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UNIFICATION OF SOME ADAPTIVE CONTROL SCHEMES
PART I. CONTINUOUS TIME
PART II. DISCRETE TIME

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Stochastic Design

The deterministic design procedure can of course be used also when disturbances are acting on the plant. The choice of observer polynomial will, however, be of importance not only during an initial transient period. If it is assumed that w(t) is given by (3), then it is well-known that the optimal choice of observer polynomial is

$$T(q^{-1}) = C(q^{-1}),$$

in the sense of minimum variance. This is explicitely demonstrated in [5] as a generalization of the result on minimum variance regulators in [8].

III. CLASS OF ADAPTIVE CONTROLLERS

A general adaptive control scheme will be defined in this section. The scheme is a self-tuning version of the controller described in section II. It will be shown to include earlier proposed MRAS and STR as special cases.

The plant is still assumed to satisfy (1). The following assumptions are also introduced.

A1) The number of plant poles n and zeros m are known.

A2) The time delay k is known and the sign of b_0 is known. Without loss of generality b_0 is assumed positive.

A3) The plant is minimum phase (cf. section II).

Remark. Notice that some coefficients in $A(q^{-1})$ or $B(q^{-1})$ may be zero. Therefore, it suffices to know an upper bound on the polynomial degrees to put the equation into the form of (1) with known n and m. The condition on k in A2) is the counterpart of the continuous time condition, that the pole excess (i.e. the difference between number of poles and number of zeros) is known.

The objective of the controller is the same as in section II, i.e. to minimize the error defined by

$$e(t) = y(t) - y^M(t).$$

The controller to be described uses an implicit identification [7]. This means that the controller parameters are estimated instead of the parameters of the model (1). The first step in the development of the algorithm is therefore to obtain a model of the plant, expressed in the unknown controller parameters. Thus, use the identity (4) and the equations (1) and (2) to write for the error:

$$TA^M e(t) = TA^M y(t) - TA^M y^M(t) = (AS + q^{-(k+1)}R)y(t) - TA^M y^M(t) =$$

$$= q^{-(k+1)} [b_0 BSu(t) + Ry(t) - TB^M u^M(t)] + Sw(t). \qquad (5)$$

To obtain some flexibility of the model structure, a filtered version of the error will be considered. Let Q and P be stable polynomials, defined by

$$Q(q^{-1}) = 1 + q_1 q^{-1} + \ldots + q_{n_Q} q^{-n_Q}$$

$$P(q^{-1}) = P_1(q^{-1}) P_2(q^{-1}) = 1 + p_1 q^{-1} + \ldots + p_{n_P} q^{-n_P},$$

where P_1 and P_2 are factors of P of degree n_{P_1} and n_{P_2} respectively. It is assumed that $P_1(0) = P_2(0) = 1$.

Define the filtered error by

$$e_f(t) = \frac{Q(q^{-1})}{P(q^{-1})} e(t)$$

Using (5), $e_f(t)$ can be written as

$$e_f(t) = \frac{Q}{TA^M} q^{-(k+1)} \left[\frac{b_o BS}{P} u(t) + \frac{R}{P} y(t) - \frac{TB^M}{P} u^M(t)\right] + \frac{QS}{TA^M P} w(t) = \tag{6}$$

$$= \frac{Q}{TA^M} q^{-(k+1)} \left[b_o \frac{u(t)}{P_1} + b_o(BS - P_2) \frac{u(t)}{P} + R \frac{y(t)}{P} - \frac{TB^M}{P} u^M(t)\right] + \frac{QS}{TA^M P} w(t)$$

Remark. The polynomials Q and P give the necessary flexibility to cover both MRAS and STR. The exact choices of the polynomials and their degrees will be commented in the examples in section IV. It should also be noted that instead of polynomials Q and P, one could consider rational functions. We will not, however, elaborate this case.

The general adaptive controller will first be given for the deterministic design case.

Deterministic Design

The observer polynomial T is now determined a priori. Let θ be a vector, containing the unknown parameters of the polynomials $BS-P_2$ and R/b_o and the constant $1/b_o$ as the last element. Note that θ contains the parameters of the controller, described in section II.

Furthermore, define the vector $\varphi(t)$ from

$$\varphi^T(t) = \left[\frac{u(t-1)}{P}, \frac{u(t-2)}{P}, \ldots , \frac{y(t)}{P}, \frac{y(t-1)}{P}, \ldots , -\frac{TB^M}{P} u^M(t)\right] \tag{7}$$

where the numbers of u and y-terms are compatible with the definition of θ. Note that the elements of φ are known signals.

