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Adaptive Friction and Load Compensation Based on Shaft Angle Measurement in DC-Motor Servos

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Abstract

A DC-Servo is controlled by an adaptive controller. Only the shaft angle of the servo is measured. The parameters of a continuous time friction and load model are estimated recursively. All other parameters of the process are known.

The method eliminates the nonlinear effects due to friction and the performance of the DC servo is improved. Simulations show this.

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1. INTRODUCTION

Background

Friction causes a great deal of trouble in servos. It is important to take care of friction effects when performance should be improved. It is difficult to make friction models that are valid under all the different operating conditions we have. The parameters of the models vary due to wear and load.
A way to solve the problem with different friction models under different conditions is to estimate the parameters of a friction model under operation and compensate for the effects. How this can be done is shown in [C&Å&B] where the velocity measurement is the base for the friction compensation.

Purpose

This report deals with the problem how to compensate for as well static and viscous friction as constant load disturbances in a DC-motor servo when only the input voltage and the shaft-angle are measurable.
The torque due to friction depends on the angular velocity and therefore it is necessary to estimate the angular velocity to be able to handle the nonlinear friction.

The Process

The torque balance with respect to the shaft for the servo gives the following differential equation

$$J\ddot{y}(t) = -T_f(\dot{y}(t)) + \frac{k_m}{R}(u_m(t) - k_e\dot{y}(t)) + V(t)$$

where J is the moment of inertia, $y(t)$ is the shaft angle, $T_f(\dot{y}(t))$ is the friction torque, k_m is the torque constant, R is the rotor resistance, $u_m(t)$ is the voltage to the motor, k_e is the back e.m.f. constant and $V(t)$ is a constant load disturbance.

The Friction Model

The friction model is the same as in [C&Å&B]. The friction torque is non-symmetric with respect to the angular velocity and includes static and viscous friction. See figure 1.

2. REGULATOR DESIGN

Compensation for Load and Static Friction

The selected friction model gives the following nonlinear description of the plant

$$\begin{aligned} J\ddot{y}(t) &= -(\mu\dot{y}(t) + \beta) + \frac{k_m}{R}(u_m(t) - k_e\dot{y}(t)) + V(t) \\ &= -(\mu + \frac{k_m k_e}{R})\dot{y}(t) + \frac{k_m}{R}u_m(t) + (V(t) - \beta) \\ &= -\alpha\dot{y}(t) + k u_m(t) + V_\beta \end{aligned}$$

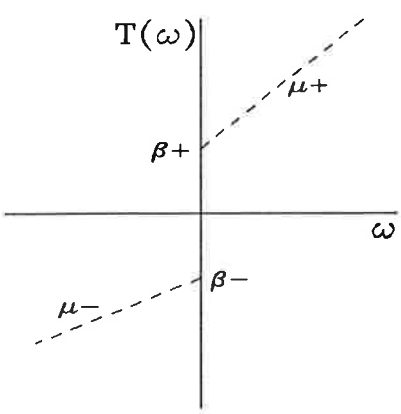

Figure 1. Friction torque vs. angular velocity

where $\alpha = \alpha(sign(\dot{y}(t)))$, $\beta = \beta(sign(\dot{y}(t)))$ and $V_\beta = V_\beta(sign(\dot{y}(t)))$. The term V_β is introduced because it is not possible to separate the contributions from the torque disturbance $V(t)$ and the static friction torque β. Under the assumption that these terms are constant V_β is a function of $sign(\dot{y})$ only. For the same reason the factor α is introduced.
It is possible to eliminate the constant terms in the equation above with the control law

$$u_m(t) = u(t) - \frac{1}{k} V_\beta(sign(\dot{y}(t)))$$

Now we have a piecewise linear process which we can control in a traditional way.

$$J\ddot{y}(t) = -\alpha\dot{y}(t) + ku(t)$$

A hysteresis band eliminates unwanted switching between the different parameter sets. The measurement noise determines the hysteresis width.

