sigma}{s^3} + \frac{3\sigma^2}{s^4} + \frac{4\sigma^3}{s^5} + \frac{5\sigma^4}{s^6} + \ldots\right]$$

$$+ k_d\left[\frac{1}{s} + \frac{\sigma}{s^2} + \frac{\sigma^2}{s^3} + \frac{\sigma^3}{s^4} + \frac{\sigma^4}{s^5} + \ldots\right] = 0$$

The series within brackets converge if $|\sigma/\omega|<1$. Summing the series we get

$$1 + \frac{k}{s^2}\,\frac{1}{(1-\sigma/s)^2} + \frac{k_d}{s}\,\frac{1}{1-\sigma/s} = 0$$

or

$$(s-\sigma)^2 + k_d(s-\sigma) + k = 0$$

Using three terms in the series expansion gives

$$k = \frac{\omega^4(\omega^2-\sigma^2)}{\omega^4-2\sigma^2\omega^2+3\sigma^4} \approx \left(\omega^2+\sigma^2\right)\left(1-\frac{\sigma^4}{\omega^4}\right)$$

$$k_d = \frac{2\sigma\omega^4}{\omega^4-2\sigma^2\omega^2+3\sigma^4} \approx 2\sigma\left(1+\frac{2\sigma^2}{\omega^2}\right)$$

Similarly the following gains are obtained if four terms of the series expansion are used.

$$k = \frac{\omega^6}{\omega^4-\omega^2\sigma^2-4\sigma^4} \approx \left(\omega^2+\sigma^2\right)\left(1+\frac{5\sigma^4}{\omega^4}\right)$$

$$k_d = \frac{2\sigma(\omega^2-2\sigma^2)\omega^4}{(\omega^2-\sigma^2)(\omega^4-\omega^2\sigma^2-4\sigma^4)} \approx 2\sigma\left[1 + \frac{4\sigma^4}{\omega^4}\right]$$

These numbers are correct up to terms of the order of σ^4/ω^4.

Next a plant with the transfer function

$$G_p(s) = \frac{1}{(s+1)^3} \tag{4.6}$$

will be investigated. Since the plant is of third order it is clear that exact pole placement cannot be obtained with PI, PD or PID-control. First consider PI-control.

EXAMPLE 4.2 - PI control of $(s+1)^{-3}$

With PI-control of the system (4.6), the loop transfer function is

$$G(s) = \frac{k}{(s+1)^3} + \frac{k_i}{s(s+1)^3}$$

Hence

$$G'(s) = -\frac{3k}{(s+1)^4} - \frac{3k_i}{s(s+1)^4} - \frac{k_i}{s^2(s+1)^3}$$

The design equation (3.2) becomes

$$1 + \frac{k}{(s+1)^3} + \frac{k_i}{s(s+1)^3} + \frac{3\sigma k}{(s+1)^4} + \frac{3\sigma k_i}{s(s+1)^4} + \frac{\sigma k_i}{s^2(s+1)^3} = 0$$

Hence

$$s^2(s+1)^4 + ks^2(s+1+3\sigma) + k_i\left[s^2+s(1+4\sigma)+\sigma\right] = 0$$

or

$$\begin{cases} k(1+3\sigma)\omega^2 + k_i(\omega^2-\sigma) = -\omega^6 + 6\omega^4 - \omega^2 \\ -k\omega^2 + k_i(1+4\sigma) = -4\omega^4 + 4\omega^2 \end{cases}$$

Solving for k and k_i gives

$$k = \frac{\sigma(-4\omega^4+20\omega^2) + 3\omega^4 + 2\omega^2 - 1}{\omega^2 + 12\sigma^2 + 6\sigma + 1}$$

$$k_i = \frac{-\omega^6 + 2\omega^4 + 3\omega^2 - 12\sigma(\omega^4-\omega^2)}{\omega^2 + 12\sigma^2 + 6\sigma + 1}$$

□

EXAMPLE 4.3 - PD control of $(s+1)^{-3}$

Consider PD-control of the system (4.6). The loop transfer function becomes

$$G(s) = \frac{k+k_d s}{(s+1)^3}$$

Hence

$$G'(s) = -\frac{3(k+k_d s)}{(s+1)^4} + \frac{k_d}{(s+1)^3}$$

The design equation becomes

$$1 + \frac{k+k_d s}{(s+1)^3} + \frac{3\sigma(k+k_d s)}{(s+1)^4} - \frac{\sigma k_d}{(s+1)^3} = 0$$

