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Lundh, Michael

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Michael Lundh

Department of Automatic Control
Lund Institute of Technology
September 1985

Department of Automatic Control
Lund Institute of Technology
P.O. Box 118
S-221 00 Lund Sweden

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Author(s)
Michael Lundh

Supervisor

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Abstract

A controller can be implemented in many different ways. It is investigated how different realizations influence the variance of the output of the controller, due to the round-off noise which is generated in the multiplications. Three different controllers have been studied.

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A SHORT STUDY ON IMPLEMENTATION OF CONTROLLERS

Michael Lundh

Department of Automatic Control
Lund Institute of Technology
1985-09-09

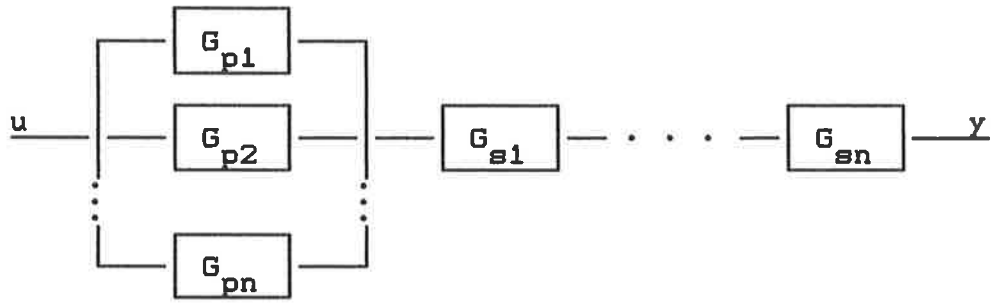


Figure 1. Structure of Controller.

1. INTRODUCTION.

A controller with a given transfer-function can be realized in many different ways. The sensitivity to quantization in coefficients and to round-off errors in the multiplications depend on the chosen realization.

Purpose.

In this paper we will study how the variance in the output from the controller due to the noise from round-off-effects in the multiplications in the controller depends on the realization of the controller.

Limitations.

This paper only treats single-input single-output continuous time controllers. A study is made of those realizations where it is possible to split the controller into a number of first order sub-systems. Some of these sub-systems are connected in parallel and others are connected serially. See figure 1. The observable and controllable canonical forms will not be treated here because they are badly conditioned, and therefore sensitive to changes in the coefficients.

Assumptions.

The controller is on the form

$$\dot{x}(t) = A \cdot x(t) + B \cdot u(t) + Q \cdot e(t)$$
$$y(t) = C \cdot x(t)$$

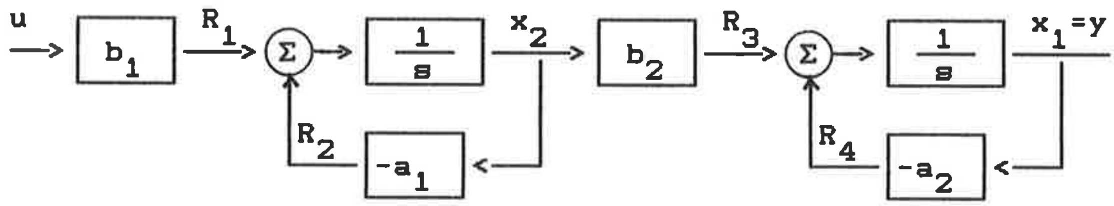
where A, B and C are matrixes and e(t) is a white noise process with unit variance.

The covariance-matrix Q is diagonal and its values are representing the sum of variances of noises that are thought as being generated by the analog multipliers. Each diagonal-element of Q is a sum of two variances that come from the multiplications with a_i and b_i. It is possible to add the variances becauses the noises are independent.

A multiplication whose product belongs to the range $[-1,1]$ generate noise with the incremental variance $\varepsilon^2 \cdot dt$ and a multiplication that gives a result in the range $[-q_i,q_i]$ generates noise with the incremental variance $(q_i \cdot \varepsilon)^2 \cdot dt$.