Using the definitions of θ and φ, it is possible to write (6) as

$$e_f(t) = \frac{Q}{TA^M} q^{-(k+1)} \left[b_o \frac{u(t)}{P_1} + b_o \theta^T \varphi(t)\right] + \frac{QS}{TA^M P} w(t). \tag{8}$$

This model, which involves the unknown parameters b_o and θ, can be taken as a basis for a class of adaptive controllers. The intention is to estimate the unknown parameters b_o and θ, and to use these estimates in the control law. Taking the different possibilities of choosing e.g. estimation algorithm and control law into consideration, a class of controllers can be characterized in the following way.

Basic control scheme:-

- o Estimate the unknown parameters b_o and θ (or some combination of these) in the model (8).

- o Use these estimates to determine the control signal.

A natural requirement on the controller is that it performs as the controller in section II, if the parameter estimates are equal to the true parameters.

Stochastic Design

The algorithm described above can of course be used also when $w \neq 0$. However, if $w(t)$ is given by (3) with an unknown C-polynomial, it was seen in section II that the choice $T = C$ is optimal. Since C is unknown it can be estimated. Some minor changes are then needed. Concatenate the θ-vector with a vector whose elements are the unknown parameters of C/b_o. Also, redefine the φ-vector as

$$\varphi^T(t) = \left[\frac{u(t-1)}{P}, \frac{u(t-2)}{P}, \ldots, \frac{y(t)}{P}, \frac{y(t-1)}{P}, \ldots, -\frac{B^M}{P} u^M(t), -\frac{B^M}{P} u^M(t-1), \ldots\right] \tag{9}$$

The filtered error can then be written as

$$e_f(t) = \frac{Q}{CA^M} q^{-(k+1)} \left[b_o \frac{u(t)}{P_1} + b_o \theta^T \varphi(t)\right] + \frac{QS}{A^M P} v(t) \tag{10}$$

which constitutes the model for a class of algorithms in the same way as in the deterministic case.

The class of algorithms described above contains many different schemes. Apart from the choice between fixed or estimated observer polynomial, the choices of control law and estimation algorithm generate different schemes. The choice of estimation algorithm will be commented upon in connection with some examples in section IV and further discussed in section V. To proceed, it is, however, convenient to specify one particular method.

A Special Parameter Estimator

A characteristic feature of the model reference methods is that the estimation is based on a model like (8), where the parameters b_o and θ enter bilinearly. The estimation scheme will be described in the deterministic design case.

Let $\hat{b}_o(t-1)$ and $\hat{\theta}(t-1)$ denote estimates at time t-1 of b_o and θ. Using the model (8), a one step ahead prediction of $e_f(t)$ is defined as

$$\hat{e}_f(t|t-1) = \frac{Q}{TA^M}\left[\hat{b}_o(t-1)\,\frac{u(t-k-1)}{P_1} + \hat{b}_o(t-1)\,\hat{\theta}^T(t-1)\varphi(t-k-1)\right]. \qquad (11)$$

The prediction error $\varepsilon(t)$ is defined as

$$\varepsilon(t) = e_f(t) - \hat{e}_f(t|t-1), \qquad (12)$$

where $e_f(t)$ is given by (8), and is usually used in the parameter updating. The following expression is obtained for $\varepsilon(t)$ if it is assumed that the disturbance w(t) is equal to zero:

$$\varepsilon(t) = \frac{Q}{TA^M}\Big[(b_o - \hat{b}_o(t-1))\Big(\frac{u(t-k-1)}{P_1} + \hat{\theta}^T(t-1)\varphi(t-k-1)\Big)+$$

$$+\, b_o(\theta-\hat{\theta}(t-1))^T\varphi(t-k-1)\Big]. \qquad (13)$$

In analogy with the continuous time case, the following parameter updating is used in the constant gain case:

$$\begin{bmatrix}\hat{b}_o(t)\\ \hat{\theta}(t)\end{bmatrix} = \begin{bmatrix}\hat{b}_o(t-1)\\ \hat{\theta}(t-1)\end{bmatrix} + \Gamma\begin{bmatrix}\frac{u(t-k-1)}{P_1} + \hat{\theta}^T(t-1)\varphi(t-k-1)\\ \varphi(t-k-1)\end{bmatrix}\varepsilon(t), \qquad (14)$$

where Γ is a constant, positive definite matrix.

