CODEN: LUTFD2/(TFRT-7427)/1-06/(1989)

Adaptive Control–A Perspective

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August 1989

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Document name
Report

Date of issue
August 1989

Document Number
CODEN: LUTFD2/(TFRT-7427)/1-06/(1989)

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Supervisor

Sponsoring organisation

Title and subtitle
Adaptive Control–A Perspective.

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Key words
Adaptive control; automatic tuning; robust control; industrial control.

Classification system and/or index terms (if any)

Supplementary bibliographical information

ISSN and key title

ISBN

Language
English

Number of pages
6

Recipient's notes

Security classification

The report may be ordered from the Department of Automatic Control or borrowed through the University Library 2, Box 1010, S-221 03 Lund, Sweden, Telex: 33248 lubbis lund.

Paper presented at International Symposium on
Adaptive Systems in Control and Signal Processing,
19-21 April 1989, Glasgow, UK.

Adaptive Control – A Perspective

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Abstract. The paper presents a perspective on adaptive control. Use of adaptive control in current industrial products is discussed. The technique of averaging is discussed. It is shown how it can be used to make an assessment of the relative merits of different adaptive algorithms. Properties of robust and adaptive control is discussed. Suggestions for future research directions are then given.

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1. Introduction

Adaptive control, which has been a research topic for over 30 years, is now starting to have some industrial impact. A rough estimate is that adaptation and automatic tuning is used in more than 100 000 loops. Although this is an impressive number, it represents only a small fraction of the total number of industrial feedback loops. The drivers of the development are advances in computer and information technology, control theory, demand from users, and challenging problems.

The purpose of this paper is to give a perspective on the development to reflect upon the state of the art and to discuss key research problems. The paper is organized as follows. Section 2 is a brief discussion of different commercial adaptive regulators and their uses. Some theoretical aspects are treated in Section 3 with particular emphasis on averaging methods. In Section 4 averaging is used to make a reassessment of the MIT and SPR tuning rules for a simple adaptive problem. Robust high gain control is another way to deal with plant uncertainty. This is discussed in Section 5. In Section 6, finally, we give an assessment of future research needs.

2. The Industrial Scene

Adaptive control is starting to have industrial impact. There are now many industrial products based on adaptive control. They are used both as general purpose controllers and in special products. Some adaptive systems have been in operation for over 10 years. See Goodwin and Sin (1984), Narendra (1986), Seborg et al. (1986), Åström (1987), and Åström and Wittenmark (1989). There are several ways to use adaptive techniques: as tuning devices, to generate gain schedules and for true adaptive control.

Algorithms

Because of the work done on unification (Egardt 1979) most adaptive systems can be represented by the block diagram shown in Fig. 1. Both parameter estimation and control design can be done in many different ways which means that there is a large variety of adaptive systems. Practically all algorithms used are based on the *certainty equivalence principle.* This means that the control design is carried out as if the estimates represent the true parameter values.

The system shown in Fig. 1 is called *indirect* adaptive control, because the regulator parameters are obtained indirectly via the control design. In some cases it is possible to reparameterize the system so that the regulator parameters are estimated. The design block in Fig. 1 then vanishes. Such algorithms are called *direct* adaptive control. In recent work (Narendra and Duarte, 1988) it has also been suggested to have mixed direct and indirect schemes.

An important aspect of adaptive control is that it is possible to model disturbances and to estimate such models. It is also possible to tune feedforward control. Since feedforward critically depends on a correct model adaptation, it is almost a prerequisite for using of feedforward control. Very good results have

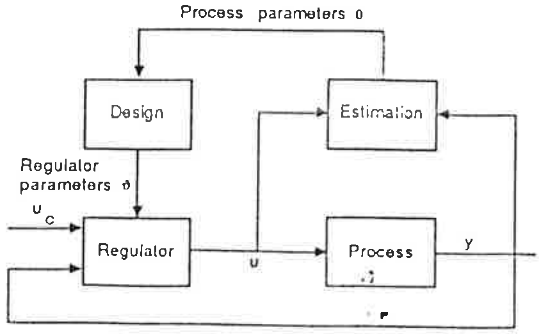

Figure 1. Block diagram of an adaptive control system.

also been reported from industrial use of adaptive feedforward, see Åström and Wittenmark (1989).

