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UNIFICATION OF SOME ADAPTIVE CONTROL SCHEMES - PART I, CONTINUOUS TIME

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Abstract

Model reference adaptive regulators based on input-output descriptions are examined. By reinterpreting the concept of "augmented error", it is shown that there are no essential differences between the model reference adaptive algorithms and the self-tuning regulators. Both types of schemes can be thought of as composed of a parameter estimator and a control law, based on the parameter estimates. It is shown that many schemes proposed are special cases of a general algorithm. The positive real condition for model reference adaptive systems is also examined. It is shown that this condition is a consequence of the choice of estimator and that it is not crucial for stability.

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I. INTRODUCTION

Considerable theoretical work has been devoted to adaptive control systems in the recent years. In particular, the problem of controlling a plant with unknown but constant parameters has been widely discussed. Two approaches seem to attract the most interest, namely the model reference adaptive systems (MRAS) and the self-tuning regulators (STURE).

The two approaches are based on completely different ideas. The MRAS are motivated by stability considerations, whereas the STURE are based on a separation of identification and control. The connections between the two schemes were not recognized in the beginning. However, recently it has been shown in [1] that a positive real condition plays a crucial role for the convergence of a self-tuning regulator. Such a condition is also essential in the analysis of MRAS. The question naturally arises, whether the two schemes are more closely related than earlier thought. Results indicating this were given in [2] and [3]. The purpose of the present paper is to provide a unified treatment of some earlier proposed MRAS and to clarify the relationships with the STURE approach. In the problem to be considered there is no noise and only the output of the plant is available for measurement. The MRAS by Monopoli [4], which is perhaps the most well-known in this area, belongs to the algorithms covered by the analysis. The concept of "augmented error", introduced by Monopoli, is given a new interpretation. This interpretation leads to the possibility of regarding the MRAS as being STURE of a special type. As an interesting side result, it is shown that the positive real condition for MRAS can be dispensed with. A preliminary version of this paper was first published in [5].

The paper is organized as follows. First, in section II, the MRAS by Monopoli is shortly described and the augmented error is given an interpretation. A design method for known plants is described in section III. In section IV a class of adaptive algorithms, based on the design method in section III, is defined. Several MRAS are treated as special cases of the general algorithm in section V. The positive real condition is examined in section VI. Finally, a summary of the results and some concluding remarks are given in section VII.

II. MONOPOLI'S SCHEME AND THE AUGMENTED ERROR

One reason why Monopoli's scheme has been so frequently discussed is that it provides a technique to solve the adaptive control problem when the "pole excess" of the plant (i.e. the difference between the number of poles and the number of zeros) is greater than two. The situation where the pole excess is less than or equal to two is much simpler. The solution was given in [6] and reformulated in [7]. To treat the general case, Monopoli introduced the concept of "augmented error", motivated by the results on adaptive observers.

Monopoli's Scheme

To be consistent with the rest of this paper, the notation is different from Monopoli's [4]. A cross reference between the notations is given in table 1.

TABLE 1

Present Notation Compared to Monopoli's

This Paper	Monopoli's
$y(t)$	$x(t)$
$y_m(t)$	$x_m(t)$
$u_m(t)$	$r^1(t)$
$B^m(p)u_m(t)$	$r(t)$
$e(t)$	$-e(t)$
$e_1(t)$	$y(t)$
$\eta(t)$	$-\eta(t)$
$A(p)$	$D_p(p)$
$b_oB(p)$	$D_u(p)$
$A^m(p)$	$D_m(p)$
$B^m(p)$	$D_r(p)$
$-D(p)$	$A(p)$
$G(p)$	$B(p)$
$b_oB(p)F(p)$	$C(p)$

The plant is described by the differential equation

$$y(t) = \frac{b_o B(p)}{A(p)} u(t) = \frac{b_o(p^m+b_1 p^{m-1}+\ldots+b_m)}{p^n+a_1 p^{n-1}+\ldots+a_n} u(t) \tag{1}$$

where p denotes the differential operator. The following assumptions are made:-

A 1) The degrees n and m are known and $m \leq n-1$. Notice that it is sufficient to know the pole excess and an upper bound on the number of poles to write the differential equation in the form of (1) with known n and m.

A 2) The parameter b_o is nonzero and its sign is known. Without loss of generality b_o is assumed to be positive.

A 3) The plant is minimum phase.

The reference model is given by

$$y_m(t) = \frac{B^m(p)}{A^m(p)} u_m(t) = \frac{b_o^m p^m+\ldots+b_m^m}{p^n+a_1^m p^{n-1}+\ldots+a_n^m} u_m(t) \tag{2}$$

Thus, it is assumed that the pole excess of the reference model is greater than or equal to the pole excess of the plant. This assumption is natural to avoid differentiators in the control law.

