Automatic Tuning of Simple Regulators for Phase and Amplitude Margins Specifications

Åström, Karl Johan; Hägglund, Tore

1983

CODEN:LUTFD2/(TFRT-7253)/1-07/(1983)

AUTOMATIC TUNING OF SIMPLE REGULATORS FOR PHASE AND AMPLITUDE MARGINS SPECIFICATIONS

KARL JOHAN ÅSTRÖM
TORE HÄGGLUND

DEPARTMENT OF AUTOMATIC CONTROL
LUND INSTITUTE OF TECHNOLOGY
APRIL 1983

LUND INSTITUTE OF TECHNOLOGY
DEPARTMENT OF AUTOMATIC CONTROL
Box 725
S 220 07 Lund 7 Sweden

Document name
Report

Date of issue
April 1983

Document number
CODEN:LUTFD2/(TFRT-7253)/1-07/(1983)

Supervisor

Sponsoring organization

Author(s)
Karl Johan Åström
Tore Hägglund

Title and subtitle
AUTOMATIC TUNING OF SIMPLE REGULATORS FOR PHASE AND AMPLITUDE MARGINS SPECIFICATIONS.

Abstract

The paper describes procedures for automatic tuning of regulators of the PID type to specifications on phase and amplitude margins. The key idea is a simple method for estimating the critical gain and the critical frequency. The procedure will automatically generate the appropriate test signals. The method is non-parametric and insensitive to modeling errors and disturbances. It may be used for automatic tuning of simple regulators as well as initialization of more complicated adaptive regulators.

Key words
Adaptive control; Control nonlinearities; Describing function; Identification; Limit cycles; Nyquist criterion; PID control; Relay control.

Classification system and/or index terms (if any)

Supplementary bibliographical information

ISSN and key title

ISBN

Language
English

Number of pages
7

Security classification

Recipient's notes

AUTOMATIC TUNING OF SIMPLE REGULATORS

FOR PHASE AND AMPLITUDE MARGINS SPECIFICATIONS

K.J. Aström and T. Hägglund

Department of Automatic Control, Lund Institute of Technology, Lund, Sweden.

Abstract. The paper describes procedures for automatic tuning of regulators of the PID type to specifications on phase and amplitude margins. The key idea is a simple method for estimating the critical gain and the critical frequency. The procedure will automatically generate the appropriate test signals. The method is non-parametric and insensitive to modeling errors and disturbances. It may be used for automatic tuning of simple regulators as well as initialization of more complicated adaptive regulators.

Keywords. Adaptive control; Control nonlinearities; Describing function; Identification; Limit cycles; Nyquist criterion; PID control; Relay control.

1. INTRODUCTION

Adaptive techniques may be used in many different ways. In the applications discussed in this paper the purpose is to obtain techniques for automatic tuning of regulators. The word auto-tuning is used to emphasize this.

Many of the proposed adaptive regulators require considerable a priori information. It is often necessary to specify the sampling period, desired closed loop bandwidth, forgetting factors and desired regulator complexity. This means that it may be a considerable engineering effort to commission such adaptive regulators.

This paper proposes a simple method for automatic tuning of simple regulators of the PID type, which was first suggested in Aström (1981). The basic idea is the observation that many simple regulators require information about the point where the open loop Nyquist curve of the process transfer function intersects the negative real axis. This point is commonly described in terms of the critical gain k_c and the critical frequency ω_c. See Ziegler and Nichols (1943).

A very simple estimation procedure which gives these parameters is proposed. The method has the advantage that a perturbation signal is generated automatically. This perturbation signal is in fact close to an optimal input signal for the particular estimation problem.

Simple regulators can also be tuned using conventional self-tuners or model reference adaptive control. See Wittenmark and Aström (1980), Aström and Wittenmark (1973,1980) and Landau (1979). The method proposed in this paper has two advantages over these approaches.