Hence

$$(s+1)^4 + k(s+1+3\sigma) + k_d(s^2+s+2\sigma s-\sigma) = 0$$

Putting $s = i\omega$ gives

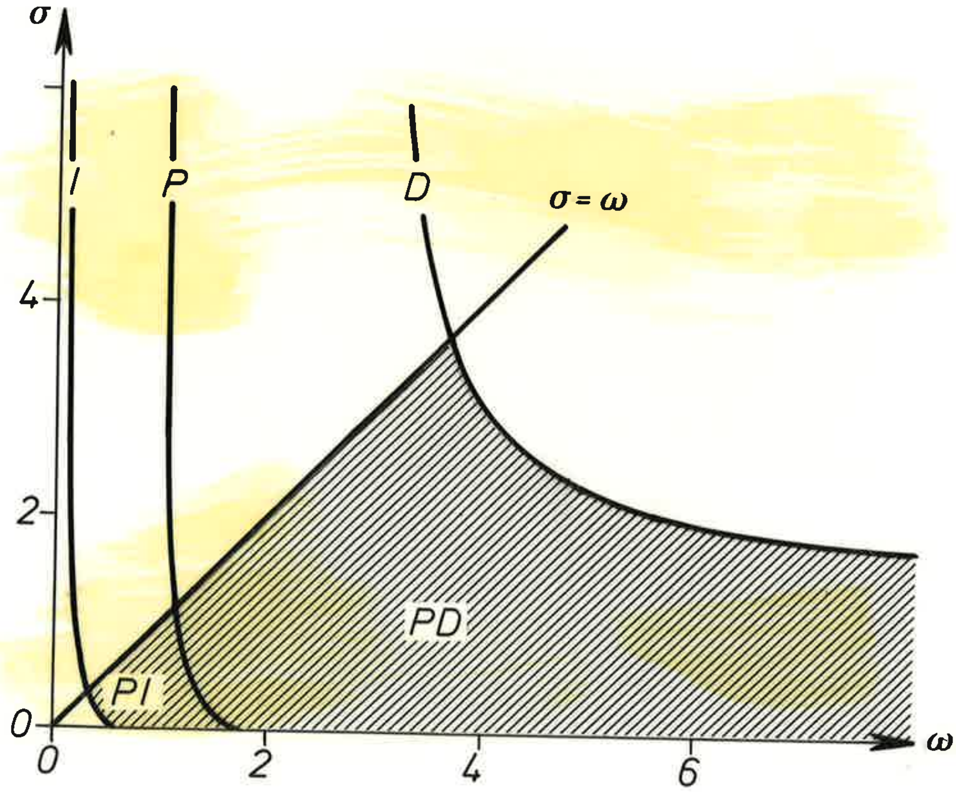

Fig.5 Areas of positive gains in the PI and PD regulators.

$$(1+3\sigma)k - (\omega^2+\sigma)k_d + \omega^4 - 6\omega^2 + 1 = 0$$

$$k + (1+2\sigma)k_d - 4\omega^2 + 4 = 0$$

These equations have the solution

$$k = \frac{-2\sigma\omega^4 + 3\omega^4 + 16\sigma\omega^2 + 2\omega^2 - 6\sigma - 1}{\omega^2 + 6\sigma^2 + 6\sigma + 1}$$

$$k_d = \frac{\omega^4 + 12\sigma\omega^2 - 2\omega^2 - 12\sigma - 3}{\omega^2 + 6\sigma^2 + 6\sigma + 1}$$

□

In the Examples 4.2 and 4.3, regulators with positive gains can be found only if the specifications on the dominant poles are restricted to certain areas. Fig. 5 shows those combinations of σ and ω which give positive gains for the PI and the PD regulators respectively. The border-lines are given by the pure P, I and D regulators. Notice that the approximative formulas are only valid if $\sigma < \omega$. From this figure it is seen that the bandwidth ω cannot be chosen too high if only a PI controller is used.

The next two examples demonstrate PI and PD control of processes with the transfer functions

$$G_p(s) = \frac{1}{(s+1)^n} \tag{4.7}$$

where n is an integer greater than 3. Hence, Examples 4.2 and 4.3 are special cases of the following examples.

EXAMPLE 4.4 - PI control of $(s+1)^{-n}$

When the system (4.7) is controlled by a PI-controller, the loop transfer function becomes

$$G(s) = \frac{k}{(s+1)^n} + \frac{k_i}{s(s+1)^n}$$

Hence

$$G'(s) = -\frac{nk}{(s+1)^{n+1}} - \frac{nk_i}{s(s+1)^{n+1}} - \frac{k_i}{s^2(s+1)^n}$$

The design equation (3.2) becomes

$$1 + \frac{k}{(s+1)^n} + \frac{k_i}{s(s+1)^n} + \frac{n\sigma k}{(s+1)^{n+1}} + \frac{n\sigma k_i}{s(s+1)^{n+1}} + \frac{\sigma k_i}{s^2(s+1)^n} = 0$$

Hence

$$s^2(s+1)^{n+1} + ks^2(s+1+n\sigma) + k_i\left[s^2+s[1+(n+1)\sigma]+\sigma\right] = 0$$

Putting $s = i\omega$ gives the following two equations

$$\begin{cases} k(1+n\sigma)\omega^2 + k_i(\omega^2-\sigma) = -\omega^2 \mathrm{Re}(1+i\omega)^{n+1} \\ -k\omega^2 + k_i[1+(n+1)\sigma] = \omega\, \mathrm{Im}(1+i\omega)^{n+1} \end{cases}$$

Solving for k and k_i gives

$$k = \frac{-\omega[1+(n+1)\sigma]\mathrm{Re}(1+i\omega)^{n+1} + (\sigma-\omega^2)\mathrm{Im}(1+i\omega)^{n+1}}{\omega(\omega^2 + n(n+1)\sigma^2 + 2n\sigma + 1)}$$

$$k_i = \frac{-\omega^2\mathrm{Re}(1+i\omega)^{n+1} + \omega(1+n\sigma)\mathrm{Im}(1+i\omega)^{n+1}}{\omega^2 + n(n+1)\sigma^2 + 2n\sigma + 1}$$

where

$$\mathrm{Re}(1+i\omega)^{n+1} = \left[1+\omega^2\right]^{\frac{n+1}{2}} \cos(n+1)\varphi$$

$$\mathrm{Im}(1+i\omega)^{n+1} = \left[1+\omega^2\right]^{\frac{n+1}{2}} \sin(n+1)\varphi$$

and

$$\varphi = \mathrm{atan}(\omega)$$ □

EXAMPLE 4.5 - PD control of $(s+1)^{-n}$

When the system (4.7) is controlled by a PD-controller, the loop transfer function becomes

$$G(s) = \frac{k+k_d s}{(s+1)^n}$$

Hence

$$G'(s) = -\frac{n(k+k_d s)}{(s+1)^{n+1}} + \frac{k_d}{(s+1)^n}$$

The design equation (3.2) becomes

$$1 + \frac{k+k_d s}{(s+1)^n} - \frac{\sigma k_d}{(s+1)^n} + \frac{\sigma n(k+k_d s)}{(s+1)^{n+1}} = 0$$

Hence

$$(s+1)^{n+1} + k(s+1+n\sigma) + k_d\left[s^2+s[1+(n-1)\sigma]-\sigma\right] = 0$$

Putting $s = i\omega$ gives the following two equations

$$\begin{cases} k(1+n\sigma) + k_d(-\omega^2-\sigma) = -\operatorname{Re}(1+i\omega)^{n+1} \\ k\omega + k_d\omega[1+(n-1)\sigma] = -\operatorname{Im}(1+i\omega)^{n+1} \end{cases}$$

Solving for k and k_d gives

$$k = \frac{-\omega[1+(n-1)\sigma]\operatorname{Re}(1+i\omega)^{n+1} + (\omega^2-\sigma)\operatorname{Im}(1+i\omega)^{n+1}}{\omega(\omega^2 + n(n-1)\sigma^2 + 2n\sigma + 1)}$$

$$k_d = \frac{\omega\operatorname{Re}(1+i\omega)^{n+1} - (1+n\sigma)\operatorname{Im}(1+i\omega)^{n+1}}{\omega^2 + n(n-1)\sigma^2 + 2n\sigma + 1}$$

□

As n increases, it becomes of course more and more difficult to control the process $(s+1)^{-n}$ with a PID controller. If a figure corresponding to Fig. 5 were drawn for different values of n, it would be seen that the admissible areas of combinations of σ and ω decrease with increasing n. It means that only a very low closed loop bandwidth can be obtained with a PID controller when n is large.