The objective of the adaptive controller is to force the error $e(t) = y(t) - y_m(t)$ to zero. This is possible to achieve in a simple way with a bounded control signal if condition A 3) above is fulfilled. In summary, Monopoli's algorithm is as follows. Polynomials

$$D_w(p) = p^{n-1} + d_1^w p^{n-2} + \ldots + d_{n-1}^w \tag{3}$$

$$D_f(p) = p^{n-m-1} + d_1^f p^{n-m-2} + \ldots + d_{n-m-1}^f \tag{4}$$

are chosen. Furthermore, the polynomials

$$D(p) = d_o p^{n-2} + d_1 p^{n-3} + \ldots + d_{n-2} \tag{5}$$

$$F(p) = f_o p^{n-m-2} + f_1 p^{n-m-3} + \ldots + f_{n-m-2} \tag{6}$$

$$G(p) = g_o p^{n-1} + g_1 p^{n-2} + \ldots + g_{n-2} \tag{7}$$

are solved from the identities

$$D_w(p) = B(p)D_f(p) + D(p)/b_o \tag{8}$$

$$(A(p) - A^m(p))D_f(p) = A(p)F(p) + G(p). \tag{9}$$

Also, define the augmented error $\eta(t)$ as

$$\eta(t) = e(t) - e_1(t) \tag{10}$$

where

$$e_1(t) = \frac{D_w(p)}{A^m(p)} w(t) \tag{11}$$

and $w(t)$ is an auxiliary signal to be specified later. With these definitions and the polynomials above, it is possible to write the augmented error as

$$\eta(t) = \frac{D_w(p)}{A^m(p)} \left[\frac{b_o}{D_f(p)} u(t) - \frac{D(p) + b_o B(p)F(p)}{D_f(p)D_w(p)} u(t) - \right.$$

$$\left. - \frac{G(p)}{D_f(p)D_w(p)} y(t) - \frac{B^m(p)}{D_w(p)} u_m(t) - w(t)\right] \tag{12}$$

Collect the unknown parameters of the numerators (divided by b_o) into the vector θ and define the vector $\varphi(t)$ as

$$\varphi^T(t) = \left[\frac{p^{n-2}}{D_f(p)D_w(p)} u(t),\dots, \frac{1}{D_f(p)D_w(p)} u(t), \right.$$

$$\left. \frac{p^{n-1}}{D_f(p)D_w(p)} y(t),\dots, \frac{1}{D_f(p)D_w(p)} y(t), -\frac{B^m(p)}{D_w(p)} u_m(t)\right]. \tag{13}$$

This gives (12) the alternative form

$$\eta(t) = \frac{D_w(p)}{A^m(p)} \left[\frac{b_0}{D_f(p)} u(t) + b_0\theta^T\varphi(t) - w(t)\right]. \tag{14}$$

Now, let $\hat{b}_0(t)$ and $\hat{\theta}(t)$ denote estimates of b_0 and θ. Let the extra signal $w(t)$ be chosen as

$$w(t) = \hat{b}_0(t)w_1(t) \tag{15}$$

and determine $w_1(t)$ and $u(t)$ so that

$$\frac{u(t)}{D_f(p)} + \hat{\theta}^T(t)\ \varphi(t) = w_1(t) \tag{16}$$

Equation (14) then transforms into

$$A^m(p)\eta(t) = D_w(p)\left[(b_0-\hat{b}_0(t))w_1(t)+b_0(\theta-\hat{\theta}(t))^T\varphi(t)\right]. \tag{17}$$

If the parameter up-dating

$$\begin{bmatrix} \dot{\hat{b}}_0(t) \\ \dot{\hat{\theta}}(t) \end{bmatrix} = R \begin{bmatrix} w_1(t) \\ \varphi(t) \end{bmatrix} \eta(t), \quad R \text{ positive definite} \tag{18}$$

is used and the transfer operator $D_w(p)/A^m(p)$ is strictly positive real, it is possible to apply the Kalman-Yakubovich lemma to assure that $\eta(t)$ tends to zero, see [4].

However, even if $\eta(t)$ tends to zero, it does not follow that $e(t)$ tends to zero, which is the primary goal. This fact, as well as the exact choices of u and w_1 in (16), will be commented upon in section IV. This concludes the description of Monopoli's scheme.