It is however possible without loss of generality to chose coefficients so that all multiplications for computation of a certain derivative give results in the same range.

For a second order system we have the following block-diagram



For this realization the values of R_1 and R_2 lies within the range $[-q_1,q_1]$ and the values of R_3 and R_4 lies within $[-q_2,q_2]$. This gives the following covariance-matrix.

$$Q = \begin{bmatrix} 2 \cdot q_1^2 & 0 \\ 0 & 2 \cdot q_2^2 \end{bmatrix} \cdot \varepsilon^2$$

It is also assumed that multiplication by 1 or -1 in the C-matrix gives no noise. Further, the integrators are assumed to be ideal. Finally it is assumed that the signals are scaled to avoid overflow in the internal states. The scaling itself is no source for noise.

2. ANALYSIS OF SOME SYSTEMS.

Three different controllers are studied to obtain some experience of the properties of different realizations.

2.1 Controller 1.

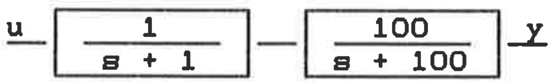
The first controller has the transfer-function

$$G(s) = \frac{100}{(s + 100)(s + 1)}$$

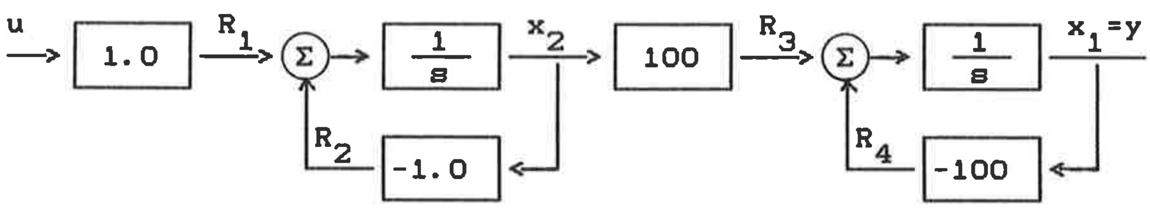
This transfer-function has been realized in five different ways. For each realization the asymptotic output-variance $E\,y^2$ is computed with the package CTRLC. See appendix 1 for the macro used.

Realization 1.1

This realization consist of two serially connected first order systems.



The A, B and C matrixes that correspond to this structure can be found in appendix 2.
The block-diagram then looks as follows.



The results R_1 and R_2 are in the interval [-1,1]. The results R_3 and R_4 belong to the interval [-100,100].

According to the assumptions of the generated noise this gives q_1=100.0 and q_2=1.0. Then we achieve the following covariance-matrix Q.

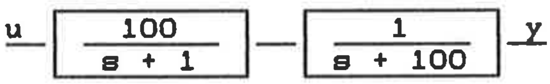
$$Q = \begin{bmatrix} 2\cdot 100^2 & 0 \\ 0 & 2\cdot 1^2 \end{bmatrix} \cdot \varepsilon^2$$

Due to round-off effects in the multiplication the asymptotic variance in the output for this realization is

$$E\ y^2 = 100.99\cdot\varepsilon^2$$

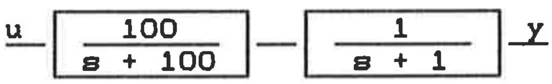
For realization 1.2 to 1.5 the domain of the multiplications have been determined in the same way as for realization 1.1. The A, B and C matrixes for all realizations can be found in appendix 2. From these the variances of the noises have been determined. The asymptotic output-variances were computed with the following results.