The early work on adaptive control naturally dealt with simple estimation schemes and simple control design techniques, e.g., least squares estimation and minimum variance control. This was partially dictated by available computational power and partially by the necessity to start with simple problems. As time progressed more elaborate design methods like pole placement, LQG and predictive control have been used.

Industrial Products

The ideas of adaptive control have been around since the mid fifties. In the beginning (the brave era) the regulators were implemented using analog devices. This met with considerable difficulties. Digital implementations are much simpler and the advent of microprocessors together with improved theory gave rise to a significant increase in the applications of adaptive control. In particular it is possible to have much more efficient estimation algorithms with computer implementations. In the beginning of the eighties a number of adaptive controllers started to appear on the process control market. A variety of approaches were used in the products. Leeds and Northrop announced an adaptive PID regulator in 1981. Asea Brown Boveri announced a general purpose adaptive controller, Novatune, in 1982. This controller is a direct self-tuner based on least squares estimation and minimum variance control. It also has adaptive feedforward. In 1984 there were several product announcements. NAF (now SattControl) announced a small DDC system called SDM20, with a relay auto-tuner. Foxboro announced an adaptive PID regulator called Exact, and Turnbull Control also introduced an adaptive PID regulator. First Control Systems introduced a general adaptive regulator in 1986. This system may be considered as a second generation of Novatune. SattControl announced a single-loop relay auto-tuner with gain scheduling in 1986. Yokogawa introduced adaptive PID regulators in 1987, which are similar to Exact. Eurotherm announced a PID regulator with auto-tuning and adaptation in 1987. SattControl announced a PID regulator with auto-tuning, gain scheduling, adaptation and adaptive feedforward in 1988. It is interesting to discuss the structure of these products. For this purpose we will group them into different categories.

Early self-tuning PID. This includes the regulators from Leeds and Northrup and Turnbull Control. A discrete time transfer function is estimated in both systems. The PID parameters are then computed from the discrete model. The key difficulty in this approach is that it requires as much prior information as a general purpose adaptive regulator. It is in particular required to know the time scale of the process. As a consequence the manufacturers added a *pretune mode* to help getting the required prior information.

Closed loop PID tuners. This category includes Eurotherm, Foxboro Exact, Yokogawa and SattControl. In these types of regulators the transient behavior of a given PID regulator is observed and empirical rules are used to adjust the parameters. The key difficulty with this approach is that sufficient prior information to design a preliminary PID regulator is needed.

Relay auto-tuners. This includes the regulators from SattControl and Fisher. The idea is to determine one point on the Nyquist curve by a relay feedback experiment. The key difficulty in this approach is to determine a suitable initial amplitude of the relay.

General purpose adaptive regulators. This includes the controllers Novatune from ABB and First Line from First Control. These regulators have a more general structure than the PID regulators including feedforward and higher order compensators. A discrete time model is estimated and some control design is applied to the model obtained. These regulators admit better performance than PID regulators. The feedforward compensation has been found particularly useful. The key drawback is that it is necessary to know the time scale of the process to initialize the identification. Prior information is also required for choices of specifications for the design. A consequence is that these regulators offer potentially better performance but also that they demand more knowledge of the user.

Special purpose adaptive regulators. Adaptation is now also becoming an important component in special purpose systems. Autopilots for ship steering is one area where adaptive control is now becoming commonplace. One advantage of the special purpose systems is that much is known a priori about the systems, which makes it easy to design the safety networks. One example from a different area is a system for hemodialysis developed by Gambro AB in Lund. There are over 4000 systems of this type in operation. The system is based on adaptive pole placement.