The Augmented Error

It is easy to give an interpretation of the augmented error from the equations given above. Thus, if equations (15) and (16) are inserted into equation (11), the following is obtained:-

$$e_1(t) = \frac{D_w(p)}{A^m(p)}\left[\hat{b}_o(t)\left(\frac{u(t)}{D_f(p)} + \hat{\theta}^T(t)\varphi(t)\right)\right]. \qquad (19)$$

Compare this with the identity

$$e(t) = \frac{D_w(p)}{A^m(p)}\left[b_o\left(\frac{u(t)}{D_f(p)} + \theta^T\varphi(t)\right)\right], \qquad (20)$$

which follows from equation (14). The conclusion is that

$$e_1(t) = \hat{e}(t), \qquad (21)$$

where $\hat{e}(t)$ is an estimate of $e(t)$ using the model (20) with the latest available parameter estimates. From equation (10) it then follows that the augmented error η is simply the estimation error, i.e. the difference between $e(t)$ and its estimate $\hat{e}(t)$.

This interpretation of the augmented error is useful to provide a bridge to the self-tuning regulators. In the identification part of these algorithms, it is precisely such an estimation error (or in discrete time prediction error) that drives the adjustment of the parameter estimates. Compare with equation (18).

III. DESIGN METHOD FOR KNOWN PLANTS

Before a unified description of several algorithms is given, the known parameter case has to be considered. A design scheme, which includes interesting special cases, will be described in this section. The scheme is presented in [8] and special cases are treated in e.g. [9] and [10].

The plant is still assumed to satisfy (1) and to be minimum phase in order to get a bounded control signal. The problem consists of designing a controller, which makes the closed-loop transfer operator equal to a given reference model transfer operator, given by (2). The approach taken here is to consider the controller configuration shown in fig. 1, where R', S', and T' are polynomials in the differential operator. It should be noted that the realization of the controller does not look exactly as in fig. 1, because this would incorporate differentiators in the control law. Instead, the transfer function [T'R']/S' is implemented, and this can of course be done with a degree (S')-dimensional state space.

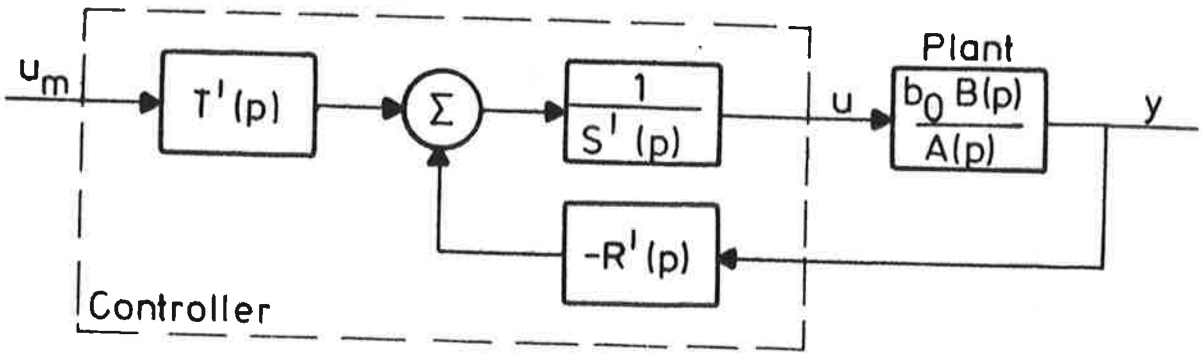

Figure 1. Controller Configuration

The desired closed-loop transfer function is obtained if the polynomials R', S', and T' are chosen to satisfy the equation

$$\frac{B^m(p)}{A^m(p)} = \frac{b_0 B(p) T'(p)}{A(p)S'(p) + b_0 B(p) R'(p)}$$

or, equivalently,

$$b_oB(p)T'(p)A^m(p) = B^m(p)[A(p)S'(p) + b_oB(p)R'(p)] \tag{22}$$

It is possible to simplify this polynomial equation by introducing polynomials R, S, and T, satisfying

$$\begin{aligned} R'(p) &= R(p) \\ S'(p) &= b_oB(p)S(p) \\ T'(p) &= B^m(p)T(p) \end{aligned} \tag{23}$$

The polynomials S' and T' must in fact look like this if it is assumed that A(p) is relatively prime to B(p) and $B^m(p)$ is relatively prime to $A^m(p)$ and B(p). This follows readily from (22). With the polynomials in (23), the identity (22) is reduced to

$$T(p)A^m(p) = A(p)S(p) + R(p) \tag{24}$$

and the controller structure is now as shown in figure 2.