Realization 1.2



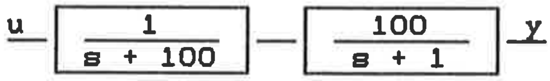
$$\Rightarrow \begin{cases} q_1 = 100\cdot\varepsilon \\ q_2 = 100\cdot\varepsilon \end{cases} \Rightarrow E\ y^2 = 100.99\cdot\varepsilon^2$$

Realization 1.3



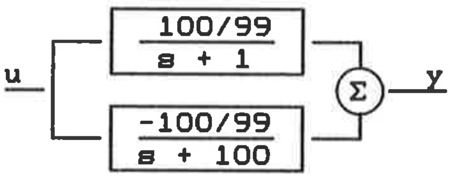
$$\Rightarrow \begin{cases} q_1 = 1\cdot\varepsilon \\ q_2 = 100\cdot\varepsilon \end{cases} \Rightarrow E\ y^2 = 1.9901\cdot\varepsilon^2$$

Realization 1.4



$$\Rightarrow \begin{cases} q_1 = 1 \cdot \varepsilon \\ q_2 = 1 \cdot \varepsilon \end{cases} \Rightarrow \quad E\, y^2 = 1.9901 \cdot \varepsilon^2$$

Realization 1.5



$$\Rightarrow \begin{cases} q_1 = 100/99 \cdot \varepsilon \\ q_2 = 100/99 \cdot \varepsilon \end{cases} \Rightarrow \quad E\, y^2 = 1.0305 \cdot \varepsilon^2$$

Summary

Realization 5, which is the parallel form, has the lowest output-variance. The realizations 3 and 4 also have low output-variance, but realization 1 and 2 are obviously unsuitable from the point of view of multiplication noise. It is also noticed that only the location of poles determine the output-variance. The influence of the location of multiplications with gain-factors is eliminated because of scaling.

From these calculations it is obvious that it is preferable to realize this controller in the parallel form. If the the first order sub-systems for some reason must be connected serially then the part with smallest bandwidth should follow after the part with wider bandwidth.

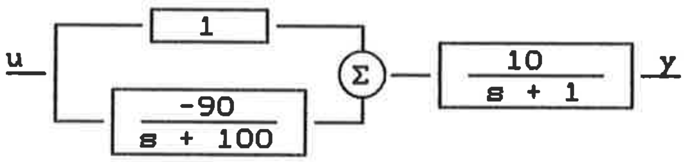
2.2 Controller 2.

The second controller has the transfer-function

$$G(s) = \frac{100\ (s + 10)}{(s + 100)\ (s + 1)}$$

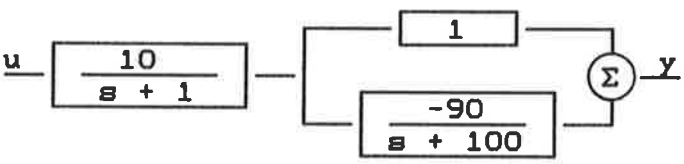
This transfer-function has been realized in five different ways. For each realization the asymptotic output-variance $E\ y^2$ is computed in the same way as was described previously.

Realization 2.1



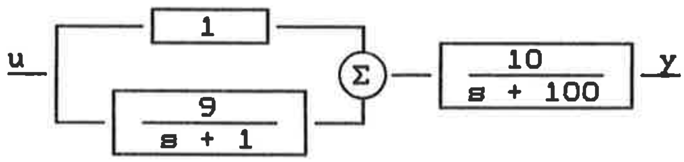
$$\Rightarrow \begin{cases} q_1 = 1 \cdot \varepsilon \\ q_2 = 90 \cdot \varepsilon \end{cases} \Rightarrow \quad E\ y^2 = 81.198 \cdot \varepsilon^2$$

Realization 2.2



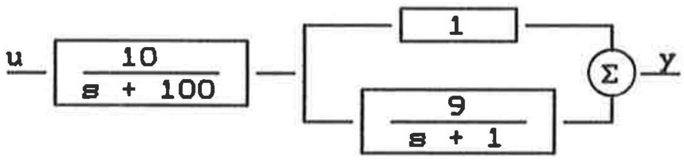
$$\Rightarrow \begin{cases} q_1 = 10 \cdot \varepsilon \\ q_2 = 900 \cdot \varepsilon \end{cases} \Rightarrow \quad E\ y^2 = 8102.0 \cdot \varepsilon^2$$

