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A Model for the Drum-Downcomer-Riser Loop

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Abstract
A fourth order model for the critical part of a boiler is described. The model starts from the fundamental partial differential equations. Approximations are developed based on the shape of the steady state solutions for the PDE.

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1. INTRODUCTION

There are many papers on models for drum boilers. In spite of this there are still no good models which describe the drum level in a realistic way. In this report a comparatively simple model is derived from first principles. A fourth order model is obtained which captures most of the features of the drum downcomer riser loop. The model has the nice characteristics that it is only one parameter representing the friction term in the circulation loop which has to be determined from plant data. The basic equations and the state equations are given in the report.

2. BASIC PHYSICAL EQUATIONS

The basic balance equations and the constitutive equations that describe the system will now be derived. The notations used are first described. The system is partitioned into drum, downcomer, and riser. The mass, energy, and momentum balances for each subsystem are given. The constitutive equations simply consist of the equations describing saturated steam and water.

Notations:

a	steam quality (volume ratio) in risers
$\bar{a}$	average steam quality
A	total cross section of riser tubes
C_p	specific heat of water
h	specific enthalpy
$h_c = h_s - h_w$	specific evaporation enthalpy
h_d	specific enthalpy of water in the drum
h_{fw}	specific enthalpy of feedwater
h_s	specific enthalpy of steam at state of saturation
h_w	specific enthalpy of water at state of saturation
K	coefficient in momentum balance
ℓ	average length of riser tubes
P	power transferred to the riser tubes
q	mass flow
q_c	condensate flow, i.e. the flow of water to the drum due to condensation in steam
q_{dc}	flow in downcomers
q_{fw}	feedwater flow
q_r	mass flow at riser outlet
q_s	steam flow out of the drum

ρ_d	density of water in the drum
ρ_s	density of steam at state of saturation
ρ_w	density of water at state of saturation
T	temperature of water in the drum
T_s	temperature of saturated steam
V_r	total riser volume
V_s	volume of steam in the drum
V_w	volume of water in the drum
x	steam quality (mass ratio) in risers
x_r	steam quality at riser outlet

Mass and Energy Balances for Steam in Drum

It is assumed that the steam in the drum is at the state of saturation and that the steam density is the same in the whole drum. The steam flow out of the drum is q_s. The steam flow out of the risers is $q_r x_r$. Part of this steam flow q_c condenses to drum water. The *mass balance* for the steam in the drum can be written as

$$\frac{d}{dt}(\rho_s V_s) = x_r q_r - q_c - q_s. \tag{2.1}$$

The energy balance for the steam in the drum is

$$\frac{d}{dt}(\rho_s h_s V_s) = x_r q_r h_s - q_c h_c - q_s h_s. \tag{2.2}$$

Since it was assumed that the steam was at the state of saturation the density ρ_s and the specific enthalpy h_s are functions of the drum pressure p.

Mass and Energy Balances for Drum Water

There are three flows of water into the drum, feedwater flow q_c, flow from the risers $q_r(1-x_r)$ and condensation of steam from the risers q_c.

From the drum there is a flow q_{dc} to the downcomers. The mass balance is

$$\frac{d}{dt}(\rho_d V_w) = q_{fw} - q_{dc} + q_r(1-x_r) + q_c. \tag{2.3}$$

The energy balance is

$$\frac{d}{dt}(h_d \rho_d V_w) = q_{fw}h_{fw} - q_{dc}h_w + q_r h_w(1-x_r) + q_c h_c. \tag{2.4}$$

The water in the drum has normally a temperature which is slightly below the critical temperature. The density ρ_d and the specific enthalpy h_d are thus functions of the drum water temperature T and the drum pressure p. It is assumed that the pressure and the temperature is the same all over the drum.

Mass Energy and Momentum Balances for Downcomer and Riser

It is assumed that the temperature of the water at the bottom of the downcomers is the same as the drum water temperature. In the risers the water is first heated to the boiling temperature. When boiling occurs the amount of steam in the risers increases along the riser tubes.

