



LUND UNIVERSITY

Evaluation of A Subroutine for Nelder and Mead Search

Mattsson, Sven-Erik

1978

Document Version:

Publisher's PDF, also known as Version of record

[Link to publication](#)

Citation for published version (APA):

Mattsson, S.-E. (1978). *Evaluation of A Subroutine for Nelder and Mead Search*. (Technical Reports TFRT-7150). Department of Automatic Control, Lund Institute of Technology (LTH).

Total number of authors:

1

General rights

Unless other specific re-use rights are stated the following general rights apply:

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

Read more about Creative commons licenses: <https://creativecommons.org/licenses/>

Take down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

LUND UNIVERSITY

PO Box 117
221 00 Lund
+46 46-222 00 00

EVALUATION OF A SUBROUTINE FOR NELDER
AND MEAD SEARCH

SVEN ERIK MATTSSON

**TILLHÖR REFERENSBIBLIOTEKET
UTLÄNAS EJ**

Department of Automatic Control
Lund Institute of Technology
September 1978

Dokumentutgivare
Lund Institute of Technology
Handläggare Dept of Automatic Control
Sven Erik Mattsson
Författare
Sven Erik Mattsson

Dokumentnamn
REPORT LUTFD2/(TFRT-7150)/1-92/
Utgivningsdatum
Sept 1978
Dokumentbeteckning
Årendebeteckning (1978)
06T6

10T4

Dokumenttitel och undertitel 18T0 Evaluation of a Subroutine for Nelder and Mead Search		
Referat (sammandrag) 26T0 This paper presents a Fortran IV subroutine called NELME that is a straightforward implementation of the simplex method for <u>function minimization</u> proposed by Nelder and Mead. NELME is written and documented according to the rules of the Scandinavian Control Library. NELME has been tested and compared with a quasi-Newton algorithm without derivatives and with the Powell-Brent algorithm. The result of the tests shows that it is hard to rank the algorithms.		
Referat skrivet av Author		
Förslag till ytterligare nyckelord Nelder-Mead search, nonlinear programming, nonlinear optimization, parametric optimization, simplex method		
Klassifikationssystem och -klass(er) 50T0		
Indextermer (ange källa) 52T0		
Omfång 92 pages	Övriga bibliografiska uppgifter 56T2	
Språk English		
Sekretessuppgifter 60T0	ISSN 60T4	ISBN 60T6
Dokumentet kan erhållas från 42T0 Department of Automatic Control Lund Institute of Technology PO Box 725, S-220 07 Lund 7, SWEDEN		Mottagarens uppgifter 62T4
Pris 66T0		

DOKUMENTTABLAD enligt SIS 62 10 12

Abstract

This paper presents a Fortran IV subroutine called NELME that is a straightforward implementation of the simplex method for function minimization proposed by Nelder and Mead. NELME is written and documented according to the rules of the Scandinavian Control Library.

NELME has been tested and compared with a quasi-Newton algorithm without derivatives and with the Powell-Brent algorithm.

The result of the tests shows that it is hard to rank the algorithms.

1. INTRODUCTION

This paper presents a Fortran IV subroutine called NELME that is a straightforward implementation of the Nelder and Mead search. NELME has been tested and compared with a quasi-Newton algorithm without derivatives and with the Powell-Brent algorithm.

Chapter 2 describes NELME and the computer codes can be found in Appendix A. Chapter 3 treats the problem of evaluating NELME and test results are presented. Appendix B contains additional test results.

2. NELME

NELME is a Fortran IV subroutine that is a straightforward implementation of a simplex method for function minimization proposed by Nelder and Mead (1964). In this implementation the reflection coefficient is chosen to 1, the contraction coefficient to 0.5 and the expansion coefficient to 2.

NELME is written and documented according to the rules of the Scandinavian Control Library and consequently the subroutine within itself contains all adequate information how it is used. A listing of the subroutine NELME can be found in Appendix A.

3. EVALUATION

In Himmelblau (1972) chapter 5 and in Brent (1973) section 7.7 the problem of evaluate algorithms for unconstrained nonlinear programming is discussed and a number of algorithms are evaluated. The inventors have of course evaluated their method and Himmelblau (1972) discusses explicitly the method of Nelder and Mead.

Five different functions were used to test NELME. The functions, whose minimum value is zero, were:

- (1) Rosenbrock's parabolic valley (Rosenbrock (1960))

$$y = 100(x_2 - x_1^2)^2 + (1 - x_1)^2,$$

starting point (-1.2, 1),

minimum point (1, 1)

- (2) Wood's function (Colville (1968))

$$y = 100(x_2 - x_1^2)^2 + (1 - x_1)^2 + 90(x_4 - x_3^2)^2 + (1 - x_3)^2 \\ + 10.1((x_2 - 1)^2 + (x_4 - 1)^2) + 19.8(x_2 - 1)(x_4 - 1),$$

starting point (-3, -1, -3, -1),

minimum point (1, 1, 1, 1)

- (3) Fletcher and Powell's helical valley (Fletcher and Powell (1963))

$$y = 100(x_3 - 100(x_1, x_2))^2 + (\sqrt{(x_1^2 + x_2^2)} - 1)^2 + x_3^2$$

$$\text{where } 2\pi\theta(x_1, x_2) = \begin{cases} \arctan(x_2/x_1), & x_1 > 0 \\ \pi + \arctan(x_2/x_1), & x_1 < 0 \end{cases}$$

starting point (-1, 0, 0)

minimum point (1, 0, 0)

(4) Powell's quartic function (Powell (1962))

$$y = (x_1 + 10x_2)^2 + 5(x_3 - x_4)^2 + (x_2 - 2x_3)^4 + 10(x_1 - x_4)^4,$$

starting point (3,-1,0,1) ×

minimum point (0,0,0,0)

(5) A quadratic function with truncated linear terms

$$y = x_1^2 + e(|x_1|) + 5x_2^2 + e(x_2),$$

where $e(z) = \text{sign}(z) \cdot n$, n is the largest integer $\leq |z|$,

starting points (± 2 , ± 2),

minimum point (0,0)

The properties of functions 1-4 are discussed in Brent (1973).

The progress of NELME on function 1-5 has been studied for different sizes of the initial simplex. The number of iterations, the number of function evaluations and the value of the test quantity (TESTQ) were noted when the function value had been reduced to 10^{-j} for $j = 1, 3, 5, 7$. TESTQ (to be compared in the algorithm with the desired accuracy in the minimum value) is given by

$$\text{TESTQ} = \sqrt{\frac{1}{n+1} \sum_{i=1}^{n+1} (f(x_i) - f(x_c))^2}$$

where n is the dimension of the optimization problem, the x_i 's are the vertices and x_c the centroid of the polyhedron and $f(\cdot)$ the function to be minimized.

The results can be found in Appendix B. The maximum number of times the loss function could be evaluated was set to 2000. If a number in the column "number of evaluations required" is 2000 or greater, it means that the polyhedron has degenerated (to a point) before reaching the minimum point.