The final example gives the result of PI and PD control of a heat process described by the model

$$G_p(s) = e^{-\sqrt{sT}} \tag{4.8}$$

EXAMPLE 4.6 - PI and PD control of a heat process

First consider PI control of the system (4.8). The open loop transfer function becomes

$$G(s) = (k + \frac{k_i}{s})e^{-\sqrt{sT}}$$

The proportional and the integral gains become

$$k = -\frac{2e^r\omega\left[2\omega\cos(r) - \sigma r\sin(r) - 2\sigma\sin(r) + \sigma r\cos(r)\right]}{4\omega^2 + \sigma^2T\omega + 4r\sigma\omega + 2r\sigma^2}$$

$$k_i = \frac{2e^r\omega^2\left[2\omega\sin(r) + \sigma r\sin(r) + \sigma r\cos(r)\right]}{4\omega^2 + \sigma^2T\omega + 4r\sigma\omega + 2r\sigma^2}$$

where

$$r = \sqrt{\frac{\omega}{2}}$$

When the system (4.8) is controlled by a PD controller, the following loop transfer function is obtained.

$$G(s) = (k + k_d s)e^{-\sqrt{sT}}$$

The proportional and the derivative gains become

$$k = -\frac{2e^r\omega\left[2\omega\cos(r) - \sigma r\sin(r) + 2\sigma\sin(r) + \sigma r\cos(r)\right]}{4\omega^2 + \sigma^2T\omega + 4r\sigma\omega - 2r\sigma^2}$$

$$k_d = -\frac{2e^r\left[2\omega\sin(r) + \sigma r\sin(r) + \sigma r\cos(r)\right]}{4\omega^2 + \sigma^2T\omega + 4r\sigma\omega - 2r\sigma^2}$$

□

5. SIMULATION EXAMPLES

In Hägglund and Åström (1984), the dominant pole design was applied to several different simulated processes. The transfer functions of these processes were chosen to demonstrate the limitations of more simple design methods. In this section, the dominant pole design is applied to some additional systems which have dynamics that is common in process control.

The dominant pole design based on difference approximation is well suited for auto-tuning. In Åström and Hägglund (1984b), a method to automatically identify two points on the Nyquist curve is presented. The identification procedure automatically determines the frequencies as well as the values of the open loop transfer function at two points in the neighborhood of the cross over frequency. This identification procedure is used in the following examples. The desired relative damping of the dominant poles is chosen to $\zeta = 0.4$.

The PID controller has the structure given by Equation (4.3), with the relation α between the integral time and the derivative time equal to 0.25. See Equation (4.2). The parameter β is chosen according to Equation (4.5).

Processes with the following transfer functions were used

$$G_1 = \frac{1}{s} \qquad G_2 = \frac{1}{s+1} \qquad G_3 = \frac{1}{(s+1)^3}$$

$$G_4 = \frac{1}{(s+1)^6} \qquad G_5 = \frac{1}{(1+s)(1+0.2s)(1+0.05s)(1+0.01s)}$$

$$G_6 = \frac{1}{s+1}\, e^{-0.2s} \qquad G_7 = \frac{1}{s+1}\, e^{-s} \qquad G_8 = \frac{1}{s+1}\, e^{-2s}$$

$$G_9 = \frac{1}{(s+1)^2}\, e^{-0.4s} \qquad G_{10} = \frac{1}{(s+1)^2}\, e^{-2s} \qquad G_{11} = \frac{1}{(s+1)^2}\, e^{-4s}$$

The PID parameters and the frequency ω of the dominant poles are presented in the following table.

	ω	K	T_i	T_d	β
G_1	15.8	21.0	0.169	0.0422	0.29
G_2	15.7	24.7	0.185	0.0464	0.26
G_3	1.71	3.71	2.28	0.571	0.20
G_4	0.578	1.23	4.61	1.15	0.29
G_5	9.52	9.62	0.492	0.123	0.16
G_6	8.61	5.79	0.347	0.0867	0.26
G_7	2.12	1.46	1.12	0.281	0.32
G_8	1.20	0.762	1.78	0.445	0.36
G_9	2.14	3.20	1.54	0.385	0.23
G_{10}	0.880	0.939	2.64	0.661	0.33
G_{11}	0.555	0.593	3.67	0.917	0.38

In Fig. 6 - 16, the result of the simulations are presented. The figures show the output signals y above the input signals u. The systems are disturbed by a set-point change followed by a constant load disturbance.

The dominant pole design manages to control all the processes satisfactory. The processes G_8 and G_{11} have time delays which are quite long compared to the time-constants of the system. These processes are known to be difficult to control with a PID regulator without dead-time compensation. Processes with several different time-constants, like the process G_5, is known to be poorly controlled when the Ziegler-Nichols design is used.