Realization 2.3



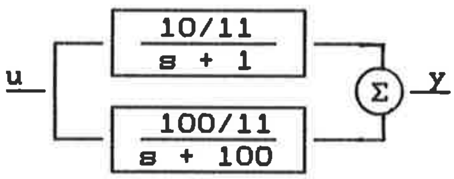
$$\Rightarrow \begin{cases} q_1 = 100 \cdot \varepsilon \\ q_2 = 9 \cdot \varepsilon \end{cases} \Rightarrow \quad E\, y^2 = 100.80 \cdot \varepsilon^2$$

Realization 2.4



$$\Rightarrow \begin{cases} q_1 = 0.9 \cdot \varepsilon \\ q_2 = 10 \cdot \varepsilon \end{cases} \Rightarrow \quad E\, y^2 = 2.7902 \cdot \varepsilon^2$$

Realization 2.5



$$\Rightarrow \begin{cases} q_1 = 10/11 \cdot \varepsilon \\ q_2 = 100/11 \cdot \varepsilon \end{cases} \Rightarrow \quad E\, y^2 = 1.6529 \cdot \varepsilon^2$$

<u>Summary</u>

The parallel realization has the lowest output-variance, as in the previous calculations. The realization 2.4 also has low output-variance. This was expected because the sub-system with the widest band-width is in the first block of the controller. The realizations 2.1, 2.2 and 2.3 have bad properties.

2.3 Controller 3.

We now will take a look at a controller which has multiple poles. It is therefore not possible to realize it completely in parallel. This controller has the transfer-function

$$G(s) = \frac{5\,(\,s + 80\,)}{(\,s + 100\,)\ \ (\,s + 2\,)^2}$$

This transfer-function has been realized in eleven different ways. belonging to three groups. For each realization the asymptotic output-variance $E\ y^2$ is computed in the same way as in the previous experiments.

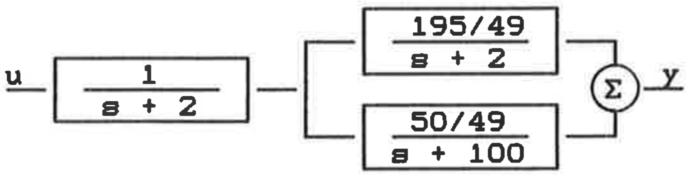
GROUP 1.

The controller has been separated in sub-systems as follows

$$G(s) = \frac{1}{s + 2}\left[\frac{195/49}{s + 2} + \frac{50/49}{s + 100}\right]$$

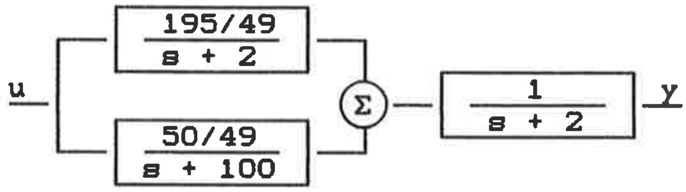
This structure can be realised in two different ways, either starting with the parallel or with the serial block.

Realization 3.11



$$\Rightarrow \quad E\ y^2 = 2.9825 \cdot \varepsilon^2$$

Realization 3.12



$$\Rightarrow \quad E\, y^2 = 2.9899 \cdot \varepsilon^2$$

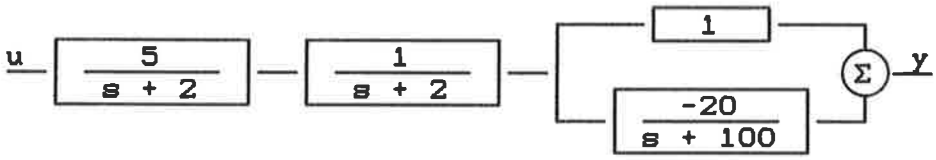
GROUP 2.