Remark. It is straightforward to define stochastic approximation (SA) or least squares (LS) versions of the algorithm (14). For LS Γ is replaced by $P(t) \triangleq (\sum^{t}\psi(s)\psi^T(s))^{-1}$ and a SA variant uses e.g. $1/\mathrm{tr}P^{-1}(t)$ instead of Γ. Here

$$\psi(t) \triangleq \begin{bmatrix}\frac{u(t-k-1)}{P_1} + \hat{\theta}^T(t-1)\varphi(t-k-1)\\ \varphi(t-k-1)\end{bmatrix}.$$

The intention with the algorithm (14) is to exploit the properties of a strictly positive real transfer function in order to establish convergence of $\varepsilon(t)$ to zero. The motivation is the successful use of the Kalman-Yakubovich lemma in continuous time, [1]. The problems which arise will be discussed next. Let us just briefly comment upon the stochastic design case. The algorithm given by (11), (12), and (14) cannot be applied directly to the model (10), because the C-polynomial is unknown. This implies that the prediction cannot be calculated according to (11). An easy modification is to replace C in front of the parenthesis with an a priori estimate of C or even with unity.

Choice of Control Law

The control law, given in section II, can be written as

$$u(t) = - P_1(q^{-1})(\theta^T\varphi(t)),$$

where θ is the vector of true parameters. Any reasonable control law should equal this one when the parameter estimates are correct. Notice that a parameter estimator like (14) has the objective to force the prediction error $\varepsilon(t)$ to zero. It would thus be desirable to choose a control such that $\hat{e}_f(t|t-1)$ is equal to zero, because convergence of $e_f(t)$ to zero would then follow from the convergence of $\varepsilon(t)$ to zero, cf. (12). This implies the use of the control law

$$u(t) = -P_1(q^{-1})(\hat{\theta}^T(t+k)\varphi(t))$$

as seen from (11). This control law is, however, non-causal. It is therefore natural to modify it in the following way:

$$u(t) = -P_1(q^{-1})(\hat{\theta}^T(t)\varphi(t)). \tag{15}$$

This control law is used in almost all control schemes of the type considered.

Difficulties with Convergence Analysis

There are two key problems in the analysis of the schemes of MRAS type described above. The first problem is that the control law (15) has to be used if a causal control law is required. This implies that $\hat{e}_f(t|t-1)$ is not equal to zero in the case $k \neq 0$. This in turn means that it is not easy to conclude that $e_f(t)$ tends to zero even if $\varepsilon(t)$ tends to zero.

The second problem is to show that $\varepsilon(t)$ tends to zero. Consider for simplicity the case $k = 0$, which is analogous to the case for continuous time systems, where the pole excess is equal to one, cf. [1]. Then $\varepsilon(t)$ is equal to $e_f(t)$ if the control law (15) is used. Contrary to the continuous time case, convergence of $e_f(t)$ to zero cannot be proved straightforwardly. The reason is the following one. If the control law (15) is used and it is assumed that $b_o = 1$, the equation (13) can be written

$$\varepsilon(t) = e_f(t) = H(q^{-1})\cdot q^{-1}\,[-\tilde{\theta}^T(t)\varphi(t)]. \tag{16}$$

Here

$$H(q^{-1}) = \frac{Q(q^{-1})}{T(q^{-1})A^M(q^{-1})}$$

and

$$\tilde{\theta}(t) = \hat{\theta}(t) - \theta.$$

In continuous time the estimation error $\varepsilon(t)$ is given by

$$\varepsilon(t) = G(p)[-\tilde{\theta}^T(t)\varphi(t)].$$

Positive realness of $G(p)$ can be used to prove the convergence of $\varepsilon(t)$ to zero. It is, however, not possible to use the same arguments in discrete time, because the transfer function $H(q^{-1})\cdot q^{-1}$ can never be made positive real. The difference appears because a discrete time transfer function must contain a feedthrough term to be strictly positive real, whereas a continuous time transfer function may be strictly proper. This difficulty is also emphasized in [2].

The problem with the time delay and also, in the case $k \neq 0$, the previously mentioned problem to relate convergence of $\varepsilon(t)$ and $e_f(t)$ are closely related to the boundedness of the signals of the closed loop system. This can be seen in e.g. [4]. It seems that no general solution to the problems has been presented so far.

IV. EXAMPLES OF THE GENERAL CONTROL SCHEME

Some special cases of the procedure proposed in section III will now be given. Both model reference adaptive systems and self-tuning regulators will be shown to fit into the general description.

