A Nonlinearity with Adjustable Phaseshift

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1. INTRODUCTION

A method for automatic tuning of simple regulators was described in Åström and Hägglund (1984). The method is based on automatic determination of the crossover frequency and the gain at that frequency. The data is obtained by analysing the limit cycle obtained when the system is connected in a feedback loop with a relay. It is of interest to make similar experiments for determination of points where the phaseshift of the plant is specified at other values than 180°. This paper will describe a nonlinearity which allows this. Apart from applications to autotuning the result is also useful for other problems like in expert control (Åström and Anton (1984)) when it is of interest to explore the properties of the open loop transfer function of a system.

The paper is organized as follows. The basic idea is given in section 2. The implementation of the nonlinearity is discussed in section 3 which also contains some simulations. Some possible uses are outlined in the conclusion section 4.

2. THE BASIC IDEA

The key problem is to construct a nonlinear system such that the inverse describing function has a constant angle to the real axis as shown in Fig. 2.1.

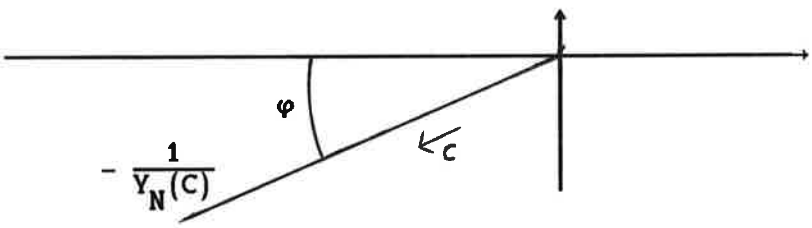


Fig. 2.1. Desired inverse describing function

Consider first a relay with hysteresis. See Fig. 2.2.

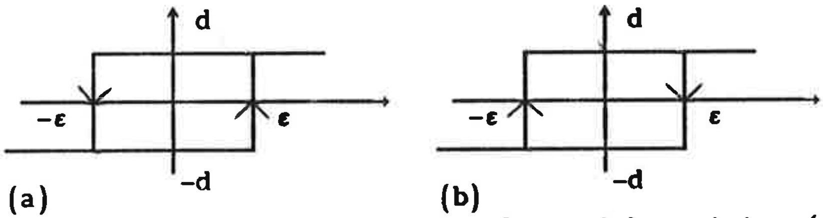


Fig. 2.2. Characteristics of relays with positive (a) and negative (b) hysteresis.

The negative inverse describing function of the relay is

$$-\frac{1}{Y_N(C)} = -\frac{\pi}{4}\cdot\frac{C}{d}\left[\sqrt{1-\left(\frac{\varepsilon}{C}\right)^2} + i\cdot\frac{\varepsilon}{C}\right] \tag{2.1}$$

and is shown in Fig. 2.3.

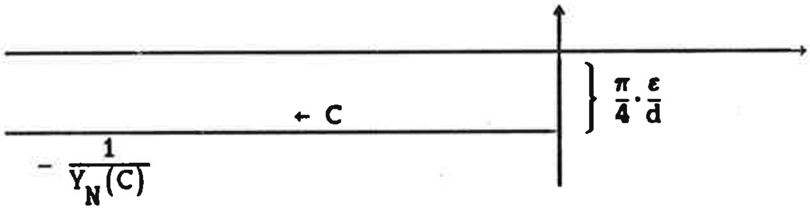


Fig. 2.3. The negative inverse describing function of a relay with hysteresis width ε and gain d

Assume that the hysteresis width ε is made proportional to the input amplitude C, then the phase of (2.1) will be constant. In particular if ε is chosen as

$$\varepsilon = C\cdot\sin(\varphi)$$

we get

$$\arg\left[\sqrt{1-\left(\frac{\varepsilon}{C}\right)^2} + i\cdot\frac{\varepsilon}{C}\right] = \arctan\left[\frac{\sin(\varphi)}{\sqrt{1-\sin^2(\varphi)}}\right] = \varphi$$

Hence, the relay with amplitude dependent hysteresis will have the desired property

$$-\frac{1}{Y_N(C)} = -\frac{\pi}{4}\cdot\frac{C}{d}\cdot e^{i\varphi} \quad\Rightarrow\quad \begin{cases} |G(i\omega)| \approx \frac{\pi}{4}\cdot\frac{C}{d} \\ \arg\{G(i\omega)\} \approx -\pi+\varphi \end{cases}$$

Clearly, the amplitude of the limit cycle can easyly be justified by just changing the relay gain. It should be noted that the system is strictly spoken not a static nonlinearity. Apart from the fact that a usual relay with fix hysteresis is not memoryless this system also needs an extra state to define the maximum of the input. The need to know the amplitude of the limit cycle will restrict our choice of phaseshift as will be discussed below.

