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UNIFYING FOUNDATIONS – TO BE SEEN IN THE PHENOMENON OF LANGUAGE

When it was objected that reality is
more fundamental than language and
lies beneath language, Bohr answered,
'We are suspended in language in such a way
that we cannot say what is up and what is down'.
(Petersen, 1968)

ABSTRACT. Scientific knowledge develops in an increasingly fragmentary way. A multitude of scientific disciplines branch out. Curiosity for this development leads into quests for a unifying understanding. To a certain extent, foundational studies provide such unification. There is a tendency, however, also of a fragmentary growth of foundational studies, like in a multitude of disciplinary foundations. We suggest to look at the foundational problem, not primarily as a search for foundations for one discipline in another, as in some reductionist approach, but as a steady revelation of presuppositions for individual scientific theories – which are bound to meet, sooner or later, in a common language. A decisive point here is our holistic conception of language, as a whole of description-interpretation processes which are entangled (complementary) in the language itself. For every language there is a linguistic complementarity. We suggest this unique form of entanglement as a unifying presupposition, ultimately foundational for all communicable knowledge. Involved is a linguistic realism, in terms of which we critically examine "language-world" problems, as exposed by Wittgenstein, and Russell, about a foundational interdependence of language and reality (world). Throughout, we attach to the development of foundational studies of mathematics, logics, and the natural sciences. In particular, we study the interpretation problem for axioms of infinity in some detail. We emphasize that the holistic concept of language contradicts Carnap's semiotic fragmentation thesis (thus, no clean cut between syntax, semantics, pragmatics).

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1. OUTLINE

Sciences are *deductive* in the sense that the *results* of scientific activity are *deductively presentable* as in descriptive theories. This is what allows wide *communication* of tentative results, necessary for their accessibility, examination and tests by other scientists, eventually to be intersubjectively accepted.

Scientific *activity*, on the other hand, is in general a process which is beyond full deductive description. In other words, scientific processes may well be beyond scientific description. This is sometimes understood with reference to the *inductive* nature of producing scientific hypotheses and a general agreement on *induction as beyond deduction*.

This *activity/result* distinction (or induction/deduction distinction) for science is not always appreciated, however, which easily causes confusion as to what science really is. It frequently happens that scientific thinking becomes identified with deductive thinking – leaving the inductive activities out from a domain of proper scientific inquiry. Instead they are approached in *foundational research*. Here, sometimes, the full question is addressed of where the axioms, on which the deductive scientific processes build, do come from.

The fact that scientific processes may well be beyond scientific description, or that scientific describability is not universal, makes science develop in a multitude of fragmented disciplines. Although foundational research may help to provide unification of the disciplinary diversity, it frequently happens that also foundational research develops by fragmentation. Compare mathematical foundations for physics, logical foundations for mathematics, etc. Again, a reason for this second order fragmentation is a preoccupation, also in foundational research, with deductive processes at the cost of inductive.

Involved in the unification problem is a general understanding of fragmentation-unification as a *part-whole* issue. Compare the following three quotes, representing such insights from (meta-)physics, systems research, and philosophy.

PART-WHOLE VIEW IN PHYSICS (CHEW, 1968, pp. 762-763). “Conventional science requires the a priori acceptance of certain concepts, so that ’questions’ can be formulated and experiments performed to give answers. ..
The number of a priori concepts has lessened as physics has progressed, but it would seem that science, as we know it, requires a language based on some unquestioned framework. Semantically, therefore, an attempt to explain *all* concepts can hardly be called ’scientific’. ..
A key discovery of Western culture has been the discovery that different aspects of nature can be individually ‘understood’ in an approximate sense without everything’s being understood at once. All phenomena ultimately are

interconnected, so an attempt to understand only a part necessarily leads to some error, but the error often is sufficiently small for the partial approach to be meaningful. Save for this remarkable and far from obvious property of nature, scientific progress would be impossible. Current examples of the partial approach in science are a cosmology that ignores quantum effects; a biology that ignores almost all hadrons; a particle physics that ignores gravitation; a natural science that ignores the mechanism underlying consciousness. Supporting the partial approach is the unavoidable error in every experiment. Does it make sense, in other words, to speak of absolute precision in a theory when we cannot conceive of an absolutely precise experiment?"

Notice, in particular, Chew's observation of the necessity to accept, a priori, certain concepts for science, i.e., *scientifically relevant* concepts not themselves fully accounted for as *scientific results*. In section 10.1 we develop this view to conclude that in every (*holistic*) *language*, there are *presuppositions* hidden in the language, namely hidden from full description in the language. Underneath is our *holistic conception of language* as a whole of complementary description-interpretation processes, developed in section 3.

Chew's view of fragmented knowledge as "approximation" to nature, presupposes nature as independent (in some unspecified sense) of language or of communicable knowledge. In *linguistic realism* (section 8), based on the holistic conception of language, there is a further possibility, depending upon that the linguistic complementarity is tensioned (section 4). Here reality (nature) is itself a linguistic phenomenon, complementaristically conceived as a whole of description-interpretation processes – with a characteristically maximal "interpretation content" and minimal residual "description content". We use the expression *interpretation-like* in the characterization of reality in linguistic realism.

In Chew's above list of examples of "the partial approach in science", he mentions "a natural science that ignores the mechanism underlying consciousness". In our view, a still more fundamental example would be *a science that ignores holistic language*. This follows from our understanding of consciousness, in the form of existential perceptions (section 8.2), as a phenomenon in linguistic realism which requires *induction beyond deduction* for its comprehension.

Part-Whole View in Systems Research (Ashby, 1972, pp. 78-79). "For two hundred years (after Newton) this method [analytic science] yielded such an abundance of discoveries and advances that most workers felt little inclination to complain. ..
Then, in the 1930's, general systems theory arose, mostly through the work of Ludwig von Bertalanffy, who saw not only that the study of parts (in 'classic' science) must be supplemented by the study of wholes, but also that there *ex-*

ists a science of wholes, with its own laws, methods, logic, and mathematics."

It would seem plausible to assume that Ashby here thinks of "science", in the postulated "science of wholes", as a systemic whole of scientific activity and scientific results. We have not, however, seen this line of thinking more fully developed by Ashby, or by von Bertalanffy.

PART-WHOLE VIEW OF THE PHILOSOPHICAL LANGUAGE-WORLD PROBLEM (RUSSELL, 1940, p. 21). "Finally there is the question: how far, if at all, do the logical categories of language correspond to elements in the nonlinguistic world that language deals with? Or, in other words: does logic afford a basis for any metaphysical doctrines?"
(RUSSELL, 1940, p. 341). "I propose to consider whether anything, and, if so, what, can be inferred from the structure of language as to the structure of the world. There has been a tendency, especially among logical positivists, to treat language as an independent realm, which can be studied without regard to non-linguistic occurrences. To some extent, and in a limited field, this separation of language from other facts is possible; the detached study of logical syntax has undoubtedly yielded valuable results. But I think it is easy to exaggerate what can be achieved by syntax alone. There is, I think, a discoverable relation between the structure of sentences and the structure of the occurrences to which the sentences refer. I do not think the structure of non-verbal facts is wholly unknowable, and I believe that, with sufficient caution, the properties of language may help us to understand the structure of the world."

Russell here tends to use "language" in a non-holistic sense, presupposing a distinguishability between "language" and "world" allowing understandings of a possible relation. Leaving out the problems of where (in or beyond language, world) to formulate (describe), interpret, and discover such a relation.

By comparison (see section 9), in exploring *holistic language* we find *it*, with its characteristic complementaristic conceivability, rather than for example logic, a foundational category allowing understanding of interrelations with "the real world". In particular we stress the *inductive* nature of the linguistic processes, which is beyond access as deductive logical result (section 7).

In section 9 we also look into Wittgenstein's language-world conception, with its explicitly stated *presuppositions*, and compare with the *presuppositions* behind holistic language in linguistic realism. We find Wittgenstein's presuppositions too strong to be acceptable in present-day philosophy of mathematics.

Substantial parts of foundational research have developed towards understandings of basic concepts, like *truth* in terms of *verifiers* – in mathematics as well as in the sciences. With holistic language as a foundational category, foundational research obtains a further direction. Namely towards *revelation of presuppositions* behind concepts. Such *language-based foundational research* is illustrated in section 10.

Throughout, we attempt a *unifying* linguistic insight, notably into problems from foundations of mathematics. In section 5 we study the interpretation problem for axioms of infinity, revealing wide insights into consequences of our basic presuppositions for the concept of language.

2. SELECTED FOUNDATIONAL STUDIES. WHEREFROM THE AXIOMS?

Foundational studies are comparatively well exhibited in foundational studies for mathematics. For example, set theory is sometimes called upon as a branch of mathematics, whose concepts bear close relationship to underlying foundational reflections.

Mostowski writes as follows in his book Thirty Years of Foundational Studies: Lectures on the Development of Mathematical Logic and the Study of the Foundations of Mathematics in 1930-1964.

A Fundamental Question (Mostowski, 1966, p. 140). "The abstract set theory has contributed more than any other branch of mathematics to the development of foundational studies. The reasons for this phenomenon are numerous.

One of the basic assumptions of set theory is the axiom of infinity which says that there exist infinite sets. This assumption implies that the scale of infinte cardinals is itself infinite. Thus the axiom of infinity leads us out of the mathematical domains which are close to everyday practice and even to scientific experience. We are thus faced at the very beginning of set theory with the fundamental question of the philosophy of mathematics: which mathematical objects are admissible and why? ...

Most mathematicians do not perceive the problem which is posed by the abstractness of set theory. They prefer to take an aloof attitude and pretend not to be interested in philosophical (as opposed to purely mathematical) questions. In practice this simply means that they limit themselves to deducing theorems from axioms which were proposed to them by some authorities."

We will return to the axiomatizability of infinity as a genuine foundational issue in section 5. Let us for the moment expand on the last sentence in the quote, where Mostowski cautiously hints at the process of forming axioms (and rules of inference) as a foundational subject for mathematical authorities, somewhat distinct from the preoccupation of most mathematicians.

Because of the affinity between mathematics and parts of metamathematics, large parts of accepted foundational results for mathematics are developed without explicit reference to the induction/deduction distinction. In *Principia Mathematica*, a classic which is sometimes referred to as foundational for mathematics, there is however a distinction somewhat in this direction.

(Whitehead and Russell, 1962, p. v). "We have, however, avoided both controversy and general philosophy, and made our statements dogmatic in form. The justification for this is that the chief reason in favour of any theory on the principles of mathematics must always be inductive, i.e., it must lie in the fact that the theory in question enables us to deduce ordinary mathematics. In mathematics, the greatest degree of self-evidence is usually not to be found quite at the beginning, but at some later point; hence the early deductions, until they reach this point, give reasons rather for believing the premisses because true consequences follow from them, than for believing the consequences because they follow from the premisses."

The authors here look upon ordinary mathematics as somehing already given and provide, by systematizing efforts, basic statements from which ordinary mathematics follows deductively. They deliberately focus on a theory of mathematical principles for ordinary mathematics, and refrain from philosophical issues involved in the truly inductive question: Where do the axioms come from? How has (ordinary) mathematics evolved? How is mathematics evolving?

Gödel had early a fairly complete view of foundational research for mathematics.