In order to get a feeling for the relative efficiency of NELME, the problems 1-5 were solved with the help of NUFLET and POWBRE. NUFLET is based on a quasi-Newton method without derivatives. The method is described in Fletcher (1971). POWBRE is an implementation of a version of Powell's algorithm, modified by Brent (1973). Powell's algorithm is a conjugate direction method without derivatives. The computer codes (in Fortran) can be found in Källström (1978). NUFLET and POWBRE have some additional parameters that have to be set. The values of these parameters have been chosen according to Källström (1978),
i.e.

in NUFLET DFN = -0.2 (estimate of the likely reduction
 to be obtained in $f(x)$ is $0.2|f(x_0)|$.
 DFN is only used on the first iteration
 so an order of magnitude estimate
 suffices)

 XM=[1,...,1] (no scaling wanted)

 HH= 10^{-3} (the step length used when calculating
 the gradient is 10^{-3})

 EPS= 10^{-5} (the accuracy required in x_i is 10^{-5})

 MODE=1 (the initial estimate of the Hessian
 matrix is set to the unit matrix)

and

in POWBRE DIST=1 (estimated distance from initial
 approximation to minimum)

 SCALE=1 (no scaling wanted)

 TOL= 10^{-6} (wanted relative accuracy in x)

 MODE=1 (the algorithm is started with the
 coordinate axes as search directions)

 ILLCO=,TRUE. (the problem is supposed to be illcondi-
 tioned)

 NSTOP=1 (number of iterations without progress
 before termination)

The results for NELME with VDIST=1 (initial size of the simplex), NUFLET and POWBRE on functions 1-4 are given in Table 1-4.

Table 1

Number of iterations (n_i) and number of function evaluations (n_f) to reduce Rosenbrock's function to $< 10^{-j}$.

j	NELME		NUFLET		POWBRE	
	n_i	n_f	n_i	n_f	n_i	n_f
1	59	185	23	99	37	102
2	75	231	29	140	47	127
3	82	253	31	152	52	140
4	83	256	32	158	52	140
5	86	268	33	164	52	140
6	88	276	34	170	57	152
7	89	279	35	176	57	152

Table 2

Number of iterations (n_i) and number of function evaluations (n_f) to reduce Wood's function to $< 10^{-j}$.

j	NELME		NUFLET		POWBRE	
	n_i	n_f	n_i	n_f	n_i	n_f
1	30	105	68	664	253	681
2	35	125	72	704	265	709
3	35	125	73	714	270	719
4	81	269	74	724	287	758
5	94	310	75	734	287	758
6	103	344	76	744	294	777
7	108	358	77	754	299	787

Table 3

Number of iterations (n_i) and number of function evaluations (n_f) to reduce Fletcher and Powell's helical valley to $< 10^{-j}$.

j	NELME		NUFLET		POWBRE	
	n_i	n_f	n_i	n_f	n_i	n_f
1	154	480	18	102	43	110
2	172	533	24	158	57	143
3	185	576	28	191	63	158
4	189	590	29	199	63	158
5	193	606	29	199	67	166
6	196	616	30	207	67	166
7	200	632	31	215	73	180

Table 4

Number of iterations (n_i) and number of function evaluations (n_f) to reduce Powell's quartic function to $< 10^{-j}$.

j	NELME		NUFLET		POWBRE	
	n_i	n_f	n_i	n_f	n_i	n_f
1	31	99	9	61	38	91
2	56	177	10	67	43	101
3	68	206	12	79	48	111
4	70	218	22	161	55	130
5	76	238	23	171	65	152
6	78	246	25	191	72	171
7	85	276	27	211	77	182

The results speak for themselves and only a few comments will be given here. As seen from the results in Appendix B the size of the initial simplex has a significant effect on the speed of the convergence. Wood's function is an illustrative example. The result shown in Table 2 (the size of the initial simplex is 1) is very good, but NELME fails for some sizes of the initial simplex. POWBRE too had difficulties and stopped at $(-0.988, 0.986, -0.949, 0.913)$ and declared that this point is a minimum. But when NSTOP was increased to 5, POWBRE found the minimum point and it is these results that are given in Table 2. Brent (1973) reports that with DIST=10 POWBRE after 191 linear searches and 452 function evaluations had reduced the function value to $6 \cdot 10^{-14}$.

This dependence, which is not *a priori* known, makes it difficult to rank NELME, NUFLET and POWBRE. Function 5 is discontinuous when x_1 or x_2 is an integer and one may suspect that NUFLET and POWBRE might fail. As seen from the results in Table 5 this is the case.

If no special information is available that can confirm the result of a "minimization algorithm" to be a close approximation to the solution, how can it then be decided whether the solution of the problem is found or not? This is a hard question. This problem is discussed in Murray (1972). He gives the following advice:

- 1) Check the rate of convergence
- 2) Restart the routine
- 3) Try other starting points, input parameters, rescale the variables
- 4) Try a different method

Table 5

Number of iterations (n_i) and number of function evaluations (n_f) to reduce function 5 to 10^{-7} for different starting points.

starting point	NELME		NUFLET		POWBRE	
	n_i	n_f	n_i	n_f	n_i	n_f
(2, 2)	28	94	8	44	12	34
(2, -2)	24	86	failed	1)	failed	2)
(-2, 2)	31	104	8	69	12	34
(-2, -2)	26	90	12	87	failed	3)

1) stopped at (1.9996, -2.0098)

2) stopped at $(-1.53 \cdot 10^{-3}, -2)$

3) stopped at $(1.01 \cdot 10^{-3}, -2)$

4. REFERENCES

- Brent, R.P. (1978): Algorithms for minimization without derivatives, Prentice-Hall, Inc. Englewood Cliffs, N.Y. (7.7).
- Colville, A.R. (1968): A comparative study of nonlinear programming codes, IBM New-York Scientific Center, Report 320-2949.
- Elmqvist, Tyssö, Wieslander (1976): Programming and documentation rules for subroutine libraries - designed for the Scandinavian Control Library, Department of Automatic Control, Lund Institute of Technology, Lund, Sweden. CODEN:LUTFD2/(TFRT-3139)/1-041/(1976).
- Fletcher, R., and Powell, M.Y.D. (1963): A rapidly convergent descent method for minimization, The Computer Journal, Vol. 6, p. 163,
- Fletcher. R. (1971): Fortran subroutines for minimization by quasi-Newton methods, Report AERE-R7125, Harwell.
- Himmelblau, D.M. (1972): Applied nonlinear programming, Mc Gray-Hill Book Company, New York (p. 148-157, 190-217).
- Källström, C.G. (1978): LISPID-user's manual, Department of Automatic Control, Lund Institute of Technology, Lund, Sweden. CODEN: LUTFD2/(TFRT-7147)/1-147/(1978)
- Murray, W. (1972): Numerical methods for unconstrained optimization, Academic Press, London (p. 119-122).
- Nelder, Y.A. and Mead, R. (1964): A simplex method for function minimization, The Computer Journal, Vol. 7, p. 308.
- Powell, M.Y.D. (1962): An iterative method for finding stationary value of a function of several variables, The Computer Journal, Vol. 5. p. 147.
- Rosenbrock, H. (1960): An automatic method for finding the greatest or least value of a function, The Computer Journal, Vol.3, p.175.

APPENDIX A

Computer Codes

NELME	16
NEWX	24
PNELME	25

NETWORK(FUNC,X,FMIN,N,MODE,VDIST,MAXFN,EPS,ITER,IPRINT,PRINT,WX,WL)
 = NAME OF SUBROUTINE WHICH COMPUTES THE LOSS FUNCTION
 X = REAL VECTOR, SIZE(N); (1/D)
 SEE NOTE 1 AND 21
 FMIN = REAL CONTAINING, IF SUCCESS, THE MINIMUM VALUE; (D)
 SEE NOTE 21
 N = ACTUAL DIMENSION OF THE OPTIMIZATION PROBLEM; (I)
 MODE = INDICATOR TO CONTROL THE CALCULATION OF THE STARTING
 POLYHEDRON; (1)
 HAS TO BE 0 OR 1. SEE NOTE 11
 = DISTANCE BETWEEN TWO VERTICES IN THE STARTING
 POLYHEDRON; (1)
 SEE NOTE 11
 MAXFN = MAXIMUM NUMBER OF TIMES FUNC CAN BE CALLED; (I)
 MAXFN+1
 EPS = STOPPING CRITERION; (1)
 SEE NOTE 2 AND 31
 IERR = ERROR INDICATOR; (D)
 -1: THE SEARCH IS TO BE CONTINUED, USED ONLY IN PRINT
 0: SUCCESS
 1: INCORRECT CALL OF NETWORK
 N<2
 INVALID MODE-VALUE

ARGUMENTS:

PROGRAM TYPE: SUBROUTINE

=====

TO FIND THE MINIMUM OF A FUNCTION F(X) OF SEVERAL VARIABLES
 WITHOUT CALCULATING DERIVATIVES.