Conventional adaptive control based on parameter estimation needs a priori knowledge of the dominating time constants. This is required in order to obtain a guess of the sampling period. There are techniques to adjust the sampling period automatically, see Kurz (1979) and Aström and Zhaoying (1981). These techniques will however not work if the initial guess is off by an order of magnitude. The methods proposed in this paper will not require prior knowledge of the time scale of the process. Therefore they may also be used to initialize more sophisticated self-tuners.

Conventional approaches to self-tuning PID control result in a microprocessor code of a few K bytes. The code for the proposed schemes is at least an order of magnitude smaller. The proposed methods may therefore conveniently be incorporated even in very simple regulators.

The major drawback of the methods proposed in this paper compared to conventional adaptive techniques is that they are limited to tuning of simple control laws of the PID type.

The paper is organized as follows: The estimation method is described in Section 2. Simple algorithms for automatic tuning to amplitude margin and phase margin specifications are given in Sections 3 and 4. Results from laboratory and industrial experiments with the algorithms are presented in Section 5.

2. THE BASIC IDEA

The Ziegler-Nichols rule for tuning PID regulators was based on the observation that the regulator parameters could be determined from knowledge of one point on the Nyquist curve of the open loop system. This point is traditionally described in terms of the critical gain and the critical frequency.

In the original Ziegler-Nichols scheme, described in Ziegler and Nichols (1943), the critical gain and the critical frequency are determined in the following way. A proportional regulator is connected to the system. The gain is gradually increased until an oscillation is obtained. It is difficult to perform this experiment in such a way that the amplitude of the oscillation is kept under control. Another method for automatic determination of specific points on the Nyquist curve is therefore proposed.

The method is based on the observation that a system with a phase lag of at least π at high frequencies may oscillate with frequency ω_c under relay control. To determine the critical gain and the critical frequency the system is connected in a feedback loop with a relay as is shown in Fig. 1. The error e is then a periodic signal and the parameters k_c and ω_c can be determined from the first harmonic component of the oscillation.

Let d be the relay amplitude and let a be the amplitude of the first harmonic of the error signal. A simple Fourier series expansion of the relay output then shows that the relay may be described by the equivalent gain

$$k_r = \frac{4d}{\pi a} . \tag{1}$$

It follows from the principle of harmonic balance that this gain is equal to the critical gain k_c. It is possible to modify the procedure to determine other points on the Nyquist curve. An integrator may be connected in the loop after the relay to obtain the point where the Nyquist curve intersects the negative imaginary axis. Other points on the Nyquist curve can be determined by repeating the procedure with linear systems introduced into the loop.

An exact expression for the period of oscillation is given by the following theorem.

Theorem 1. Consider a linear time-invariant system under relay control. Let H(T,z) be the pulse transfer function of the linear system with a sample and hold. Assume that there is a limit cycle with period t_p. Then t_p is given by

$$H(t_p/2,-1) = 0 \qquad \square$$

The proof is given in Åström (1983).

A simple relay control experiment thus gives the information about the process which is needed in order to apply the design methods. This method has the advantage that it is easy to control the amplitude of the limit cycle by an appropriate choice of the relay amplitude. Notice also that the estimation method will automatically generate an input signal to the process which has a significant frequency content at ω_c. This ensures that the critical point can be determined accurately. See Mannerfelt (1981).

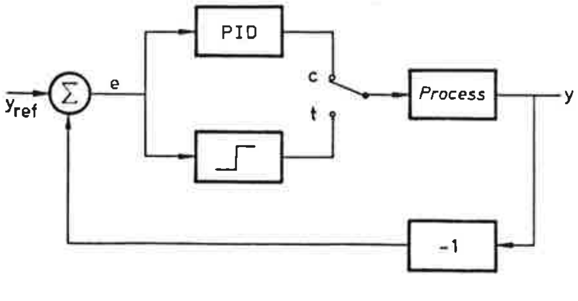

Fig. 1. Block diagram of the auto-tuner. The system operates as a relay controller in the tuning mode (t) and as an ordinary PID regulator in the control mode (c).