Gödel on the Problem of Giving a Foundation (Feferman, 1998, p. 165). "The aim of Gödel's lecture [referring to a handwritten, unpublished, text for an invited lecture to a 1933 meeting of the Mathematical Association of America] is announced in his first paragraph with admirable clarity: 'The problem of giving a foundation for mathematics ... can be considered as falling into two different parts. At first [the] methods of proof [actually used by mathematicians] have to be reduced to a minimum number of axioms and primitive rules of inference, ... and then secondly a justification in some sense or other has to be sought for these axioms'."

Gödel's comment on the second part of the problem of foundation, the justification problem, is: "it must be said that the situation is extremely unsatisfactory." And so it seems to remain – as long as the induction/deduction distinction is not fully recognized. Or, cannot be accounted for *within* mathematics. Proper foundational studies are bound to transcend deductive disciplines.

Beth, in his book *The Foundations of Mathematics*, does not refrain from illustrating the more truly foundational issues.

(Beth, 1959, p. 409) [in referring to Brouwer]. "It is not possible to penetrate the foundations of mathematics without paying attention to the conditions under which the mental activity proper to mathematicians takes place... Research which does not give proper attention to this side of the problem is unable to reveal the essence of mathematical thinking; it can give information only as to its external appearance."

We find this an expression of mathematical thinking, or mathematical activity, conscious or unconscious, which is clearly beyond the deductive form of mathematical results.

Concerning the deductive sciences, Russell is more outspoken on induction and the induction/deduction distinction.

(Russell, 1948, p. 700). "What these arguments [referring to Hume] prove – and I do not think that the proof can be controverted – is, that induction is an independent logical principle, incapable of being inferred [deductively] either from experience or from other logical principles, and that without this principle science is impossible."

Compare also (Russell, 1961, pp. 43, 225).

3. TOWARDS COMPLEMENTARISTIC COMPREHENSION OF LANGUAGE

In a disciplinary account of logic, as in mathematical logic, the concept of language is either not defined at all, or is considered as partly outside the domain of the discipline. Compare Shoenfield's book on mathematical logic:

A Non-Holistic Conception of Language (Shoenfield, 1967, p. 4). "We consider a language to be completely specified when its symbols and formulas are specified. This makes a language a purely syntactical object. Of course, most of our languages will have a meaning (or several meanings); but the meaning is not considered to be part of the language."

Shoenfield's honest account of his disciplinary approach indicates how the fragmentation into mathematical logic "makes" language devoid of meaning. This is a clearly distortive approach, or a high price to be paid for mathematical clarity.

Also in a wider, philosophical linguistic context, admitting meaning, a fragmentation problem is apparent.

(Putnam, 1975, pp. 215-216). "Analysis of the deep structure of linguistic forms gives us an incomparably more powerful description of the *syntax* of natural languages than we have ever had before. But the dimension of language associated with the word 'meaning' is, in spite of the usual spate of heroic if misguided attempts, as much in the dark as it ever was."...
In my opinion, the reason that so-called semantics is in so much worse condition than syntactic theory is that the *prescientific* concept on which semantics is based – the prescientific concept of meaning – is itself in much worse shape than the prescientific concept of syntax."

In semiotics, sometimes referred to as the science of language, there is an explicit recognition of syntax and semantics, as well as pragmatics. This emphasis on very central parts of language is no doubt a step towards a general understanding of language – taken without explicitly recognizing a lurking fragmentation problem. In our opinion, the following quote from Carnap, where he clearly proposes that the whole science of language can be fragmented into three individually understood parts, is to be conceived as a *fragmentation hypothesis* for language.

Carnap's Fragmentation Thesis (Carnap, 1968, p. 9). "If we are analyzing a language, then we are concerned, of course, with expressions. But we need not necessarily also deal with speakers and designata. Although these factors are present whenever language is used, we may abstract from one or both of them in what we intend to say about the language in question. Accordingly, we distinguish three fields of investigation of languages. If in an investigation explicit reference is made to the speaker, or, to put it in more general terms, to the user of the language, then we assign it to the field of **pragmatics**. (Whether in this case reference to designata [what the expressions refer to] is made or not makes no difference for this classification.) If we abstract from the user of the language and analyze only the expressions and their designata, we are in the field of **semantics**. And if, finally, we abstract from the designata also and analyze only the relations between the expressions, we are in (logical) **syntax**. The whole science of language, consisting of the three parts mentioned, is called **semiotic**."

Carnap here seems to take it for granted that the three parts mentioned can be individually understood – at the same time that they, as individually understood, constitute the whole science of language.

By contrast, we have come to the conclusion that language has to be comprehended by *complementaristic conception*:

> **Language**, in its general conception, is a *whole* of complementary description-interpretation processes. The meaning of "complementary" is that of *the linguistic complementarity* given below. In particular, Carnap's fragmentation thesis does not hold for language in its general conception.

The complementaristic conception of language will sometimes also be referred to as *holistic* language, or *systemic* language. The property of being complementary, in a language, will also sometimes be referred to as being *entangled* in the language.
Many authors usually write and talk as if the language – then in use – could be detached (as if their ideas were independent of the language in use). To use the word "complementary" in such contexts, easily causes confusions. Complementarity in the sense of the present paper, where language cannot be detached, obtaines its very meaning just from this fact (cf view iv, below, of the linguistic complementarity).

In comparing this complementaristic conception of holistic language with Carnap's fragmentation of language, we may look at the syntax and semantics parts as corresponding to the complementary description-interpretation processes, and at pragmatics as corresponding to the processual nature of the description-interpretation processes. There is then an incompatibility between the two conceptions of language. Whereas Carnap assumes fragmentability of syntax and semantics (also within the language in which "the whole science of language" is expressed), syntax and semantics are *entangled* in the complementaristic conception of language.

Our general conception of language admits species such as genetic language, programming language, formal language, observation language, inner cerebral language, external communication language. Such phenomena of language are at the bottom of all human activity and are, indeed, at the roots of all forms of life as genetic processes. The phenomena are extremely rich, and exceedingly difficult to conceptualize (classically, i.e., noncomplementaristically) without distorting them in the act. Yet, at the same time, our communication languages are so natural and easy for us to use that we hardly notice them. It is as if they were universal, as if what we are saying had an absolute meaning independent of the language in use. As if the language could be detached from the ideas we are talking about. Such impressions fade away, however, when we come to understand language, in nondistorted objectification, as a complementaristic whole.

The linguistic complementarity. In general, complementarity refers to holistic situations where (a classical) fragmentation into parts does not succeed. In its *complementaristic conception*, the phenomenon of language is such a whole of description and interpretation processes, yet a whole which has no such parts fully expressible within the language itself. Instead, within the language, the parts are complementary or tensioned. There are various related ways of looking at the complementarity:

(i) as descriptional incompleteness: in no language can its interpretation process be completely described in the language itself;

(ii) as a tension between describability and interpretability within a language: increased describability implies decreased interpretability, and conversely;

(iii) as degrees of partiality of self–reference (introspection) within a language: complete self–reference within a language is impossible;

(iv) as a principle of "nondetachability of language".

Languages may change and evolve, and with them their capacities for describing and interpreting. Yet, at each time that we want to communicate our actual knowledge, even on the evolution of language, we are in a *linguistic predicament*, namely to be confined to a language with its inescapable complementarity.

The linguistic closure. Our thinking abilities are usually looked upon as free and unbounded. But when it comes to *communicable thought*, we are confined to some shared communication language. The systemic wholeness, or the complementaristic nature, of this language implies a closure, or circumscription, of our linguistic abilities – be they creation of "pure thoughts" communicable in a formal mathematical language, or constructive directions for an experimental interpretation-domain of a physics language. The nature of this closure is not that of a classical boundary of a capacity, like describability, or interpretability. It is a *tensioned and hereditary* condition on the systemic capacity of *describability-and-interpretability* admitting *potentialities* in two directions:

(a) The closure is TENSIONED. Within the language there is a tension between describability and interpretability (view (ii) of the linguistic complementarity), whereby it may be possible to increase the describability at the cost of a lowered interpretability, and conversely.

In other words, what the closure bounds off is neither describability, nor interpretability, but their interactive whole as a linguistic unit of *describability-and-interpretability*.

(b) The closure is HEREDITARY. Languages may evolve, and at a later time we can have access to another shared communication language of greater capacity for communication. However, we are then back to the *linguistic predicament*: at each time that we try to communicate thoughts – even introspective thoughts about language and its evolution – we are confined to a shared language, however evolved, and the linguistic complementarity of that language restricts our communicability in the tensioned way according to (a).

4. DEEPER INTO THE LINGUISTIC COMPLEMENTARITY

As explained in (Löfgren, 1992) we have argued the validity of the linguistic complementarity from the functional role of any language, namely to admit communication or control. This requires that the descriptions are finitely representable, as well as locally independent of time (static). Presuppositions for our arguments are revealed as follows.

4.1. *Presuppositions for the Linguistic Complementarity*

PRESUPPOSITION I. *Descriptions* (sentences, theories) are always *finitely representable* (in order to be terminately transmittable in a communication) and *locally independent of time* (remaining fixed for as long as the description is being used as description, i.e., is being transmitted in communication, or being analyzed and interpreted). For *interpretations* (meanings, models) no such restrictions apply.

In the idea of *communication* lies the presupposition that a language is *shared* among the participants of a communication.

PRESUPPOSITION II. Communication presupposes that a language be *shared* among participants of the communication. This means that they all have *inherited* (genetically; i.e., a phenomenon in genetic language) some basic description-interpretation processes of the language, and have *acquired* (by learning the language in its acquisition phase) certain other commonly held properties, permitting them to further explore, by the description-interpretation processes constituting the language, a linguistic domain of common interest (which may contain conceptions of the language itself).

4.2. *Argument for View (i) of the Linguistic Complementarity*

By way of a first reason, by *simple plausibility*, for view (i), consider the opposite case where all interpretations (meanings) *are* reducible to descriptions (sentences) within language. Why would we then at all use meanings the way we do in our linguistic performance, sometimes even with impressions of being able to think without words (which we certainly can do to a certain extent but not purely so; cf view ii of the linguistic complementarity).

Our non-reduced use of menings is there as an observable fact. A fact that is compatible with deeper understandings of the role of conscious meanings, as in existential perceptions for example, where this role is tied with our *inductive* linguistic capacities *beyond* pure syntactic deductions in a meaningless language.

What allows us to proceede to a *real argument* for view (i), is the sharp contrast (according to presupposition I) between the *restrictions on descriptions* – finiteness and local independendence of time – and the *freedome from restrictions upon interpretations*. Interpretations may be dynamic as well as infinite, whereas corresponding descriptions always are static and finite.

OUTLINE OF **argument for view (i)** OF THE LINGUISTIC COMPLEMENTARITY, BASED ON THE CONTRAST BETWEEN THE FINITENESS-RESTRICTION FOR DESCRIPTIONS AND THE NONRESTRICTED NATURE OF INTERPRETATIONS. Consider an interpretation process in a language, L that starts out from a description (a finite, constant, string of symbols), and ends in what the description is intended to descibe, say an infinite set. Let us face the problem of describing this interpretation process. Somewhere in the process, the intended infinity must *emerge* from non-infinity. Let us refer to a subprocess, where the emergence takes place, as a *creative process*. The interpretation process must contain at least one such creative process, and our problem reduces to its describability.

If the creative process is incomprehensible in the language L, the interpretation process is, to say the least, not fully describable in L.