The controller has been separated in sub-systems as follows

$$G(s) = \frac{5}{(s+2)^2} \left[1 - \frac{20}{s+100} \right]$$

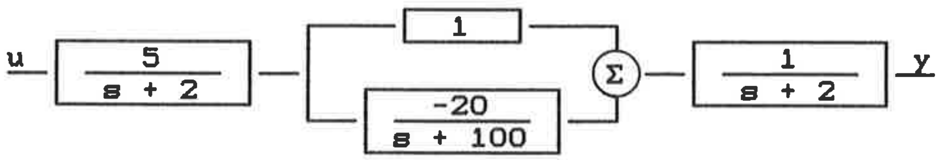
This structure can be realised in three different ways, depending on the location of the parallel part.

Realization 3.21



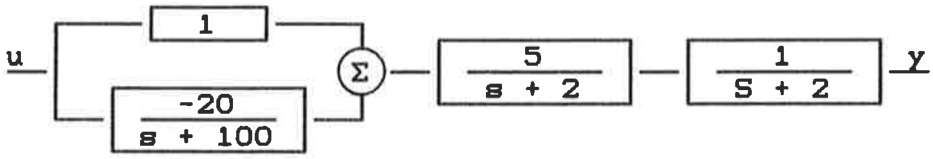
$$\Rightarrow \quad E\, y^2 = 9.2723 \cdot \varepsilon^2$$

Realization 3.22



$$\Rightarrow \quad E\, y^2 = 3.1228 \cdot \varepsilon^2$$

Realization 3.23



$$\Longrightarrow \quad E\,y^2 = 3.0625 \cdot \varepsilon^2$$

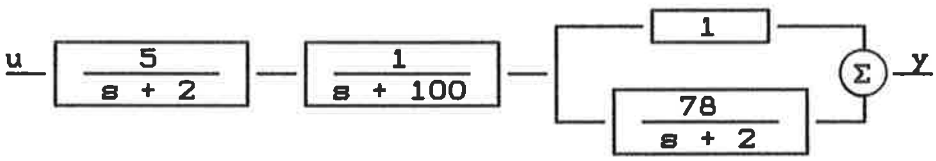
GROUP 3.

The controller has been separated in sub-systems as follows

$$G(s) = \frac{5}{(s + 2)\,(s + 100)} \left[1 + \frac{78}{s + 2} \right]$$

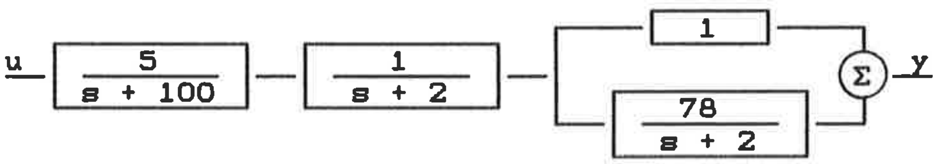
This structure can be realised in six different ways, depending on the location of the parallel part.

Realization 3.31



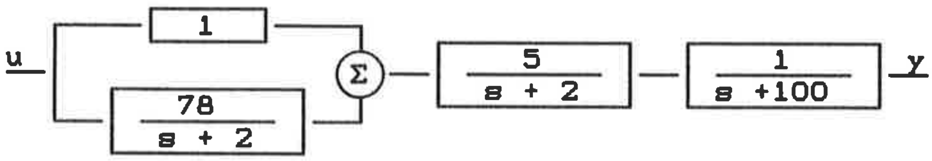
$$\Longrightarrow \quad E\,y^2 = 4.9235 \cdot \varepsilon^2$$