An Assessment

It is interesting to observe that the industrial products are often based on the PID structure and that they also use nonstandard estimation methods. See Åström and Hägglund (1988). This can be contrasted with the theoretical work that often has followed a standardized path of estimating parameters of a generic structure and to apply some control design method. Another observation is that automatic tuning is sufficient and that true adaptation is not always needed. A third observation is that adaptation has paved the way for wide spread use of feedforward control.

Adaptive systems also have parameters that must be supplied by the user. It is clear that we must tell the system what we expect it to do, e.g., in terms of desired bandwidth or a specified loss function, or the weighting between state deviations and control signals in an LQG based design. It is useful to introduce such *performance related knobs* that have a strong intuitive appeal for the user.

Experience has also shown that it is necessary to provide more information, typically some knowledge about time scales sampling period, dead time, model structure, and signal ranges. Most adaptive controllers do in fact have more than three parameters that are chosen by the user. This also means that a reasonable knowledge on the part of the user is required. When it was tried to use adaptive control practically, the need for *pre-tuning* to help the user acquire the prior information needed therefore emerged quickly.

3. Adaptive Control Theory

Some resons for developing adaptive control theory are: to understand specific algorithms, to determine performance limits, and to develop new algorithms. Theory has developed slowly, because adaptive systems are complicated and difficult to analyze. After much work there is, however, now a body of results that can help us to understand adaptive systems. Stability theory and averaging are powerful techniques. To obtain strong results it is necessary to exploit the particular structure of adaptive systems.

Special Structure of Adaptive Systems

Consider the system shown in Fig. 1. Assume that the system and the controller are linear. Let ϑ denote the regulator parameters and ν the external driving signals, i.e., the command signal u_c and nonmeasurable disturbances acting on the process. With constant regulator parameters the closed-loop system can then be written as

$$\begin{aligned} \frac{d\xi}{dt} &= A(\vartheta)\xi + B(\vartheta)\nu \\ \eta = \begin{pmatrix} e \\ \varphi \end{pmatrix} &= C(\vartheta)\xi + D(\vartheta)\nu \end{aligned} \tag{1}$$

The state vector ξ includes the states of the system, the reference model, and the auxiliary state variables that may have to be introduced in order to calculate the error (e) and the regression vector (φ) used in the parameter adjustment mechanism. Since this system is linear, it can also be characterized by the differential operators $G_{e\nu}$ and $G_{\varphi\nu}$, which relate e and φ to ν. These operators depend on the regulator parameters (ϑ).

Furthermore, let θ denote the process parameters: We will consider a simple gradient scheme for adjusting the parameters:

$$\frac{d\theta}{dt} = \gamma\varphi(\vartheta,\xi)e(\vartheta,\xi) \tag{2}$$

This equation can also be written as

$$\frac{d\theta}{dt} = \gamma\left(G_{\varphi\nu}\nu\right)\left(G_{e\nu}\nu\right) \tag{3}$$

The control design can be represented by a nonlinear function $\vartheta = \chi(\theta)$, which maps the estimated parameters into regulator parameters. This map becomes the identity for direct algorithms. The adaptive system is thus described by Eqs. (1) and (2). Notice that the equations have a very special structure. Equation (1) is linear in the states and the external driving signals. Nonlinearities appear only in the product φe in Eq. (2), in the design map χ, and in the functions $A(\vartheta)$, $B(\vartheta)$, $C(\vartheta)$, and $D(\vartheta)$ in Eq. (1). These functions are actually affine in ϑ.

Averaging

The dynamic analysis is generally quite complicated, because the complete system is often of high order. Analysis of a direct algorithm for a second-order system with four unknown parameters using a gradient method leads to a differential equation of order 10 (2 states of the system, 4 parameters, and 4 differential equations to generate the regression variables). Ten more equations are obtained if a least-squares estimation algorithm is used.