When the critical point on the Nyquist curve is known it is straightforward to apply the classical Ziegler-Nichols tuning rules. It is also possible to devise many other design schemes based on the knowledge of the critical point. Algorithms for automatic tuning of simple regulators based on the amplitude and phase margin criteria will be given in Sections 3 and 4.

Determination of amplitude and period

Methods for automatic determination of the frequency and the amplitude of the oscillation will be given to complete the description of the estimation method.

The period of an oscillation can easily be determined by measuring the times between zero-crossings. The amplitude may be determined by measuring the peak-to-peak values. These estimation methods are easy to implement because they are based on counting and comparisons only.

Since the describing function analysis is based on the first harmonic of the oscillation the simple estimation techniques require that the first harmonic dominates. If this is not the case it may be necessary to filter the signal before measuring. See e.g. Åström (1975).

The precision of the method can be improved considerably by introducing a simple correlation with the relay amplitude. This can be implemented simply by up-down counting.

More elaborate estimation schemes like least squares estimation and extended Kalman filtering may also be used to determine the amplitude and the frequency of the limit cycle oscillation. This is discussed in Åström (1982) and Kai Siew (1982).

3. AMPLITUDE MARGIN AUTO-TUNERS

When the critical point is known it is straight forward to find a regulator which gives a desired amplitude margin. The simplest way is to choose a proportional regulator with the gain

$$k = k_c/A_m \tag{2}$$

where A_m is the desired amplitude margin and k_c is the critical gain.

Sometimes this solution is not satisfactory because integral action may be required. Since the frequency response of a PID regulator can be written as

$$G_R(i\omega) = k \left[1 + \frac{1}{i\omega T_i}(1 - \omega^2 T_i T_d)\right] \tag{3}$$

it follows that any PID regulator with the gain given by (2) and

$$T_d = \frac{1}{\omega_c^2 T_i} \tag{4}$$

also gives the desired amplitude margin. The integration time can then be chosen arbitrarily, e.g. inversely proportional to ω_c, and the derivation time is then given by Equation (4).

4. PHASE MARGIN AUTO-TUNERS

Consider a situation when one point on the Nyquist curve for the open loop system is known. With PI, PD or PID control it is then possible to move the given point on the Nyquist curve to an arbitrary position in the complex plane. This is indicated in Fig. 2. By changing the gain it is possible to move the Nyquist curve in the direction of $G(i\omega)$. The point A may be moved in the orthogonal direction by changing integral or derivative gain. It is thus possible to move a specified point to an arbitrary position. This idea can be used to obtain design methods. By moving A to a point on the unit circle it is e.g. possible to obtain systems with a prescribed phase-margin. An example is given below.

Example 1 Consider a process with the transfer function $G(s)$. The loop transfer function with PID control is

$$k\left(1 + sT_d + \frac{1}{sT_i}\right) G(s)$$

Assume that the point ω_c where the Nyquist curve of G intersects the negative real axis is known. Requiring that the argument of the loop transfer function at ω_c is $\phi_m - \pi$ the following condition is obtained

$$\omega_c T_d - \frac{1}{\omega_c T_i} = \tan \phi_m \tag{5}$$

There are many T_d and T_i which satisfy this condition. One possibility is to choose T_i and T_d so that

$$T_i = \alpha T_d \tag{6}$$

Equation (5) then gives a second order equation for T_d which has the solution

$$T_d = \frac{\tan \phi_m + \sqrt{\frac{4}{\alpha} + \tan^2 \phi_m}}{2\omega_c}. \tag{7}$$

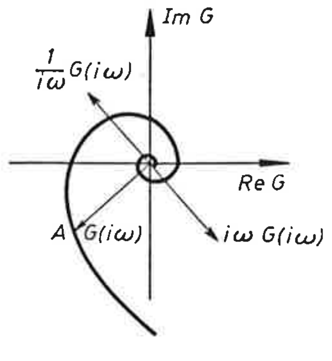

Fig. 2. Shows that a given point on the Nyquist curve may be moved to an arbitrary position in the G-plane by PI, PD or PID control. The point A may be moved in the directions $G(i\omega)$, $G(i\omega)/i\omega$ and $i\omega G(i\omega)$ by changing proportional, integral and derivative gain respectively.