But if it is comprehensible, how do we conceive the emergence of such a creative step? Kronecker and Brouwer admitted the infinite set of the positive integers to be a creation of God; cf our quote of von Neumann in (Löfgren, 1988, page 129). But here, restricting ourselves to scientific activity, where epistemological induction is indispensable (Russell, 1948, page 700), let us next assume that the creative process is inductive (in the epistemological sense). Then we can conclude, as demonstrated in section 7, that epistemological induction, as it occurs in the description-interpretation process of a

language L, is not fully describable in L.
Finally, let the creative process be *inductive in the mathematical sense.* This is the case when the initial description is formulated as an axiom of infinity *utilizing an inductive rule.* Also in this case, the creative process is *not fully describable* in L. The reason is that, in order to describe the creation process, one has to use the principle of mathematical induction – which *presupposes* an infinite set. This makes the intended interpretation of the axiom of infinity circular. This is argued in detail in section 5.
Thus, in all cases, the interpretation process is not fully describable in the language where it occurs.

In section 5 we also conclude on the necessity for us to *share* a language (cf presupposition II), and illuminate our *complementaristic conceivability.*

4.3. *References to Further Arguments*

We have here emphasized the finite-infinite opposition. For arguments emphasizing the constancy-change opposition, see Löfgren (2000).

Concerning all four views of the linguistic complementarity (i–iv; section 3), they are argued in several papers, a selection of which is:

(i	incompleteness):	(Löfgren, 1992, pp 121-132),
		(——— , 1988, pp 138-140),
		(——— , 1981);
(ii	tension):	(——— , 1992, pp 121-132);
		(——— , 1994);
(iii	partiality):	(——— , 1990);
(iv	nondetachability):	(——— , 1993, pp 310-313).

As to our use of the term *complementarity*, in “linguistic complementarity” and in “complementaristic comprehension of language”, our terminological choice is influenced by Bohr’s primary concept of complementarity (Bohr, 1928) (as opposed to Bohr’s further uses of the term, as well as other’s). Notably in (Löfgren, 1994, pp 159-160), we compare, in clarifying detail, our view ii of the linguistic complementarity (tension between describability and interpretability within a language) with Bohr’s primary view of complementarity (tension between definability and observability). Bohr advanced his primary view of complementarity without reference to some objectified language – at a time well before Tarski’s and Gödel’s work on language and formal system – and was perhaps not well understood at the time. Later on, Bohr's view of complementarity in terms of phenomena turned out sufficiently comprehensible to arouse well known discussions. The linguistic complementarity, which essentially relates to the holistic conceivability of language, may well help enlighten Bohr's primary view.

To avoid possible misconceptions (due to unintended meanings of "complementarity"), we use in this paper the term "complementary", interchangeably with "systemic" and "entangled", namely with *the linguistic complementarity* (section 3) as the paradigmatic case of complementarity. Misconceptions often do occur when concepts are considered complementary *without* reference to language (as if complementarity were language-independent; cf view (iv) as a principle of "nondetachability of language").

5. FULL LINGUISTIC VIEW ON AXIOMATIZABILITY OF INFINITY

We want here to confront axiomatizations of infinite sets, notably the problem of interpreting them, with the linguistic complementarity, primarily view (i) saying that in no language its interpretation processes can be fully described. This suggests that difficulties are to be expected in attempts at fully describing the interpretation of an axiom of infinity.

In the *general linguistic* setting, there is an interplay between attempted descriptions and attempted interpretations (objects) that eventually converges on a concept of infinity. The starting point may be an *intuition* which, in the linguistic description-interpretation process turns into, or generates, an acceptable concept of infinity. As such, we consider a complementaristic conception, involving both description and interpretation in the entangled way.

Looking at the problem of describing the interpretation of an axiom of infinity in set theory, that is with the axiom given, we are only considering *half of the general linguistic* problem. The full problem also involves describability of the formation of the descriptions, answering the question of where the axiom comes from (cf sections 2 and 7).

Now what, if any, intuition do we have of infinity? In connection with counting the natural numbers, we find it natural to *enumerate* them by successively adding 1 to an already produced number (thus using a recursively given rule, or *generating clause*):

$$0,\, 1, ...,\, n,\, n+1, ...$$

This is what Aristotle characterized as a *potential infinity.* It is a *steadily continuing process* with potential for being completed to generate *all* natural numbers, the collection of which is denoted N. That is an *actual* infinity. It is a *denumerable* infinity, enumerated by the generating clause. Notice that, since *every* natural number n is finite, we could not have the number of steps in the completed enumerating process to be represented by a natural number. In changing our interest from potential to actual infinity we are bound to take a nontrivial transcending step.

As we are about to see, this step is (comparatively) easy to formulate as an axiom, but intrinsically difficult to interpret in a complete sense.

5.1. *Axiom of Infinity and its Interpretation Problem*

Let us start out from the following formulation (there are several) of the axiom of infinitey in Zermelo-Fraenkel set theory (ZF); cf (Fraenkel et al, 1973, page 46), or (Mendelson, 1979, page 182).

Axiom of Infinity:

$$\exists w \ (\emptyset \in w \ \ \& \ \ \forall x \ (x \in w \Rightarrow x \cup \{x\} \ \in w)).$$

(There exists a set w, with the empty set $\emptyset$ among its elements, such that with every contained element x, also $x \cup \{x\}$ is a contained element.)

The "intended" interpretation of this *finite string of symbols* (this finite description) is that an *infinite* object exists as a set. At first, we tend to think of the meaning of the axiom as *not* depending on some further (nondescribed) features, but to objectively describe an infinity. In other words, we tend to think that there is a specifiable interpretation process, which operates objectively on the finite description and as result yields the infinite object as a set. Even though the interpretation process does *not only* operate on the axiom but *also* on the whole formal set theory, ZF, in which it is embedded, our main point remains, namely that it operates on a finite string of symbols (formal axiomatic theories are finitely representable).

We have here exposed the interpretation problem for the axiom in a form that makes it confront view (i) of the linguistic complementarity. Namely, if the interpretation process really is specifiable so as to ensure its objectivity, this specification cannot, according to view (i), be a description within the set-theory language where the interpretation occurs.

Now, how do we interpret the axiom of infinity? It has an unmistakable inductive formulation, i.e., it contains a recursively given implication, namely the inductive *generating clause* $\forall x \ (x \in w \Rightarrow$

$x \cup \{x\} \in w$). Obviously, the intension is that we from the formulation are able to infer an actual infinity, the set w. Let us examine this inductive step, from the axiom to the property of being infinte.

From the axiom we see that w contains the elements $\emptyset$, $\{\emptyset\}$, $\{\emptyset, \{\emptyset\}\}$, etc, which are all distinct (according to the axiom of extensionality in ZF). For convenience we use the von Neumann set model for the natural numbers, namely with 0 for $\emptyset$ and $n = \{0, 1, 2, ..., n-1\}$ for $n > 0$. Then $0 \in w$, and $n \in w \Rightarrow n+1 \in w$, because $n \in w$ implies according to the axiom that $n + 1 = n \cup \{n\}$ belongs to w. By *mathematical induction* on the base clause $0 \in w$ and the generating clause $n \in w \Rightarrow n+1 \in w$, we obtain $\forall n \in N$: $n \in w$. Here N is the set of *all* natural numbers, which is an *actual infinity.* Hence, $N \subset w$, i.e., N is a subset of w, meaning that w *is an infinite set.*

The axiom of infinity is formulated by means of an induction rule (a generating clause), a formulation which as demonstrated needs an induction principle for its intended interpretation – and the induction principle presupposes the *infinite well ordered set of natural numbers, N.* In other words, our attempt to fully describe the interpretation process turns out circular and fails as being acceptable as a full description.

5.2. *Compatibility with the Linguistic Complementarity and with the Presupposition of a Shared Language*

The exhibited failure of fully describing the interpretation of the axiom of infinity compares well with the *descriptional incompleteness* according to view (i) of the linguistic complementarity.

Descriptional incompleteness does not, however, prevent *complementaristic conceivability* of infinity. On the contrary, the analysis thus far is compatible with conceiving infinity, in a language, as a description-interpretation whole, whereby not all of the interpretation (the infinite set as an existing object) can be described in the language but is left there as a residual interpretation. Such a residuum may take the form of an image, like a picture impression, or an existential perception (cf section 8). The residuum may well be accessible to mathematicians communicating in a *shared* language. By inherited genetic linguistic dispositions, as well as by inductive learnings in play during communication processes, they may have come to share a residual interpretation, like in grasping N, whereby they are able to understand what it means to say that an infinite set (w) exists.

5.3. *More on Complementaristic Conceivability of Sets*

Let us for a moment go back to intuitions hinted at by founders of set-theory. That may be illuminating for the reason that the more developed a set-theory becomes – as theory – the less traces of the germinating intuitions will in general remain.

(MOSTOWSKI, 1967, p. 82). "... there are several essentially different notions of set which are equally admissible as the intuitive basis for set theory.

The notion of set seems to have never been understood in a unique way by mathematicians. We find in Becker .. an instructive account of a conversation which took place between Cantor and Dedekind. Whereas Dedekind compared sets to bags which contain unknown things, Cantor took a much more metaphysical position: he said that he imagined a set as an abyss.
... the divergence of opinion about the nature of sets is very important for the foundations of mathematics."

Recall as well Cantor's intuition for a set as: a collection of definite, distinguishable objects of our intuition or of our intellect to be conceived as a whole. With reference to early insights into the deep nature of the part-whole problem, it would seem reasonable to view a set as an abyss, or bottomless gulf. In our linguistic perspective, Cantor's allusion to our intuition, or intellect, clearly implies reference to language, to our inner cerebral language as well as to our external communication language where a set-conception is communicated and tested for acceptability. At this point we see how complementaristic conceivability may illuminate Cantor's "to be conceived as a whole" in a way that avoids an otherwise endless quest for more complete descriptions.

Complementaristic conceivability is tied with the *tension* view (ii) of the linguistic complementarity. This is explained in some detail in (Löfgren, 1992, pp. 123-127). Aiming towards a fuller descriptional account of a set (diminishing an unavoidable residual interpretation), we may talk of the complementaristic conception of the set as "description-like" ("syntax-like"). Or, as "interpretation-like" ("object-like", "semantic-like"), when aiming at a fuller interpretational account (diminishing an unavoidable residual description).

Compare how these two choices are reflected in the symbolisms of set-theory. The expression $S = \{x : Px\}$ is understandable as the set S of all elements that satisfy the predicate Px, formulated in the set-language. This is a description-like conception of S, emphasizing the defining predicate Px, and simply viewing the set S as extensional interpretation of the description P.

By contrast, an expression like $S = \{a, b, c\}$ calls for an interpretation-like (semantic-like) conception of S as a set of elements a, b, c that are *selected*, or *choosen*, from some already existing collection of set-elements. No essential description, or predicate Px, is needed here: the selected elements present themselves as a, b, c. In this understanding, $S = \{a, b, c\}$ is a one-dimensional Euler-Venn diagram representing S. Euler-Venn diagrams are usually drawn as two-dimensional pictures of sets, with a set represented by a circle (as a closing up of the one-dimensional $\{\ \}$). The encircled points represent the elements of the set.