In an adaptive system it is natural to separate between the states of the system and the process parameters. The process parameters are typically changing more slowly than the states. Averaging theory is one technique that uses this feature. To describe the averaging methods, consider the adaptive system described by Eqs. (1) and (2). The rate of change of the parameter θ can be made arbitrarily small by choosing the adaptation gain γ sufficiently small. Now consider Eq. (2). The product φe in the right-hand side depends on ϑ and ξ, where $\vartheta = \vartheta(\theta)$ varies slowly and ξ varies fast. The key idea in the averaging method is to approximate the product φe by

$$G(\theta) = \operatorname{avg}\left\{\varphi\big(\vartheta(\theta), \xi(\vartheta(\theta), t)\big) e\big(\vartheta(\theta), \xi(\vartheta(\theta), t)\big)\right\}$$

where avg denotes the average and $\xi(\vartheta(\theta), t)$ is computed under the assumption that the parameters θ are constant. The average can be computed in many ways. Time averages can be used if the signals are periodic and ensemble averages can be used when the signals are stationary stochastic processes. The calculation of $\xi\big(\vartheta(\theta), t\big)$ is a straightforward exercise in linear system analysis. The expressions may, however, be complex for high-order systems. Symbolic calculation is a useful tool for carrying out the calculations.

The use of averaging thus results in the following averaged nonlinear differential equation for the parameters:

$$\frac{d\bar\theta}{dt} = \gamma \operatorname{avg}\left\{\varphi\big(\vartheta(\bar\theta), \xi(\vartheta(\bar\theta), t)\big) e\big(\vartheta(\bar\theta), \xi(\vartheta(\bar\theta), t)\big)\right\} \tag{4}$$

This equation can also be written as

$$\frac{d\bar\theta}{dt} = \gamma \operatorname{avg}\left\{\left(G_{\varphi\nu}\nu\right)\left(G_{e\nu}\nu\right)\right\} \tag{5}$$

Notice that the transfer functions $G_{e\nu}$ and $G_{\varphi\nu}$ depend on the averaged parameter $\bar\theta$. When the averaged equations are obtained, the behavior of the state variables ξ can be obtained by linear analysis. There are theorems that give conditions for $\bar\theta$ being close to θ. The conditions typically require smoothness conditions of the functions involved and periodicity or near periodicity of the time functions. There are also stochastic averaging theorems. See Guckenheimer and Holmes (1983), Kumar and Varaiya (1986) and Anderson et al. (1986).

A significant advantage of averaging theory is that it reduces the dimensions of the problem. The theorems require that the adaptation gain should be small, but experience has shown that averaging often gives a good approximation, even for large adaptation gains.

4. Reassessment of the MRAS

The insight that can be derived from the analysis will now be illustrated by a classical case, namely adaptation of a feedforward gain.

Consider a process with the transfer function $G(s)$ and an adjustable feedforward gain. Find a feedforward gain θ such that the input-output behavior matches the transfer function $G_m(s)$ as well as possible. The algorithms obtained by the MIT and the SPR rules are

$$\begin{aligned} \frac{d\theta}{dt} &= -\gamma y_m e \qquad \text{(MIT)} \\ \frac{d\theta}{dt} &= -\gamma u_c e \qquad \text{(SPR)} \end{aligned} \tag{6}$$

where u_c is the command signal, $y_m = \theta^0 G_m u_c$ the model output, and e the error defined by

$$e(t) = y - y_m = G(p)\big(\theta(t)u_c(t)\big) - \theta^0 G_m(p)u_c(t) \tag{7}$$

Notice that the parameter adjustment rules give very similar adaptation laws.

Perfect Models

The calculation of the error given by (7) requires that a model G_m of the system G is known apart from a gain factor. When the model is known, i.e., $G_m = G$ there is a drastic difference between the systems obtained by the MIT and the SPR rules. Figure 2 shows the stability diagram for the system with the MIT rule when the plant has the transfer function $G(s) = 1/(s+1)$ and the input signal is a sinusoid with frequency ω. The figure illustrates the complex behavior of simple adaptive systems. The solution corresponding to a command signal of one frequency can be stable but the one corresponding to another can be unstable. Notice, however, that the system is stable for low adaptation gains.