PURPOSE

=====

ACCEPTED:

VERSION:

INSTITUTE: DEPARTMENT OF AUTOMATIC CONTROL
 LUND INSTITUTE OF TECHNOLOGY, SWEDEN

IMPLEMENTOR: SVEN ERIK WATSSON

DATE: 1978-06-19

KEYWORDS: NONLINEAR PROGRAMMING, NONLINEAR OPTIMIZATION,
 PARAMETRIC OPTIMIZATION, NELDER-MEAD SEARCH

LANGUAGE: FORTRAN IV

SUBTITLE: THE FLEXIBLE POLYHEDRON SEARCH OF NELDER AND MEAD

NAME: NELME

NUMBER:

LIBRARY SUBROUTINES REQUIRED: RMACON, RMOVE

PDF 15: 1716
PDF 11: 1578

SIZE:

CHARACTERISTICS

1. NELDER, J.A. AND MEAD, R. (1964) / A SIMPLEX METHOD FOR FUNCTION MINIMIZATION, THE COMPUTER JOURNAL, VOL. 7, P. 398
2. HIMMELBLAU, D.M. (1972) / APPLIED NONLINEAR PROGRAMMING, MC GRAM HILL BOOK COMPANY, NEW YORK, P. 148-157

REFERENCES:

HIMMELBLAU, D.M. (1972).
 THE METHOD OF CALCULATING A STARTING POLYHEDRON CAN BE FOUND IN
 COEFFICIENT TO 1/2 AND THE EXPANSION COEFFICIENT TO 2.
 THE REFLECTION COEFFICIENT IS CHOSEN TO 1, THE CONTRACTION
 POLYHEDRON SEARCH DESCRIBED IN NELDER, J.A. AND MEAD, R. (1964).
 THIS ROUTINE IS A STRAIGHTFORWARD IMPLEMENTATION OF A FLEXIBLE

METHOD

THE ACTUAL SUBROUTINE NAME MUST BE DECLARED EXTERNAL IN THE
 PROGRAM THAT CALLS NEMLE. IF IPRINT IS SET TO ZERO AND A DUMMY
 IS GOING TO BE USED AS PRINT AND IF THIS DUMMY IS NOT DECLARED
 EXTERNAL, IT NEED NOT BE INCLUDED AT LOADING.
 THE SUBROUTINE NEMLE IN THE LIBRARY CAN BE USED AS THE ACTUAL
 SUBROUTINE PRINT.

- ERRR - ERROR INDICATOR OF NEMLE, (I)
 - IPRINT - PRINT INDICATOR, (I)
 - TEST0 - OBTAINED STOPPING CRITERION, (I)
 - NFN - NUMBER OF CALLS OF FUNC, (I)
 - NITE - ITERATION NUMBER, (I)
 - N - ACTUAL DIMENSION OF THE OPTIMIZATION PROBLEM, (I)
 - WF - VECTOR CONTAINING THE VALUES OF THE OBJECTIVE FUNCTION AT THE VERTICES, SIZE(N+4), (I)
 - MX - DEFINES THE POLYHEDRON, SIZE(N,N+4), DIMENSIONED(N, N, I)
- THE FIRST N+1 COLUMNS OF MX CONTAIN THE VERTICES OF THE POLYHEDRON.

AT THE BEGINNING OF THE FIRST ITERATION, ON EVERY SUBSEQUENT
 ABS(IPRINT) ITERATIONS AND ON EXIT OF NEMLE PRINT IS CALLED BY
 NEMLE IN ORDER TO PRINT A MESSAGE. IF IPRINT=9, PRINT IS NOT
 CALLED AND PRINT MAY BE A DUMMY.

SUBROUTINE PRINT(MX,WF,N,NITE,NFN,TEST0,IPRINT,ERRR)

TO CALL FUNC WITH DUMMIES.
 (FXX) MUST NOT BE USED IN FUNC IN ORDER TO MAKE IT POSSIBLE
 PROGRAM THAT CALLS NEMLE. IF ICONT IS 0 (1) THEN FX AND FXX

50 CONTINUE

NOW THE INITIAL POLYHEDRON EXISTS. CALCULATE THE VALUE OF THE PROJECTIVE FUNCTION AT THE VERTICES OF THE INITIAL POLYHEDRON.

40 CONTINUE

WX(I,I)=A+D1

30 CONTINUE

WX(I,J)=B

DO 30 J=1,N

WX(I,NPLUS1)=A+D3

E=A+D2

A=X(I)

DO 40 I=1,N

D1=C*(B+FLN)+D3

D2=C*B+D3

B=B-1.

C=VDIST1/(A*FLN)

D3=-VDIST1/(A*B)

E=SQRT(FLNPL1)

A=SQRT(2./E)

IF(ABS(VDIST1).LE.A) VDIST1=A

VDIST1=VDIST

IN THE FIRST N+1 COLUMNS OF WX

VDIST THE DISTANCE BETWEEN TWO VERTICES AND STORE THE VERTICES CALCULATE THE INITIAL POLYHEDRON. LET X BE THE CENTROID AND

IF(MODE.EQ.1) GO TO 50

IF(MODE.NE.0) RETURN

IS THE INITIAL POLYHEDRON GIVEN OR HAS IT TO BE CALCULATED?

IF(N.LT.2 .OR. MAXFN.LE.NPLUS1) RETURN

IERR=1

IF(EPS.LT.A) EPS=A

A=100.*RMACD(3)

FLNPL1=FLOAT(NPLUS1)

FLN=FLOAT(N)

NPLUS4=N+4

NPLUS3=N+3

NPLUS2=N+2

NPLUS1=N+1

VALIDITY TEST AND CALCULATION OF INITIAL VALUES

DATA ICNT /0/

DATA ALFA,BETA,GAMMA/1.,-0.5,-2./

--GAMMA: EXPANSION COEFFICIENT

--BETA: CONTRACTION COEFFICIENT

ALFA: REFLECTION COEFFICIENT

DIMENSION X(1),WX(N,1),WF(1)

* IPRINT,PRINT,WX,WF

SUBROUTINE NEMEX(FUNC,X,FMIN,N,MODE,VDIST,MAXFN,EPS,IERR)