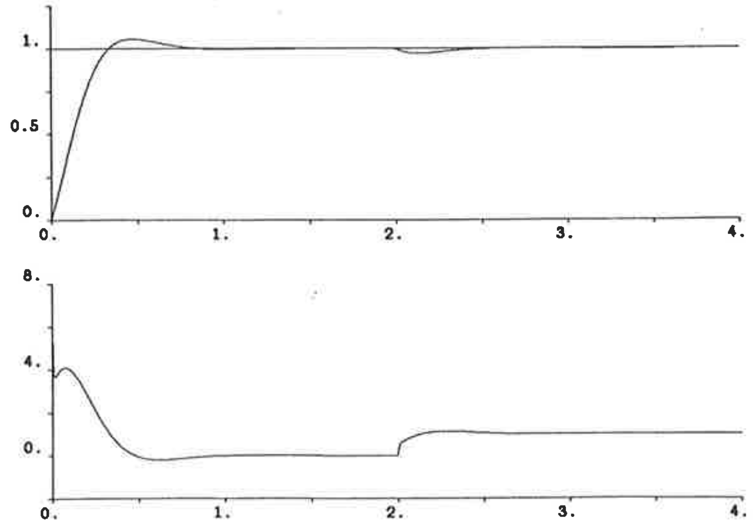

Fig. 6 $G_1 = \frac{1}{s}$

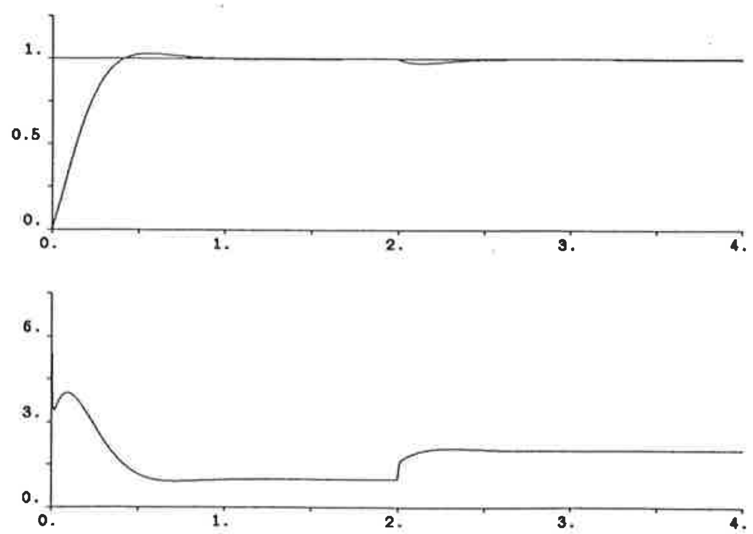

Fig. 7 $G_2 = \frac{1}{s+1}$

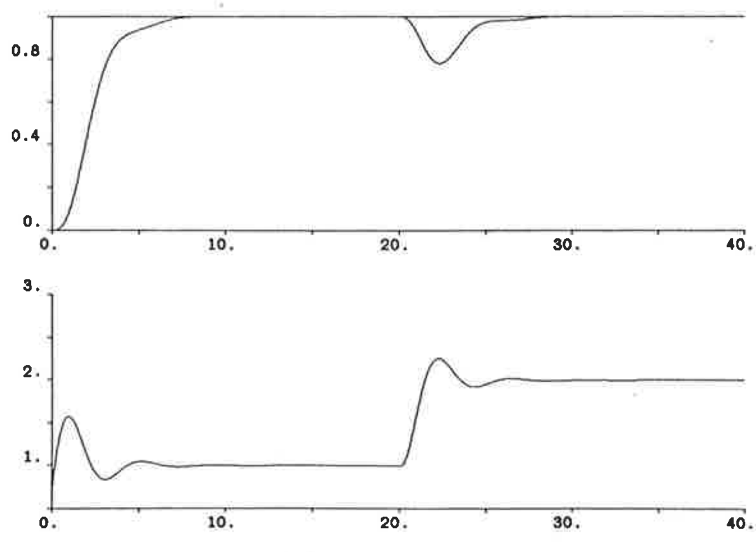

Fig. 8 $G_3 = \frac{1}{(s+1)^3}$

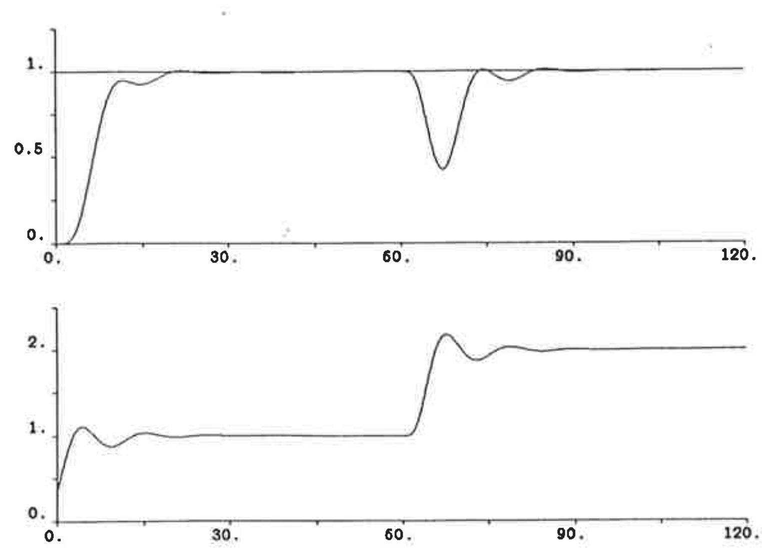

Fig. 9 $G_4 = \frac{1}{(s+1)^6}$

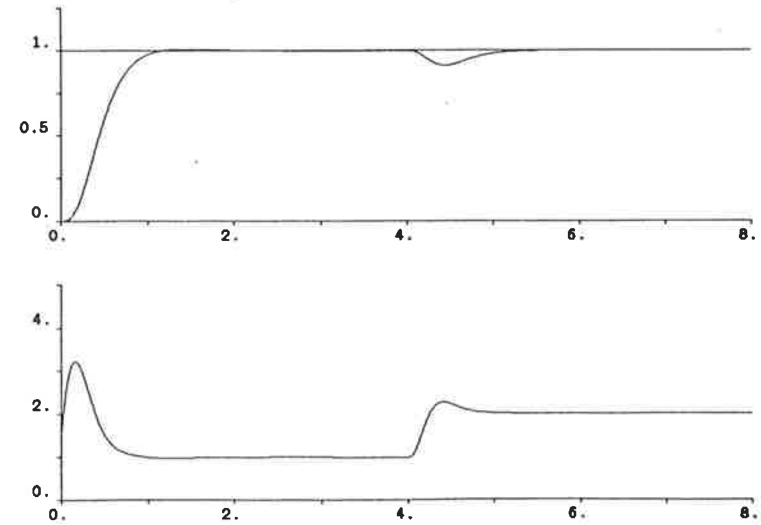

Fig. 10 $G_5 = \frac{1}{(1+s)(1+0.2s)(1+0.05s)(1+0.01s)}$

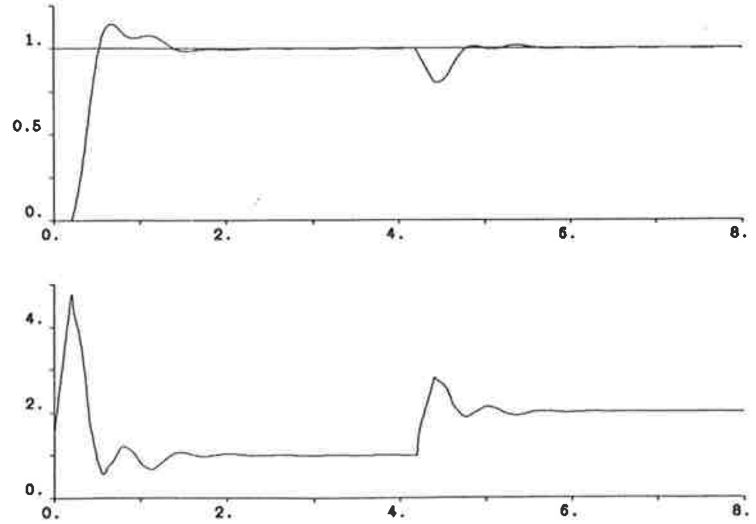

Fig. 11 $G_6 = \frac{1}{s+1} e^{-0.2s}$

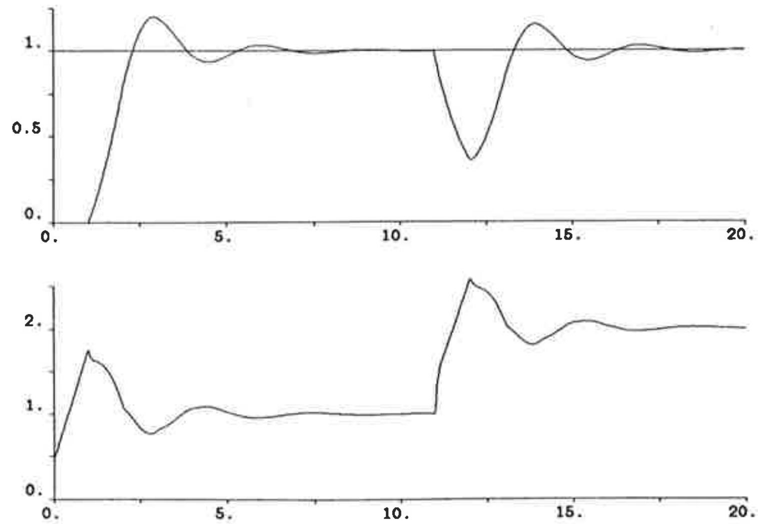

Fig. 12 $G_7 = \frac{1}{s+1} e^{-s}$

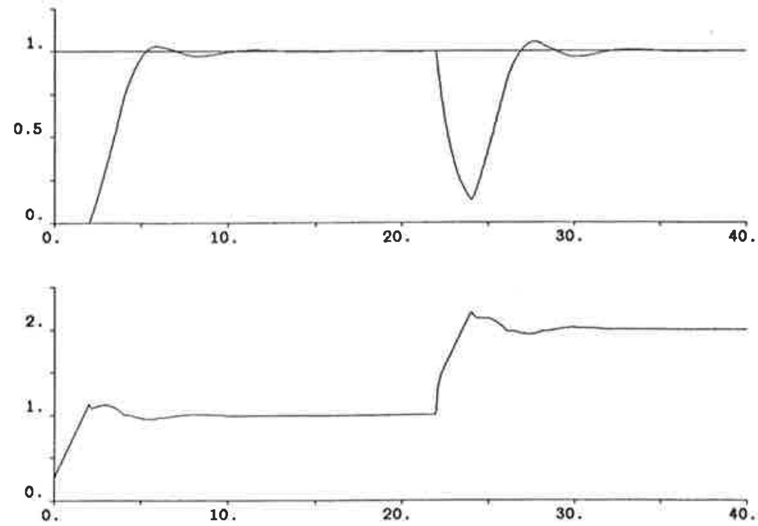

Fig. 13 $G_8 = \frac{1}{s+1} e^{-2s}$

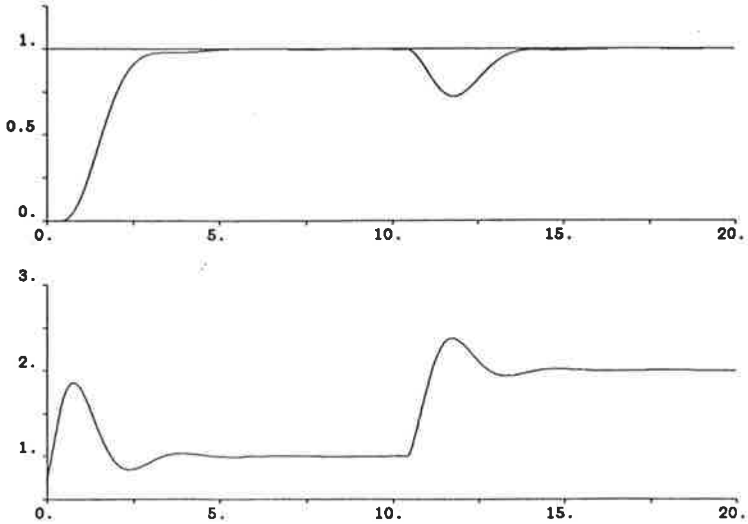

Fig. 14 $G_9 = \frac{1}{(s+1)^2} e^{-0.4s}$

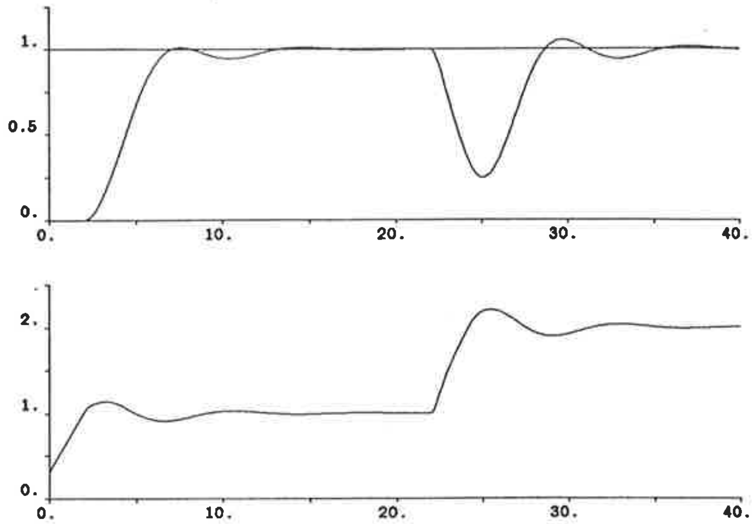

Fig. 15 $G_{10} = \frac{1}{(s+1)^2} e^{-2s}$

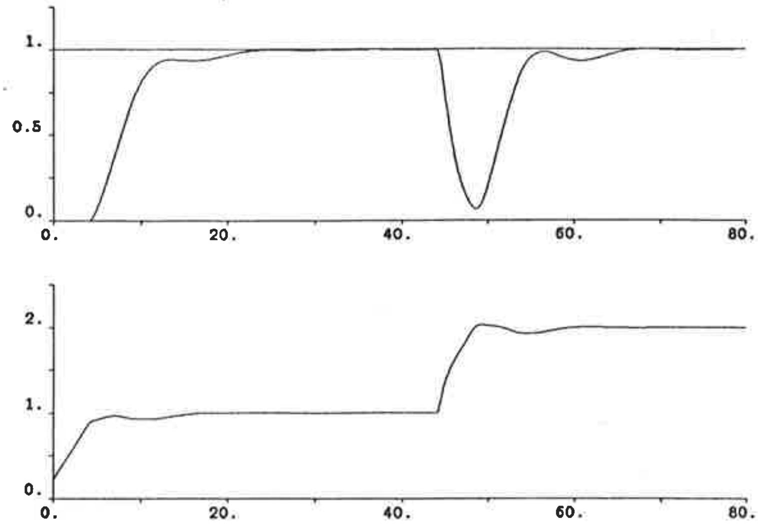

Fig. 16 $G_{11} = \frac{1}{(s+1)^2} e^{-4s}$

6. CONCLUSIONS