Energy balance for heating zone

In the heating zone the water temperature is described by

$$\frac{d}{dt}(\rho_d\ C_p\ A\ T\ dz) = \frac{Pdz}{\ell}.$$

Neglecting that ρ_d and C_p depend on temperature this equation can be integrated

$$T_s - T_d = \frac{P}{\ell\ \rho_d\ C_p\ A} \cdot t,$$

where t is the time it takes to pass the heating zone. If the length of the heating zone is z we get

$$t = \frac{z\ \rho_d\ A}{q_{dc}}$$

or

$$z = \frac{q_{dc}\ t}{\rho_w\ A}.$$

Hence

$$\frac{z}{\ell} = \frac{q_{dc}\ C_p(T_s - T_d)}{P}. \tag{2.5}$$

An energy balance for the heating zone thus makes it possible to determine where the boiling starts.

Mass and energy balances for boiling zone

The mass balance is given by

$$\frac{d}{dt}\left[\rho_s\ \bar{a}\ V_r + \rho_w(1-\bar{a})\ V_r\right] = q_{dc} - q_r \tag{2.6}$$

and the energy balance is

$$\frac{d}{dt}\left[\rho_s\ h_s\ \bar{a}\ V_r + \rho_w\ h_w(1-\bar{a})\ V_r\right] = P\left(1 - \frac{z}{\ell}\right) + h_w\ q_{dc} - h_r\ q_r \tag{2.7}$$

where

$$h_r = x_r\ h_s + (1-x_r)\ h_w. \tag{2.8}$$

In these equations the density ρ_s and the specific enthalpy h_w will be functions of the pressure. It is assumed that the pressure is equal to the drum pressure at all points along the risers. This is clearly an approximation. It is not known how serious it is. If the approximation is not made it will be necessary to determine the pressure from a momentum balance. This is very complicated due to the two phase flow.

The steam quality distribution

In the equations appear both the mean steam quality $\bar{a}$ (volume ratio) and the steam quality at the riser outlet x_r (mass ratio). To obtain the relationships between those variables it is necessary to know the steam distribution. The steam distribution can be obtained from a distributed model. In this case a distribution of the steam quality along the risers is simply postulated. It is assumed that the steam quality expressed as a mass ratio is linear from the point where the boiling starts i.e.

$$x(\zeta) = \begin{cases} 0 & 0 \leqslant \zeta \leqslant z \\ x_r \dfrac{\zeta - z}{\ell - z} & z \leqslant \zeta \leqslant \ell. \end{cases}$$

The relation between the steam quality expressed as a volume ratio and as a mass ratio is given by

$$a = \frac{\rho_w x}{\rho_w x + [1-x] \rho_s} .$$

We thus find that the average steam quality $\bar{a}$ (volume ratio) is related to the steam quality ar the riser outlet expressed x_r (mass ratio) as

$$\bar{a} = \frac{1}{\ell} \int_0^\ell a(\zeta)\, d\zeta = \frac{1}{\ell} \int_0^\ell \frac{\rho_w x(\zeta)}{\rho_w x(\zeta) + [1-x(\zeta)] \rho_s} .$$

Simple calculations give

$$\bar{a} = \frac{\rho_w}{\rho_w - \rho_s} \left(1 - \frac{z}{\ell}\right) \left[1 - \frac{\rho_s}{x_r(\rho_w - \rho_s)} \ln \frac{x_r(\rho_w - \rho_s) + \rho_s}{\rho_s} \right]. \tag{2.9}$$

Momentum balance

A momentum balance for circulation loop is given by

$$(\rho_d - \rho_s)\, \bar{a}\, V_r = \frac{1}{2} k\, q_{dc}^2 . \tag{2.10}$$

Constitutive Equations

The constitutive equations are the equations which describe how the densities ρ_s, ρ_w, the specific heat C_p, and the boiling temperature T_s depend on the thermodynamical state of water and steam.