INTERNAL SUBROUTINES REQUIRED: NEMX


```

DO 149 I=1,N
HIGH=NPLUS1
WF(HIGH)=WF(NPLUS2)
CALL MOVE(WX(I,NPLUS2),WX(I,HIGH),N,I,N)
C
C
REPLACE WX(I,HIGH) BY WX(I,NPLUS2)
C
C
OF THE POLYEDRON,
C
NO, W(NPLUS2) < W(HIGH), ACCEPT WX(I,NPLUS2) AS A NEW VERTEX
C
C
IF(W(NPLUS2).GE.W(HIGH)) GO TO 109
C
C
IS W(NPLUS2) > W(HIGH) ?
C
YES, W(NPLUS2) > W(I) FOR ALL I=1,1,1,N+1 AND I=HIGH, BUT
C
C
139 CONTINUE
GO TO 119
IF(IE,HIGH) GO TO 139
IF(W(NPLUS2).GT.W(I)) GO TO 139
DO 139 I=1,NPLUS1
129 CONTINUE
C
C
I NOT EQUAL TO HIGH ?
C
W(ILOW) BUT IS W(NPLUS2) > W(I) FOR ALL I=1,1,1,N+1 AND
C
C
AT THIS POINT OF THE PROGRAM IT IS KNOWN THAT W(NPLUS2) >
C
C
GO TO 129
WF(HIGH)=W(NPLUS2)
CALL MOVE(WX(I,NPLUS2),WX(I,HIGH),N,I,N)
119 CONTINUE
C
C
REPLACE WX(I,HIGH) BY WX(I,NPLUS2)
C
C
ACCEPT WX(I,NPLUS2) AS A NEW VERTEX OF THE POLYEDRON.
C
C
GO TO 129
WF(HIGH)=W(NPLUS4)
CALL MOVE(WX(I,NPLUS4),WX(I,HIGH),N,I,N)
C
C
REPLACE WX(I,HIGH) BY WX(I,NPLUS4)
C
C
ACCEPT WX(I,NPLUS4) AS A NEW VERTEX OF THE POLYEDRON.
C
C
IF(W(NPLUS4).GE.W(ILOW)) GO TO 119
C
C
TEST FUNCTION VALUE AT EXPANSION POINT.
C
C
IF(IERR1.NE.0) GO TO 509
CALL FUNC(WX(I,NPLUS4),W(NPLUS4),DUM1,DUM2,N,ICONT,IERR1,IDUM2)
IX=NPLUS4
NFN=NFN+1
CALL NEWX(WX(I,NPLUS2),WX(I,NPLUS3),GAMMA,N,WX(I,NPLUS4))
C
C
AND VALUE (W(NPLUS4)) AT THAT POINT,
C
WX(I,NPLUS4)=WX(I,NPLUS2)+GAMMA*(WX(I,NPLUS2)-WX(I,NPLUS3))
C
C
TRY EXPANSION! CALCULATE EXPANSION POINT
C
W(NPLUS2) < W(ILOW)
C
C
IF(W(NPLUS2).GE.W(ILOW)) GO TO 129
C
C
TEST FUNCTION VALUE AT REFLECTION POINT
C
C
IF(IERR1.NE.0) GO TO 509
CALL FUNC(WX(I,NPLUS3),W(NPLUS3),DUM1,DUM2,N,ICONT,IERR1,IDUM2)
IX=NPLUS3
NFN=NFN+1
CALL NEWX(WX(I,NPLUS2),WX(I,HIGH),ALFA,N,WX(I,NPLUS3))

```

```

      IF(WF(I).GT.WF(IHIGH)) IHIGH=I
140 CONTINUE
C
C   TRY CONTRACTION! CALCULATE CONTRACTION POINT
C   WX(.,NPLUS4)=WX(.,NPLUS2)+BETA*(WX(.,NPLUS2)+WX(.,IHIGH))
C   AND VALUE AT (WF(NPLUS4)) THAT POINT.
C
      CALL NEWX(WX(1,NPLUS2),WX(1,IHIGH),BETA,N,WX(1,NPLUS4))
      NFN=NFN+1
      IX=NPLUS4
      CALL FUNC(WX(1,NPLUS4),WF(NPLUS4),DUM1,DUM2,N,ICONT,IERR1,IDUM2)
      IF(IERR1.NE.0) GO TO 500
C
C   TEST FUNCTION VALUE AT CONTRACTION POINT.
C
      IF(WF(NPLUS4).GT.WF(IHIGH)) GO TO 150
C
C   WF(NPLUS4) < WF(IHIGH). ACCEPT WX(.,NPLUS4) AS A NEW VERTEX OF
C   THE POLYHEDRON.
C   REPLACE WX(.,IHIGH) BY WX(.,NPLUS3).
C
      CALL MOVE(WX(1,NPLUS4),WX(1,IHIGH),N,1,0,N,N)
      WF(IHIGH)=WF(NPLUS4)
      GO TO 170
C
C   REDUCTION: REPLACE ALL WX(.,I) BY WX(.,ILOW)-0.5*(WX(.,ILOW)-
C   WX(.,I))
C
150 CONTINUE
      NFN=NFN+N
      DO 160 IX=1,NPLUS1
      IF(IX.EQ.ILOW) GO TO 160
      CALL NEWX(WX(1,ILOW),WX(1,IX),-0.5,N,WX(1,IX))
      CALL FUNC(WX(1,IX),WF(IX),DUM1,DUM2,N,ICONT,IERR1,IDUM2)
      IF(IERR1.NE.0) GO TO 500
160 CONTINUE
C
C   TEST OF CONVERGENCE
C
C   CALCULATE THE TEST QUANTITY
      TESTQ=SQRT(SUM((WF(I)-WF(NPLUS2))**2/(N-1)))
C
170 CONTINUE
      A=0
      DO 180 I=1,NPLUS1
      B=WF(I)-WF(NPLUS2)
      A=A+B*B
180 CONTINUE
      TESTQ=SQRT(A/FLNPL1)
      IF(TESTQ.LT.EPS) GO TO 200
C
C   THE CONVERGENCE CRITERION IS NOT FULFILLED.
C   TEST NUMBER OF FUNCTION EVALUATIONS
C
      IF(NFN.GE.MARFN) GO TO 190
C
C   PRINT A MESSAGE AND DO A NEW ITERATION.
C
      NPRINT=NPRINT+1
      IF(NPRINT.GT.0 .OR. IPRINT.EQ.0) GO TO 70
      CALL PRINT(WX,WF,N,NITE,NFN,TESTQ,IPRINT,IERR)
      NPRINT=IABSPR
      GO TO 70
C

```

```

C      END OF AN ITERATION LOOP
C-----
C
C      FUNC HAS BEEN CALLED MORE THAN MAXFN TIMES.
C
199  CONTINUE
      TERR=3
      GO TO 205
C
C      THE CONVERGENCE CRITERION IS FULFILLED, BUT THIS DOES NOT
C      CERTAINLY MEAN THAT A LOCAL MINIMUM IS FOUND.
C
200  CONTINUE
      IERR=0
C
C      OUTPUT THE RESULT, PRINT A MESSAGE AND RETURN.
C
205  CONTINUE
      DO 210 I=1,NPLUS1
      IF(WF(I).LT.WF(ILOW)) ILOW=I
210  CONTINUE
      CALL MMOVE(WX(1,ILOW),X,N,1,0,N,N)
      FMIN=WF(ILOW)
      IF(IPRINT.NE.0) CALL PRINT(WX,WF,N,NITE,NFN,TEST0,IPRINT,IERR)
      RETURN
C
C      FUNC HAS BEEN CALLED WITH AN ILLEGAL X (WX(,IX)) IN THE
C      INITIALIZATION PART.
C
400  CONTINUE
      TEST0=0.
      DO 410 J=IX,NPLUS1
      WF(J)=0.
410  CONTINUE
      DO 420 J=NPLUS2,NPLUS4
      WF(J)=0.
      DO 420 I=1,N
      WX(I,J)=0.
420  CONTINUE
C
C      FUNC HAS BEEN CALLED WITH AN ILLEGAL X (WX(,IX)),
C      MOVE THIS X TO WX(,NPLUS2).
C
500  CONTINUE
      IERR=2
      IF(IX.NE.NPLUS2) CALL MMOVE(WX(1,IX),WX(1,NPLUS2),N,1,0,N,N)
      GO TO 205
      END