However, if the expression $S = \{a, b, c\}$ is considered as a *formula* (sentense), rather than as a one-dimensional Euler-Venn diagram representing S, then a, b, c are descriptions (names) of corresponding elements. Furthermore, since every description must be finite, we *cannot this way* describe an infinite set (which we can by using a predicate Px, like $x \in N$ – provided that we have residual access to the interpretation of N).

We have here emphasized two weightings of the tension in the complementaristic conception of a set: a description-like and an interpretation-like conception.

In a *description-like set-conception* we construct, or image, sets by emphasizing their descriptions, formulated as in an axiomatic set theory in a set-language. The aim is to capture the sets, with extensively formulated descriptions – thereby allowing us to diminish the necessary residual interpretations.

In an *interpretation-like set-conception* we construct, or image, sets by emphasizing their semantic-like *selections* or *choices* in the semantics of the set-language – thereby allowing us to diminish the necessary residual syntactic operations.

Instrumental for interpretation-like set-conceptions is the possibility of axiomatizing the existence of selection or choice functions – as functions beyond rules (rules would go against the here intended diminishing of syntactic operations).

Consider a collection obtained by a process of simultaneously choosing – without some specified rule – for every non-void element S of a given set M an element s of S, and considering this collection of choices a set. This choice or fragmentation process in the semantical domain is purified in the *axiom of choice*.

Axiom of choice. For every set M there exists a function ("choice function") f on M which assigns to each non-void element S of M an element of S: $f(S) \in S$.

For other, but equivalent formulations, see (Fraenkel et al, 1973); (Moore, 1982). The axiom of choice is consistent with the core axioms of set theory (ZF), but is *independent* of them (Fraenkel et al, 1973, sect 4.2). (In the finite case, choice functions exist in trivially demonstrable ways.)

The axiom merely postulates the *existence of a choice function*. The function is *semantically* conceived as an infinite set of ordered pairs – without any rule or law specifiable that would allow its describability.

Levy makes a remark that invites comparison with our separation of the above two set-comprehensions.

(Levy, 1979, p. 159). "What led mathematicians to adopt the axiom of choice as an axiom of set theory, in addition to the opportunistic reason that it enables them to prove many theorems, is the following consideration. The basic idea of the axiom of comprehension is that every collection of given objects should be a set (or at least a class). The only collections that we can handle easily are those we called .. specifiable, namely those collections which can be described as the collection of all x's such that $\phi(x)$, where $\phi(x)$ is a formula of the (basic) language. However, there is little reason to assume that only the specifiable collections should be admitted as sets. One may choose to introduce also other collections as sets, such as the collection f obtained by the mental process of simultaneously choosing for every non-void $x \in a$ a member y of x and putting all these pairs $\langle x, y \rangle$ in f (together with the pair $\langle 0, 0 \rangle$, if $0 \in a$); the f thus obtained is a choice function on a."

In Löfgren (1992, pp. 124-125), where we discuss an axiom of relative comprehension in terms of the linguistic complementarity, we similarly argue that no comprehension axiom, with the predicate $\phi(x)$ specified as a well-formed formula of the set-language, can capture all set comprehensions.

5.4. *Foundational Inadequacy of Indirect Interpretations of the Axiom of Infinity*

Instead of directly facing the difficult full interpretation process working on the axiom of infinity, as in subsection 5.1, it may seem plausible to try an *indirect* interpretation. For example, to argue that the set w of the axiom cannot be finite, and from there, by *tertium non datur*, conclude that it must be infinite. However, such an indirect inference is not acceptable on a foundational level. This is made clear, for our actual example, by Fraenkel's further analysis of finite and infinite sets.

(Fraenkel, 1966, p. 40). "It should be stressed again that the axiom of choice is needed for a full analysis of *the concepts of finite set and finite number (cardinal, ordinal). ...*
Only by using the axiom of choice can we prove that *mediate sets and cardinals do not exist*, and hence that any set or cardinal is either finite or infinite."

Compare as well Moore (1982, sections 1.3 and 4.2).

Accordingly, the suggested indirect reasoning is not complete, but requires foundational support for accepting the independent axiom of choice, and thus of the required *tertium non datur*. This brings us back to our linguistic view, again indicating its indispensability for the problem of how to comprehend infinity.

5.5. *Some "Principles" for Generating Axioms of Infinity and Their Shortcomings; Inaccessibility*

In Mostowski (1967), we find a discussion of some ways in which axioms of infinity can be, or have been, generated. The topic of how we do form axioms is in general inductive (epistemological sense) and not fully accessible by purely deductive formal methods. Although Mostowski's discussion does not address the full inductive problem, it is somewhat wider than a formal deductive reasoning, and could perhaps be characterized as *informal* in a mathematical-like understanding; cf informality as exposed in other contributions to the book (Lakatos, 1967).

Principle of transition from potential to actual infinity (Mostowski, 1967, p. 84). "In recent years set theoreticians have formulated and advocate several new assumptions of an existential character. These assumptions are known as 'axioms of infinity'...
There are two general principles which allow us to formulate infinitely many such axioms.
The first of them may be called the *principle of transition from potential to actual infinity...*
In a more sophisticated form the principle of transition from potential to actual infinity is used in the formulation of inaccessible numbers. The Zermelo-Fraenkel axioms state that the set-theoretical universe is closed with respect to certain operations and hence that it is 'potentially closed' in a certain sense of this word. According to the general principle we assume an axiom stating that not only the universe but also a set (i.e. an object of the universe) is closed with respect to these operations."

Among axioms belonging to this first principle, Mostowski refers to Tarski's axiom of inaccessible cardinals and Lévy's stronger scheme of inaccessibility. Although the set of natural numbers N satisfies both the axiom of infinity of subsection 5.1, and Tarski's axiom of inaccessibility, it is often not counted among the inaccessibles. The reason being that the axiom of infinity is already accepted as one of the basic axioms of ZF and, thus, that N is accessible in ZF – although inaccessible in ZF-AI (Zermelo-Fraenkel set theory without AI, the axiom of infinity).

PRINCIPLE OF EXISTENCE OF SINGULAR SETS (MOSTOWSKI, 1967, pp. 85-87). "Still stronger axioms of infinity can be obtained by the use of the second principle; we shall call it the *principle of existence of singular sets*. This principle, which is much less sharply defined than the previous one, is concerned with the following situation. Let us assume that in constructing sets by means of the operations described by those set-theoretical axioms which we have accepted so far, we obtain only sets with a property P. If there are no obvious reasons why all sets should have the property P, we adjoin to the axioms an existential statement to the effect that there are sets without the property P. In this form the principle is certainly far too vague to be admissible. It is an historical fact, however, that several axioms of infinity were accepted with no other justification than that they conform to this this vague principle.

...

While it is not difficult to show the independence of the axioms of infinity, proofs of their relative consistency are as good as hopeless. A straightforward application of Gödel's second incompleteness theorem shows that no such proof can be formalized within set theory. In view of what has been said above about the reconstruction of mathematics in set theory it is hard to imagine what such a non-formalizable proof could look like. Thus there does not exist any rational justification of the strong axioms of infinity."

It is interesting to see here, in an analytic context of mathematics, that when hypothetical assumptions are formulated according to more or less lawful principles, their justification problem is deemed beyond rationality.

In sections 7 and 8 we return to the general problem of how axioms come to be formulated and accepted, in mathematics as well as in science. In general, both formulation and acceptance are entangled processes beyond description in the language where they occur.

In this context of inaccessibility, we want to mention (Löfgren, 1966), where we provide an information-based model for *(in)explicability* of sets, and prove this concept equivalent to Tarski's strong (in)accessibility. In a sense, our starting point, that of (in)explicability, is quite close to the general linguistic realm where principles for hypothesis formation are contemplated.

6. EXTENDED INFORMALITY; INDUCTIVE REASONINGS IN EXTENDED LOGICS

6.1. *Feferman on the Result/Activity Opposition in Mathematics*

Feferman observes the contrast between the way mathematical results can be formally described, and the mathematical activity behind the results, an activity which is exceedingly more difficult to comprehend and describe.

(FEFERMAN, 1981). "Mathematics offers us a puzzling contrast. On the one hand it is supposed to be the paradigm of certain and final knowledge: not fixed, to be sure, but a steadily accumulating coherent body of truths obtained by successive deduction from the most evident truths. By the intricate combination and recombination of elementary steps one is led incontrovertibly from what is trivial and unremarkable to what can be nontrivial and surprising.
On the other hand, the actual development of mathematics reveals a history full of controversy, confusion, and even error, marked by periodic reassessments and occasional upheavals. The mathematician at work relies on surprisingly vague intuitions and proceeds by fumbling fits and starts with all too frequent reversals. In this picture the actual historical and individual processes of mathematical discovery appear haphazard and illogical. ... Clearly, logic as it stands fails to give a direct account either of the historical growth of mathematics or the day-to-day experience of its practitioners. It is also clear that the search for ultimate foundations via formal systems has failed to arrive at any convincing conclusion."

These remarks seem compatible with our view that proper foundational research has to transcend logics into language. Again, the "puzzling" contrast between mathematical knowledge and mathematical activity, may rather seem "natural" in the light of the linguistic complementarity. "Puzzling", however, in the (widespread) view that mathematics and logics could be detached from language.

We want to compare this insight of Feferman with two other views, one earlier by Russell, and one by van Benthem.

6.2. *Russell on Widenings of Logic, Taking Induction into Account*

In (Russell, 1961, Lecture II), Russell is concerned with widenings of the scope of logic with specific reference to induction – somewhat like van Benthem (next subsection).

(RUSSELL, 1961, p. 43). "The first extension [of logic] was the introduction of the inductive method by Bacon and Galileo – by the former in a theoretical and largely mistaken form, by the latter in actual use in establishing the foundations of modern physics and astronomy. This is probably the only extension of the old logic which has become familiar to the general educated public. But induction, important as it is when regarded as a method of investigation, does not seem to remain when its work is done: in the final form of a perfected science, it would seem that everything ought to be deductive. If induction remains at all, which is a difficult question, it will remain merely as one of the principles according to which deductions are effected. Thus the ultimate result of the introduction of the inductive method seems not the creation of a new kind of non-deductive reasoning, but rather the widening of the scope of deduction by pointing out a way of deducing which is certainly not syllogistic, and does not fit into the mediaeval scheme."

The following characterization of "modern" logic can perhaps be understood as a result of inductive thinking, broadly conceived as in the above quote.

(Russell, 1961, p. 68). "Modern logic, as I hope is now evident, has the effect of enlarging our abstract imagination, and providing an infinite number of possible hypotheses to be applied in the analysis of any complex fact. In this respect it is the exact opposite of the logic practiced by the classical tradition. In that logic, hypotheses which seem *prima facie* possible are professedly proved impossible, and it is decreed in advance that reality must have a certain special character. In modern logic, on the contrary, while the *prima facie* hypotheses as a rule remain admissible, others, which only logic would have suggested, are added to our stock, and are very often found to be indispensable if a right analysis of the facts is to be obtained. The old logic put thought in fetters, while the new logic gives it wings. It has in my opinion, introduced the same kind of advance into philosophy as Galileo introduced into physics, making it possible at last to see what kinds of problems may be capable of solution, and what kinds must be abandoned as beyond human powers."

Although conceived inductively, does the above characterization of modern logic maintain direct traces of induction? Or, are these only to be seen, retrospectively, in the awareness of new hypotheses that only logic would have suggested?