Example 1. Ionescu's and Monopoli's Scheme

The scheme by Ionescu and Monopoli is presented in [4] and is a straightforward translation of the continuous time MRAS by Monopoli [9] into discrete time. The continuous time procedure was commented upon in detail in [1]. In the same way it is possible to treat the scheme in [4] as a special case of the general algorithm. Choose the polynomials as

$P_1 = T$ of degree k
P_2 of degree $n-1$
$Q = P = P_1P_2$ of degree $n+k-1$.

The equation (6) then transforms into

$$e_f(t) = e(t) = \frac{P_2}{A^M} q^{-(k+1)} \left[b_o \frac{u(t)}{P_1} + b_o(BS-P_2) \frac{u(t)}{P} + R \frac{y(t)}{P} - \frac{B^M}{P_2} u^M(t) \right], \quad (17)$$

where the disturbance w has been assumed to be zero as in [4]. This is the model used by Ionescu and Monopoli and the estimation scheme is similar to the one in (14). The polynomial P_2 is chosen to make P_2/A^M strictly positive real. Some modifications of the estimation scheme are done to handle the problems discussed at the end of the preceeding section, although no complete solution is presented. The concept of 'augmented error', introduced by Monopoli in [9], is translated into discrete time. It can be shown that the augmented error $\eta(t)$ is given by

$$\eta(t) = \varepsilon(t) - \frac{P_2}{A^M}(K_\eta\, \eta(t)|\varphi(t-1)|^2), \quad K_\eta \text{ constant.}$$

It is shown in [4] that $\eta(t)$ tends to zero, but a boundedness assumption is needed in order to establish convergence of $\varepsilon(t)$ or $e_f(t)$. Finally, it should be noted that P_1 and P_2 are called Z_f and Z_w in [4].

□

Example 2. Bénéjean's Scheme

Bénéjean presented both a continuous time and a discrete time algorithm in [3]. It was shown in [1] that the continuous time scheme is closely related to Monopoli's scheme. In the same way it can be shown that the discrete time controller is very similar to Ionescu's and Monopoli's scheme. The model used by Bénéjean is obtained by reparametrizing (17) as follows:-

$$e_f(t) = e(t) = \frac{P_2}{A^M} q^{-(k+1)} \left[b_o \frac{u(t)-u^M(t)}{P_1} + b_o(BS-P_2)\frac{u(t)-u^M(t)}{P} + \right.$$

$$\left. + R \frac{y(t)}{P} + (b_oBS-B^MP_1)\frac{u^M(t)}{P}\right]$$

The estimation algorithm is similar to the one used by Ionescu and Monopoli. As in the continuous time case, more parameters have to be estimated because of the reparametrization.

□

In the two MRAS examples above the natural choice Q = P has been used. This implies that the filtered error $e_f(t)$ equals the error $e(t)$. Another possibility is to choose the polynomials so that the transfer function Q/TA^M becomes very simple. This is done below.

Example 3. Self-tuning Pole Placement Algorithm

A pole placement algorithm with fixed observer polynomial is described in [7]. It can be generated from the general structure in the following way. Choose the polynomials $Q = TA^M$ and $P = 1$, i.e. $e_f(t) = TA^M e(t)$. This implies that (6) has the simple form

$$e_f(t) = q^{-(k+1)} [b_o u(t) + b_o \theta^T \varphi(t)], \tag{18}$$

where the elements of φ are simply lagged input and output signals. The noise has been assumed to be zero. The model (18) is used in [7] with a minor modification. The parameters which are estimated by a least squares algorithm are b_o and $b_o\theta$. As the last element in θ is $1/b_o$, the effect is that one parameter is known to be equal to one. If θ and φ are redefined not to include the last known element, the equation (18) can be written as

$$e_f(t) = TA^M(y(t) - y^M(t)) = q^{-(k+1)} \left[b_0 u(t) + b_0 \theta^T \varphi(t)\right] - TA^M y^M(t),$$

which is the model used in [7]. □

In the three examples above the choice of observer polynomial T was made in advance. However, if there is noise of the form given by (3), the optimal choice of observer polynomial is T = C, see section II. Since C is unknown, it must be estimated as described in section III. Below some schemes of this type will be described.