3. STATE SPACE EQUATIONS

The equations given in the previous section give a complete description of the system. The equations will now be reduced to standard state space form. To do so it is necessary to decide which variables should be regarded as inputs and state variables.

Input Signals

It is natural to consider the following variables as inputs to the system.

P	power supplied from the fuel
q_s	steam flow
q_{fw}	feed water flow
h_{fw}	specific enthalpy of feedwater.

State Variables

The state variables describe the storage of mass energy and momentum.

The mass of water in the drum can be described by V_w or equivalently the drum level. Since the total drum volume is known, the steam volume is then given by

$$V_s = V_d - V_w.$$

The mass of the steam stored in the drum is then uniquely characterized by the steam density ρ_s or the steam pressure p.

The energy stored in the drum is uniquely given by V_w and the drum water temperature. The energy stored in the steam is characterized by V_w and ρ_s (or p).

The mass of water in the riser tubes is uniquely given by the steam quality $\bar{a}$ or the steam quality x_r at the riser outlet. Either of these variables will also describe the energy stored in the risers.

The state variables are thus chosen as

p steam pressure

V_w volume of water in drum

x_r steam quality (mass ratio) at riser outlet

T drum water temperature.

There are thus 4 state variables. Since the physical system was characterized by 6 differential equations this means that two differential equations can be eliminated. This is done as follows. The condensate flow q_c can be eliminated between equations (2.1) and (2.2) and the mass flow out of the risers q_r can be eliminated from equations (2.6) and (2.7).

Outputs

Apart from the state variables there are several other variables that are of interest. The following variables are of prime interest:

x drum level

T_s temperature of steam in drum.

It is also of interest to know the variables

q_c condensate flow

q_r flow at riser outlet

q_{dc} criculation flow

z/ℓ position along riser tubes where boiling starts.

Parameters

It is of interest to know the parameters that have to be determined to obtain the model:

K coefficient in equation for circulation flow

A_d area of water in drum

V_d drum volume

V_r riser volume

It is thus a remarkable small number of parameters that have to be determined. Since the energy storage in the metal masses have been neglected, it may be useful to make a correction. If this is done it will be necesssary to know also the metal masses and their heat capacities.

State Equations

The state equations will now be derived. Elimination of q_c between equations (2.1) and (2.2) gives

$$\frac{d}{dt}(\rho_s h_s V_s) - h_c \frac{d}{dt}(\rho_s V_s) = x_r q_r (h_s - h_c) - q_s (h_s - h_c).$$

Since

$$h_c = h_s - h_w$$

and

$$V_s = V_d - V_w$$

we get

$$h_w V_s \frac{d\rho_s}{dt} + \rho_s V_s \frac{dh_s}{dt} - \rho_s h_w \frac{dV_w}{dt} = x_r q_r h_w - q_s h_w.$$

Since the steam in the drum is in saturated state its enthalpy is a function of its density. Furthermore using equation (2.6) to eliminate q_r we get

$$\left[\left(h_w V_s + x_r h_w \bar{a} V_r\right) \frac{d\rho_s}{dp} + \rho_s V_s \frac{dh_s}{dp}\right] \frac{dp}{dt} - \rho_s h_w \frac{dV_w}{dt} -$$

$$- x_r h_w (\rho_w - \rho_s) V_r \frac{d\bar{a}}{dx_r} \cdot \frac{dx_r}{dt} = x_r q_{dc} h_w - q_s h_w . \qquad (3.1)$$

Elimination of the condensate flow q_c between (2.1) and (2.3) gives

$$\frac{d}{dt} (\rho_d V_w) + \frac{d}{dt} (\rho_s V_s) = q_r + q_{fw} - q_{dc} - q_s .$$