Realization 3.32



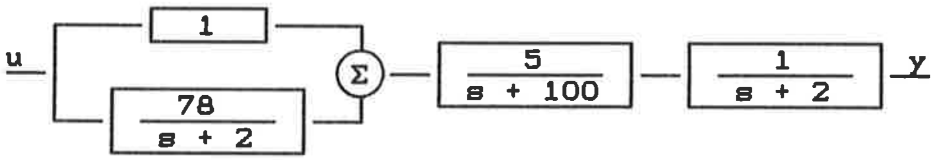
$$\Longrightarrow \quad E\,y^2 = 3.9021 \cdot \varepsilon^2$$

Realization 3.33



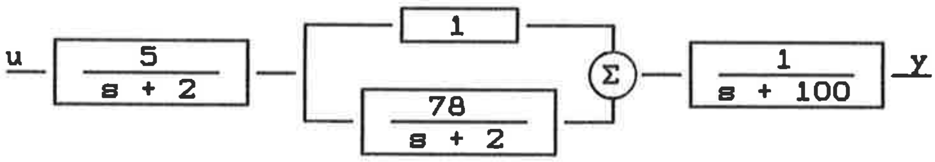
$$\Rightarrow \quad E\, y^2 = 102.91 \cdot \varepsilon^2$$

Realization 3.34



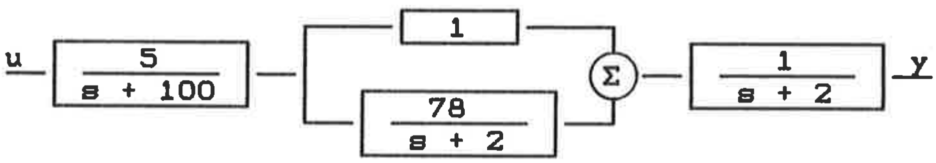
$$\Rightarrow \quad E\, y^2 = 4.9110 \cdot \varepsilon^2$$

Realization 3.35



$$\Rightarrow \quad E\, y^2 = 102.86 \cdot \varepsilon^2$$

Realization 3.36



$$\Rightarrow \quad E\, y^2 = 3.9508 \cdot \varepsilon^2$$

Summary

The realizations 3.11 and 3.12 both have dynamics in all branches of the parallel part. They have the lowest output-variance in this experiment. It is of less importance whether the parallel block is placed before or after the remaining sub-system. The realizations 3.22, 3.23, 3.31, 3.32, 3.34 and 3.36 also have low variances. This depends on the fact that the sub-system closest to the output of the controller is a narrow low-pass filter. The realizations 3.33 and 3.35 have much higher output-variance. This because the sub-system closest to the output is not as good as low-pass filter as in the other realizations.

3. SIMULATION OF SOME SYSTEMS.

In order to investigate whether the theoretically computed variances were realistic three of the previous treated realizations have been simulated in SIMNON.

The realizations 1.1, 1.3 and 1.5 were implemented as shown in appendix 3. For these realizations the multiplications to compute the derivatives have been computed with finite precision in order to generate noise. The result of each multiplication done with finite precision is quantized in ±10 levels. Scaling was done so that the available levels were fully used.

These realization were driven by the simple input-signal u(t)=sin(t) for 30 seconds. This simple signal was chosen so that it was possible to analytically compute a "true" output signal from the controller.

A controller where the multiplications are executed in full precision was also simulated, in order to estimate the noise generated by the integration-routine in SIMNON. The algorithm RKFIX was used with the step-length 0.0001. The differences between the output signal from this controller and the "true" output signal is considered as noise from the integration. (This signal is denoted ds.) The differences between the output from the realizations with finite precision and the "true" signal are considered as being a sum of truncation-noise and integration-noise. (These signals are denoted d1, d3 and d5.) The variances of the truncation-noises are achieved by subtracting the variance of ds from the variances of d1,d3 and d5.

The differences were logged with the frequency 1000 Hz. The variances of the difference-signals ds, d1, d3 and d5 were computed in IDPAC after the simulation. The variances of the truncation-noise for the different realizations are shown in the table below. To make the comparisons easy the previously computed variances also are presented.