<u>Example 4. Åström's and Wittenmark's Self-tuning Regulator</u>

The self-tuning regulator by Åström and Wittenmark is described in [10]. It is based on a minimum variance strategy, which minimizes the output variance. This is a special case of the problem considered in section II with $A^M = 1$ and $u^M = y^M = 0$. Inserting this into (6) and using the polynomials Q = P = 1, the following is obtained

$$e_f(t) = y(t) = \frac{1}{C} q^{-(k+1)} \left[b_0 u(t) + b_0(BS-1)u(t) + Ry(t)\right] + Sv(t)$$

This model can be written analogously with (10) as

$$e_f(t) = y(t) = \frac{1}{C} q^{-(k+1)} \left[b_0 u(t) + \theta^T \varphi(t)\right] + Sv(t) \tag{19}$$

and is the basis for the self-tuning regulator. Since C is unknown, the prediction is chosen as in (11) with T = C replaced by unity, cf. the discussion in section II:-

$$\hat{e}_f(t|t-1) = \hat{y}(t|t-1) = \hat{b}_0(t-1)u(t-k-1) + \hat{\theta}^T(t-1)\varphi(t-k-1). \tag{20}$$

The fact that C is included in (19) but not in (20) makes it somewhat unexpected that the algorithm really converges to the optimal minimum variance regulator. It is shown in [11] that the scheme (with a stochastic approximation estimation algorithm) converges if 1/C is strictly positive real. If instead a least squares estimation algorithm is used, convergence holds if 1/C - 1/2 is SPR. The condition on 1/C and its relation to the positive real condition for MRAS will be further examined in the following section. □

Example 5. Clarke's and Gawthrop's Self-tuning Controller

Clarke and Gawthrop consider a 'generalized output'

$$\Phi(t) = P(q^{-1})y(t) + Q(q^{-1})u(t-k-1) - R(q^{-1})u^M(t-k-1)$$

and applies the basic self tuner to the system generating this output, [12]. It is possible to treat the algorithm within the general structure in the special case Q = 0 in their notation. Thus, change the notation into:

$$\Phi(t) = A^M(q^{-1})y(t) - q^{-(k+1)}B^M(q^{-1})u^M(t).$$

Then it follows that $\Phi(t)$ equals $e_f(t) = A^Me(t)$ if $P = 1$ and $Q = A^M$. If the noise is given by (3) and T is chosen to be equal to C, the equation (6) can be written as

$$e_f(t) = \frac{1}{C} q^{-(k+1)} \left[b_o u(t) + b_o(BS-1)u(t) + Ry(t) - CB^M u^M(t)\right] + Sv(t)$$

This is the model used in [12], where also the fact that the first parameter in C is known to be unity is exploited. The prediction is calculated as in example 4, i.e. C is replaced by 1. The estimation part consists of a least squares algorithm. □

V. THE POSITIVE REAL CONDITION

A special model structure and a specific estimation scheme were described in section III. The structure was obtained from an analogy with the model reference adaptive systems in continuous time. The intention was to use the properties of positive real transfer functions to establish convergence. It was noted in example 4, section IV, that a positive real condition also appears in the analysis of a self-tuning regulator in the presence of noise. The relations between the conditions in the two cases will be treated below.

First consider the deterministic design case and for simplicity assume that $k = 0$ and $b_o = 1$. If the control law (15) is used, we have from (16)

$$\varepsilon(t) = -H(q^{-1})[\tilde{\theta}^T(t-1)\varphi(t-1)]. \tag{21}$$

We want to show in a simple way that a positive real condition really appears in the analysis in a natural way. To do so, assume that a modified version of the parameter updating (14) is used:-

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \frac{\varphi(t-1)}{|\varphi(t-1)|^2}\varepsilon(t). \tag{22}$$

This algorithm is similar to stochastic approximation schemes and is used in e.g. [4].

Subtract the true parameter vector θ from both sides, multiply from the left by its transpose and use (21) to get

$$|\tilde{\theta}(t)|^2 = |\tilde{\theta}(t-1)|^2 + 2\frac{\tilde{\theta}^T(t-1)\varphi(t-1)}{|\varphi(t-1)|^2}\varepsilon(t) + \frac{\varepsilon^2(t)}{|\varphi(t-1)|^2} =$$

$$= |\tilde{\theta}(t-1)|^2 - 2\frac{\varepsilon(t)\left(\dfrac{\varepsilon(t)}{H(q^{-1})}\right)}{|\varphi(t-1)|^2} + \frac{\varepsilon^2(t)}{|\varphi(t-1)|^2} =$$

$$= |\tilde{\theta}(t-1)|^2 - 2\frac{\varepsilon(t)[1/H-1/2]\varepsilon(t)}{|\varphi(t-1)|^2}. \tag{23}$$

It can be seen that the positive real condition enters in a natural way. If $1/H - 1/2$ is positive real, the parameter error will eventually decrease. Moreover, $\varepsilon(t)/|\varphi(t-1)|$ tends to zero if $1/H - 1/2$ is SPR.