Using (2.6) to eliminate q_r we get

$$\left\{(V_s + \bar{a} V_r) \frac{d\rho_s}{dp} + V_w \frac{\partial \rho_d}{\partial p} + (1-\bar{a}) V_r \frac{d\rho_w}{dp}\right\} \frac{dp}{dt} + (\rho_d - \rho_s) \frac{dV_w}{dt} -$$

$$- (\rho_w - \rho_s) V_r \frac{d\bar{a}}{dx_r} \cdot \frac{dx_r}{dt} + V_w \frac{\partial \rho_d}{\partial T} \cdot \frac{dT}{dt} = q_{fw} - q_s . \qquad (3.2)$$

Elimination of q_r between (2.6) and (2.7) gives after some calculations

$$\left[h_c (1-x_r) \bar{a} \frac{d\rho_s}{dp} + h_c x_r \bar{a} \frac{d\rho_w}{dp} + \rho_s \bar{a} \frac{dh_s}{dp} + \rho_w (1-\bar{a}) \frac{dh_w}{dp}\right] V_r \frac{dp}{dt} +$$

$$+ h_c [(1-x_r) \rho_s + x_r \rho_w] V_r \frac{d\bar{a}}{dx_r} \cdot \frac{dx_r}{dt} = - h_c x_r q_{dc} + \left(1 - \frac{z}{\ell}\right) P . \qquad (3.3)$$

Elimination of q_c between (2.2) and (2.4) gives

$$\frac{d}{dt} (\rho_s h_s V_s) + \frac{d}{dt} (\rho_d h_d V_w) = q_r h_r + q_{fw} h_{fw} - q_{dc} h_d - q_s h_s .$$

Notice that the density of the drum water ρ_d and the specific enthalpy of the water in the drum are functions of temperature and pressure

$$\rho_d = \rho_d (p, T)$$

$$h_d = h_d (p, T) .$$

Using (2.6) to eliminate q_r we get

$$\Big[(h_s V_s + h_r \bar{a} V_r) \frac{d\rho_s}{dp} + h_d V_w \frac{\partial \rho_d}{\partial p} + h_r (1-\bar{a}) V_r \frac{d\rho_w}{dp} + \rho_s V_s \frac{dh_s}{dp} +$$
$$+ \rho_d V_w \frac{\partial h_d}{\partial p} \Big] \frac{dp}{dt} + (\rho_d h_d - \rho_s h_s) \frac{dV_w}{dt} - h_r (\rho_w - \rho_s) V_r \frac{d\bar{a}}{dt} +$$
$$+ \left(\frac{\partial \rho_d}{\partial T} h_d + \rho_d \frac{\partial h_d}{\partial T} \right) V_w \frac{dT}{dt} = (h_r - h_d)\, q_{dc} + q_{fw}\, h_{fw} - q_s\, h_s. \quad (3.4)$$

The equations (3.1), (3.2), (3.3), and (3.4) thus gives a set of linear equations for the time derivatives of the state variables. Notice that the equations have the following structure:

$$\begin{pmatrix} e_{11} & e_{12} & e_{13} & 0 \\ e_{21} & e_{22} & e_{23} & e_{24} \\ e_{31} & 0 & e_{33} & 0 \\ e_{41} & e_{42} & e_{43} & e_{44} \end{pmatrix} \begin{pmatrix} dp/dt \\ dV_w/dt \\ dx_r/dt \\ dT/dt \end{pmatrix} = \begin{pmatrix} x_r h_w q_{dc} - h_w q_s \\ q_{fw} - q_s \\ (h_w - h_r)\, q_{dc} + P(1 - z/\ell) \\ (h_r - h_w)\, q_{dc} + q_{fw} h_{fw} - q_s h_s \end{pmatrix} \quad (3.5)$$

Auxiliary Equations

Several auxiliary variables were introduced when the state-equations were derived. The equations for these variables are summarized below for easy reference.

Thermodynamical equations for saturated steam

$$\begin{cases} h_s = h_s(p) \\ \dfrac{dh_s}{dp} = \dfrac{dh_s}{dp}(p) \\ \rho_s = \rho_s(p) \\ \dfrac{d\rho_s}{dp} = \dfrac{d\rho_s}{dp}(p) \\ T_s = T_s(p) \\ \dfrac{dT_s}{dp} = \dfrac{dT_s}{dp}(p) \end{cases} \quad (3.6)$$

These functions are obtained from the steam table for saturated steam.