```



```
      SUBROUTINE NEWX(X1, X2, ALFA, N, X3)
C
C      CALCULATES  $X3=X1+ALFA*(X1-X2)$ 
C
      DIMENSION X1(1), X2(1), X3(1)
      DO 10 I=1, N
      X3(I)=X1(I)+ALFA*(X1(I)-X2(I))
10  CONTINUE
      RETURN
      END
```

AND ITS FUNCTION VALUE IS PRINTED.
VALUES IS SUPPRESSED AND ONLY THE VERTEX WITH THE LOWEST VALUE
IS PRINTED. THE PRINTING OF ALL VERTICES AND THEIR FUNCTION
COORDINATES OF THE VERTICES (X(I)) HAVE THE FORMAT 616.7.
THE FUNCTION VALUES (F(X)) HAVE THE FORMAT 614.7 AND THE

```
.....  
/ F(XNPLDST) / .....  
.....  
.....  
/ F(X2) / X(2) / .....  
.....  
.....  
/ F(X1) / X(1) / X(2) / .....  
.....  
/ F(X) / X / .....  
.....  
/ F(X1) / X(1) / X(2) / .....  
.....  
/ F(X8) / X(8) / .....  
/ F(X9) / X(9) / .....  
/ F(X14) / X(14) / .....  
.....
```

ITERATION NO.=19 NO. OF CALLS OF FUND=19 STOPPING CRITERION=66.3
THE PRINTING IS IN THE FORM
/ PRINT FROM NETWE /
THE PRINTING STARTS ON A NEW PAGE WITH THE TEXT
PRINT IN NETWE IS CONTROLLED.
IN THE DOCUMENTATION OF NETWE IT IS DESCRIBED HOW THE CALLING OF

NOTE:

ARGUMENTS: SEE THE DOCUMENTATION OF THE SUBROUTINE NETWE.

PROGRAMTYPE: SUBROUTINE

USAGE

TO BE USED AS THE ACTUAL SUBROUTINE PRINT IN THE SUBROUTINE NETWE.
SEE THE DOCUMENTATION OF THE SUBROUTINE NETWE.

PURPOSE

ACCEPTED: VERSION:

INSTITUTE: DEPARTMENT OF AUTOMATIC CONTROL
FOND INSTITUTE OF TECHNOLOGY, SWEDEN

IMPLEMENTOR: SVEN ERIK WATSSON DATE: 1978-06-19

KEYWORDS:

LANGUAGE: FORTRAN IV

SUBTITLE: I/O ROUTINE OF THE SUBROUTINE NETWE

NAME: FNELME NUMBER:

C

```
50 CONTINUE
  IF(IERR.NE.2) GO TO 60
  NPLUS2=N+2
  WRITE(LOUT,550) (MX(I,NPLUS2), I=1,N)
550 FORMAT(/ /1X,40HFUNC HAS BEEN CALLED WITH THE ILLEGAL X=
*          / (1X,8015.7))
  WRITE(LOUT,560)
560 FORMAT(1H1)
  RETURN
```

C

C

C

```
  IERR=3,
60 WRITE(LOUT,570)
570 FORMAT(/ /1X,23HFUNC CALLED MAXFN TIMES/1H1)
  RETURN
  END
```

APPENDIX B

Results from the minimization of five functions using
NELME.

Function 1 (Rosenbrock's function)	29
2 (Wood's function)	37
3 (Fletcher and Powell's helical valley)	45
4 (Powell's quartic function)	53
5 (A quadratic function with truncated linear terms)	61

If a number in the column "number of evaluations required" is 2000 or greater, it means that NELME failed. The polyhedron has degenerated before reaching the minimum point.

PRINT FROM NELME

MINIMIZATION OF $F(X)=100*(X(2)-X(1)**2)**2+(1-X(1))**2$
 UNTIL $F(X) < 1.E-1$ FOR DIFFERENT SIZES OF THE INITIAL POLYHEDRON.
 STARTING POINT IS $(-1.2, 1.0)$.

SIZE OF THE INITIAL POLYHEDRON	NUMBER OF ITERATIONS REQUIRED	NUMBER OF EVALUATIONS REQUIRED	TESTO OF NELME
0.1	64	204	0.141E-01
0.2	51	159	0.213E-01
0.3	70	214	0.294E-01
0.4	60	191	0.406E-01
0.5	66	209	0.288E-01
0.6	59	191	0.184E-01
0.7	72	220	0.150E-01
0.8	47	151	0.443E-01
0.9	70	220	0.285E-02
1.0	59	185	0.195E-01
1.1	63	200	0.547E-02
1.2	63	198	0.536E-02
1.3	51	164	0.712E-01
1.4	64	202	0.651E-01
1.5	48	149	0.207E-01
1.6	58	187	0.181E-01
1.7	63	198	0.498E-01
1.8	63	192	0.155E-01
1.9	55	175	0.359E-02
2.0	72	229	0.925E-02
2.1	59	183	0.338E-01
2.2	53	171	0.342E-01
2.3	69	214	0.133E-01
2.4	58	187	0.249E-01
2.5	64	202	0.464E-01
2.6	86	271	0.161E-01
2.7	89	272	0.242E-01
2.8	80	252	0.668E-01
2.9	81	255	0.538E-01
3.0	92	284	0.457E-01
3.1	8	32	0.110
3.2	10	38	0.743E-01
3.3	6	26	0.877
3.4	1	8	145.
3.5	6	28	0.311
3.6	5	24	2.38
3.7	102	321	0.115E-01
3.8	99	309	0.990E-02
3.9	30	98	0.171E-01
4.0	43	143	0.345E-01

PRINT FROM NELME

MINIMIZATION OF $F(x) = 100x(2) - x(1)**2 - x(2)**2 + (1 - x(1))**2$
 UNTIL $F(x) < 1.E-4$ FOR DIFFERENT SIZES OF THE INITIAL POLYHEDRON,
 STARTING POINT IS (-1.2, 1.0).

SIZE OF THE INITIAL POLYHEDRON	NUMBER OF ITERATIONS REQUIRED	NUMBER OF EVALUATIONS REQUIRED	TESTO OF NELME
4.1	113	354	0.371E-01
4.2	95	297	0.495E-01
4.3	111	353	0.411E-02
4.4	96	306	0.295E-01
4.5	84	271	0.198E-01
4.6	88	280	0.274E-01
4.7	93	296	0.261E-01
4.8	112	344	0.133E-01
4.9	92	288	0.162E-01
5.0	92	286	0.133E-01
5.1	87	277	0.122E-01
5.2	94	295	0.205E-01
5.3	105	321	0.241E-01
5.4	87	275	0.733E-01
5.5	105	330	0.861E-02
5.6	101	316	0.413E-01
5.7	111	356	0.354E-02
5.8	95	296	0.304E-01
5.9	96	309	0.720E-02
6.0	90	295	0.113E-01
6.1	105	326	0.506E-01
6.2	87	276	0.620E-01
6.3	67	213	0.215E-01
6.4	57	184	0.335E-01
6.5	73	236	0.302E-01
6.6	113	354	0.409E-01
6.7	95	296	0.129E-01
6.8	108	343	0.258E-01
6.9	102	325	0.744E-01
7.0	103	327	0.737E-01
7.1	92	283	0.412E-01
7.2	96	305	0.264E-01
7.3	109	341	0.313E-01
7.4	122	387	0.666E-02
7.5	95	306	0.259E-01
7.6	102	316	0.391E-01
7.7	116	361	0.232E-01
7.8	111	346	0.116E-01
7.9	110	343	0.153E-01
8.0	100	307	0.174E-01

PRINT FROM NELME

MINIMIZATION OF $F(X) = 100 * (X(2) - X(1)) * (X(2) + (1 - X(1))) * X(2)$
UNTIL $F(X) < 1.E-3$ FOR DIFFERENT SIZES OF THE INITIAL POLYHEDRON.
STARTING POINT IS (-1.2, 1.0).