It should be noted that the boundedness condition mentioned at the end of section III appears because (23) only proves convergence of $\varepsilon(t)/|\varphi(t-1)|$.

It is straightforward to use the ideas in [1] to show that the positive real condition can be avoided. Thus, let $\bar{x}$ denote the signal obtained by filtering x by Q/TA^M and rewrite (8) as

$$e_f(t) = q^{-1}\left[\frac{\bar{u}(t)}{P_1} + \theta^T\bar{\varphi}(t)\right],$$

where the same assumptions as above are used. Now consider this as being the model instead of (8). The prediction (11) is then replaced by

$$\hat{e}_f(t|t-1) = \frac{\bar{u}(t-1)}{P_1} + \hat{\theta}^T(t-1)\bar{\varphi}(t-1) \tag{24}$$

and instead of (21) we have

$$\varepsilon(t) = -\tilde{\theta}^T(t-1)\bar{\varphi}(t-1).$$

If the parameter updating (22) is replaced by

$$\tilde{\theta}(t) = \tilde{\theta}(t-1) + \frac{\bar{\varphi}(t-1)}{|\bar{\varphi}(t-1)|^2}\,\varepsilon(t) \tag{25}$$

the following is obtained:-

$$|\tilde{\theta}(t)|^2 = |\tilde{\theta}(t-1)|^2 + 2\,\frac{\tilde{\theta}^T(t-1)\bar{\varphi}(t-1)}{|\bar{\varphi}(t-1)|^2}\,\varepsilon(t) + \frac{\varepsilon^2(t)}{|\bar{\varphi}(t-1)|^2} =$$

$$= |\tilde{\theta}(t-1)|^2 - \frac{\varepsilon^2(t)}{|\bar{\varphi}(t-1)|^2}\,.$$

It thus follows that $\varepsilon(t)/|\bar{\varphi}(t-1)|$ tends to zero without any positive real condition. Of course, the boundedness of the closed loop signals mentioned in section III is still a problem. The conclusion is that it is possible to eliminate the positive real condition in the deterministic design case if a modified estimation scheme is used.

Now, consider the stochastic design, where the observer polynomial C is estimated. The transfer function $H(q^{-1})$, which was previously known, now contains the unknown C-polynomial. This implies that the filtering in (24) and (25) cannot be done with the true C-polynomial. The positive real condition then enters in the same way as in ex. 4, section IV. The positive real condition on $H(q^{-1}) = 1/C(q^{-1})$ and a boundedness condition are in fact sufficient to assure convergence for the self-tuning regulator in ex. 4, see [11]. A natural modification in order to weaken the condition on C is to filter with $1/\hat{C}(t)$, where $\hat{C}(t)$ is the time-varying estimate of C. This is discussed in [11].

The conclusion of the discussion above is that the positive real condition, which appears in the analysis of both deterministic MRAS and stochastic STR, are of a similar technical nature. There is, however, an important difference. The condition can be eliminated for the deterministic case by choosing another estimation algorithm, which includes filtering by the transfer function $H(q^{-1})$. In the stochastic case, the positive real condition is not possible to be dispensed with, because the filter is unknown.

VI. CONCLUDING REMARKS

An attempt to describe both model reference adaptive regulators and self-tuning regulators in a unified manner has been made. It has been shown that MRAS can be derived from the STR point of view and that both types of schemes can be thought of as composed of two parts. The first is a parameter estimator based on a model obtained from analysis of the known parameter case. The second part consists of a feedback law, which is identical to the known parameter case but with the estimates replacing the unknown parameters.

The basic structure of the two approaches is the same, but some differences appear. For example, the optimal observer polynomial is estimated in several STR, whereas MRAS use a fixed observer. There is also an important difference in the estimation structure. The estimation in MRAS is based on a model, which is bilinear in the parameters. Furthermore, positive realness of a certain transfer function plays an important role in the design of estimation scheme. The relation between the choice of estimation algorithm and the positive real condition was examined. The conclusion was that the condition can be dispensed with in the deterministic design, but that the condition is important when the observer is estimated in the stochastic design situation.