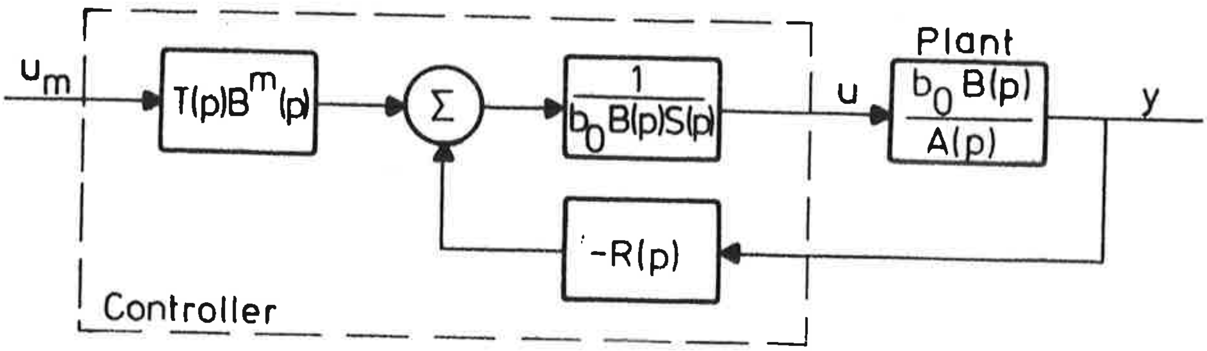

Figure 2. Controller Configuration with Polynomials in (23)

The polynomials $b_oB(p)$ and T(p) are cancelled in the closed-loop transfer function. These polynomials represent unobservable or uncontrollable modes. The polynomial T(p) thus can be chosen freely without changing the closed-loop transfer function. When T(p) has been determined, the equation (24) has many solutions S(p) and R(p). However, it will be assumed that the degree of R(p) is less than or equal to n-1, which assures that the equation has a unique solution, [8]. Since the polynomials A(p) and $A^m(p)$ both have degree n, T(p) and S(p) will have the same degree n_T. In order to assure that the control law does not contain any derivatives of the output, n_T is chosen greater than or equal to n-m-1. Furthermore, S and T are scaled so that they are monic.

In summary then, the design scheme consists of the following steps:-

1) Choose the monic polynomial T(p)

$$T(p) = p^{n_T} + t_1 p^{n_T - 1} + \dots + t_{n_T}, \quad n_T \geq n-m-1 \tag{25}$$

2) Solve the equation

$$T(p)A^m(p) = A(p)S(p) + R(p) \tag{26}$$

for the unique solutions R(p) and S(p), defined by

$$R(p) = r_o p^{n-1} + \dots + r_{n-1} \tag{27}$$

$$S(p) = p^{n_T} + s_1 p^{n_T - 1} + \dots + s_{n_T} \tag{28}$$

The first step, the choice of T(p) (including its degree) does not affect the closed-loop transfer function. However, it is of importance for the transient properties and the effect of disturbances. In fact, if n_T in (25) and (28) is chosen to be equal to n, it can be shown, see [8], that the described controller is nothing but a standard solution consisting of a Kalman filter and state estimate feedback, augmented with a zero cancellation and adding of new zeros. It also follows that T(p) can be interpreted as the characteristic polynomial of the Kalman filter and thus should be chosen according to the noise colour. In the same way the choice n_T = n-1 makes the controller an input-output counterpart to a state space solution with a Luenberger observer and state estimate feedback.

The importance of the noise colour for the choice of observer will not influence the discussion in this paper, since only the noiseless case is considered. However, this problem is treated in detail in the companion paper on discrete time systems, [13].

IV. CLASS OF ADAPTIVE CONTROLLERS

As described in section II, the augmented error used in the MRAS approach has an important interpretation. It consists of the difference between one quantity, the error, and its estimate. This difference is then used in the parameter estimation. The implication is that there is a strong relationship between the MRAS and the self-tuning regulators.

In order to make this relationship even more clear, the idea behind the self-tuning regulators will be used to define a general class of adaptive regulators. These regulators will be adaptive versions of the controller described in section III. The class of algorithms will later be shown to include several MRAS schemes as special cases.

The plant is still assumed to satisfy (1) and assumptions A 1) - A 3) in section II. The desired closed-loop transfer function is given by (2). The first step in the development is to use the results in section II to obtain expressions for the error or, more generally, the error filtered by some transfer function.