SIZE OF THE INITIAL POLYHEDRON	NUMBER OF ITERATIONS REQUIRED	NUMBER OF EVALUATIONS REQUIRED	TESTS OF NELME
--------------------------------------	-------------------------------------	--------------------------------------	-------------------

0.1	83	263	0.105E-02
0.2	70	216	0.201E-03
0.3	85	261	0.859E-03
0.4	77	246	0.191E-02
0.5	81	255	0.786E-03
0.6	72	233	0.179E-02
0.7	85	259	0.219E-02
0.8	67	212	0.149E-02
0.9	89	273	0.519E-03
1.0	82	253	0.366E-03
1.1	82	261	0.734E-03
1.2	80	251	0.192E-02
1.3	77	242	0.506E-03
1.4	80	252	0.117E-02
1.5	61	188	0.357E-02
1.6	75	240	0.933E-03
1.7	79	247	0.310E-03
1.8	74	226	0.112E-01
1.9	73	231	0.131E-02
2.0	84	266	0.766E-03
2.1	89	274	0.672E-03
2.2	68	218	0.845E-03
2.3	84	257	0.199E-02
2.4	68	217	0.633E-02
2.5	73	231	0.334E-02
2.6	102	321	0.656E-02
2.7	101	311	0.198E-02
2.8	94	298	0.455E-03
2.9	90	284	0.354E-02
3.0	117	363	0.448E-03
3.1	29	98	0.114E-02
3.2	50	164	0.117E-02
3.3	38	123	0.993E-03
3.4	32	111	0.881E-03
3.5	33	111	0.787E-04
3.6	28	96	0.606E-03
3.7	115	360	0.320E-02
3.8	121	375	0.332E-03
3.9	51	162	0.149E-03
4.0	65	210	0.191E-02

PRINT FROM NELME

MINIMIZATION OF $F(X)=100*(X(2)-X(1)**2)**2+(1-X(1))**2$
 UNTIL $F(X) < 1.E-3$ FOR DIFFERENT SIZES OF THE INITIAL POLYHEDRON.
 STARTING POINT IS (-1.2, 1.0).

SIZE OF THE INITIAL POLYHEDRON	NUMBER OF ITERATIONS REQUIRED	NUMBER OF EVALUATIONS REQUIRED	TESTO OF NELME
4.1	121	381	0.487E-02
4.2	111	350	0.260E-03
4.3	141	444	0.679E-03
4.4	110	349	0.258E-02
4.5	102	326	0.839E-03
4.6	101	320	0.202E-02
4.7	109	343	0.846E-03
4.8	121	371	0.350E-02
4.9	101	317	0.131E-02
5.0	109	340	0.592E-03
5.1	100	317	0.120E-02
5.2	113	351	0.132E-02
5.3	118	364	0.134E-02
5.4	103	326	0.387E-03
5.5	120	379	0.201E-02
5.6	116	364	0.441E-03
5.7	134	422	0.341E-03
5.8	111	346	0.893E-03
5.9	113	362	0.515E-03
6.0	105	331	0.161E-02
6.1	129	400	0.601E-03
6.2	101	321	0.152E-02
6.3	78	248	0.140E-02
6.4	76	243	0.160E-02
6.5	101	322	0.675E-03
6.6	133	415	0.102E-02
6.7	112	346	0.696E-03
6.8	129	409	0.164E-02
6.9	111	356	0.976E-03
7.0	119	381	0.156E-02
7.1	112	349	0.173E-02
7.2	109	346	0.996E-03
7.3	137	430	0.869E-04
7.4	148	467	0.107E-02
7.5	111	355	0.175E-02
7.6	119	373	0.560E-03
7.7	130	407	0.433E-03
7.8	129	402	0.965E-03
7.9	126	390	0.153E-02
8.0	123	380	0.151E-02

PRINT FROM NELME

MINIMIZATION OF $F(X)=100*(X(2)-X(1)**2)**2+(1-X(1))**2$
 UNTIL $F(X) < 1.E-5$ FOR DIFFERENT SIZES OF THE INITIAL POLYHEDRON.
 STARTING POINT IS (-1.2, 1.0).

SIZE OF THE INITIAL POLYHEDRON	NUMBER OF ITERATIONS REQUIRED	NUMBER OF EVALUATIONS REQUIRED	TESTO OF NELME
0.1	90	286	0.168E-04
0.2	82	254	0.691E-05
0.3	93	285	0.591E-05
0.4	85	271	0.176E-04
0.5	85	270	0.270E-04
0.6	76	248	0.684E-04
0.7	90	276	0.308E-04
0.8	72	230	0.103E-04
0.9	99	304	0.579E-04
1.0	86	268	0.726E-05
1.1	87	279	0.135E-04
1.2	84	266	0.114E-04
1.3	83	259	0.109E-04
1.4	86	273	0.762E-05
1.5	69	214	0.159E-04
1.6	80	257	0.192E-04
1.7	86	271	0.121E-04
1.8	80	248	0.555E-05
1.9	78	249	0.191E-04
2.0	90	285	0.184E-04
2.1	102	316	0.683E-04
2.2	77	249	0.127E-04
2.3	90	277	0.499E-04
2.4	75	240	0.278E-04
2.5	80	255	0.408E-04
2.6	109	345	0.212E-04
2.7	105	326	0.530E-04
2.8	100	318	0.236E-04
2.9	95	303	0.638E-05
3.0	123	382	0.164E-04
3.1	35	118	0.472E-05
3.2	55	180	0.138E-03
3.3	43	139	0.823E-04
3.4	41	139	0.193E-04
3.5	42	140	0.184E-04
3.6	36	122	0.913E-05
3.7	123	386	0.198E-04
3.8	123	392	0.624E-03
3.9	63	201	0.672E-05
4.0	72	234	0.104E-04

PRINT FROM NELME

MINIMIZATION OF $F(X)=100*(X(2)-X(1)**2)**2+(1-X(1))**2$
 UNTIL $F(X) < 1.E-5$ FOR DIFFERENT SIZES OF THE INITIAL POLYHEDRON,
 STARTING POINT IS (-1.2, 1.0).