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UNIFICATION OF SOME ADAPTIVE CONTROL SCHEMES - PART II, DISCRETE TIME

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Abstract

Adaptive control can be approached from many different points of view. In recent years there have been much progress made both on model reference adaptive systems (MRAS) and on self-tuning regulators (STR). The two approaches are here treated in a general framework. It is shown that MRAS can be derived from the STR point of view. Special attention is paid to the positive real condition, which appears in the analysis of both types of schemes. It is shown that this condition can be dispensed with in the deterministic case. Some problems, specific for the discrete time case, are also examined.

I. INTRODUCTION

Adaptive control of constant but unknown plants in continuous time was considered in an accompanying paper, [1]. The purpose of that paper was to show that two common approaches, the model reference adaptive systems (MRAS) and the self-tuning regulators (STR), are essentially equivalent. This paper gives the corresponding treatment of discrete time systems.

The MRAS philosophy has been applied to the discrete time case in [2 - 4]. Stability theory is the major design tool. The STR approach has been used almost exclusively for discrete time systems. The basic idea is to use a certainty equivalence structure, i.e. to use a control law for the known parameter case and just replace the unknown parameters by their estimates. Since the control algorithms obtained by the MRAS and the STR approaches are very similar, it is of interest to investigate the connections between the two approaches. Results in this direction are given in [5] and [6].

The purpose of the present paper is to provide a unified treatment of both MRAS and STR for problems with output feedback. It will be shown that MRAS can be derived using the STR approach.

The nature of the positive real condition, associated with both types of schemes, will be examined in detail. It is shown that this condition can be avoided in the deterministic case.

The paper is organized as follows. A design procedure for known plants is described in section II. Using this design method as a basis, a class of adaptive controllers is defined in section III. Some problems, which are associated with the design and analysis of the schemes, are also discussed.

In section IV, it is shown that both MRAS and STR can be treated as special cases of the algorithm proposed in section III. The positive real condition is examined in section V. Concluding remarks are finally given in section VI.

II. DESIGN METHOD FOR KNOWN PLANTS

A design method, which will be the basis for the general adaptive algorithm in the next section, is described below. It is a straightforward translation of the procedure described in [1] into discrete time. It consists essentially of a pole placement. Related schemes can be found in e.g. [3 - 5] and [7].

The plant is assumed to satisfy the difference equation

$$A(q^{-1})y(t) = q^{-(k+1)}b_oB(q^{-1})u(t) + w(t), \tag{1}$$

where q^{-1} is the backward shift operator, k is a nonnegative integer and $A(q^{-1})$ and $B(q^{-1})$ are polynomials defined by

$$A(q^{-1}) = 1 + a_1q^{-1} + \ldots + a_nq^{-n}$$

$$B(q^{-1}) = 1 + b_1q^{-1} + \ldots + b_mq^{-m}.$$

Furthermore, w(t) is a nonmeasurable disturbance. The parameter b_o is not included in the B-polynomial, because it will be treated in a special way in the estimation part of the adaptive controller in section III.

The objective of the controller design is to make the difference between the plant output y(t) and the reference model output $y^M(t)$ as small as possible in some sense. The reference output y^M is related to the command input u^M by the reference model given by

$$y^M(t) = \frac{q^{-(k+1)}B^M(q^{-1})}{A^M(q^{-1})}u^M(t) = \frac{q^{-(k+1)}(b_o^M+b_1^Mq^{-1}+\ldots+b_m^Mq^{-m})}{1+a_1^Mq^{-1}+\ldots+a_n^Mq^{-n}}u^M(t) \tag{2}$$

where coefficients in the polynomials may be zero. It is seen that the time delay in the reference model is greater than or equal to the time delay in the plant. This is a natural assumption to avoid non-causal control laws.

The problem will be approached by assuming a controller configuration as seen in fig. 1.