The polynomial identity (26) and the equations (1) and (2) are used to get the following expression for the error $e(t) = y(t) - y_m(t)$:-

$$\begin{aligned} TA^m e(t) &= TA^m y(t) - TA^m y_m(t) = \\ &= (AS + R)y(t) - TA^m y_m(t) = \\ &= b_0 BSu(t) + Ry(t) - TB^m u_m(t) \end{aligned} \tag{29}$$

Let $P_1(p)$, $P_2(p)$, and $Q(p)$ be stable, monic polynomials of degree $n-m-1$, $m+n_T$, and $n+n_T-1$ respectively, and let $P(p)$ be given by

$$P(p) = P_1(p)P_2(p). \tag{30}$$

Now define the filtered error

$$e_f(t) = \frac{Q(p)}{P(p)} e(t). \tag{31}$$

Using (29), $e_f(t)$ can be written as

$$e_f(t) = \frac{Q}{TA^m} \left[\frac{b_o BS}{P} u(t) + \frac{R}{P} y(t) - \frac{TB^m}{P} u_m(t)\right] =$$

$$= \frac{Q}{TA^m} \left[\frac{b_o(P_2+BS-P_2)}{P} u(t) + \frac{R}{P} y(t) - \frac{TB^m}{P} u_m(t)\right] =$$

$$= \frac{Q}{TA^m} \left[b_o \frac{u(t)}{P_1} + b_o(BS-P_2) \frac{u(t)}{P} + R \frac{y(t)}{P} - \frac{TB^m}{P} u_m(t)\right] \tag{32}$$

Define

$$u_f(t) = \frac{u(t)}{P_1} \tag{33}$$

and let θ be a vector containing the unknown parameters of the polynomials $(BS-P_2)$ (degree $m+n_T-1$) and R/b_o (degree $n-1$) and the constant $1/b_o$ as the last element. Note that the vector θ essentially contains the parameters of the controller, described in section III. Furthermore, define the vector

$$\varphi^T(t) = \left[\frac{p^{m+n_T-1}}{P} u(t), \ldots, \frac{1}{P} u(t), \frac{p^{n-1}}{P} y(t), \ldots, \frac{1}{P} y(t), - \frac{TB^m}{P} u_m(t)\right]. \tag{34}$$

It is then possible to rewrite the expression (32) for the filtered error $e_f(t)$ as

$$e_f(t) = \frac{Q}{TA^m}\left[b_o u_f(t) + b_o \theta^T \varphi(t)\right]. \tag{35}$$

Using this model, the estimate of $e_f(t)$, i.e. $\hat{e}_f(t)$, is defined as

$$\hat{e}_f(t) = \frac{Q}{TA^m}[\hat{b}_o(t)u_f(t) + \hat{b}_o(t)\hat{\theta}^T(t)\varphi(t)] \tag{36}$$

where $\hat{b}_0(t)$ and $\hat{\theta}(t)$ are estimates of b_o and θ. Define the difference between the filtered error and its estimate (cf. the augmented error $\eta(t)$) as

$$\varepsilon(t) = e_f(t) - \hat{e}_f(t). \tag{37}$$

The following equation is then obtained for $\varepsilon(t)$:-

$$\varepsilon(t) = \frac{Q}{TA^m}\left[(b_o - \hat{b}_o(t))(u_f(t) + \hat{\theta}^T(t)\varphi(t)) + b_o(\theta - \hat{\theta}(t))^T\varphi(t)\right] \tag{38}$$

This equation is of the same form as equation (17). When using a parameter up-dating similar to (18), it thus follows that $\varepsilon(t)$ tends to zero, provided that the transfer function Q/TA^m is strictly positive real. In this case it is seen that the polynomials Q and P serve two purposes, namely to define the filtered error $e_f(t)$ in (31) and to make the transfer function Q/TA^m strictly positive real (SPR). The transfer function Q/TA^m can always be made SPR because all polynomials are known and the degree of Q is one less than the degree of TA^m.

The parameter estimation described above, which is used in the MRAS schemes, is by no means the only possible solution. In presence of noise it could e.g. be suitable to have a gain of the parameter adjustment which decreases with time. Furthermore, there is a lot of freedom in the choice of control law. Taking these facts into consideration, the development done so far thus proposes a class of adaptive algorithms, consisting of two parts:-

- parameter estimator using the model (35);
- a control law based on the parameter estimates.

We will return to the first part, the parameter estimator, in section VI. Below the choice of control law will be examined.

Choice of Control Law

The choice of control law contains one difficulty. It is natural to determine the control signal so that the estimate of the error, i.e. $\hat{e}_f(t)$, is equal to zero. According to equation (36) this means that $u_f(t) + \hat{\theta}^T(t)\varphi(t) = 0$ or, using (33),

$$u(t) = -P_1(p)[\hat{\theta}^T(t)\varphi(t)]. \qquad (39)$$

This control law is identical to the one described in section III if $\hat{\theta}(t)$ is equal to θ in (35). This can be seen from (32).