We think that Russell's view, at least to some extent, is open for both possibilities. Notably with the here very broadly sketched understanding of induction. Compare, again, Mostowski's *Principle of existence of singular sets*, generating hypotheses from an introspective view of logic itself (or rather set theory) at a point of its development.

Russell, here, simply avoids further explanation of how hypotheses come into being "which only logic would have suggested", and does not formulate principles in this respect. He is aware of the difficulties involved, perhaps with experiences like those of Feferman.

When facing the problem of formulating a principle of induction, Russell suggests as follows.

EPISTEMOLOGICAL INDUCTION AS AN INDEPENDENT LOGICAL LAW (RUSSELL, 1961, p. 225). "The principle involved is the principle of induction, which, if it is true, must be an *a priori* logical law, not capable of being proved or disproved by experience. It is a difficult question how this principle ought to be formulated; but if it is to warrant the inferences which we wish to make by its means, it must lead to the following proposition: 'If, in a great number of instances, a thing of a certain kind is associated in a certain way with a thing of a certain other kind, it is probable that a thing of the one kind is always similarly associated with a thing of the other kind; and as the number of instances increases, the probability approaches indefinitely near to certainty.' It may well be questioned whether this proposition is true; but if we admit it, we can infer that any characteristic of the whole of the observed past is likely to apply to the future and to the unobserved past."

In comparing this principle with Mostowski's *Principle of existence of singular sets*, we see the danger in proposing logic-like laws for inductive phenomena.

In section 7 we take stance to Russell's formulation and give a very different account of induction, based on the systemic concept of language, rather than on logic.

6.3. *Van Benthem's Broad View on Current and Future Logic*

Van Benthem considers logic in a very wide sense, as well as a "logical re-orientation" of science.

(VAN BENTHEM, 1982, p. 435). "Logic I take to be the study of reasoning, wherever and however it occurs. Thus, in principle, an ideal logician is interested both in that *activity* and its *products*, both in its *normative* and its *descriptive* aspects, both in *inductive* and *deductive* argument...
An enligthened logician like Beth, for instance, realized the danger of intellectual sterility in a standard gambit like separating the *genesis* of knowledge in advance from its *justification* ..."
(VAN BENTHEM, 1982, p. 450). "... theories as scientific *activities* rather than *products* of such activities are not irrevocably outside the scope of logic...
In the semantic perspective too, there is room for pragmatic studies. E.g., model theory presupposes that successful interpretation has taken place already. How?
... these references are only the first landmarks in a hopefully fruitful new area

of logic."

We find van Benthem's views interesting and mostly agreeable, notably in their holistic character.

However, without a clearer understanding of induction than the one presented, it is not obvious how to think of van Benthem's proposal of a widened *logic* – rather than *language* (as we propose) – as the right category for understanding *reasoning* in its widest linguistic sense.

A concept like "inductive *argument*", in a widened logic, would seem difficult unless communicable in some shared language. Thereby *language*, in its holistic conception, appears as the more fundamental category. As demonstrated in section 7, holistic language does allow complementaristic understandings of induction beyond deduction. No specific concept of logic is involved in the argument.

Both van Benthem and Feferman refer to Lakatos' view of mathematics as well as to Polya's and Popper's views. Let us expand somewhat, along these lines, on the activity/result distinction viewed as induction/deduction opposition.

7. INDUCTION AS A LINGUISTIC PHENOMENON BEYOND LOGIC

In our view, it is not logic that should be widened to encompass epistemological induction. It is in language that induction occurs – beyond what can be understood in logical terms. In its full systemic conception, language is a whole of entangled description-interpretation processes which are inductive in the language, and can be known to be so by complementaristic conceivability. Recalling the tension view (ii) of the linguistic complementarity, as well as subsection 5.3, we can conceive of the description-interpretation processes as "description-like" with an unavoidable residual interpretation – identifiable as induction!

This conforms well with the general view of an induction/deduction distinction: *induction* is about how we form general statements (like axioms in scientific and mathematical theories) – which cannot be reduced to *deduction* which is about how we draw conclusions from premises (like axioms) by following *algorithmic rules.*

Furthermore, the linguistic view of induction explains the difficulties that are associated with classical attempts at "induction principles" like Russell's (section 6.2). Induction as a linguistic phenomenon is *not* some inference from a great number of instances (as in Russell's principle) – but a process bringing forth hidden properties of the language in use.

Several writers, like (Wittgenstein, 1961), (Quine, 1963), (Kemeny, 1953), (Popper, 1959), (Feigl, 1949), have elaborated on induction in terms of the simplicity principle (Ockham's razor).

(WITTGENSTEIN, 1961, statement **6.363**). "The procedure of induction consists in accepting as true the *simplest* law that can be reconciled with our experiences."
(WITTGENSTEIN, 1961, statement **6.3631**). "This procedure, however, has no logical justification but only a psychological one.
It is clear that there are no grounds for believing that the simplest eventuality will in fact be realized."

Neither Wittgenstein, nor the other above writers, provide but very general arguments for their views. Feigl's following view is interesting in that it focuses on the difficulty of *measuring* simplicity, which certainly is a principal question.

(FEIGL, 1949, p. 303). "Now the ultimate goal of science is not the achievement of a loosely connected miscellany of descriptions, but the establishment of a systematic structure of laws as a basis for explanation and prediction. The prescriptive rule, which is a direct consequence of this objective, is then the real principle of induction. It reads: *'Seek to achieve a maximum of order by logical operations on elementary propositions. Generalize this order (whatever its form be: causal, statistical or other), with a minimum of arbitrariness, that is, according to the principle of simplicity.'* The condition of simplicity is essential, because it restricts the ambiguity of the procedure. But, since simplicity is measurable, if at all, only with great difficulty, there will usually be several ways of generalizing. This explains the case of competing scientific theories. Only when new experimental evidence is supplied, can it be determined that the one or the other theory is more complicated in that it employs more arbitrary hypotheses.
There can be no guaranty for the validity of generalizations, be they simple enumerative inductions or hypotheses of the more advance scientific type. At any stage of scientific progress (as we know it) there will be outstanding premises from which the more specific statements can be derived with – indeed – (*deductive*) certainty; but those premises in themselves are *assumptions*, ever ready for revision, valid only 'until further notice'."

In subsection 7.3 we will explain "the great difficulty" of measuring simplicity, namely in terms of its nonrecursive properties. This is central for our induction/deduction distinction and for the necessity of conceiving induction as a linguistic phenomenon (beyond logic).

Let us emphasize that in the above Feigl-quote there is an

ARGUMENT FOR SCIENTIFIC FRAGMENTARY DEVELOPMENT. "But, since simplicity is measurable, if at all, only with great difficulty, there will usually be several ways of generalizing. This explains the case of competing scientific theories."

This argument becomes amplified by our results in subsections 7.2 and 7.3.

7.2. *Argument for the Simplicity Principle as Induction-Principle*

In Löfgren (1977, p. 199), we give a structure of a general linguistic learning (description) process (Löfgren, 1973), based on the simplicity principle. We there *give a reason* for the simplicity principle as an induction principle.

(LÖFGREN, 1977, p. 199) "Involved is the idea that the more regularities have been found, the more can they be utilized (be referred to) in the description to make it shorter (than a lenghty listing of uncorrelated facts). Furthermore, the predictive power of the description will increase with the number of regularities found. Hence, the shorter the description can be made, the more communicable will it be, and the more genuine will the learning be. Then more reliable predictions can be made on the basis of the learned description, and more safe inferences can be made of how to behave in the surrounding."

The context is here that of a shared language, adapted (cf subsection 8.3) to the nature we are trying to learn.

7.3. *Argument for Induction as Beyond Deduction*

In the case of a programming language, where shortness is mathematically expressible as "the shortest description function" $s(z,u)$, a proof of the induction/deduction distinction can be given in terms of the nonrecursive (nonalgorithmic) properties of $s(z,u)$, as follows.

The context is a programming language with a universal Turing machine u as interpreter. $s(z,u)$ is the length of the shortest description (program) that makes u compute z (representing observed facts). The nonrecursive properties of $s(z,u)$ were first proved in (Löfgren, 1967). Later proofs by Kolmogorov, and Chaitin, appeared in so called algorithmic information theory.

Notice how $s(z,u)$ also depends on u – *bringing interpretation and thus language into the picture*. This is not elaborated on by Kolmogorov or Chaitin. Now, our Theorem 1 in (Löfgren, 1967, pp 170-171) states that, for no universal Turing machine u, the shortest form function $s(z, u)$ is recursive in z. Hence, for no programming language (no u), can there be an algorithm for induction.

Corollary. Also within programming languages, where simplicity *is* definable (as $s(z, u)$), *induction according to the simplicity principle is deductively irreducible to deduction.* In other words, induction cannot be neutrally reduced to deduction, i.e., not without using inductive processes in the reduction.

7.4. *Deductive Fragments of Induction; the Theory of Supports*

Although induction is beyond deduction, it is possible to give deductive accounts of *very* partial aspects of the phenomenon of induction. For example of *support* relations as capturing possible (microscopic) steps of the confirmation process (Löfgren, 1978).

Suppose that we, in a particular stage of investigating a proposed hypothesis H, make the observation B, and ask: does B support H against an accepted background theory T in a language L? The intended meaning of "support" is not that of confirmation. Only that the observation B is a possible step, however small, towards a possible future confirmation of H.

This intended meaning is made precise in terms of the concept of *hypothetical content*, $Inf_T H$, of H relative to T. An hypothesis is, as such, neither true nor false, but hypothetical. Its hypothetical content, or meaning, $Inf_T H$, is expressible as a description-like interpretation, namely as the set of all deductions from H in T that are not T-theorems (not already known in the background knowledge).

Now, B *supports* H *in* T precisely when:

$$(\emptyset \neq) Inf_{TB} H \subset Inf_T H$$

that is, when the hypothetical content of H with respect to the background knowledge TB *after* the observation B (T augmented with the new observation B) is *diminshed* (becoming a proper subset) relative to the hypothetical content of H *before* the observation B.

We use the notation $Inf_T H$ for the hypothetical content because this content is the same as the intralinguistic *information* which H gives if considered, not as an hypothesis, but as a new piece of knowledge (for example communicated, as H, by another researcher being able to make the full *inductive* decisions).

In Löfgren (1978), we argue that the simplicity principle, as a metaprinciple for inductive theory-formation, is compatible with the, yet incomparably much narrower, purely deductive theory of supports for single observations.

In that paper, we also show how the theory of supports resolves the well known, and much attempted, Hempel paradox concerning supports.

In Löfgren (1994), we use the deductive theory of linguistic support relations to settle a dispute on how to understand a recent quantum mechanical double-prism experiment.

7.5. *Complementaristic Resolution of the Popper-Carnap Controversy on Induction*

An illuminating controversy on induction, the "Popper-Carnap controversy" (Michalos, 1971) occured in the 50's. It originated in an apparent clash between two probability considerations concerning the acceptance of hypotheses. For corroboration, Popper accepts bold, improbable hypotheses. For confirmation, Carnap accepts highly confirmed hypotheses, with degrees of confirmation developed as degrees of probability – i.e., probable hypotheses.

In Löfgren (1981), we argue that Popper's program aims at *modelling*, Carnap's at *describing* induction. Thus, if both programs are confronted in a language, the linguistic complementarity of the language implies that the two programs in fact are entangled.