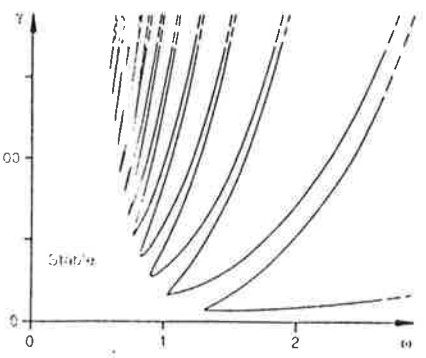

Figure 2. Stability region for adjustment of a feedforward gain with the MIT rule. The plant has the transfer function $G(s) = 1/(s+1)$ and the command signal is $u_c = sin(\omega t)$.

The system obtained with the SPR rule differs significantly from the one obtained by the MIT rule, because the SPR rule gives a system that is stable *for arbitrarily high adaptation gain* γ. Hence there seems to be no reason not to use the SPR rule. This conclusion depends, however, on the fact that the problem is contrived because of the assumption that $G_m = G$.

Effects of Modelling Errors

The case when the model G_m used in the adaptive controller differs from the plant dynamics G will now be investigated. Inserting the expressions for y_m and e into the equations for the parameters, we get

$$\begin{aligned} \frac{d\theta}{dt} &= \gamma(G_m u_c)\left(\theta^0 G_m u_c - G(\theta u_c)\right) \\ \frac{d\theta}{dt} &= \gamma u_c\left(\theta^0 G_m u_c - G(\theta u_c)\right) \end{aligned} \tag{8}$$

where the first equation holds for the MIT rule and the second for the SPR rule. The corresponding averaging equations are

$$\begin{aligned} \frac{d\bar\theta}{dt} &= \gamma\left(\theta^0 \operatorname{avg}\{(G_m u_c)^2\} - \bar\theta \operatorname{avg}\{(G_m u_c)(G u_c)\}\right) \\ \frac{d\bar\theta}{dt} &= \gamma\left(\theta^0 \operatorname{avg}\{u_c(G_m u_c)\} - \bar\theta \operatorname{avg}\{u_c(G u_c)\}\right) \end{aligned} \tag{9}$$

The equilibrium parameters are

$$\begin{aligned} \bar\theta_{\text{MIT}} &= \theta^0 \frac{\operatorname{avg}\{(G_m u_c)^2\}}{\operatorname{avg}\{(G_m u_c)(G u_c)\}} \\ \bar\theta_{\text{SPR}} &= \theta^0 \frac{\operatorname{avg}\{u_c(G_m u_c)\}}{\operatorname{avg}\{u_c(G u_c)\}} \end{aligned} \tag{10}$$

If $G = G_m$ the equilibrium values are equal to the true parameters for all command signals u_c. When $G \neq G_m$, the equilibrium obtained will depend on the command signal as well as on the unmodeled dynamics.

The stability conditions for the averaged equations (10) are

$$\begin{aligned} &\gamma \operatorname{avg}\{(G_m u_c)(G u_c)\} > 0 \qquad \text{(MIT)} \\ &\gamma \operatorname{avg}\{u_c(G u_c)\} > 0 \qquad \text{(SPR)} \end{aligned}$$

This implies that for sinusoidal input signals the MIT rule gives a stable equilibrium if G_m and G differ at most by 90°. With the SPR rule the equilibrium is stable only if the phase lag of the process is at most 90°. The calculations are illustrated by an example.