Design methods based on knowledge of only one point on the Nyquist curve, like the Ziegler-Nichols method and specifications on phase and amplitude margins, have the advantage of being simple to use. Ziegler and Nichols also proposed a simple method to identify one point on the Nyquist curve.

In Hägglund and Åström (1984), the limitations of design methods based on knowledge of only one point on the Nyquist curve was demonstrated. The dominant pole design method, which is based on the knowledge of two points on the Nyquist curve, is a method to approximately position those poles which have a dominant influence on the transient behaviour of the system. In Hägglund and Åström (1984) and in this report, this method is shown avoid the problems associated with the simpler methods mentioned above, and to manage to control many models of processes which are common in the process industry.

The dominant pole design method is primarily intended to be used combined with the autotuning method presented in Åström and Hägglund (1984b). This enables an automatic determination of the two points on the Nyquist curve.

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Title and subtitle

Dominant Pole Design

Abstract

This paper describes a new method for designing simple feedback systems. The idea is to position only those closed loop poles which have a dominant influence on the closed loop response. Techniques for approximate determination of the dominant poles are derived, as well as design methods based on these formulas. Apart from applications to design, the results may also be used to assess the complexity of the compensators needed to satisfy different specifications. The method works for systems where the closed loop dynamics can be approximated by dominant poles.

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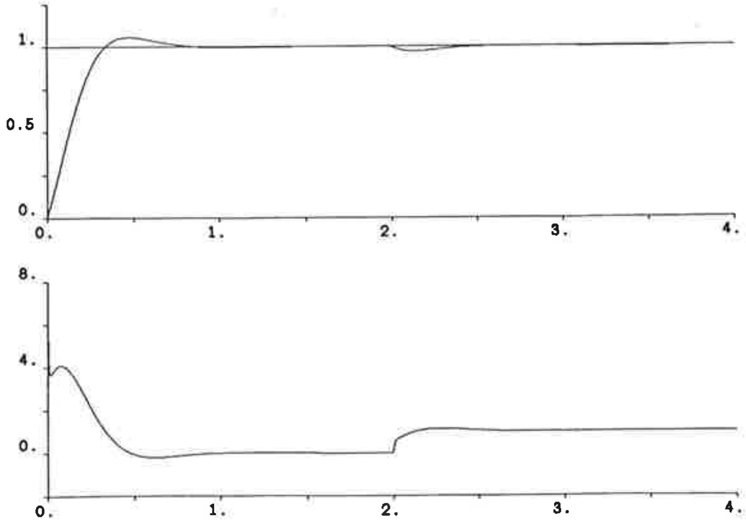

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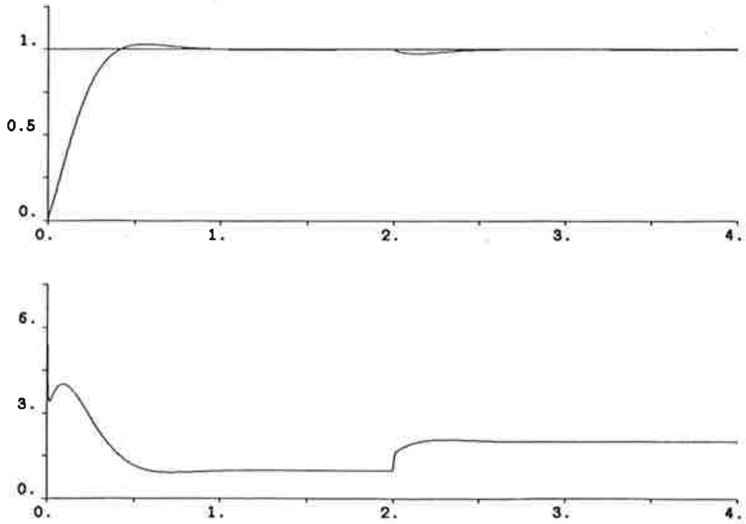