Simple calculations show that the loop transfer function has unit gain at ω_c if the regulator gain is chosen as

$$k = \frac{\cos \phi_m}{|G(i\omega_c)|} = k_c \cos \phi_m \tag{8}$$

where k_c is the critical gain. The design rules are thus given by the equations (5), (6), (7) and (8).

There are many other possibilities. The parameter T_i may e.g. be chosen so that $\omega_c T_i$ has a given value.

□

So far, it has been assumed that the nonlinearity introduced in the feedback loop is a relay, and the point where the Nyquist curve intersects the negative real axis has been identified. Other nonlinearities can also be used. In Hägglund (1981) a relay with hysteresis is used to tune the system to a desired phase margin. The inverse describing function of a relay with hysteresis is

$$-\frac{1}{N(a)} = -\frac{\pi}{4d}\sqrt{a^2 - h^2} - i\frac{\pi h}{4d} \tag{9}$$

where d is the relay amplitude and h is the hysteresis width. In the complex plane this function may be described as a straight line parallel to the real axis, see Fig. 3. By choosing the relation between h and d it is therefore possible to determine a point on the Nyquist curve with a specified imaginary part. In the next example, this property is used to obtain a regulator which gives a desired phase margin of a system.

Example 2 Consider a process with transfer function $G(s)$, controlled by a proportional regulator. The loop transfer function is thus $k \cdot G(s)$. Assume that the design goal is to obtain a closed loop system with the phase margin ϕ_m. Choose the relay characteristics so that the negative inverse describing function goes through the point p defined in Fig. 3. The parameters are then

$$d = \frac{\pi a^*}{4} \qquad h = a^* \sin(\phi_m)$$

where a^* is the desired amplitude of the oscillations. The desired phase margin is

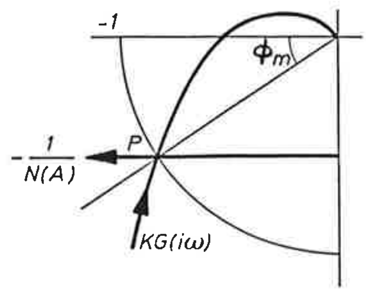

Fig. 3. The position of the inverse describing function and the desired location of the Nyquist curve.

obtained if the Nyqvist curve goes through the point p in Fig. 3. Since the intersection between $-1/N(a)$ and $k \cdot G(i\omega)$ can be determined from the amplitude of the oscillation, this point can be reached e.g. by iteratively changing the gain k. The Regula falsi method gives the formula

$$k_{n+1} = k_n - (a_n - a^*) \frac{k_n - k_{n-1}}{a_n - a_{n-1}} \qquad (10)$$

which has a quadratic convergence rate near the solution. Integral and derivative action can be included, using the methods proposed in Example 1.

□

There are many possible variations of the given design methods for PID-regulators. All methods are closely related because they are based on information about the process to be controlled in terms of one point on the Nyquist curve of the process. The points where the Nyquist curve intersects the real axes or straight lines parallel to the real axes are simple choices. The design methods may be modified. Other relations between T_i and T_d than those given by (6) may e.g. be used. Other criteria like damping or bandwidth may be chosen instead of the phase- or amplitude-margins. It is also possible to have design methods which are based on knowledge of more points on the Nyquist curve.

5. EXPERIMENTS

A number of simulations and experiments have been performed in order to find out if a useful auto-tuner can be designed based on the ideas described in the previous sections. The results are summarized in this section. Some representative examples are also presented.