EXAMPLE 1

Consider a reference model with the transfer function

$$G_m(s) = \frac{a}{s+a}$$

Assume that the process has the transfer function

$$G(s) = \frac{ab}{(s+a)(s+b)}$$

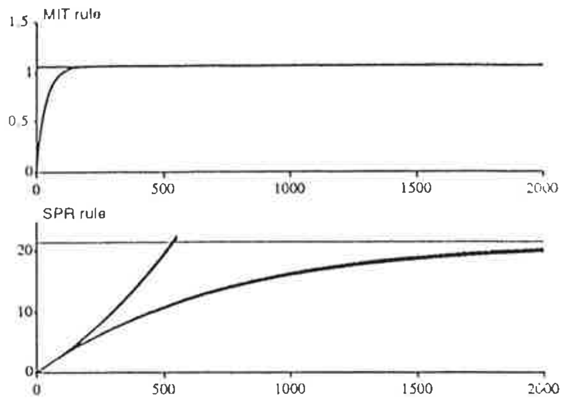

Figure 3. Feedforward gains obtained by (a) the MIT rule and (b) the SPR rule, for a system with $G_m = \frac{1}{s+1}$ and $G = \frac{16}{(s+1)(s+16)}$ with sinusoidal input signals having frequencies 3.9 and 4.1 rad/s. The straight lines are the equilibrium values.

Let the command signal be a sinusoid with frequency ω. Equation (10) gives the equilibria

$$\bar{\theta}_{\text{MIT}} = \frac{b^2 + \omega^2}{b^2}$$

$$\bar{\theta}_{\text{SPR}} = \frac{b^2 + \omega^2}{b(ab - \omega^2)} \qquad \omega < \sqrt{ab}$$

With the MIT rule the equilibrium is stable for all ω, but with the SPR rule it is stable only if $\omega < \sqrt{ab}$. Figure 3 shows the behavior of the systems for $a = 1$ and $b = 16$ when the input signals have frequencies $\omega = 3.9$ and $\omega = 4.1$. The equilibrium values predicted by the averaging theory are also shown in the figure. The SPR is unstable for $\omega > 4$. Notice the drastic difference in the equilibria. For $\omega = 3.9$, the MIT rule gives an equilibrium gain of 1.059 while the SPR rule gives an equilibrium gain of 21.5. This should be compared with the correct value of 1.0. For $\omega = 4.1$ the MIT rule gives an equilibrium gain of 1.066 while the SPR rule gives an unstable solution. □

In conclusion, we find that averaging analysis gives useful insights. It shows that analysis of the ideal case can be quite misleading. Even in the simple case of adjustment of a feedforward gain, unmodeled dynamics together with high-frequency excitation signals may lead to instability of the equilibrium. The equilibrium analysis also illustrates interesting properties of the MIT and the SPR rules. First, the equilibrium of the MIT rule has a good physical interpretation as the parameter that minimizes the mean square error. Secondly, in the presence of unmodeled dynamics, the SPR value works for a much smaller class of inputs than the MIT rule.

5. Robust Control

Robust control is an alternative to adaptive control dealing with plant uncertainties. There are techniques to design constant gain regulators that can cope with processes having variable dynamics. One powerful method, which originated in Bode's classical work on feedback amplifiers, has been developed by Horowitz (1963). The controller has the form

$$u = G_{fb}(G_{ff}u_c - y) \tag{11}$$

which is called a *two-degree-of-freedom* system, because the controller has two transfer functions a feedback G_{fb} and a feedforward G_{ff}. The key idea is to determine the feedback G_{fb} so that the inner loop is insensitive to load disturbances and *plant uncertainty*. This typically requires both high gain and high bandwidth. The feedforward transfer function G_{ff} is then determined so that the given response to command signals is obtained. The key design trade-off is that a high gain feedback amplifies measurement noise so that the actuators may saturate. There are systematic graphical techniques to obtain the transfer functions provided that specifications on the uncertainty and the closed loop system are given.