The control law, however, uses derivatives of the parameter estimates, except for the trivial case when $m = n-1$, i.e. the pole excess is equal to one. In this case P_1 is a constant. Since $\hat{\theta}(t)$ is in general obtained by integration of known signals as in (18), it is in fact possible to use (39) without differentiators also in the case $m = n-2$.

However, in the general case the control law must be modified in order not to include differentiators. There are different solutions proposed in the literature. For example, Monopoli [4] choses a control signal for the algorithm in section II, which will be seen to correspond to the choice

$$u(t) = - \hat{\theta}^T(t)\ [P_1(p)\varphi(t)]. \qquad (40)$$

It is clear that the control law (40) is asymptotically equivalent to the control law (39). Note that it follows from the definition of $\varphi(t)$, (34) that $P_1(p)\varphi(t)$ contains filtered input, output, and reference signals without any derivatives. The choice (40) does not guarantee that $\hat{e}_f(t) = 0$, and it remains to conclude that $e_f(t)$ tends to zero from the fact that $\varepsilon(t) = e_f(t) - \hat{e}_f(t)$ tends to zero. This problem is closely related to the problem of boundedness of the closed-loop signals. It seems that the only solution to this problem so far is a complicated control law by Feuer and Morse [11].

V. EXAMPLES OF THE GENERAL CONTROL SCHEME

Some special cases of the procedure proposed in section IV will now be given. Several MRAS schemes proposed in the literature will be shown to fit into the general description. A new algorithm is also given as an example of the large number of schemes that are possible to derive from the general algorithm.

Example 1. Monopoli's Scheme

The scheme by Monopoli was given in section II. It follows from equations (10) - (12) that

$$e(t) = \frac{D_w}{A^m}\left[b_o \frac{u(t)}{D_f} - b_o(D/b_o + BF)\frac{u(t)}{D_f D_w} - G\frac{y(t)}{D_f D_w} - \frac{B^m}{D_w} u_m(t)\right] \tag{41}$$

Compare this with the expression (32), i.e.

$$e_f(t) = \frac{Q}{TA^m}\left[b_o \frac{u(t)}{P_1} + b_o(BS - P_2)\frac{u(t)}{P} + R\frac{y(t)}{P} - \frac{TB^m}{P} u_m(t)\right] \tag{42}$$

It is straightforward to verify that (41) coincides with (42) if the degree of T, n_T, is chosen to be n-m-1. The polynomials are related as follows:-

$$\begin{aligned} Q &= P \\ D_f &= P_1 = T \\ D_w &= P_2 \\ D &= b_o(P_2 - BP_1) \\ F &= P_1 - S \\ G &= -R \end{aligned} \tag{43}$$

Thus, it follows that Monopoli's scheme uses a model which is a special case of the general one. The "observer" order is n-m-1 and the filtered error is equal to the error itself. The ordinary MRAS estimation scheme in (18) is used. Furthermore, Monopoli chooses the control signal according to (40) and the extra signal w_1 is chosen to satisfy (16). It can be seen from (16) that the control law (39) corresponds to the choice $w_1(t) = 0$. If $n-m \leq 2$ it is thus possible to set the extra signal w_1 to zero and this means that the augmented error $\eta(t)$ is simply equal to the error $e(t)$.

□

Example 2. Bênêjean's Scheme

The Bênêjean scheme is presented in [10] and can be shown to be a variant of Monopoli's. The model used is obtained from (42) after a reparametrization:-

$$e_f(t) = \frac{Q}{TA^m}\left[b_o \frac{u(t) - u_m(t)}{P_1} + b_o(BS - P_2)\frac{u(t) - u_m(t)}{P} + \right.$$

$$\left. + R\frac{y(t)}{P} - (TB^m - b_o BS)\frac{u_m(t)}{P}\right] \qquad (44)$$

The choices of polynomials are identical to Monopoli's and so are the estimation algorithm and the choice of control law. The reparametrization implies that the number of parameters estimated increases considerably, because the coefficients of $TB^m/b_o - BS$ are estimated instead of just $1/b_o$.