8. LINGUISTIC REALISM

Linguistic realism (Löfgren, 1993) is a critical form of realism according to which "the real world", intended to refer to a world that exists independent of us human beings and our languages, in this very intention is a complementaristic conception *within* the language in which it is conceived as such.

It is most natural for us, sharing a language, to develop a common conception of an external world that does not depend on us. What this means is that we then form an hypothesis, namely that an "independent" external world exists. We feel the hypothesis strongly supported by our experiences and accept it as confirmed, as true. This view, however, suggests that "the real world", in spite of its intended interpretation, does in fact depend on us and our common, notably *inductive*, linguistic capacities.

How is a concept like a basic, or fundamental, truth to be conceived in linguistic realism?

Recall Descartes' *Cogito* (Descartes, 1986) as a safe assurance against the critical attitude that everything can be doubted. Descartes found it well possible to advance doubts as to the existence of physical objects around him. But, what was immune to doubt was — doubting. For if he doubted doubting, it would certainly be true that he was doubting. And, with doubting a form of thinking, he had found at least one indubitable proposition: "I am thinking".

From this, he reasoned, there follows another, "I exist", for it was self-evident that nothing could think without existing (cf below). Thus Descartes could be certain of his own existence because he was thinking. He concluded "cogito, ergo sum", i.e., "I am thinking, therefore I exist".

Apart from possible objections (Löfgren, 1977e) as to understanding the "I", which binds premise and conclusion, what is it in "cogito, ergo sum", which makes it so convincing?

Well, realistically speaking, it is in the fact that it is a *sentence in a language*, a sentence which can be interpreted and confirmed by everyone with the actual language (or translations of it) in common. The sentence, which has remained intact for over 350 years, satisfies Presupposition I for descriptions of a language – and it undoubtedly has a meaning (contemplated by philosophers as well as laymen ever since its conception).

This means that we can propose "cogito, ergo lingua est" as an example of a still safer existence. Here, "lingua" refers to the actual external communication language.

Notice that the inference expressed in "cogito, ergo lingua est", i.e., from the existence of this sentence, as a string of symbols, to the existence of a language, follows from the existence of the *meaning* of the string, entangled as it is with the string as description – no doubt a genuine linguistic phenomenon,

Admittedly, the understanding of this linguistic Cogito may not be *immediately* obvious to people mostly acquainted with contexts where they use to talk as if language could be detached, i.e., without reflecting upon that a language *is* involved. But among the smaller group of people that have come to appreciate the fundamental nature of languages, or else with this reminder, the linguistic Cogito would seem at least as obviously convincing as Descarte's Cogito (for him implying his own existence – as a linguistic creature among others we would like to add).

8.2. *Existential Perceptions*

In Löfgren (1977e) we think of real concrete existence, as subsistence (compare mathematical existence intimately connected with consistency) with existential perception as *complementum possibilitatis*.

We explain how existential perceptions may occur as the result of an inner cerebral linguistic activity, operating on the data-flow that enters our brains via our receptors.

This data-flow is very complex, but the inner cerebral inductive description-interpretation process manages to produce a comprehensive view of it in terms of objects with properties and relations. The inductive nature of the processes is understood according to the simplicity principle (in which a *naming process*, familiar from logics, plays a central simplifying role). And simplicity is really what results. Instead of "seeing" an enormous data-flow, we "see" a managable picture with consciously perceived objects with (less consciously perceived) more abstract relations and properties. These latter are expressed in terms of the names for the objects that are produced in the naming process.

An existential perception (of a nameable object) exemplifies a complementaristic conception weighted towards the object-like side.

Involved is a thesis that the cerebral processes which generate *conscious* perceptions are directed from the inductive confirmation processes in the inner language. Whereby an existential perception arizes with the step from very strong support to the *experience* of certainty.

8.3. *"The Real World"*

In linguistic realism, concepts like *environment*, *context*, *reality*, *the real world*, are all linguistic phenomena. That is, phenomena which cannot be understood as if language could be detached. This observation is important in a context of foundations of science, where we try to come to grips with a concept like *truth* as relating to an "independent" reality.

The "real world" is a complementaristic conception in the form of an hypothesis, described like "a real world exists independent of us", *with* an intended interpretation, or hypothetical content, that explains the meaning in terms of concepts in the shared language – that obviously cannot then be detached.

What then, is the "independence" in that it must not be confused with an independence (detachment) of the language in use?

In linguistic realism, *"independence"*, so characteristic for the very idea of reality and the real world, refers to a highly repressed influence of describable syntactical properties of the language. That is, the "real world" is complementaristically conceived as "semantic-like", with a full interpretational content together with a correspondingly diminished, but unavoidable, residual descriptional or syntactical content.

Notice how a "full interpretational content" can be consciously conceived, as in *existential perceptions* with a diminished syntactical content, reduced to a mere name.

Also when thinking in more abstract contexts, where *conscious* existential perceptions do not occur (but well thoughts on subsistence), we do not fully cut away intended interpretations, even though we like to think that we do so, for purposes of rigor.

What about the real world as it existed long before us and our languages! Isn't that completely independent of our language? Some reflection reveals that we also here have an understanding behind the expression "the real world as it existed long before us" which is, often unintensionally, coloured by presuppositions and inductive capacities in our present language. Even intensionally so, like when we form hypotheses about being able to see it, as it existed long ago, by means of present day observations, aided by telescopes as well as cosmological theories.

In a realm of quantum mechanics and its conceptual foundations, d'Espagnat develops reality views. For example, in arguing that Hume's view concerning justifiability of induction is in favor of "the realist".

> (d'Espagnat, 1989, p. 248). "Moreover, he [the realist] can understand that 'simple' and 'easily expressible by men' should be synonymous also under the realist viewpoint, since men are themselves a part of reality and therefore structures of their minds may well reflect rather fundamental structures of reality itself".

At the end of this quote, d'Espagnat adds a footnote with a remark that this argument should be, and could be, developed in order to gain convincing power.

In our view, such arguments could be established by replacing "men" and their mind-structures by the more tractable concept of language (complementaristically conceived). Compare Löfgren (1992), and a discussion part (contained in the including book), with points notably by Stapp (for example on p. 241).

8.4. *Evolution of Language*

In Löfgren (1981a), we develop life as a linguistic phenomenon in genetic language, permitting us to understand evolution of language as we understand evolution of life. Like species of life evolve in adaptation to particular surroundings, languages evolve in adaptation to particular surroundings, making it possible to describe essential features of the surrounding by managable, short descriptions. The shortness is, via the simplicity principle, tied with *predictive* power, most valuable for survival of the biological individuals that share the language and make it survive.

To have predictive power means to have a model of one's own behaviors in the surrounding. The effects of *intended* behaviors can be foreseen (the better the larger the predictive power), and realistic decisions as how to behave can be made. Thus, by the adaptation of the *language*, its linguisitc properties come to reflect basic properties of the surrounding.

In our general interactive setting, allowing constructive interaction with the surrounding, it may be found advantageous to modify the surrounding according to our inherited and acquired linguistic structures. Recall the reflecion: "we form our houses, and the houses help form us" (allegedly attributed to Winston Churchill).

Again, with a surrounding that also contains other linguistic individuals, we are in a situation where a language evolves with high introspective capacities. Allowing for example one individual to conceive of another individual in the acts of conceiving the first.

If presuppositions are shared concerning our surrounding world as strongly independent of us, we will try to adapt ourselves to it. Whereby our interests will be centered on descriptional efforts directed towards interpretations (somehow fixed in the language according to the presupposition). We may then get the impression that "language adapts towards the world".

If, instead, our interests are directed towards shared presuppositions (possibly as a result of observed shortcomings with "universal" scientific theories), our language will contain less shared presuppositions, because some earlier such presuppositions will now occur as objects of investigation. We may then get the impression of doing foundational research, sharing a somewhat less presuppositional, or more basic, language with our fellow researchers.

In section 10, we develop *revelation of presuppositions* as a basic foundational activity based on language (rather than logic).

9. WITTGENSTEIN'S VIEW OF A LANGUAGE-WORLD CONNECTION

In his famed *Tractatus Logico–Philosophicus*, Wittgenstein is philosophizing about thought and the expressibility of thought in language. He suggests that language (throughout non-holistically conceived as a collection of sentences with logical syntax) and reality must have something in common, namely logical form, in order for propositions to be able to represent reality. In the preface of the book he provides a characterization of it.

(Wittgenstein, 1961, p. 3). "The whole sense of the book might be summed up in the following words: what can be said at all can be said clearly, and what we cannot talk about we must consign to silence.
Thus the aim of the book is to set a limit to thought, or rather – not to thought, but to the expression of thoughts: for in order to be able to set a limit to thought, we should have to find both sides of the limit thinkable (i.e., we should have to be able to think what cannot be thought).
It will therefore only be in language that the limit can be set, and what lies on the other side of the limit will simply be nonsense."

We understand this as follows. Thoughts are more free than our expressions of them (than our descriptions of them in language). In order to say (describe in language) something clearly, we must restrict (set a limit to) the syntax of the language to sort away nonsense.

This raises the question whether the proposed syntactic restrictions will not also affect our thoughts, such that some of them, that we do perceive as sensefull, may in fact be eliminated from discussion for artificial reasons.

This question is but indirectly touched at in the book. For example in statement 4.116.

(Wittgenstein, 1961, statement **4.116**). "Everything that can be thought at all can be thought clearly. Everything that can be put into words can be put clearly."

But can a thought be put into words? Is it possible to put a clear thought clearly into a statement of words?

This question is not explicitly dealt with. And how could it within Wittgenstein's aim at clarity? Nevertheless, an attempt in this direction is made in his non-representability statement 4.12, which we will examine in a following subsection with respect to presuppositions and clarity.

For comparison with linguistic realism, and holistic language, it must be remembered that Wittgenstein is conceiving of thought as a proposition with sense (4), indicating that it can be perceived by the senses (3.1) (sometimes even clearly so; cf our subsection 8.1 on Cogitos). But that is not developed by Wittgenstein, as it is for example in our inductive explanation of an existential perception. On the contrary, in his aim at a logic, Wittgenstein stops at objects, which can only be named (cf 3.203: A name means an object. The object is its meaning).

Thereby Wittgenstein, in his aim at logics rather than holistic language, as usual works with propositions expressing logical facts, disregarding any meanings of objects but to be nameable.

In linguistic realism, with “thought” as a description-interpretation whole, the two occurrences of “clearly” in 4.116 must be distinguished. Furthermore, as we are about to see, Wittgenstein’s restrictions on syntax, in order to separate sense from non-sense, are untenable in modern logic.

9.1. *On Wittgenstein’s Restrictions of Syntax*

A central point of the book is the view that sense can be distinguished from nonsense by the aid of syntactical criteria. Wittgenstein develops the view by considering how to restrict the syntax of *language*, conceived as a totality of propositions (4.001), in order to get at propositions with *sense*, understood as thoughts (4).

Let us examine the arguments given for the following proposed restrictions.

> **3.332** “No proposition can make a statement about itself, because a propositional sign cannot be contained in itself (that is the whole of the ’theory of types’).”

> **3.333** “The reason why a function cannot be its own argument is that the sign for a function already contains the prototype of its argument, and it cannot contain itself. ...
> That disposes of Russell’s paradox.”