Practical aspects

There are several practical problems which must be solved in order to implement an auto-tuner. It is necessary to account for measurement noise, level adjustment, saturation of actuators and automatic adjustment of the amplitude of the oscillation.

Measurement noise may give errors in detection of peaks and zero crossings. A hysteresis in the relay is a simple way to reduce the influence of measurement noise. Filtering is another possibility. The estimation schemes based on least squares and extended Kalman filtering can be made less sensitive to noise.

When the regulator is switched on it may happen that the process output is far from the desired equilibrium condition. It would be desirable to have a system which reaches the equilibrium automatically. For a process with finite low-frequency gain there is no guarantee that the desired steady state will be achieved with relay control unless the relay amplitude is sufficiently large. To guarantee that the output can actually reach the reference value it may be necessary to introduce manual or automatic reset.

It is desirable that the relay amplitude is adjusted automatically. A reasonable approach is to require that the oscillation is a given percentage of the admissible swing in the output signal.

Regulator complexity

The consequences of using estimation schemes of different complexity have been explored by simulation. See Åström (1982) and Kai Siew (1982). In these experiments processes having different dynamics have been regulated with different types of auto-tuners. The effects of measurement noise and load disturbances have been investigated. Although work still remains to be done the experiments have indicated that the simple estimation method based on zero-crossing and peak detection works very well. The experiments also indicate that simple minded level adjustment methods often are satisfactory.

Implementation

The auto-tuners have been implemented on several different computers. A DEC LSI 11/03 was used in some early experiments. See Elfgren (1981). The algorithms were coded in Pascal with a real time kernel. Small laboratory processes were controlled. The experiments showed that the simple algorithms were robust and that they worked well.

The algorithms were also coded in Basic on an Apple II. This implementation was very easy to use because of the graphics and the interactive man machine interface.

Experiment on a laboratory process

An experiment made with the Apple II implementation will now be presented. Fig. 4 shows the result when the auto-tuner was applied to a tank with a free outlet. The inlet valve to the tank was controlled from measurements of the water level in the tank. The tuning procedure can be divided into two phases, see Fig. 4. The first phase is an initial phase which moves the process to equilibrium, i.e. to the desired reference level. The second phase is the final tuning phase. The two phases are described in more detail below.

Phase 1. When the process dynamics is totally unknown, is is natural to make small step changes of the control signal to determine the magnitude of the gain and the dominating time constant of the system. This is done initially in phase 1. Based on this rough characterization of the process, a

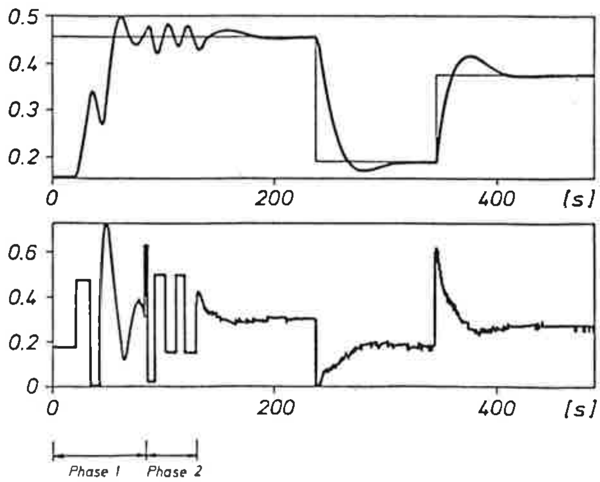

Fig. 4. Experiments made on the tank process.

conservative PI controller is designed which ramps the system to the equilibrium with a slope determined from the estimated time constant. This first phase can be omitted if the process is manually moved to the equilibrium.

Phase_2. When the desired level is reached, the estimation procedure starts. A relay with a small hysteresis is introduced in the loop as shown in Fig. 1. The relay amplitude is adjusted automatically so that an oscillation with desired amplitude is obtained. The amplitude and the frequency of the oscillation are estimated by peak detection and determination of the times between zero crossings of the control error.