Sampling of the Linear Process

The control law will be implemented in a computer. Thus it is necessary to have a sampled description of the linear process. The natural choice of states are the shaft angle x_2 and the angular velocity x_1. The state space description for the process then is

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} -\alpha/J & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} k/J \\ 0 \end{pmatrix} u(t)$$

$$y(t) = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Sampling of this process with the interval h gives the following discrete time state space description

$$\begin{pmatrix} x_1(k+1) \\ x_2(k+1) \end{pmatrix} = \begin{pmatrix} \varphi_{11} & 0 \\ \varphi_{21} & 1 \end{pmatrix} \begin{pmatrix} x_1(k) \\ x_2(k) \end{pmatrix} + \begin{pmatrix} \gamma_{11} \\ \gamma_{21} \end{pmatrix} u(k)$$

$$y(t) = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} x_1(k) \\ x_2(k) \end{pmatrix}$$

where

$$\begin{aligned}\varphi_{11} &= e^{-\alpha h/J}\\ \varphi_{21} &= J(1-\varphi_{11})/\alpha\\ \gamma_{11} &= K(1-\varphi_{11})/\alpha\\ \gamma_{21} &= K((\varphi_{11}-1)J+\alpha h)/\alpha^2\end{aligned}$$

Notice that α is a function of $sign(\dot{y}(t))$. This implies that the coefficients of the state space model are time varying and depend on $sign(\dot{y}(t))$.

Observer for Angular Velocity

To be able to do friction compensation we must either measure or estimate the angular velocity. In both cases it is necessary to obtain an unbiased estimate of the angular velocity. A full order observer must include all constant disturbances in order not to give bias in the estimated angular velocity. Another way to achieve an unbiased estimate of the angular velocity is to make a full order observer for the process described in differential form. With Δ defined as $\Delta z(k) = z(k) - z(k-1)$ the observer is

$$\begin{pmatrix}\Delta\hat{x}_1(k+1)\\ \Delta\hat{x}_2(k+1)\end{pmatrix} = \begin{pmatrix}\varphi_{11} & 0\\ \varphi_{21} & 1\end{pmatrix}\begin{pmatrix}\Delta\hat{x}_1(k)\\ \Delta\hat{x}_2(k)\end{pmatrix} + \begin{pmatrix}\gamma_{11}\\ \gamma_{21}\end{pmatrix}\Delta u(k) + K(\Delta y - \Delta\hat{x}_2)$$

Calculation of $\hat{x}_1(k)$ with $\hat{x}_1(k) = \hat{x}_1(k-1) + \Delta\hat{x}_1(k)$ is not good because we then require unbiased initial value of $\hat{x}_1$. Another better way to achieve an unbiased estimate of the angular velocity $x_1(k)$ is

$$\hat{x}_1(k) = \Delta\hat{x}_2(k)/h$$

The poles of the observer are described in terms of frequency and relative damping for the corresponding continuous time system. This system has the characteristic polynomial

$$p_c(s) = s^2 + 2\varsigma_o\omega_o s + \omega_o^2$$

and the discrete time observer has the characteristic polynomial

$$\begin{aligned}p_d(q) &= q^2 - 2e^{-\varsigma_o\omega_o h}\cos(\omega_o h\sqrt{1-\varsigma_o^2})q + e^{-2\varsigma_o\omega_o h}\\ &= q^2 + p_{o1}q + p_{o2}\end{aligned}$$

From $\det[qI - \Phi + KC] = p_d(q)$ the observer gain is given.

$$\begin{aligned}K_2 &= p_{o1} + 1 + \varphi_{11}\\ K_1 &= \frac{p_{o2} + \varphi_{11}(K_2 - 1)}{\varphi_{21}}\end{aligned}$$

State Feedback Design

After eliminating the constant terms of the nonlinear system we have a piecewise linear system. Within each region we can design a controller based on linear system theory. An integrating state feedback control law gives the servo the desired performance. Extending the process model with an extra state $\hat{x}_3(k)$ gives the integrating control law. The reference input is denoted $r(t)$.

$$\begin{pmatrix} x_1(k+1) \\ x_2(k+1) \\ \hat{x}_3(k+1) \end{pmatrix} = \begin{pmatrix} \varphi_{11} & 0 & 0 \\ \varphi_{21} & 1 & 0 \\ 0 & -h & 0 \end{pmatrix} \begin{pmatrix} x_1(k) \\ x_2(k) \\ \hat{x}_3(k) \end{pmatrix} + \begin{pmatrix} \gamma_{11} \\ \gamma_{21} \\ 0 \end{pmatrix} u(k) + \begin{pmatrix} 0 \\ 0 \\ h \end{pmatrix} r(k)$$

$$y(t) = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x_1(k) \\ x_2(k) \\ \hat{x}_3(k) \end{pmatrix}$$