SIZE OF THE INITIAL POLYHEDRON	NUMBER OF ITERATIONS REQUIRED	NUMBER OF EVALUATIONS REQUIRED	TESTO OF NELME
4.1	131	415	0.222E-05
4.2	117	370	0.121E-04
4.3	145	459	0.404E-04
4.4	119	379	0.232E-05
4.5	107	344	0.159E-04
4.6	106	337	0.375E-04
4.7	115	364	0.123E-04
4.8	128	395	0.561E-04
4.9	107	337	0.306E-04
5.0	113	355	0.249E-04
5.1	104	331	0.205E-03
5.2	119	372	0.840E-05
5.3	123	380	0.549E-05
5.4	116	367	0.523E-04
5.5	123	389	0.170E-03
5.6	121	381	0.912E-05
5.7	144	455	0.734E-05
5.8	117	367	0.877E-05
5.9	116	369	0.378E-03
6.0	110	347	0.443E-04
6.1	137	427	0.196E-04
6.2	107	341	0.142E-04
6.3	85	270	0.243E-04
6.4	76	243	0.160E-02
6.5	106	336	0.149E-03
6.6	140	439	0.483E-05
6.7	119	368	0.922E-05
6.8	137	436	0.580E-05
6.9	117	377	0.806E-05
7.0	121	389	0.807E-04
7.1	122	382	0.396E-05
7.2	117	373	0.954E-05
7.3	145	454	0.652E-04
7.4	153	484	0.299E-05
7.5	118	375	0.119E-03
7.6	126	395	0.333E-04
7.7	138	433	0.599E-05
7.8	132	414	0.154E-04
7.9	133	411	0.810E-05
8.0	131	405	0.950E-05

PRINT FROM NELME

MINIMIZATION OF $F(X)=100*(X(2)-X(1)**2)**2+(1-X(1))**2$
 UNTIL $F(X) < 1.E-7$ FOR DIFFERENT SIZES OF THE INITIAL POLYHEDRON.
 STARTING POINT IS (-1.2, 1.0).

SIZE OF THE INITIAL POLYHEDRON	NUMBER OF ITERATIONS REQUIRED	NUMBER OF EVALUATIONS REQUIRED	TESTO OF NELME
0.1	97	308	0.273E-06
0.2	89	276	0.731E-07
0.3	96	296	0.138E-06
0.4	94	303	0.113E-06
0.5	88	281	0.289E-05
0.6	81	267	0.574E-07
0.7	95	293	0.611E-06
0.8	76	244	0.808E-07
0.9	105	324	0.347E-06
1.0	89	279	0.259E-06
1.1	90	291	0.289E-06
1.2	88	280	0.889E-06
1.3	87	274	0.211E-06
1.4	89	285	0.472E-07
1.5	74	233	0.463E-07
1.6	83	269	0.447E-06
1.7	89	281	0.130E-05
1.8	88	274	0.748E-07
1.9	83	265	0.924E-07
2.0	94	300	0.262E-06
2.1	109	341	0.133E-06
2.2	82	266	0.239E-06
2.3	96	299	0.131E-06
2.4	86	276	0.432E-07
2.5	86	276	0.108E-06
2.6	113	360	0.767E-07
2.7	109	342	0.341E-06
2.8	105	336	0.147E-06
2.9	98	315	0.145E-06
3.0	128	401	0.979E-07
3.1	38	129	0.223E-06
3.2	60	200	0.554E-07
3.3	46	151	0.244E-05
3.4	45	153	0.114E-06
3.5	47	158	0.199E-06
3.6	40	137	0.708E-07
3.7	126	398	0.354E-06
3.8	129	404	0.297E-06
3.9	65	209	0.420E-06
4.0	77	252	0.434E-06

PRINT FROM NELME

MINIMIZATION OF $F(X)=100*(X(2)-X(1)**2)**2+(1-X(1))**2$
 UNTIL $F(X) < 1.E-7$ FOR DIFFERENT SIZES OF THE INITIAL POLYHEDRON,
 STARTING POINT IS (-1.2, 1.0).

SIZE OF THE INITIAL POLYHEDRON	NUMBER OF ITERATIONS REQUIRED	NUMBER OF EVALUATIONS REQUIRED	TESTO OF NELME
4.1	137	435	0.206E-06
4.2	123	392	0.790E-07
4.3	151	480	0.205E-07
4.4	124	396	0.214E-06
4.5	121	390	0.233E-07
4.6	112	358	0.843E-07
4.7	118	375	0.727E-06
4.8	133	413	0.128E-06
4.9	113	358	0.997E-07
5.0	118	373	0.451E-07
5.1	110	353	0.624E-07
5.2	129	404	0.838E-07
5.3	125	388	0.127E-05
5.4	122	387	0.798E-07
5.5	141	449	0.466E-07
5.6	126	400	0.302E-07
5.7	151	478	0.162E-06
5.8	121	382	0.843E-07
5.9	123	395	0.264E-07
6.0	116	367	0.209E-06
6.1	142	445	0.272E-06
6.2	114	364	0.113E-06
6.3	91	290	0.113E-06
6.4	83	271	0.813E-07
6.5	114	364	0.100E-06
6.6	143	451	0.210E-06
6.7	124	385	0.773E-06
6.8	142	453	0.978E-06
6.9	118	381	0.223E-05
7.0	126	408	0.290E-07
7.1	125	394	0.582E-07
7.2	122	390	0.670E-07
7.3	149	468	0.831E-06
7.4	158	502	0.139E-06
7.5	122	390	0.975E-06
7.6	131	415	0.481E-07
7.7	143	451	0.100E-06
7.8	137	432	0.230E-06
7.9	138	429	0.200E-06
8.0	136	423	0.186E-06

PRINT FROM NELME

MINIMIZATION OF $F(x) = 100x(1-x)^2 + (x(1-x)^2 + x(1-x))^2 + 90x(x(1-x)^2 - x(1-x)^2) + 10.1(x(1-x)^2 - x(1-x)^2) + 19.8(x(1-x)^2 - x(1-x)^2)$
 UNTIL $F(x) < 1.E-1$ FOR DIFFERENT SIZES OF THE INITIAL POLYHEDRON,
 STARTING POINT IS $(-3, -1, -3, -1)$,

SIZE OF THE INITIAL POLYHEDRON	NUMBER OF ITERATIONS REQUIRED	NUMBER OF EVALUATIONS REQUIRED	TESTO OF NELME
0.1	92	248	0.488E-02
0.2	183	541	0.951E-02
0.3	48	146	0.355E-01
0.4	50	161	0.592E-01
0.5	46	150	0.926E-02
0.6	86	276	0.153E-01
0.7	34	119	0.176
0.8	49	152	0.142
0.9	32	107	0.209
1.0	30	105	0.397E-01
1.1	40	135	0.404E-01
1.2	33	108	0.677E-01
1.3	33	125	0.259E-01
1.4	670	2001	0.105E-05
1.5	863	2000	0.148E-05
1.6	951	2000	0.477E-06
1.7	565	1656	0.197E-01
1.8	452	1324	0.190E-03
1.9	316	944	0.877E-02
2.0	250	748	0.244E-01
2.1	264	764	0.160E-01
2.2	954	2000	0.369E-06
2.3	950	2000	0.369E-06
2.4	956	2001	0.674E-06
2.5	950	2000	0.640E-06
2.6	947	2000	0.302E-06
2.7	286	821	0.100E-01
2.8	101	305	0.190E-01
2.9	70	226	0.207E-01
3.0	59	192	0.368E-01
3.1	72	233	0.729E-02
3.2	346	1018	0.161E-01
3.3	250	733	0.145E-01
3.4	123	398	0.252E-01
3.5	136	415	0.414E-01
3.6	157	477	0.109E-01
3.7	105	337	0.844E-02
3.8	101	329	0.148E-01
3.9	57	184	0.860E-02
4.0	37	131	0.930E-01

PRINT FROM NELME

MINIMIZATION OF $F(X) = 100*(X(2)-X(1)**2)**2 + (1-X(1))**2 +$
 $90*(X(4)-X(3))**2 + (1-X(3))**2 + 10.1*((X(2)-1)**2) + (X(4)-1)**2)$
 $19.8*(X(2)-1)*(X(4)-1)$
 UNTIL $F(X) < 1.E-1$ FOR DIFFERENT SIZES OF THE INITIAL POLYHEDRON,
 STARTING POINT IS (-3,-1,-3,-1),