□

Example 3. Feuer's and Morse's Scheme

Feuer's and Morse's scheme is given in [11]. A minor change of the design method described in section II will be needed in order to treat the algorithm within the general framework. To this end, write the reference model transfer function as

$$\frac{B^m(p)}{A^m(p)} = \frac{1}{\gamma_o(p)\gamma_1(p)}\, h(p) \tag{45}$$

where $\gamma_o(p)$ and $\gamma_1(p)$ are monic polynomials of degree 1 and n-m-1 respectively. Thus h(p) is a proper transfer operator. Now, consider $h(p)u_m(t)$ as being the input to the reference model with transfer function $1/\gamma_o\gamma_1$. The effect is that the development proceeds as if $B^m = 1$ and $A^m = \gamma_o\gamma_1$. The modification necessary is seen from the identity corresponding to equation (26), i.e.

$$T(p)\gamma_o(p)\gamma_1(p) = A(p)S(p) + R(p) \tag{46}$$

If n_T is chosen to be equal to n, it follows that S(p) has the same degree as $\gamma_o\gamma_1$, i.e. n-m. This is different from the other schemes as seen from equation (28). The control law for the known parameter case is now

$$b_oB(p)S(p)u(t) = T(p)h(p)u_m(t) - R(p)y(t) \tag{47}$$

which is identical to the one in figure 2 with $B^m = 1$ and u_m replaced by $h(p)u_m$. With the above modifications, the equation (32) becomes

$$e_f(t) = \frac{Q}{T\gamma_o\gamma_1}\left[b_o\,\frac{u(t)}{P_1} + b_o(BS - P_2)\,\frac{u(t)}{P} + R\,\frac{y(t)}{P} - \frac{T}{P}h\,u_m(t)\right] \tag{48}$$

where Q and P now should be of degree n_T+n-m-1, because A^m has been replaced by $\gamma_o\gamma_1$.

Now choose the polynomials according to

$$\begin{aligned} P_1 &= \gamma_1 \\ P_2 &= T \\ Q &= P = P_1P_2 \end{aligned} \tag{49}$$

Then (48) transforms into

$$e(t) = \frac{1}{\gamma_0}\left[b_0 \frac{u(t)}{\gamma_1} + b_0(BS - T)\frac{u(t)}{\gamma_1 T} + R\frac{y(t)}{\gamma_1 T} - \frac{h}{\gamma_1} u_m(t)\right] \tag{50}$$

which is the model used by Feuer and Morse. The estimation algorithm is the ordinary MRAS scheme. The control law is, however, a special and complicated one, derived to obtain stability of the closed loop, compare the discussion in section IV.

□

Example 4. Narendra's and Valavani's Scheme

The scheme by Narendra and Valavani is described in [9]. By choosing the "observer" order n_T to be equal to n-m-1 and the polynomials according to

$$\begin{aligned} P_1 &= L \\ P_2 &= TB^m/b_0^m \\ Q = P &= P_1P_2 \end{aligned} \tag{51}$$

the equation (32) transforms into

$$e(t) = \frac{B^mL}{b_0^mA^m}\left[b_0 \frac{u(t)}{L} + b_0\,(BS - TB^m/b_0^m)\frac{u(t)}{LTB^m/b_0^m} + \right.$$

$$\left. + R\frac{y(t)}{LTB^m/b_0^m} - \frac{1}{b_0^mL} u_m(t)\right] \tag{52}$$

This is the model used in [9] and the parameter estimation is analogous to the other MRAS described. The polynomial L is chosen to make the transfer function $B^mL/b_0^mA^m$ strictly positive real.

□

The above four examples describe algorithms that are proposed in the literature. However, the general algorithm presented in section IV has a lot of freedom in the choices of polynomials, estimation algorithm etc. Thus, many algorithms can easily be generated. A specific algorithm will be given below, which does not require the positive real condition. Further comments on the positive real condition are given in the following section.

Example 5. A New Algorithm

When using the model (32) and the ordinary MRAS estimation scheme, the positive real condition on Q/TA^m enters. A modification of the polynomial degrees, so that degree $(P_1) = n-m$ and degree (Q) = degree $(P) = n+n_T$, makes it very natural to choose

$$Q = P = TA^m \tag{53}$$

The transfer function $Q/TA^m = 1$ is automatically strictly positive real. However, since the degree of P_1 is n-m, it is possible to set $\hat{e}_f(t)$ to zero by the control law only in the case $n-m = 1$. Compare the discussion in section IV. The ordinary MRAS estimation scheme of course can be used in this case too.

□

VI. THE POSITIVE REAL CONDITION

The positive real condition has been seen to be an essential condition in order to guarantee convergence of the estimation error. The condition has been an attribute of the MRAS algorithms ever since Parks introduced the idea in [12]. However, it has been demonstrated in example 5 in the preceeding section that the condition can be removed if the polynomials Q and P are chosen in a special way. It is, in fact, possible to use a slightly different estimation algorithm in all the MRAS described, and eliminate the positive real condition in all cases discussed. Similar modifications to improve convergence properties of identification algorithms are discussed in [1].