The control law uses the reconstructed angular velocity $\hat{x}_1(k)$ from the observer and looks then like

$$u(k) = -l_1\hat{x}_1(k) - l_2 y(k) - l_3\hat{x}_3(k) + mr(k)$$

Determining the state feedback gain is a separate problem and does not involve observer dynamics. It is here assumed that the state $x_1(k)$ is used and not $\hat{x}_1(k)$. The desired closed loop transfer function is

$$H(q) = \frac{(\gamma_{21}q - \varphi_{11}\gamma_{21} + \varphi_{21}\gamma_{11})(q - 1 - \frac{hl_3}{m})m}{(q + p_x)(q^2 + p_1 q + p_2)}$$

In this we find two zeros, one that can be placed arbitrarily by choosing m and the one that comes from the sampling. The latter is not canceled. The system has three poles that can be placed arbitrarily with the control law above.

$$\begin{aligned} p_1 &= -2e^{-\varsigma\omega h}\cos(\omega h\sqrt{1-\varsigma^2}) \\ p_2 &= e^{-2\varsigma\omega h} \\ p_x &= -e^{-\alpha\omega h} \end{aligned}$$

We see that the poles are described for the corresponding continuous time system.

3. PARAMETER ESTIMATION

Filtering of Input and Output from the Process

To be able to estimate continuous time parameters it is necessary to filter the input and output from the system in order to form the regression vector. Perform the following filtering

$$\begin{aligned} Y_2(s) &= \frac{c^3 s^2}{(s+c)^3} Y(s) \\ Y_1(s) &= \frac{c^3 s}{(s+c)^3} Y(s) \\ U_0(s) &= \frac{c^3}{(s+c)^3} U(s) \end{aligned}$$

With laplace transforms the process model can be written

$$Js^2 Y(s) + \alpha s Y(s) = kU(s) + \frac{V_\beta}{s}$$

Remember that we in fact have two models, one for positive and one for negative angular velocity. Multiplying every term with $\frac{c^3}{(s+c)^3}$ it is then possible to substitute with the filtered signals.

$$JY_2(s) + \alpha Y_1(s) = kU_0(s) + \frac{c^3}{s(s+c)^3}V_\beta$$

Transform this expression to the time domain. After an initial transient we have the following relation between the signals and the parameters.

$$Jy_2(t) + \alpha y_1(t) = ku_0(t) + V_\beta$$

This relation is the ground for the parameter estimation. Because the relation is static we can pick the filtered signals at arbitrary occasions to form the regressors. Therefore the time for the sampling k of the estimator can replace the time t in the expression above.
By low pass filtering $y(t)$ and $u(t)$ the filtered signals are formed by linear combinations of the states in a state space description. For more details see [Can]. These filters are either analog or digital. We will implement them in a computer with the sampling interval h_c which is shorter than h in the previous section. The sampled state space description is

$$\begin{pmatrix} \xi_1(k+1) \\ \xi_2(k+1) \\ \xi_3(k+1) \end{pmatrix} = \begin{pmatrix} f_{11} & f_{12} & f_{13} \\ 0 & f_{22} & f_{23} \\ 0 & 0 & f_{33} \end{pmatrix} \begin{pmatrix} \xi_1(k) \\ \xi_2(k) \\ \xi_3(k) \end{pmatrix} + \begin{pmatrix} g_{11} \\ g_{21} \\ g_{31} \end{pmatrix} y(k)$$

$$\begin{pmatrix} y_0(k) \\ y_1(k) \\ y_2(k) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -c & c & 0 \\ c^2 & -2c^2 & c^2 \end{pmatrix} \begin{pmatrix} \xi_1(k) \\ \xi_2(k) \\ \xi_3(k) \end{pmatrix}$$

$$\begin{pmatrix} \vartheta_1(k+1) \\ \vartheta_2(k+1) \\ \vartheta_3(k+1) \end{pmatrix} = \begin{pmatrix} f_{11} & f_{12} & f_{13} \\ 0 & f_{22} & f_{23} \\ 0 & 0 & f_{33} \end{pmatrix} \begin{pmatrix} \vartheta_1(k) \\ \vartheta_2(k) \\ \vartheta_3(k) \end{pmatrix} + \begin{pmatrix} g_{11} \\ g_{21} \\ g_{31} \end{pmatrix} u(k)$$

$$u_0(k) = \vartheta_1(k)$$

with the coefficients

$$\begin{aligned}
f_{11} &= e^{-ch_c} & f_{12} &= ch_c e^{-ch_c} \\
f_{13} &= c^2 h_c^2 e^{-ch_c}/2 & f_{22} &= e^{-ch_c} \\
f_{23} &= ch_c e^{-ch_c} & f_{33} &= e^{-ch_c} \\
g_{11} &= 1 - (c^2h_c^2 + 2ch_c + 2)e^{-ch_c}/2 & g_{21} &= 1 - (ch_c + 1)e^{-ch_c} \\
g_{31} &= 1 - e^{-ch_c}
\end{aligned}$$