Recalling a previous statement,

> **3.318** “Like Frege and Russell I construe a proposition as a function of the expressions contained in it”,

we think that 3.333 is meant as a further explanation of 3.332.

Thus, what Wittgenstein excludes is the applicability of a proposition(al function) to itself. The question, whether a function can contain itself in its domain, has been discussed over the years in terms of various presuppositions.

In Löfgren (1968), we prove that the existence of functions which are elements in their own domain, as well as of functions which are elements of their own range, is independent of, but consistent with, NF (Quine’s New Foundations) and likewise with respect to NBG (the von Neumann-Bernays-Gödel set theory). In other words, such functions can be axiomatized in NF as well as in NBG.

Furthermore, in his work on *lambda calculus* Church writes as follows, with a clarifying distinction between conceptualizations of functions.

(Church, 1941). “In particular it is not excluded that one of the elements of the range of arguments of a function f should be the function f itself. This possibility has been frequently denied, and indeed, if a function is defined as a correspondence between two previously given ranges, the reason for this denial is clear. Here, however, we regard the operation, or rule of correspondence, which constitutes the function, as being first given, and the range of arguments then determined as consisting of the things to which the operation is applicable.”

It is in line with Church’s conception of function that the whole domain of *computability*, based on the concept of Turing machine, has developed. Furthermore (cf subsection 10.2), computability has proved presuppositional for the general concept of *formal system*.

In Löfgren (1992), we use computability theory to illustrate the *tension aspect of the linguistic complementarity*.

By consequence of the 3.332 restriction, Wittgenstein would not allow self-referential syntactical sentences like:

this sentence contains fortynine letters and eight words
this sentence contains fortynine letters but fewer words

which we use in Löfgren (1990) to illustrate varying amounts of *partiality* of self-reference.

What Wittgenstein excludes by 3.332 is also the partially self-refering sentences of Gödel and Rosser, and thereby the modern insights into *incompleteness* phenomena in pure mathematical contexts. In (Löfgren, 1990, p. 52) we compare the Gödel and Rosser sentenses with respect to amount of partiality of self-reference.

Although total self-reference is impossible by view (iii) of the linguistic complementarity, *partial* self-reference, like in the above examples, is not only possible, but can be as clear as the understanding of any sentence. Sometimes very clear to the extent that it so to speak brings an otherwise external context into the sentence.

The early fear for admitting self-reference may be due to a lacking distinction between total and partial forms of self-reference. That a too embracing self-rerence rightly should be excluded is well known. But this does not imply exclusion of all self-reference (cf view iii of the linguistic complementarity).

9.2. *On Wittgenstein's (Non-)Representability Statements in 4.12*

In 4.12, Wittgenstein states both a positive and a negative view on what propositions can represent.

(Wittgenstein, 1961, statement **4.12**) "Propositions can represent the whole of reality, but they cannot represent what they must have in common with reality in order to be able to represent it – logical form.
In order to be able to represent logical form, we should have to be able to station ourselves with propositions somewhere outside logic, that is to say outside the world."

In the statement, a concept of *logical form* occurs, which is assumed to apply to reality as well as to propositions.

We think of the logical form of a proposition as given by the form of occurrence in it of the logical connectives "or" ($\vee$), "and" ($\wedge$), "not" ($\neg$), "imply" ($\Rightarrow$).

We understand the proposed applicability of "logical form" also to reality, in terms of the way we may perceive of reality, notably by its presumed *fragmentability*; compare (Chew, 1968, page 763), quoted in section 1.

Since Wittgenstein thinks of the world, and reality, as collection of already existing "*facts*" (thereby positively deciding Chew's question whether nature really is fragmentable), we may, *by way of example*, consider a familiar set-reality with *facts* about the way sets are naturally fragmented in subsets (symbolised by $\subset$), united (symbolised by $\cup$), etc.

In the case where sets, S, are extensional interpretations of propositional functions, Px, i.e., $S = \{x : Px\}$, there is the well known structural similarity between the form of a set-fact and the form of the corresponding propositional statement. In the former case, the structure is identifiable by the form of occurence of the basic set-connectives $(\cup, \cap, \sim, \subset)$. In the latter case, the structure (logical form) is identifiable by the form of occurrence of *corresponing* logical connectives $(\vee, \wedge, \neg, \Rightarrow)$.

For example, if the two sets S and R are extensional interpretations of the propositional functions Px and Qx, respectively, the form of the set-fact that:

$$S \subset (R \cup S),$$

is the same as the form of the logical proposition which describes this set-fact, namely that for all x:

$$Px \Rightarrow (Qx \vee Px).$$

The positive statement in 4.12, namely that "*propositions can represent the whole of reality*", we explore as follows. With reality as part of the world (2.063), and the world as the totality of facts (1.1), we know today that, say, the totality of arithmetical facts, which is not recursively enumerable, cannot be described by a formal syntactical theory and, in that sense, cannot be represented by propositions forming a communicable theory.

Does this contradict Wittgenstein's positive statement?

In our understanding, since what is represented is "the whole of reality", i.e., a totality, then what represents it ought to be, not infinitely many propositions (which, as such, could not be communicated) – unless all these could be expressed in a formal theory, i.e., by one complex, but finite, sentence or proposition. But since that is impossible, we would have a contradiction against the positive statement.

Compare a critical remark, made by Russell in his introduction to Wittgenstein's Tractatus.

(RUSSELL, 1922, preface, p. xxi). "There is one purely logical problem in regard to which these difficulties are peculiarly acute. I mean the problem of generality. In the theory of generality it is necessary to consider all propositions of the form fx where fx is a given propositional function. This belongs to the part of logic which can be expressed, according to Mr. Wittgenstein's system. But the totality of possible values of x which might seem to be involved in the totality of propositions of the form fx is not admitted by Mr. Wittgenstein among the things that can be spoken of ..."

On the other hand, if Wittgenstein simply means that each fact (here each aritmetical truth) is represented by a proposition, this is of course true in the sense that each fact (arithmetical truth) is expressed by an (arithmetical) proposition.

In our view, the first (positive) part of 4.12 could be rewritten as something like:

Although each fact of (an arithmetical) reality can be represented by a proposition, there is no proposition which can represent the whole of the (arithmetical) reality.

Thereby, already Wittgenstein's positive statement fragments into a *non-representability* statement.

The negative statement in 4.12, and its argument, namely that:

"propositions cannot represent what they must have in common with reality in order to be able to represent it – logical form.
In order to be able to represent logical form we should be able to station ourselves with propositions somewhere outside logic, that is outside the world",

is somewhat nested. Let us examine the proposed argument. Assume, for contradiction, that there is a proposition P which can represent *its* particular logical form. Then the proposition P is making a statement *about* itself – which contradicts 3.332.

Again, let us assume that logical form is more abstractly conceived as a form-function, F, such that $F(P)$ is the logical form of P (whether P is a proposition or what it represents). Then, for contradiction, assume that there is a particular proposition, P_o, which represents logical form, F. Then $F(P_o)$ is the logical form of P_o as well as of what P_o represents, namely F. Since the logical form of F is $F(F)$, we are in a situation where $F(P_o) = F(F)$.

But this situation need not yield a contradiction. With reference to our remarks against 3.332 in subsection 9.1, functions which belong to their own domain (as well as range) do not contradict set theories like NF and NBG.

We do not find Wittgenstein's argument for the non-representability statement in 4.12 convincing – any more than his argument against self-reference in 3.332 (cf subsection 9.1)

9.3. *Comparison of 4.12 with View (i) of the Linguistic Complementarity*

Nowhere does Wittgenstein reveal presuppositions comparable to our I and II in subsection 4.1. These are what allow our statements of the linguistic complementarity, like its view (i):

in no language can its interpretation process be completely described in the language itself,

i.e., as a limitation-statement for languages in general (conceived holistically).

In comparing 4.12 with with view (i), we may look at 4.12 as stating a limitation of representing by some proposition [i.e., a limitation of describing] a postulated particular property of representations [i.e., of interpretations], namely to preserve logical form.

Since languages in general do not have this particular property, we understand view (i) as a more general limitation statement than 4.12.

(The particular property of preserving logical form would imply a very particular compositionality postulate.)

View (i) of the linguistic complementarity holds in a much more general case than that considered by Wittgenstein for 4.12. View (i) does not involve any concept like “logical form”, in fact no particular concept of logic at all, no compositionality postulate, no presupposition of “reality”, or the “world”, as collections of “facts”, etc.

This is what makes it plausible to consider the possibility of holistic language as a more basic category than logics. Let us explain in terms of Russell’s view, as expressed in (Russell, 1940, p. 21), quoted in section 1.

9.4. *Further Proposed Language-World Connections*

The question whether language and real world must have *something* in common for language to be able to represent the world, is certainly *raised* by Wittgenstein as a possibility. But conceived with language restricted in a non-holistic meaning.

Recalling our quotes from (Russell, 1940, p. 341) in section 1, Russell conceives of a language-world connection as follows:

> there is, I think, a discoverable relation between the structure of sentences and the structure of occurrences to which the sentences refer. I do not think the structure of non-verbal facts is wholly unknowable, and believe that, with sufficient caution, the properties of language may help us to understand the structure of the world.

This may seem a somewhat different view of a language-world connection than that of Wittgenstein. While Russell looks at the possible connection as a “discoverable relation”, Wittgenstein conceives of it “as having in common a logical form that is not representable by propositions”. (Thereby “not speakable of”, or “to be consigned to scilence” – in the black-white scale of representation which Wittgenstein thinks of as an ideal – but certainly does not follow in his writings; witness the very statement in 4.12.)

In Russell (1940, p. 341), there is a kind of classification of philosophers “with regard to the relation of words to non-verbal facts”:

> A. Those who infer properties of the world from properties of language. These are a very distinguished party; they include Paramenides, Plato, Spinoza, Leibniz, Hegel, and Bradly.
>
> B. Those who maintain that knowledge is only of words. Among these are the Nominalists and some of the Logical Positivists.

C. Those who maintain that there is knowledge not expressible in words, and use words to tell us what this knowledge is. These include the mystics, Bergson, and Wittgenstein; also certain aspects of Hegel and Bradley.

Russell, Wittgenstein, and the philosophers mentioned, use "language" in the non-holistic sense – which is why they in the first place consider the connectability problem as a question of a language-world connection (with obvious difficulties).

We, using language in its holistic conception, would add a forth class:

D. Those who maintain that every communicable knowledge is expressed in words-with-meanings in a necessarily *shared* holistic language; holistic language cannot be detached from knowledge.

In the first place, we would find it natural to conceive of the connection problem as one for the description-interpretation processes which constitute a language. The answer would then be:

There is, and must be, a connection between descriptions (sentence structures) and interpretations (meaning structures, world structures); the nature of this connection is that of entanglement.

Secondly, in linguistic realism, we can refer to "the real world" and to "evolution of language", and understand how a language can evolve in adapting to a fixed reality and thereby obtain linguistic properties (coded in the genes of people who thereby are able to *share* the language). Such linguistic properties thus in some way come to *reflect properties of the reality.* These reflected linguistic properties are of a much more general kind than the connections considered by Wittgenstein and Russell. They are what allow us our inductive linguistic capacities, like of having conscious existential perceptions, etc.