Realization	Computed		Simulated	
	Variance	Normed	Variance	Normed
1.1	$100.99\cdot\varepsilon^2$	50.7	7.715E-3	8.27
1.3	$1.9901\cdot\varepsilon^2$	1.00	9.334E-4	1.00
1.5	$1.0305\cdot\varepsilon^2$	0.52	7.175E-4	0.98

The ratios between the variances from the simulation and the theoretical variances do not correspond well. However it is seen that the realization with the lowest measured truncation variance in the output also has the lowest computed truncation variance according to section 2. The reason for poor agreement between calculation and simulation is not understood.

4. CONCLUDING REMARKS.

The following principles can be suggested. Factorize the continuous controller G(s). Determine the pole location. Divide the poles into groups according to the absolute value ($|s_i|$). Poles that are located close to each other (e. g. multiple poles) belong to the same group. Take one pole from each group and let sub-systems with these poles form the parallel block in the controller. Sort the remaining poles after their magnitude. Then connect the sub-systems serially so that the absolute magnitude of their poles are decreasing. (We only consider stable controllers.) The parallel block also is connected in this series. The location of this block is determined by the pole which is closest to the origin. It is placed with the same rule as the remaining sub-systems above.

5. FUTURE TOPICS.

A most urgent topic is to explain the differencies between the theoretically computed variances and the variances achieved in the simulation. When this is done one can continue as follows. This paper only deals with continuous time controllers with real poles and zeroes. A possible extension would be a study on controllers with both real and complex poles and zeroes. Another and more important subject that could be studied is the noise generated by truncation in the multiplications in discrete time controllers. This is most interesting since this problems occurs when a controller is implemented in a signal-processor, which uses fixed-point arithmetics.

6. REFERENCES.

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[2] Åström, K.J. and B. Wittenmark: Computer Controlled Systems. Englewood Cliffs, N.J.: Prentice Hall, Inc, 1984.

[3] Hanselmann, H: Lecture on Implementation of fast Digital Control Systems. Held from 28-jan to 2-Feb 1985 at Lund Institute of Technology.

Macro for calculating the asymptotic variance with CTRLC.

```
// [ ] = pycalc(ai,bi,ci,qi)
//
// function for calculating the variance of a controller

 px = lyap(ai, qi)     ;
 py = ci*px*ci'        ;

 print ai bi ci qi px py
```

The A, B and C matrixes of the investigated realizations.

Realization 1.1

$$A = \begin{bmatrix} -100 & 100 \\ 0 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Realization 1.2

$$A = \begin{bmatrix} -100 & 1 \\ 0 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 100 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Realization 1.3

$$A = \begin{bmatrix} -1 & 1 \\ 0 & -100 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 100 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Realization 1.4

$$A = \begin{bmatrix} -1 & 100 \\ 0 & -100 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Realization 1.5

$$A = \begin{bmatrix} -1 & 0 \\ 0 & -100 \end{bmatrix} \quad B = \begin{bmatrix} 100/99 \\ 100/99 \end{bmatrix} \quad C = \begin{bmatrix} 1 & -1 \end{bmatrix}$$

Realization 2.1

$$A = \begin{bmatrix} -1 & 10 \\ 0 & -100 \end{bmatrix} \quad B = \begin{bmatrix} 10 \\ -90 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Realization 2.2

$$A = \begin{bmatrix} -1 & 0 \\ 90 & -100 \end{bmatrix} \quad B = \begin{bmatrix} 10 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & -1 \end{bmatrix}$$

Realization 2.3

$$A = \begin{bmatrix} -100 & 10 \\ 0 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 10 \\ 9 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Realization 2.4

$$A = \begin{bmatrix} -1 & 9 \\ 0 & -100 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 10 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

Realization 2.5

$$A = \begin{bmatrix} -1 & 0 \\ 0 & -100 \end{bmatrix} \quad B = \begin{bmatrix} 10/11 \\ 100/11 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