Intuitive discussion of what happens

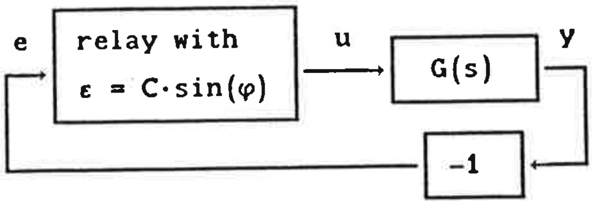
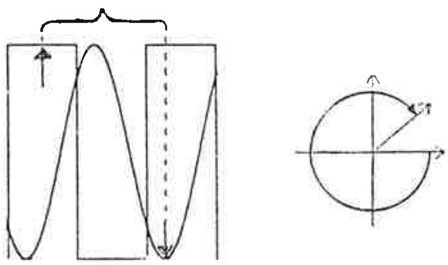
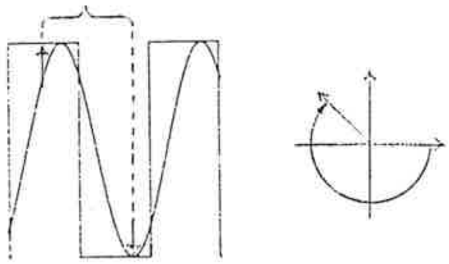


Fig. 2.4. The relay with amplitude dependent hysteresis ε connected to the system in a feedback loop

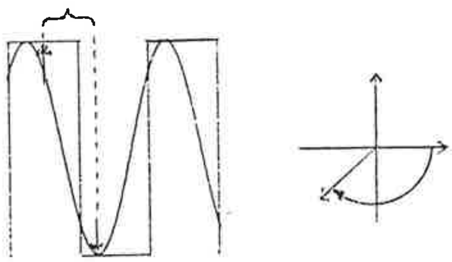
A pulse u is sent to the system and the first harmonic is returned to the relay. The amplitude of this signal C defines the hysteresis width ε. Hence it decides when the next pulse should be sent to maintain a limit cycle with a frequency corresponding to a specific phaselag.

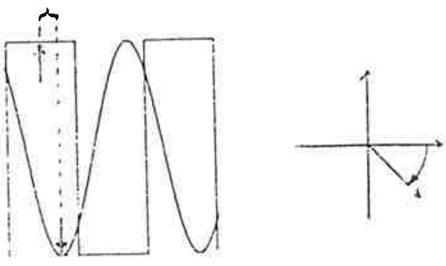


1:st quadrant: The first pulse hasn't come back yet when it is time to send the next. The information about the amplitude deciding the next pulse departure is not collected in time. Hence, here will never be any limit cycles. Clearly, the describing function interpretation of the nonstatic relay wont work here.



2:nd and 3:rd quadrant: The amplitude of the returned pulse, C, is defined in time and the next pulse can be generated.





4:th quadrant: Information about C is needed to decide the pulse width h. If the system has a monotone step response C will grow with the pulse width h and there will be no limit cycle ($h \to \infty$). However, if the step response has an overshoot, C is independent of h and a limit cycle is possible.

To summarize, limit cycles can be generated in the left half plane with the interpretation of the nonstatic relay having a describing function being a straight line emerging form the origin. This interpretation is lost in the 1:st quadrant. It is also lost in the 4:th quadrant if the system has a monotone step response.

Exact conditions for oscillations

Because the input to the system is piece-wise constant we can sample the system at the switch-times. The sample period h will be equal to half the period of the limit cycle. When there is a stable limit cycle the pulse transfer function will satisfy the following condition:

$$H(h,z=-1) = \begin{cases} \frac{\varepsilon}{d} & G(i\omega) \text{ in the upper half plane} \\ -\frac{\varepsilon}{d} & G(i\omega) \text{ in the lower half plane} \end{cases}$$

where $T = 2h \approx 2\pi/\omega$ is the period of the limit cycle.

It was mentioned in the last section that a limit cycle could be maintained in the 4:th quadrant. To illustrate this a second order system with complex poles is chosen as example.