RLS Estimation of Nonsymmetrical Friction and Load

We will only estimate those parameters in the process that are unknown i.e. α_+, $V_{\beta+}$, α_- and $V_{\beta-}$. The others (k and J) are known.
The estimator has two different parts, one for positive and one for negative angular velocity. The sign of $y_1(k)$ determines which part that should be used. For an angular velocity greater than zero the parameters α_+ and $V_{\beta+}$ and their covariance matrix are updated. The parameters indexed with minus will be updated in the opposite case. In a small band around $y_1(k) = 0$ the estimation is switched off in order to avoid the effects when the angular velocity of the

servo is zero and the forcing torque is lower than the static friction torque. This deadband also eliminates P matrix windup when excitation is poor.
The estimators are similar and each of them has the following outlook. With $\theta(k) = (\alpha \quad V_\beta)^T$ and $\varphi(k) = (-y_1(k) \quad 1)^T$ we have

$$\begin{aligned}
\varepsilon(k) &= Jy_2(k) - ku_0(k) - \varphi^T(k)\theta(k-1) \\
\theta(k) &= \theta(k-1) + K(k)\varepsilon(k) \\
K(k) &= \frac{P(k-1)\varphi(k)}{\lambda + \varphi^T(k)P(k-1)\varphi(k)} \\
P(k) &= \frac{1}{\lambda}\left[I - K(k)\varphi^T(k)\right]P(k-1)
\end{aligned}$$

Fault Detection

The friction parameters α_+ and α_- vary slowly. The parameters $V_{\beta+}$ and $V_{\beta-}$ can change abruptly because the variation in load $V(t)$. A way to deal with large changes in the parameters is shown in [Häg].
The estimator is extended by a fault detection algorithm which consists of the following equations

$$\begin{aligned}
\Delta\theta(k) &= \theta(k) - \theta(k-1) \\
w(k) &= \gamma_1 w(k-1) + \Delta\theta(k) \\
r_f(k) &= \gamma_2 r_f(k-1) + (1-\gamma_2)sign(\Delta\theta^T(k)w(k-1))
\end{aligned}$$

The parameter updates $\Delta\theta(k)$ are filtered through a first order filter to give $w(k)$. The use of the signum function on the inner product of $\Delta\theta(k)$ and $w(k-1)$ makes the fault detection insensitive to noise variance. The test quantity r_f is the filtered result from the signum function. When r_f is greater than the alarm limit r_0 is is assumed that a fault i.e. a large parameter change has occurred. Then a multiple β_{tp} of the identity matrix is added to the P matrix. This is always positive (semi) definite by the choice of β_{tp}. Thus we have

$$P(k) = \frac{1}{\lambda}\left[I - K(k)\varphi^T(k)\right]P(k-1) + \beta_{tp}I$$

where

$$\beta_{tp} = \begin{cases} \dfrac{\nu_0}{\varphi^T(k)\varphi(k)}\,\dfrac{r_f - r_0}{1 - r_0} & r_f > r_0 \\ 0 & r_f \le r_0 \end{cases}$$

$$\nu_0 = \frac{\lambda}{\lambda + \varphi^T(k)P(k-1)\varphi(k)}$$

The design parameters γ_1, γ_2 and r_0 affects how fast a fault is detected and how often false alarms occur.