Thus, write (35) as

$$e_f(t) = b_o\bar{u}_f(t) + b_o\theta^T\bar{\varphi}(t), \tag{54}$$

where '$^{-}$' denotes filtering by Q/TA^m. Let the estimate $\hat{e}_f(t)$ be given by

$$\hat{e}_f(t) = \hat{b}_o(t)\bar{u}_f(t) + \hat{b}_o(t)\hat{\theta}^T(t)\bar{\varphi}(t) \tag{55}$$

instead of (36). Introduce

$$\tilde{b}_o(t) = \hat{b}_o(t) - b_o \tag{56}$$

$$\tilde{\theta}(t) = \hat{\theta}(t) - \theta.$$

The estimation error $\varepsilon(t)$ in (37) satisfies the equation

$$\varepsilon(t) = -\tilde{b}_o(t)\left[\bar{u}_f(t) + \hat{\theta}^T(t)\bar{\varphi}(t)\right] - b_o\tilde{\theta}^T(t)\bar{\varphi}(t). \tag{57}$$

Now choose $\varepsilon^2(t)$ as a criterion. Regarding it as a function of $\tilde{b}_o$ and $\tilde{\theta}$, we have

$$\frac{\partial \varepsilon^2(t)}{\partial \tilde{b}_0} = -2\varepsilon(t)$$
(58)
$$\frac{\partial \varepsilon^2(t)}{\partial \tilde{\theta}} = -2\varepsilon(t) b_0 \bar{\varphi}(t).$$

It is natural to make the parameter adjustment in a modified steepest descent direction, i.e.

$$\begin{cases} \dot{\hat{b}}_0(t) = \frac{1}{r_0}\left[\bar{u}_f(t) + \hat{\theta}^T(t)\bar{\varphi}(t)\right]\varepsilon(t), \; r_0 \text{ positive constant} \\ \dot{\hat{\theta}}(t) = R^{-1}\bar{\varphi}(t)\varepsilon(t), \; R \text{ positive definite} \end{cases} \tag{59}$$

It is possible to verify that this estimation scheme has the desired stability property. Choose the Lyapunov function

$$V(t) = r_0 \tilde{b}_0^2(t) + \tilde{\theta}^T(t) R \tilde{\theta}(t) \tag{60}$$

Its derivative becomes

$$\dot{V}(t) = 2r_0 \tilde{b}_0(t)\dot{\hat{b}}_0(t) + 2b_0 \tilde{\theta}^T(t) R \dot{\hat{\theta}}(t) =$$

$$= 2\tilde{b}_0(t)\,[\bar{u}_f(t) + \hat{\theta}^T(t)\bar{\varphi}(t)]\,\varepsilon(t) + 2b_0\tilde{\theta}^T(t)\bar{\varphi}(t)\varepsilon(t) = -2\varepsilon^2(t) \tag{61}$$

and it follows that $\varepsilon(t)$ tends to zero.

The conclusion is that by modifying the estimation part, it is possible to eliminate the positive real condition in all the described MRAS. It should, however, be noted that the same situation occurs as in example 5, section IV. It is possible to have $\hat{e}_f(t) = 0$ without differentiators only in the case n-m = 1.

VII. CONCLUDING REMARKS

A fairly natural interpretation of the MRAS schemes has been given in the paper. They can be thought of as composed of two parts. The first is a parameter estimator based on a model obtained from analysis of the known parameter case. The second part consists of a feedback law based on the parameter estimates. A general adaptive algorithm with this two-step structure was defined and thereby several proposed MRAS algorithms could be treated within a general framework.

The most important implication of the analysis is perhaps that there are in principle no differences between the MRAS and the self-tuning regulators, which are derived from the separation principle described above. The general structure of the algorithms is the same for the two approaches and the differences appear only because of minor changes. The estimation part is perhaps the most striking difference. It has been shown in section VI that the positive real condition, which is an important condition for the MRAS, depends on the specific choice of estimation algorithm. It can be eliminated if another estimation scheme is chosen.

Finally, some extensions of the analysis will be indicated. Firstly, it is clear that the polynomial Q(p) in (30) could be replaced by Q(p) times a transfer function, which is proper, and analogously for P(p). The results would still be valid. Secondly, and more interesting, the development presented is possible to carry through for discrete time systems too. This is done in [13] and here the connections between the MRAS and the self-tuning regulators, which have most often been designed in discrete time, are examined in detail. The problem formulation in [13] includes noise and therefore the importance of the choice of observer polynomial is examined. Compare the discussion in section III. A similar development in the continuous time case should be possible to carry through, although it would require considerably more effort.

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