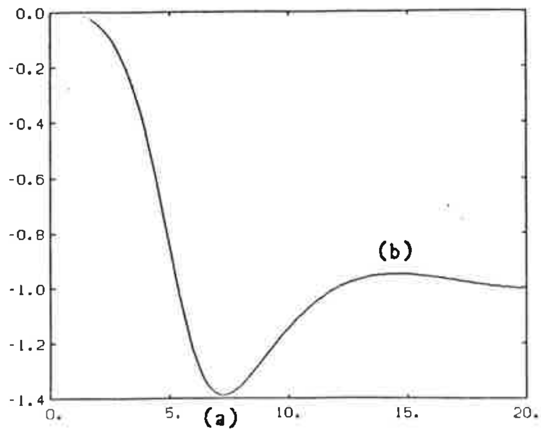
Ex 1.

$$G(s) = \frac{\omega_o^2}{s^2 + 2\zeta\omega_o s + \omega_o^2} \qquad \text{where } \begin{cases} \zeta = 0.5 \\ \omega_o = 1 \end{cases}$$

The pulse transfer function H(h,z) is evaluated in z = -1.

$$H(h,z=-1) = \frac{e^{-h} - 1 + e^{-h/2}\sin(ah)/a}{1 + 2e^{-h/2}\cos(ah) + e^{-h}} \quad , \; a = \frac{\sqrt{3}}{2}$$

Plotting H(h,z=-1) against T=2h could be helpful to examine the exact period T corresponding to a given hysteresis ε. See Fig. 2.5.



(a) (7.25, -1.39)
(b) (14.5, -0.948)

Fig. 2.5 The pulse transfer function H(h,z=-1) as a function of the period of the limit cycle T=2h.

3. SIMULATIONS

A relay with the hysteresis depending on the input amplitude C is implemented in the following simnon system.

```
DISCRETE SYSTEM RELAY
"Relay with amplitude dependent hysteresis (fix phaseshift)
"Ulf Holmberg 850717

INPUT  error
OUTPUT u
STATE sp  sn  uold  C
NEW  nsp nsn nuold nC
TIME  t
TSAMP ts

initial
uold = d
sort

"NEGATIVE HYSTERESIS ------------------------------------------------
nuold = if (e>eps and u<0) or (e<-eps and u>0) then -u else u
nsn   = if abs(e) > abs(eps) then 1 else -1
un    = if nsn*sn or nsn then uold else -uold

"POSITIVE HYSTERESIS ------------------------------------------------
up    = if e>eps then d else if e<-eps then -d else if sp then d else -d
nsp   = if u>0 then 1 else 0

"AMPLITUDE DEPENDENT HYSTERESIS
nC    = if (not nsn*sn) and nsn and fi<0 then 0 else max(abs(e),C)
eps   = abs(C*sin(fi*3.1415/180))

"RELAY OUTPUT -------------------------------------------------------
u = if fi>0 then up else un

e  = if abs(f)<90 then -error else error
fi = if abs(f)>90 then -sign(f)*(180 - abs(f)) else f

ts = t + dt

f:-135        "Phaseshift
d:1           "Relay gain
dt:.01        "Sampling interval

END
```

If the variable fi (the angle φ) is negative we get the 2:nd quadrant. Then to get the 4:th quadrant we just change sign of the returned signal e, i.e. a positive feedback.

Note that C is the amplitude of the approximate sine-wave of the limit cycle. Thus if we start in steady-state C will be zero initially and will thereafter grow until the crossing with the Nyquist curve is reached, i.e. there will be a soft self-excitation. This is with exception of the extreme case in the 4:th quadrant described earlier (φ_{min}).

Now, reconsider Ex 1. With the chosen value of the relative damping $\zeta=0.5$ the minimum phaseshift was calculated to $\varphi_{min} \approx 43^{o}$. Even though a limit cycle with this phaseshift could be maintained there will however not be an automatic startup as is the case when a larger phaseshift is chosen. This is due to the fact that the system normally starts from the origin and will therefore have a less amplitude than if it starts from $-\varepsilon$ as in Fig. 2.6. Two possible ways to reach the extreme case is now given. Both start with a larger overshoot.

1. Let the system startup itself at a higher frequency in the 4:th quadrant, i.e. a limit cycle with larger amplitude. Then shift to a less phaseshift.

2. Let the relay gain be larger initially to get a large overshoot.

The two methods will now be used to reach the limit cycle corresponding to a phaseshift of -47^o. See Fig. 3.1 a and b. The hysteresis ε approaches 0.97 which according to Fig. 2.5 corresponds to a limit cycle period of about 13, i.e. slightly to the left of point (b), where the phaseshift is $-\varphi_{min}=-43^o$.