4. SIMULATION

We will test the algorithms by simulation of a DC-motor. The parameters of the motor are

$$\begin{array}{lll} k = 1.20 & \alpha_+ = 0.50 & \beta_+ = 0.70 \\ J = 1.00 & \alpha_- = 0.70 & \beta_- = 0.20 \end{array}$$

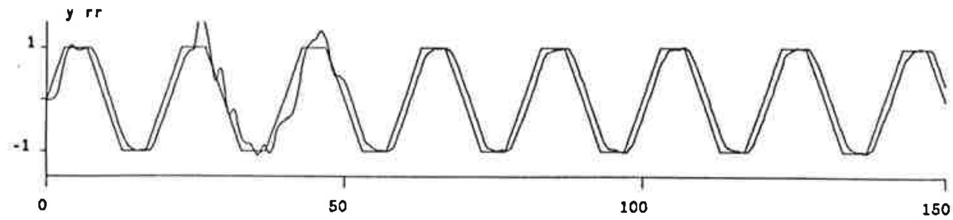

Figure 2. Reference signal and shaft angle

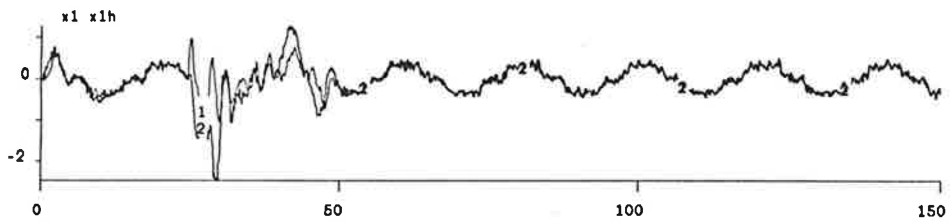

Figure 3. True and estimated angular velocity

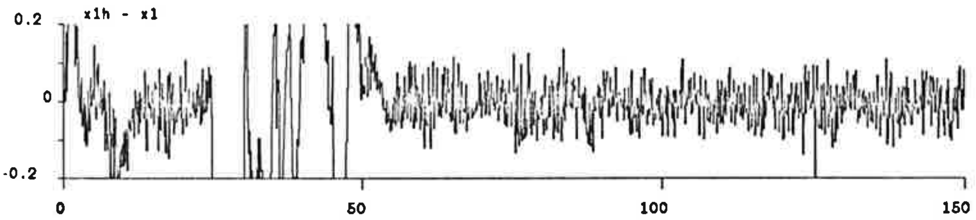

Figure 4. Difference between true and estimated angular velocity

The controller and estimator are sampled with $h = 0.1s$. The observer poles are the sampled correspondence to $s = -\varsigma_o\omega_o \pm \omega_o\sqrt{1-\varsigma_o^2}$ where $\omega_o = 1.0$ and $\varsigma_o = 0.72$. The closed loop continuous equivalent system has poles at $s = -\varsigma\omega \pm \omega\sqrt{1-\varsigma^2}$ and $s = -a\omega$ where $\omega = 2.0$, $\varsigma = 1.0$, and $a = 1.0$. The zero that can be chosen is located at the same place as the pole on the real axis. The design parameter of the filters $c = 5.0$ and the forgetting factor in the estimator $\lambda = 0.995$.

A constant load disturbance acts on the servo from $t = 25s$. The amplitude of the disturbance is $V = 3$.

Measurement noise is added to the output y from the servo. It is gaussian and white with standard deviation $\sigma = 0.03$

Figure 2 shows the reference signal r and the shaft angle y. The performance of the servo improves as the estimated parameters converge.

Figure 3 shows the true angular velocity x_1 and the observed $\hat{x}_1$, and the difference between them appears in figure 4. The estimation of the angular velocity is acceptable although the estimated parameters not have converged to their true values and moreover measurement noise is present.

The estimated parameters are found in figure 5. We see that they converge to some values. The noise in the estimator is however coloured because of the