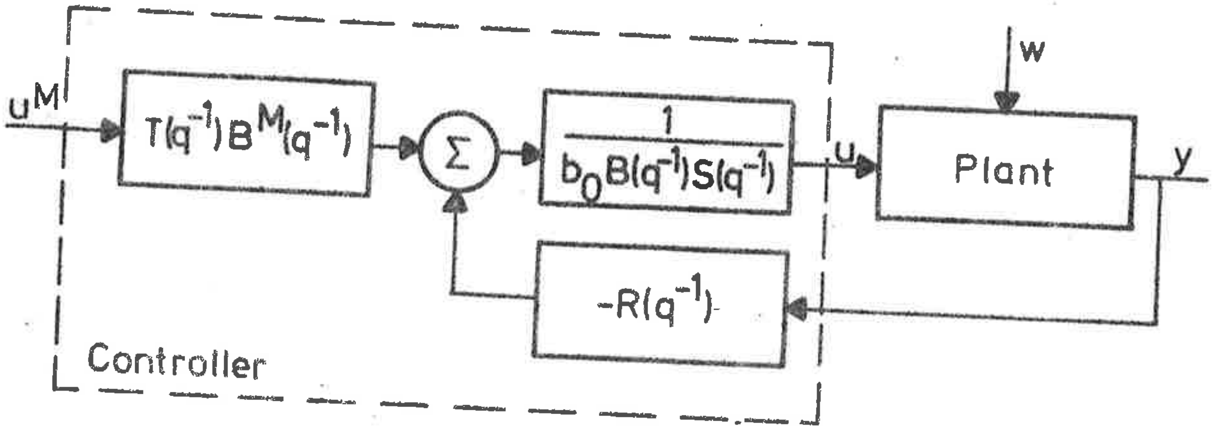

Fig. 1. Controller Configuration

Here R, S and T are polynomials in the backward shift operator. Motivation for this structure can be found in e.g. [7]. It can be shown that the controller is closely related to the solution from an ordinary state space set-up with Kalman filter and feedback from the state estimates. Notice that the process zeros are cancelled. This implies that only minimum phase systems can be considered. Other versions which allow nonminimum phase systems are discussed in [7]. The T-polynomial can be interpreted as the characteristic polynomial of an observer.

The design procedure will be given for two different problems. In the first one, the disturbances are neglected and the problem is treated as a pure servo-problem. This means that the design concentrates on tracking a given reference signal. The procedure will be referred to as a deterministic design. On the other hand, if the disturbance is considered as part of the problem, the controller should have a regulating property too and the design is stochastic.

An interesting special case of the stochastic problem is when the disturbance w(t) is a moving average given by

$$w(t) = C(q^{-1})v(t) = (1 + c_1 q^{-1} + \ldots + c_n q^{-n})v(t), \tag{3}$$

where $\{v(t)\}$ are independent, zero-mean random variables. The deterministic design is considered first.

Deterministic Design

Assuming $w(t) = 0$, it is possible to have the plant output equal to the reference model output $y^M(t)$. This is obtained by making the closed-loop transfer function equal to the reference model transfer function, i.e.

$$\frac{q^{-(k+1)}B^M(q^{-1})}{A^M(q^{-1})} = \frac{q^{-(k+1)}b_0B(q^{-1})T(q^{-1})B^M(q^{-1})}{A(q^{-1})b_0B(q^{-1})S(q^{-1})+q^{-(k+1)}b_0B(q^{-1})R(q^{-1})}$$

or, equivalently,

$$T(q^{-1})A^M(q^{-1}) = A(q^{-1})S(q^{-1}) + q^{-(k+1)}R(q^{-1}). \tag{4}$$

The observer polynomial T is cancelled in the closed-loop transfer function. Neglecting the effects of initial values, it can therefore be chosen arbitrarily. When T has been determined, the equation (4) has many solutions R and S. It will, however, be required that the degree of S is less than or equal to the time delay k. Then there is a unique solution to (4). The degree of R will depend on n, k, and the degree of T. Furthermore, it is required that $S(0) \neq 0$ in order to get a causal control law. As seen from (4) this is equivalent to $T(0) \neq 0$. Finally, the S and T-polynomials are scaled so that $T(0) = S(0) = 1$. The deterministic design procedure can thus be summarized in the following steps:

1) Choose the polynomial $T(q^{-1})$ defined by

$$T(q^{-1}) = 1 + t_1q^{-1} + \ldots + t_{n_T}q^{-n_T}.$$

2) Solve the polynomial equation

$$T(q^{-1})A^M(q^{-1}) = A(q^{-1})S(q^{-1}) + q^{-(k+1)}R(q^{-1})$$

for the unique solutions $R(q^{-1})$ and $S(q^{-1})$, defined by

$$R(q^{-1}) = r_0 + r_1q^{-1} + \ldots + r_{n_R}q^{-n_R}$$

$$S(q^{-1}) = 1 + s_1q^{-1} + \ldots + s_kq^{-k}.$$