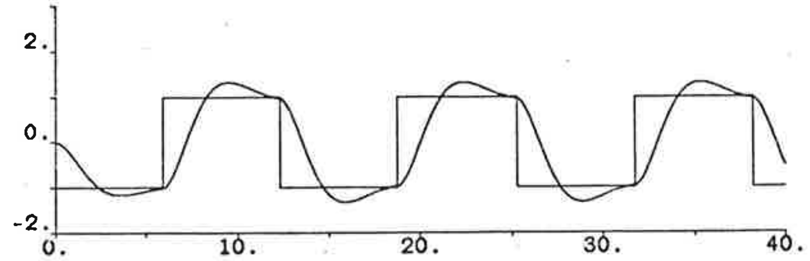


Fig. 3.1 a) The phaseshift is initially -60^o
and is at time = 10 changed to -47^o

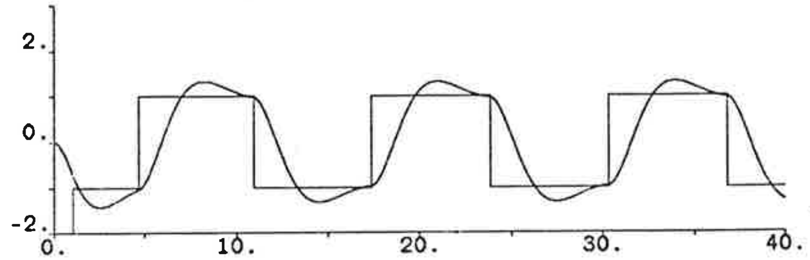


Fig. 3.1 b) The relay gain d = 2 initially and
is changed to d = 1 at time = 1

Ex 2. $$G(s) = \frac{1}{(s+1)^8}$$

This system has a monotone step response and will consequently not oscillate in the 4:th quadrant. Nevertheless the nonlinearity could be useful to explore the Nyquist curve in the left half plane. In Fig. 3.2 a and b the function $-1/Y_N(C)$ makes the angles $\pm$ 45^o to the negative real axis, causing limit cycles near frequencies ω where $\arg\{G(i\omega)\} = -\pi \pm \pi/4$.

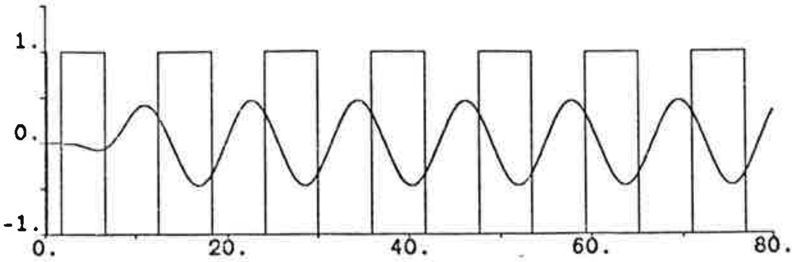


Fig. 3.2 a) Limit cycle where $\arg\{G(i\omega)\} \approx -\pi - \frac{\pi}{4}$
Compare the period with $T = 2\pi/\omega = 11.75$
and the amplitude with $|G(i\omega)| \cdot 4d/\pi = 0.47$
where $\omega = \tan(\frac{5\pi}{4} \cdot \frac{1}{8})$

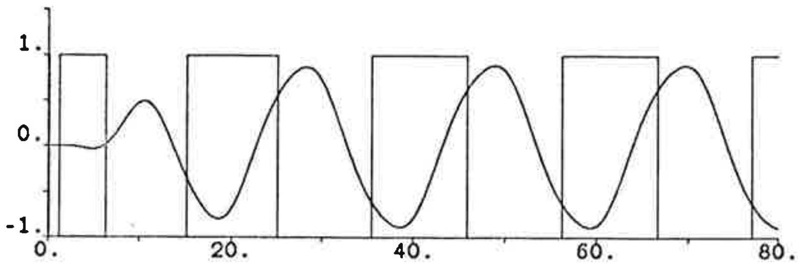


Fig. 3.2 b) Limit cycle where $\arg\{G(i\omega)\} \approx -\pi + \frac{\pi}{4}$
Compare the period with $T = 2\pi/\omega = 20.71$
and the amplitude with $|G(i\omega)| \cdot 4d/\pi = 0.90$
where $\omega = \tan(\frac{3\pi}{4} \cdot \frac{1}{8})$

In principle $-1/Y_N(C)$ could be chosen to be the positive imaginary axis. This will cause the relay to switch at the top of the returned sinusoidal signal. Of course this situation will be very sensitive to noise and to the oscillation being centered. Hence there is a practical limit to identify points on the Nyquist curve near the imaginary axis. On the other hand the relay works well near the negative real axis. This makes the relay of particular importance in areas like autotuning and model reduction where the knowledge of points on the Nyquist curve near the crossover frequency is of special interest.

4. CONCLUSIONS

A nonstatic nonlinearity acting as if it had a describing function being a straight line emerging from the origin in any direction but the first quadrant has been constructed. This nonlinearity is useful for autotuning and for exploring of the Nyquist curve of a system. When a system is connected to the nonlinearity in a feedback loop the angle could be chosen to get a specific crossing with the Nyquist curve. Valuable information of the system is then received from a stable limit cycle.

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