In some cases robust control is much superior to adaptive control, in other cases it is much inferior. The key differences are that with robust control it is necessary to specify a nominal plant and the variations a priori. This is not required for adaptive control. Another difference is that robust control achieves insensitivity to plant variations through high gain control. This implies that robust control may use unnecessarily high gain compared to adaptive control. Recently there have also been comparisons between robust and adaptive control (Åström

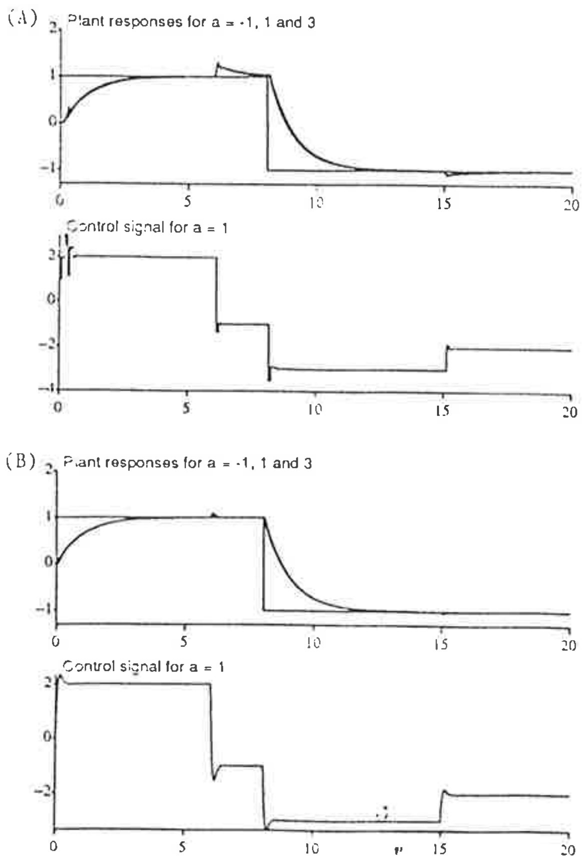

Figure 4. Comparison of adaptive (A) and robust (B) control.

et al., 1988). An illustration of a case, where robust control outperforms adaptive control, is shown in Fig. 4. The figure from Åström (1988) shows responses for three different plants and the control signal for one plant for an adaptive regulator and a robust regulator. There are severe unknown step disturbances at times 0, 6, and 15 that make estimation difficult. One reason why the adaptive regulator has poor performance is that it takes some time for the parameters to converge. Another reason is that the robust controller uses higher bandwidths. This explains why the error due to load disturbances recovers much faster for the robust controller. It is possible to use a high gain, because there is no measurement noise. The higher gain is also beneficial to reduce the effects of plant variations. The fact that the high loop gain does not result in a faster command signal response is due to the two-degree-of-freedom structure that allows different responses to load and set point perturbations. The adaptive controller uses lower gain because of the specifications on the closed loop poles used. In a case like this it would be useful to also have a two-degree of freedom structure for the adaptive controller and to have some mechanism where the adaptive controller can find out a suitable bandwidth in the inner loop. Such a feature is quite different.

6. Conclusions

It is interesting to observe that there are industrial adaptive controllers that are very close to the predominant development of control theory but perhaps even more interesting to note that there also are industrial adaptive systems that use quite different methods.

The work on robust control indicate that there is a need to replace design methods like pole placement and LQG, which are commonly used in current adaptive analyses, by robust design techniques. This is a nontrivial task for two reasons. The robust design methods available (Horowitz, 1963; Doyle, 1987) in their present form do not lend themselves well to automatic on-line use. The robust design methods also require a process description in terms of a nominal model with uncertainty regions. It is then necessary to develop estimation techniques that can give this. Initial attempts in this direction have been made by Kosut (1988a,b) and Goodwin and Salgado (1988).

There would, however, be significant advantages in combining robust and adaptive control. We would then have a system that starts with a robust regulator based on conservative bounds on the plant uncertainty. When the process is controlled the recursive estimator will reduce the plant uncertainty and the control law will be modified accordingly. In closing it is interesting to see that, in spite of all work done on adaptive control, there are still many problems that are both challenging intellectually and of great potential interest industrially.

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