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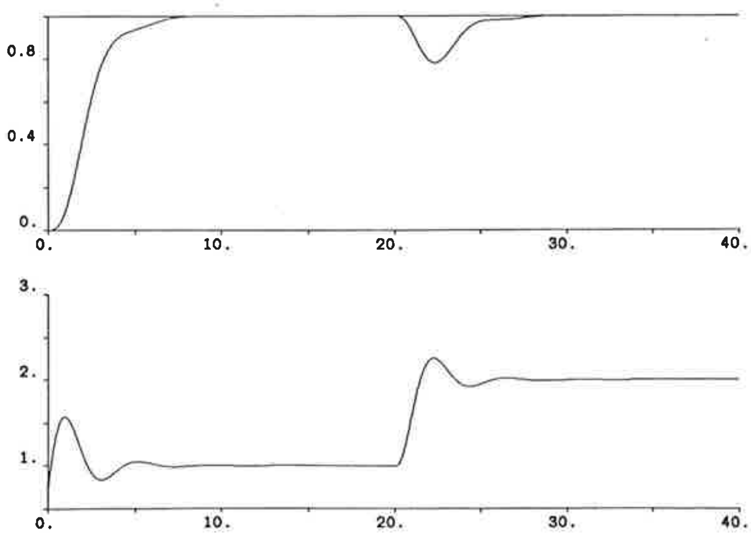

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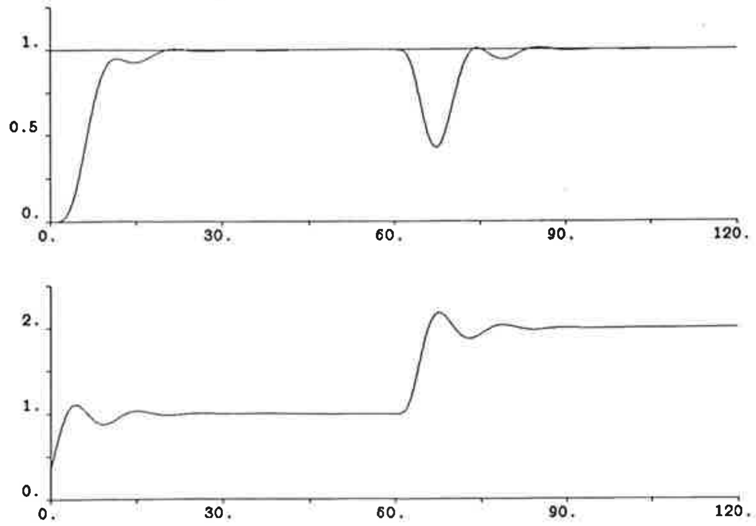

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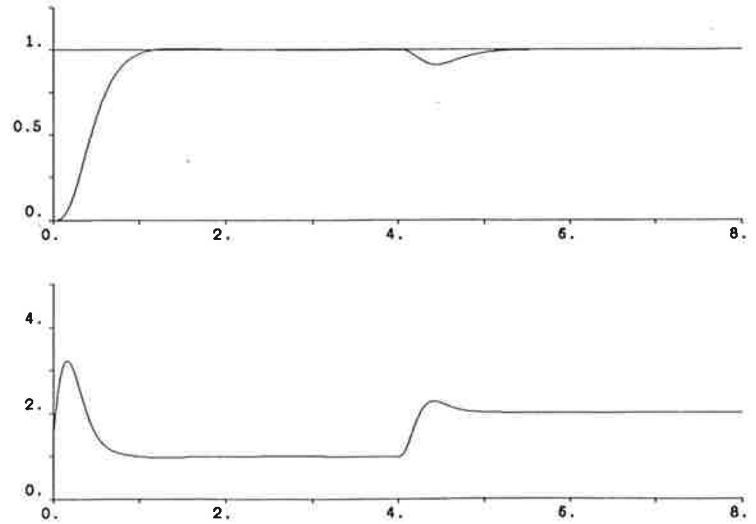

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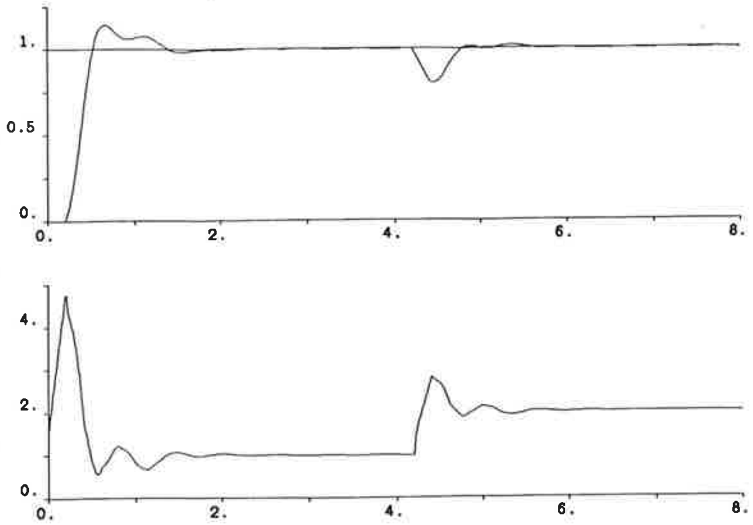