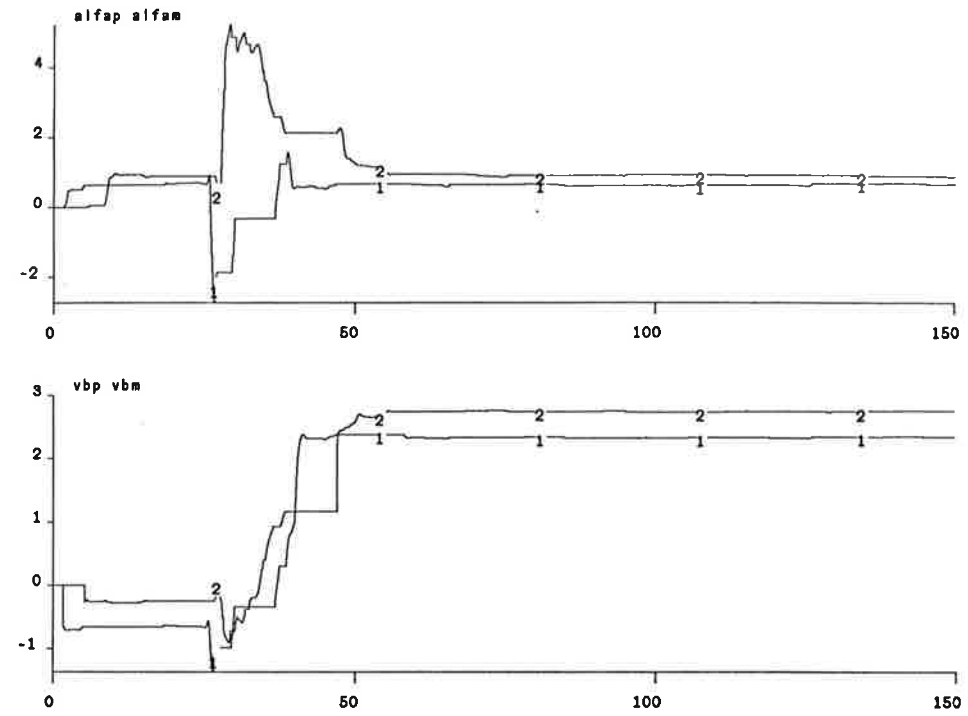

Figure 5. Estimated parameters

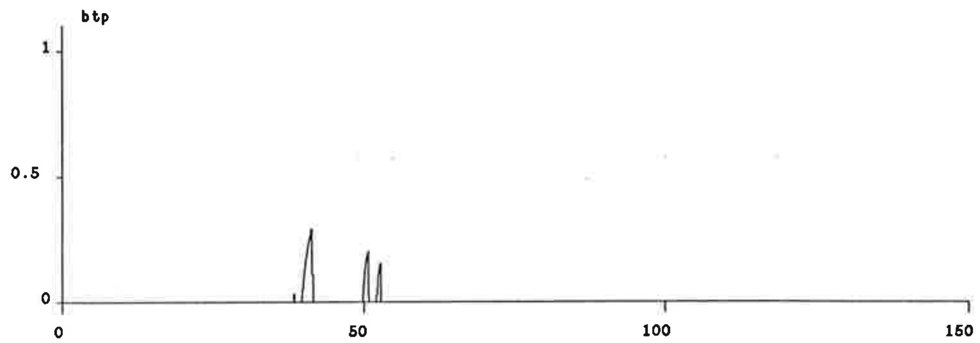

Figure 6. Additional fault detection term to P-matrix

feedback. This gives bias in the parameter estimates.
Finally the parameter β_{tp} of the fault detection algorithm is in figure 6. The parameter change occurs at time $t = 25s$ but it is not detected until $t = 41s$. This depends on the phase shift in the low-pass filters that give the regressors.

5. CONCLUSIONS

It is possible to estimate parameters of a friction and load model for a servo when only the position is measured. Then we can compensate for the effects from load and friction. With the chosen structure the servo is slightly affected by measurement noise.
The low-pass filters for the parameter estimation of the continuous time model influences the convergence speed of the estimated parameters.
A slower observer gives a less noisy estimate of the velocity. However the observer performs well although the parameters have not and will not converge to their true values.

6. FUTURE ISSUES

Two things would be interesting to do. First to make some theoretical work on the method and second to implement the method on some laboratory process.

7. REFERENCES

[Å&W] Åström, K.J. and Wittenmark, B. (1984) Computer Controlled Systems. Englewood Cliffs, N.J.: Prentice Hall, Inc.

[Can] Canudas, C. (1985) Recursive Estimation of the Continuous-Time Process Parameters. Internal Report, CODEN: LUTFD2/(TFRT-7290)/1-22/(1985). Department of Automatic Control, LTH.

[C&Å&B] Canudas, C., Åström, K.J., Braun, K. (1985) Adaptive Friction Compensation in DC Motor Drives Internal Report, CODEN: LUTFD2/(TFRT-7309)/1-21/(1985). Department of Automatic Control, LTH.

[Häg] Hägglund, T. (1983) New Estimation Techniques For Adaptive Control Doctorial Dissertation, CODEN: LUTFD2/(TFRT-1025)/1-120/(1983). Department of Automatic Control, LTH.