The design method in this example was based on a combination of phase- and amplitude-margin specification. It was required that the Nyquist curve intersects the circle with radius 0.5 at an angle of 225°. Two step responses of the system controlled by the estimated controller are shown in Fig. 4. The lack of symmetry depends on the nonlinearities. The high frequency disturbance in the control signal is caused by the low resolution in the AD-converter, 8 bits only.

Experiments_in_a_sugar_refinery

The Apple II implementation was used in a sugar refinery to test the feasibility of auto-tuning in an industrial environment. The auto-tuner was applied to several feedback loops. In Fig. 5 an example of a temperature control loop is given. Only phase 2 was used.

The experiments in the sugar refinery were very fruitful. They showed that the auto-tuner worked very well. The estimated PID parameters varied only ±10% between comparable experiments. Acceptable controllers were obtained even in loops which were considered as very hard to control.

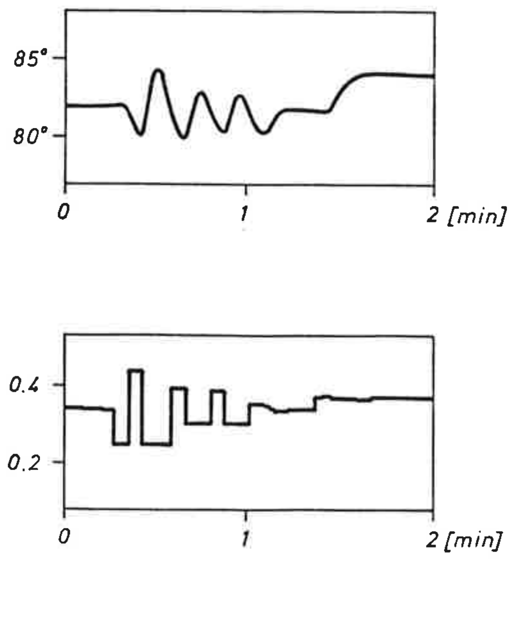

Fig. 5. Experiments on a temperature control loop in a sugar refinery. Phase 2 followed by a step response.

6. LIMITATIONS

The proposed simple scheme can obviously not work for all systems. There are several things that may go wrong. The parameter estimation may fail if there is no limit cycle. This occurs e.g. for systems which are strictly positive real. Such systems poses little problem because they are easy to control anyway. The addition of an hysteresis may also bring SPR systems to oscillations. The precise conditions for oscillation are given Theorem 1. The system may also fail to go into a limit cycle oscillation if the relay sticks to one value. This may happen for strongly unstable systems.

Phase and amplitude margins are also fairly crude design criteria. It is well-known that systems with the same margins may exhibit drastically different behaviour.

7. CONCLUSIONS

There are many possibilities to tune regulators automatically. Self-tuning regulators based on minimum variance control, pole placement or LQG design methods may be configured to give PID control. Such approaches have been considered by Wittenmark et al (1980) and Wittenmark and Aström (1980). These regulators have the disadvantage that some information about the time scale of the process must be provided a priori to give a reasonable estimate of the sampling period in the regulator. There are some possibilities to tune the sampling period automatically. Different schemes have been proposed by Kurz (1979) and Aström and Zhaoying (1981). These methods will, however, only work for moderate changes in the process time constants.

The method proposed in this paper does not suffer from this disadvantage. It may be applied to processes having widely different time scales. The test signal which is generated automatically by the algorithm will have a considerable energy at the crossover frequency of the process.

Conventional self-tuning regulators based on recursive estimation of a parametric model requires a computer code of a few K bytes. The algorithms proposed in this paper which are based on determination of zero crossings and peak detection may be programmed in a few hundred bytes. It is thus possible to use these methods even in very simple regulators.