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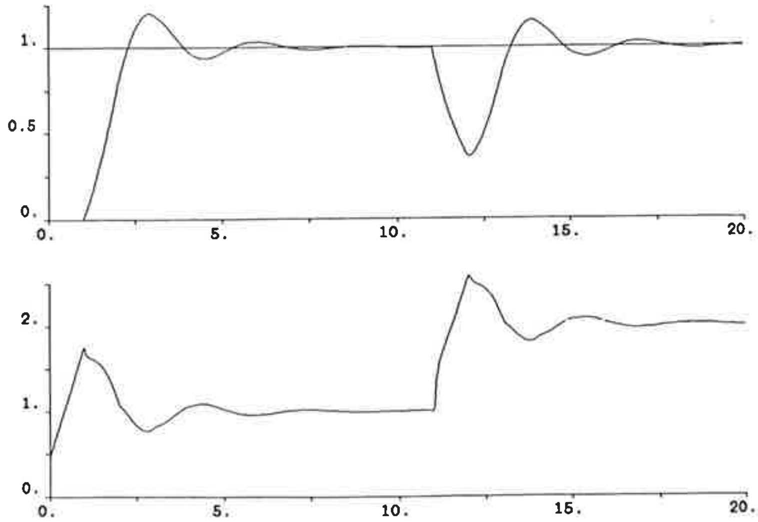

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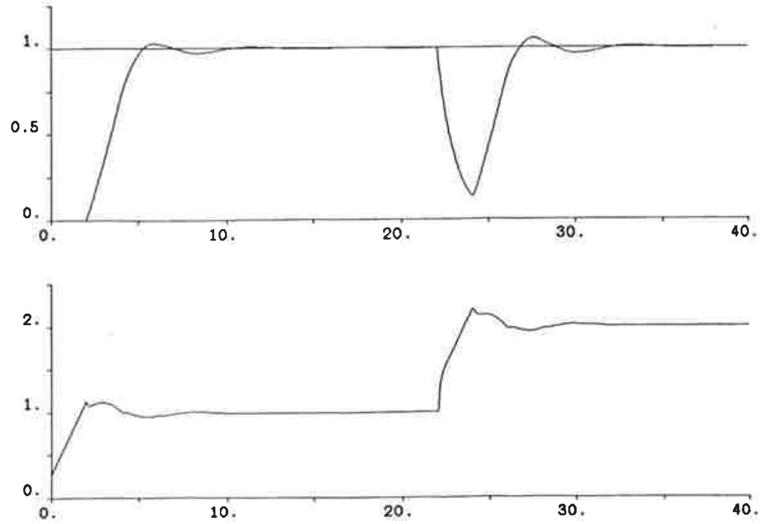

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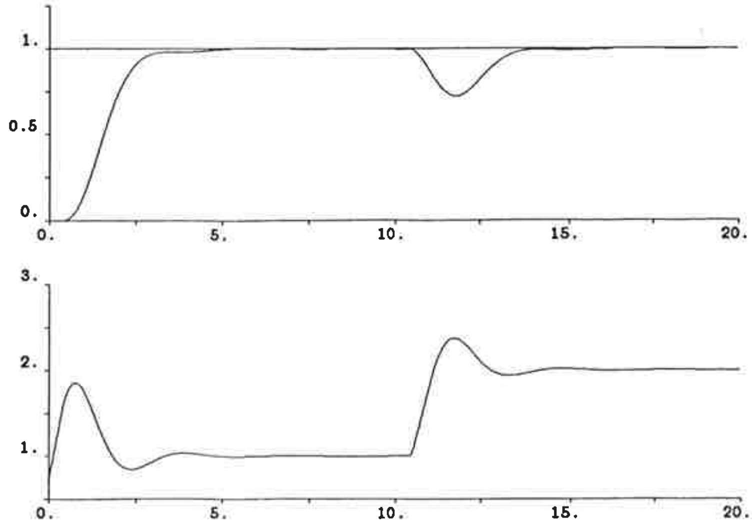

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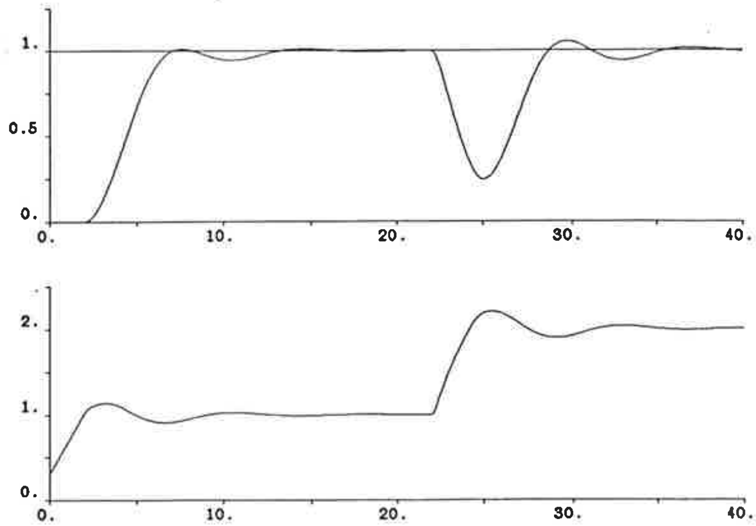

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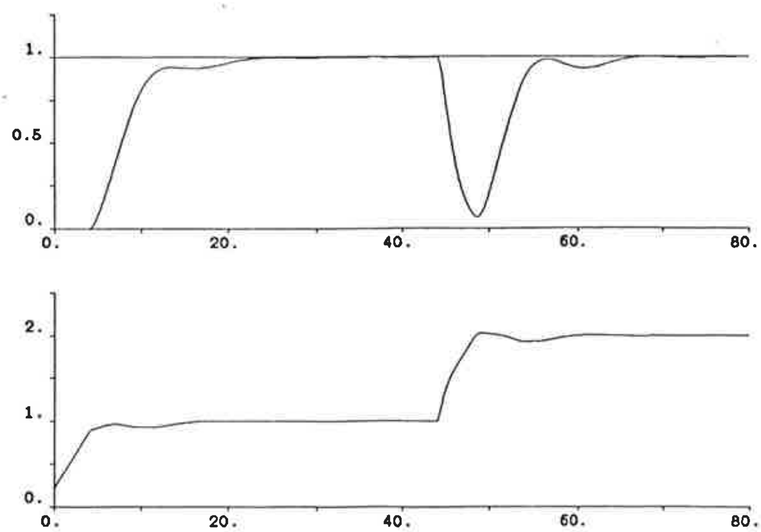

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