Realization 3.11

$$A = \begin{bmatrix} -2.00 & 0.00 & 0.00 \\ 3.97 & -2.00 & 0.00 \\ 1.02 & 0.00 & -100.00 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix}$$

Realization 3.12

$$A = \begin{bmatrix} -2. & 0. & 0. \\ 0. & -100. & 0. \\ 1. & 1. & -2. \end{bmatrix} \quad B = \begin{bmatrix} 3.97 \\ 1.02 \\ 0.00 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

Realization 3.21

$$A = \begin{bmatrix} -2. & 0. & 0. \\ 1. & -2. & 0. \\ 0. & -20. & -100. \end{bmatrix} \quad B = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix}$$

Realization 3.22

$$A = \begin{bmatrix} -2. & 0. & 0. \\ -20. & -100. & 0. \\ 1. & 1. & -2. \end{bmatrix} \quad B = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

Realization 3.23

$$A = \begin{bmatrix} -100. & 0. & 0. \\ 5. & -2. & 0. \\ 0. & 1. & -2. \end{bmatrix} \quad B = \begin{bmatrix} -20 \\ 5 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

Realization 3.31

$$A = \begin{bmatrix} -2. & 0. & 0. \\ 1. & -100. & 0. \\ 0. & 78. & -2. \end{bmatrix} \quad B = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix}$$

Realization 3.32

$$A = \begin{bmatrix} -100. & 0. & 0. \\ 1. & -2. & 0. \\ 0. & 78. & -2. \end{bmatrix} \quad B = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix}$$

Realization 3.33

$$A = \begin{bmatrix} -2. & 0. & 0. \\ 5. & -2. & 0. \\ 0. & 1. & -100. \end{bmatrix} \quad B = \begin{bmatrix} 78 \\ 5 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

Realization 3.34

$$A = \begin{bmatrix} -2. & 0. & 0. \\ 5. & -100. & 0. \\ 0. & 1. & -2. \end{bmatrix} \quad B = \begin{bmatrix} 78 \\ 5 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

Realization 3.35

$$A = \begin{bmatrix} -2. & 0. & 0. \\ 78. & -2. & 0. \\ 1. & 1. & -100. \end{bmatrix} \quad B = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

Realization 3.36

$$A = \begin{bmatrix} -100. & 0. & 0. \\ 78. & -2. & 0. \\ 1. & 1. & -2. \end{bmatrix} \quad B = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

SIMNON-code for the simulated systems.

```
continuous system exp1
" Simulation of controller 11 13 15
"
state  x11  x12  x31  x32  x51  x52  s11  s12
der    dx11 dx12 dx31 dx32 dx51 dx52 ds11 ds12
time   t

" ---------------------------- true solution
yc1 = ( 4950*sin(t) - 5050*cos(t) )/10001
yc2 = 50*exp(-t)/99
yc3 = 100*exp(-100*t)/99099
yc  = yc1+yc2+yc3

" ---------------------------- system with full word-length
u    = sin(t)

ds11 = 100 * (-s11 + s12)
ds12 =                  -s12 + u
ys   =       s11

" ---------------------------- systems with finite word-length
tc   = 10^tq

" ----------------------------------------- realization 11
dx11 = 100 * (-int(x11) + int(x12))
dx12 =                  -int(x12) + int(tc*u)
y1   =       x11/tc

" ----------------------------------------- realization 13

dx31 = -int(x31) + int(x32)
dx32 =   100.0 * (-int(x32) + int(tc*u))
y3   =     x31/tc

" ----------------------------------------- realization 15
dx51 =  -int(x51)                 + int(tc*u)
dx52 =             -int(100.0*x52) - int(tc*u)
y5   =  100.0/99.0 * (x51 + x52)/tc

" -----------------------------------------
ds =  yc-ys
d1 =  yc-y1
d3 =  yc-y3
d5 =  yc-y5

tq   :  1
pi   :  3.14159265
end
```