Density and enthalpy for drum water

The water in the drum is not in saturated state the density and the enthalpy therefore depends both on temperature and on pressure:

$$\rho_d = \rho_d(p, T) \tag{3.7}$$

$$h_d = h_d(p, T). \tag{3.8}$$

Mean steam quality, circulation flow, start of boiling

To determine the variables $\bar{a}$, q_{dc}, and z/ℓ from the state variables it is necessary to solve an algebraic equation. Equations (2.5), (2.9), and (2.10) can be written as

$$z/\ell = B\, q_{dc}$$

$$\bar{a} = C(1 - z/\ell)$$

$$q_{dc}^2 = D\, \bar{a}$$

where

$$B = C_p(T_s - T_d) / P$$

$$C = \frac{\rho_w}{\rho_w - \rho_s}\left[1 - \frac{\rho_s}{x_r(\rho_w - \rho_s)} \ln \frac{x_r(\rho_w - \rho_s) + \rho_s}{\rho_s}\right]$$

$$D = 2(\rho_d - \rho_s)\, V_r / k.$$

Elimination of $\bar{a}$ and z/ℓ gives the following second order equation for q_{dc}:

$$q_{dc}^2 + B\, C\, D\, q_{dc} - D\, C = 0.$$

Since the circulation flow must be positive the following unique solution is obtained:

$$q_{dc} = \sqrt{(BCD)^2 / 4 + DC} - BCD / 2. \qquad (3.9)$$

Then

$$z/\ell = B\, q_{dc} \qquad (3.10)$$

$$\bar{a} = C(1 - z/\ell). \qquad (3.11)$$

The function $d\bar{a}/dx_r$ is also needed. Differentiation of (2.9) gives

$$\frac{d\bar{a}}{dx_r} = \frac{\rho_w \rho_s}{x_r^2(\rho_w - \sigma_s)^2} (1 - z/\ell) \ln \frac{x_r(\rho_w - \rho_s) + \rho_s}{\rho_s} -$$
$$- \frac{\rho_w \rho_s}{x_r(\rho_w - \rho_s)[x_r(\rho_w - \rho_s) + \rho_s]} (1 - z/\ell). \qquad (3.12)$$

Flow at riser outlet

The flow at the riser outlet is given by (2.6)

$$q_r = \frac{d}{dt} [\rho_s \bar{a} V_r + \rho_w (1 - \bar{a}) V_r] + q_{dc}.$$

Hence

$$q_r = q_{dc} + \bar{a} V_r \frac{d\rho_s}{dp} \cdot \frac{dp}{dt} - (\rho_w - \rho_s) V_r \frac{d\bar{a}}{dx_r} \cdot \frac{dx_r}{dt}. \qquad (3.13)$$

Condensate flow

The condensate flow is given by (2.1)

$$q_c = x_r q_r - q_s - \frac{d}{dt} (\rho_s V_s).$$

Hence

$$q_c = x_r q_r - q_s - V_s \frac{d\rho_s}{dp} \cdot \frac{dp}{dt} + \rho_s \frac{dV_w}{dt}. \qquad (3.14)$$

Drum Level

The drum level is simply V_w / A_d, where A_d is the wet area in the drum.

4. EXTENSIONS

The simple model presented in this report can be extended in many ways. The energy storage in the metal masses can be taken into account by modifying the values of the enthalpy coefficients.

The circulation time in the downcomer riser loop can be introduced as a first order lag.

The energy storage in the riser walls can be introduced via an extra state variable.

5. VERIFICATIONS

The simple model has been used to calculate the void distribution in a BWR reactor. The results obtained by the simplified model agree surprisingly well with the simulation of the full PDE (Bergman, 1985).

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