The methods proposed will of course inherit the limitations of the PID-algorithms. They will not work well for problems where more complicated regulators are required.

The experiences reported also indicate that the simple versions of the algorithms work well and that they are robust. It thus appears worthwhile to explore these algorithms further.

The algorithms discussed in this paper may be used in several different ways. They may be incorporated in single loop controllers to provide an option for automatic tuning. They may also be used to provide the a priori information which is required by more sophisticated adaptive algorithms. When combined with a bandwidth self-tuner like the one discussed in Aström (1979) it is possible to obtain an adaptive regulator which may set a suitable closed loop bandwidth automatically.

8. ACKNOWLEDGEMENTS

This work was partially supported by research grant 82-3430 from the Swedish Board of Technical development (STU). This support is gratefully acknowledged. We would also like to thank Kalle Åström who programmed the Apple II implementation.

9. REFERENCES

Åström, K. J. (1975). Lectures on system identification. Chapter 3. Frequency response analysis. Report TFRT-7504. Dept of Automatic Control, Lund Institute of Technology, Lund, Sweden.

Åström, K. J. (1979). Simple self-tuners 1. Report TFRT-7184. Dept of Automatic Control, Lund Institute of Technology, Lund, Sweden.

Åström, K. J. (1981). More stupid. Report TFRT-7214. Dept of Automatic Control, Lund Institute of Technology, Lund, Sweden.

Åström, K. J. (1982). Ziegler-Nichols Auto-Tuners. Report TFRT-3167. Dept of Automatic Control, Lund Institute of Technology, Lund, Sweden.

Åström, K. J. (1983). Lectures on Adaptive Control - Chapter 4: Self-oscillating adaptive systems. Dept of Automatic Control, Lund Institute of Technology, Lund, Sweden.

Åström, K. J. and B. Wittenmark (1973). On self-tuning regulators. Automatica 9, 185-199.

Åström, K. J. and B. Wittenmark (1980). Self-tuning controllers based on pole-zero placement. IEE Proc. 127, 120-130.

Åström, K. J. and Z. Zhaoying (1981). Self-tuners with automatic adjustment of the sampling period for processes with time delays. Report TFRT-7229. Dept of Automatic Control, Lund Institute of Technology, Lund, Sweden.

Elfgren, L.-G. (1981). Temperaturreglering med självinställande PID-regulator. (Temperature control with a self-tuning PID-controller). MS Thesis. Report TFRT-5254. Dept of Automatic Control, Lund Institute of Technology, Lund, Sweden.

Hägglund, T. (1981). A PID tuner based on phase margin specifications. Report TFRT-7224. Dept of Automatic Control, Lund Institute of Technology, Lund, Sweden.

Kai Siew, W. (1982). Methods for automatic tuning of PID regulators. MS Thesis. Report TFRT-5288. Dept of Automatic Control, Lund Institute of Technology, Lund, Sweden.

Kurz, H. (1979). Digital parameter-adaptive control of processes with unknown constant or timevarying dead time. Preprints 5th IFAC Symposium on Identification and Parameter Estimation, Darmstadt.

Landau, I. D. (1979). Adaptive control - The model reference approach. Marcel Dekker, New York.

Mannerfelt, C. F. (1981). Robust control design for simplified models. PhD Thesis. Report TFRT-1021. Dept of Automatic Control, Lund Institute of Technology, Lund, Sweden.

Ziegler, J. G. and N. B. Nichols (1943). Optimum settings for automatic controllers. Trans. ASME 65 433-444.

Wittenmark, B., P. Hagander and I. Gustavsson (1980). STUPID - Implementation of a self-tuning PID-controller. Report TFRT-7201. Dept of Automatic Control, Lund Institute of Technology, Lund, Sweden.

Wittenmark, B. and K. J. Åström (1980). Simple self-tuning controllers in Unbehauen, H (editor). Methods and Applications in Adaptive Control, Springer, Berlin.