10. LANGUAGE-BASED FOUNDATIONAL RESEARCH

As exhibited in our selection of foundational studies in section 2, understandings of *foundations*, notably of fields like mathematics and science, depend on how these fields are conceived with respect to the activity/result (inductive/deductive) distinction (section 1). Early interests in foundational research have focused on the deductively presentable *results* of scientific and mathematical activity, and then gradually developed over informal reasonings, towards a foundational interest also for the inductive nature of scientific *activity.*

The development may also be seen as a shift in understandings of the involved concepts of language, from non-holistic conceptions to holistic.

Non-holistic conceptions, which allow a clean separation between syntax and semantics, naturally stimulated foundational studies directed towards formal systems. Gödel's basic conception of a formal system came to influence grate parts of subsequent foundational research for mathematics and logics.

Also for the natural sciences, notably physics, the non-holistic conception of language showed influential. It allowed a clean cut between observer and observed (physical) world, and classical theories developed accordingly.

However, with quantum mechanics, the presupposition of such a clean cut became object for further foundational investigations. These reflected back into, or interacted with, deeper foundational studies also for mathematics and logics. In these areas, independedent observations of shortcomings with the classical separations had been noticed (cf section 6).

With language holistically conceived, the clean cuts between syntax, semantics, and pragmatics, are no longer maintained. This may allow a deepened understanding of induction, and the influence of scientific activity on scientific results. Let us refer to corresponding foundational research as *language-based foundational research.*

Non-holistic foundational research (allowing a cut between us and an independent world) naturally stimulated revelation of basic fundamental concepts, like truth, by judgement of their consequences in the independently existing real world.

In the holistic case of language-based foundational research, there is another natural direction, namely towards *revelation of presuppositions*, which may be considered as underlying *revelation of basic truths.*

10.1. *Foundational Research Aiming at Revelation of Presuppositions*

Consider a conception of presupposition according to the following definition.

PRESUPPOSITION: what a speaker assumes in saying a particular sentence, as opposed to what is actually asserted
(*The Fontana Dictionary of Modern Thought*, Glasgow: Fontana Press, 1988)

From this definition (in a context of linguistics; semantics; pragmatics), complemented with view (i) of the linguistic complementarity, it follows that there always are presuppositions hidden in a language, namely hidden from full description in the language.

Beside *hidden* presuppositions, there are of course presuppositions that can be sufficiently well revealed in a relative sense against a presupposed shared background knowledge.

If, by foundational research, some such presuppositions are revealed, the meaning (interpretation) of a potential axiom becomes clarified, and judgement of its soundness may become possible.

Further, compare Gödel's characterization of *the problem of giving a foundation for mathematics* (in our Feferman-quote in section 2). Its second part suggests, as a foundational aim, that *"a justification in some sense or other has to be sought for these axioms"*.

Justification of axioms are inseparable from their meanings (interpretations; understandings) in the actual language. This seems to hold for hypothetical sentences (hypothetical before justification) in every language. Even for genetic language, where mutations on genotype (hypotheses) are tested on phenotype in a natural selection process.

Hence, we reach a particular view of language-based foundational research, namely as revelation of presuppositions hidden in a language. We refer to it as the PRESUPPOSITION-REVELATION VIEW.

10.2. *Presupposition of Computability Behind Formal System*

Gödel's classical conception of *formal system* from the early 30's contains a basic concept of "finite procedure". This concept may at the time have been sufficiently clear and free from presuppositions of, at the time, foundational significance. However, in a postscript from 1965 to his 1934 paper, Gödel proposes to replace "finite procedure" by that of a Turing machine computation (Gödel, 1965).

The revision may be looked at as revealing a presupposition behind the fundamental meaning of "finite procedure". Furthermore, the presupposition-revelation view suggests a still further foundational depth, with the concept of Turing machine presupposing that of holistic language (Löfgren, 1992, pp 131-2).

This remark illustrates the principal difficulty with syntax (as a formal system) fully fragmentable from language (compare our comments on Carnap's fragmentation thesis in section 3).

10.3. *Presuppositions Behind Reduction Concepts*

In sentences like "mathematics is reducible to logics", or "chemistry is reducible to physics", we may understand "reduction" in various ways with more or less explicitly understood presuppositions.

For example, it is often, but certainly not always, presupposed that the subjects, referred to in a reduction relation, are deductively presentable descriptive theories (cf section 1). This leads to concepts of *theory-reduction*, like those exposed in the following quotes.

(Aerts and Rohrlich, 1998, p. 29). "A theory S is said to be reduced to a theory T if and only if T 'implies' S. The difficulties of theory reduction lies in the explanation of the notion of 'implication' as used here. ...
It is very important that 'theory reduction' as used here refers to theories of *different levels of reality*. This is the reason for incommensurable terms...
The relationship between theories on the same level do not cause difficulties but are typically related by inclusion: S is a subtheory of T."

(Bunge, 67, p. 41). "Genuine reduction must therefore be distinguished from interlevel relation.
...And a theory is genuinely reduced to another theory if the reduced theory is proved to be a *subtheory* of the richer theory (the reducing theory)...
Every other case of intertheory relation should be regarded as one of pseudoreduction."

The first proposal contains, as a special case (theories on the same level), theory-reduction as: S is reducible to T if and only if S is a subtheory of T.

Also in the second proposal, theory reduction is conceived precisely this way, as a subtheory relation, which, furthermore, is regarded a "genuine" reduction in distinction from other intertheory relations.

Both the view of the subset relation as a "genuine", and as an "unproblematic", reduction relation reflects, however, a hidden presupposition which may be revealed by comparison with our concept of *syntactic reduction*.

(Löfgren, 1976, p. 268). "We say that a formal theory T_1 is *syntactically reducible* to a formal theory T_2 precisely when T_1 is translatable into T_2, i.e., when there exists a recursive word function, f, such that $\vdash_{T_1} W$ iff $\vdash_{T_2} f(W)$, and $W_1 \neq W_2$ implies $f(W_1) \neq f(W_2)$."

Also syntactic reduction is a theory-reduction focusing on membership and subset relations. However, it is slightly more linguistic in that a translation function is involved (operating only on sentences, not on their meanings). By consequence, the two reduction concepts (subset and syntactic reductions) become drastically different.

If S is *syntactically reducible* to T, then if S is *undecidable* so is T, and if T is *decidable* so is S. (A theory is decidable when there is an algorithm answering the question whether an arbitrarily given sentence is, or is not, a theorem of the theory; otherwise the theory is undecidable.)

However, if S is *genuinely reducible* to T, it may be the case that S is undecidable and T decidable (cf the fact that a decidable set can, like the set of natural numbers, contain undecidable subsets of numbers).

When hearing that S is genuinely *reducible* to T, in a case where T but not S is decidable, we naturally expect to be able to solve the decision problem for S by reducing it to T which is decidable. But that is impossible – unless the reduction function itself is nonalgorithmic. For if it were algorithmic we would have a contradiction against the assumption that S is not decidable.

Can a reduction relation be assumed nonalgorithmic (for example inductive)? No, some kind of *neutrality condition* must be presupposed. Without that, a given S could be said to be reducible to *any* T, simply by powering the "reduction" function accordingly.

In the present context of *theory* reduction, whith theories as deductive formal systems, a natural neutrality condition for reduction is that it be algorithmic (cf Gödel's presupposition, of Turing machine computation for finite procedure, in formal systems). In our concept of *syntactic reduction*, this neutrality condition is expressed in the stipulation that the translation function be recursive.

The proposal of the subtheory relation as a reduction relation may possibly be understood against a presupposition that all theories of interest are decidable. But that certainly is not the case in foundational contexts.

Concerning presuppositions behind elaborations of reduction as translation, like semantic reduction and even language reduction, see Löfgren (1987).

10.4. *Presuppositions Behind Holistic Language as Foundational Category*

What reasons do we have to propose holistic language as a foundational category?

Recall our discussion in subsections 9.3 and 9.4. There we illustrated different foundational attempts at the basic question of a *language-world connection*. If attempted non-holistically, as by Wittgenstein, it leads to a kind of explanation which provoked a witty remark by Russell (labeled C in 9.4). Namely, that Wittgenstein understands: "that there is knowledge not expressible in words, and use words to tell us what this knowledge is".

Instead, if this question is attempted with holistic language as basic category, the whole problem obtains a thoroughly insightful answer, expressed in the last paragraph of subsection 9.4. (Compare as well our remark in 9.4, labeled D.)

What makes all this possible is the general validity of our presuppositions, I and II (section 4.1), presupposed valid for every language. Domains like science, logics, mathematics do evolve, as do also languages. But what does not change is a requirement of communicability of knowledge, whatever domain it may refer to. Whatever, the nature of the language used in the communication, it is supposed to obey the two presuppositions I and II.

This is what allows us to talk of holistic language, as a general concept presupposing I and II, as a basic category for foundational research. (Cf also our last paragraph of section 9.3.)

11. UNITY BY FOUNDATIONAL RESEARCH

There is an undisputable fragmentary growth of science in the form of an increasing number of scientific disciplines. This is understandable, much as scientific disciplines refer to the *deductively presentable results* of scientific activity (cf section 1). Such results cannot fully account for the inductive nature of scientific activity (cf section 7). By concequence, each scientific domain (like physics; mathematics, and even logics, may here be understood as other such domains) has to be accounted for by a multitude of deductively presentable theories, each a *partial* account of the scientific domain (like in multitudes of physical or mathematical diciplines; cf section 1, and 7.1).

It is our human curiosity, and drift for understandings, that produces this development – and, when observed, takes the development itself as target for understanding or, in that sense, for unification.

By concequence, such a unifying understanding cannot itself be fully understood as a deductively presentable theory (which would have to be a partial account of unifying understanding). Instead, it must refer to our scientific *activities* – which are not short of being linguistic in the holistic sense.

With reference to section 10, where foundational research is understood as aiming at understandings of our scientific activities, notably in their presuppositions of language, we understand how unity, in foundational research, comes about.

Again, (in section 10), we have demonstrated how language-based foundational research allows a wider unifying understanding than that offered by a classical foundational concept like reduction. Here the presupposition-revelation view of foundational research proved useful.

In the presupposition-revelation view of language-based foundational research, one revealed presupposition may show to hide a presupposition on a deeper level. This leads to a process of successive revelations of presuppositions and to the question whether there is some ultimate presupposition for language-based foundational research. In a sense there is, namely in the presuppositions for language as characterized by the linguistic complementarity (section 4). These are presupposed for every language (including every language in the process of evolution).

But certainly there can be no algorithm, no describable communicable method, for such a revelation process. Foundational research is by nature an activity beyond full describability in some language. But it can be complementaristically conceived as an *activity* in some language.

The field of *general systems research*, cf our quote of Ashby in section 1, originated around basic questions on unification, but is apparently now developing by disciplinary fragmentation, and this in steadily increasing speed in spite of early warnings of von Bertalanffy. In Löfgren (2002), we suggest for "systems science", a concept used in this field, that "science" in this constellation be *systemically* understood (i.e., not be restricted to deductively presentable results as if fully separable from an underlying research activity).

12. CONCLUSIONS

In spite of the development of science into a multitude of scientific disciplines (quite understandable at that), there is no principal obstacle against their unification. That is, *not* as some unifying scientific discipline, but as a *unifying understanding* of science. Which is possible by taking holistic language as foundational category.

Yet, this insight requires some effort, some rethinking based on holistic language as a foundational conception, rather than relying on some more traditionally used foundational category, as for example logic (as if understandable as fragmentable from holistic language).

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