SIZE OF THE INITIAL POLYHEDRON	NUMBER OF ITERATIONS REQUIRED	NUMBER OF EVALUATIONS REQUIRED	TESTO OF NELME
4.1	196	604	0.224E-01
4.2	267	804	0.953E-02
4.3	24	94	0.650E-01
4.4	656	2001	0.506E-04
4.5	349	1021	0.248E-01
4.6	325	965	0.541E-02
4.7	384	1149	0.584E-02
4.8	292	877	0.862E-02
4.9	291	858	0.582E-01
5.0	268	799	0.212E-01
5.1	363	1081	0.264E-02
5.2	369	1106	0.127E-01
5.3	245	726	0.492E-01
5.4	325	961	0.161E-01
5.5	252	755	0.624E-02
5.6	895	2001	0.475E-05
5.7	320	968	0.467E-02
5.8	253	772	0.552E-01
5.9	130	409	0.118E-01
6.0	133	412	0.295E-01
6.1	138	438	0.648E-02
6.2	221	670	0.503E-01
6.3	244	716	0.336E-01
6.4	71	226	0.154E-01
6.5	83	267	0.225E-01
6.6	222	652	0.352E-01
6.7	283	854	0.230E-01
6.8	271	794	0.725E-01
6.9	67	222	0.134E-01
7.0	33	123	0.185E-01
7.1	24	91	0.332
7.2	17	69	1.31
7.3	932	2001	0.302E-06
7.4	64	219	0.175E-02
7.5	58	199	0.189E-01
7.6	67	221	0.122E-01
7.7	66	216	0.430E-01
7.8	55	193	0.217E-01
7.9	75	241	0.391E-01
8.0	152	469	0.186E-01

PRINT FROM NELME

MINIMIZATION OF $F(X)=100*(X(2)-X(1))**2**2+(1-X(1))**2+$
 $90*(X(4)-X(3))**2+(1-X(3))**2+10.1*((X(2)-1)**2)+(X(4)-1)**2)$
 $19.8*(X(2)-1)*(X(4)-1)$
 UNTIL $F(X) < 1.E-3$ FOR DIFFERENT SIZES OF THE INITIAL POLYHEDRON.
 STARTING POINT IS $(-3,-1,-3,-1)$.

SIZE OF THE INITIAL POLYHEDRON	NUMBER OF ITERATIONS REQUIRED	NUMBER OF EVALUATIONS REQUIRED	TESTQ OF NELME
0.1	211	613	0.321E-04
0.2	217	644	0.171E-02
0.3	101	306	0.673E-03
0.4	115	358	0.150E-04
0.5	129	385	0.998E-02
0.6	127	394	0.427E-03
0.7	129	395	0.600E-04
0.8	134	397	0.283E-03
0.9	101	317	0.293E-03
1.0	35	125	0.616E-02
1.1	126	393	0.412E-03
1.2	92	293	0.613E-03
1.3	128	397	0.159E-02
1.4	670	2001	0.105E-05
1.5	863	2000	0.148E-05
1.6	951	2000	0.477E-06
1.7	658	1918	0.282E-03
1.8	588	1713	0.228E-03
1.9	378	1125	0.583E-03
2.0	324	967	0.479E-03
2.1	302	884	0.304E-03
2.2	954	2000	0.369E-06
2.3	950	2000	0.369E-06
2.4	956	2001	0.674E-06
2.5	950	2000	0.640E-06
2.6	947	2000	0.302E-06
2.7	335	964	0.881E-03
2.8	146	445	0.106E-02
2.9	134	421	0.924E-04
3.0	81	254	0.851E-03
3.1	101	311	0.156E-02
3.2	389	1146	0.794E-03
3.3	320	941	0.135E-02
3.4	263	804	0.185E-03
3.5	149	458	0.897E-03
3.6	204	608	0.103E-02
3.7	161	499	0.238E-03
3.8	127	415	0.816E-03
3.9	80	256	0.138E-03
4.0	75	250	0.147E-02

PRINT FROM NELME

MINIMIZATION OF $F(X) = 100(X_1 - X_2)^2 + (1 - X_1)^2 + X_2^2 + 90(X_1(X_2 - 1) + X_1(X_3) + X_2(X_3) + X_3(X_4) - 1) + 19.8(X_2(X_3 - 1) + X_3(X_4) - 1)$
UNTIL $F(X) < 1.E-3$ FOR DIFFERENT SIZES OF THE INITIAL POLYHEDRON.
STARTING POINT IS (-3, -1, -3, -1).

SIZE OF THE INITIAL POLYHEDRON	NUMBER OF ITERATIONS REQUIRED	NUMBER OF EVALUATIONS REQUIRED	TESTS OF NELME
--------------------------------	-------------------------------	--------------------------------	----------------

4.1	217	668	0.243E-02
4.2	309	925	0.623E-03
4.3	108	349	0.563E-03
4.4	656	2001	0.506E-04
4.5	390	1139	0.452E-03
4.6	412	1214	0.563E-03
4.7	422	1255	0.934E-03
4.8	355	1058	0.156E-03
4.9	303	897	0.241E-02
5.0	339	1016	0.463E-03
5.1	458	1351	0.184E-03
5.2	391	1173	0.455E-03
5.3	284	852	0.242E-04
5.4	350	1039	0.408E-03
5.5	354	1054	0.945E-03
5.6	895	2001	0.475E-05
5.7	373	1125	0.384E-03
5.8	413	1242	0.560E-03
5.9	204	635	0.770E-04
6.0	160	495	0.623E-03
6.1	179	557	0.331E-02
6.2	282	840	0.229E-02
6.3	320	949	0.425E-03
6.4	124	389	0.129E-02
6.5	141	436	0.210E-02
6.6	332	973	0.384E-03
6.7	347	1037	0.657E-02
6.8	308	900	0.247E-02
6.9	104	334	0.647E-03
7.0	102	328	0.667E-03
7.1	86	277	0.346E-03
7.2	77	260	0.566E-03
7.3	932	2001	0.302E-06
7.4	105	332	0.110E-02
7.5	128	409	0.736E-04
7.6	116	362	0.213E-03
7.7	88	284	0.101E-02
7.8	78	263	0.733E-03
7.9	98	316	0.793E-03
8.0	186	574	0.467E-03

PRINT FROM NELME

MINIMIZATION OF $F(X) = 100*(X(2)-X(1))^2 + (1-X(1))^2 +$
 $90*(X(4)-X(3))^2 + (1-X(3))^2 + 10.1*((X(2)-1)^2 + (X(4)-1)^2)$
 $19.8*(X(2)-1)*(X(4)-1)$
 UNTIL $F(X) < 1.E-5$ FOR DIFFERENT SIZES OF THE INITIAL POLYHEDRON,
 STARTING POINT IS $(-3, -1, -3, -1)$.

SIZE OF THE INITIAL POLYHEDRON	NUMBER OF ITERATIONS REQUIRED	NUMBER OF EVALUATIONS REQUIRED	TESTO OF NELME
0.1	259	748	0.790E-05
0.2	232	695	0.436E-05
0.3	127	380	0.569E-05
0.4	146	441	0.499E-05
0.5	156	483	0.825E-06
0.6	140	437	0.108E-04
0.7	154	471	0.309E-05
0.8	154	457	0.624E-05
0.9	132	412	0.141E-04
1.0	94	310	0.709E-05
1.1	139	436	0.453E-05
1.2	104	332	0.667E-05
1.3	141	438	0.180E-04
1.4	670	2001	0.105E-05
1.5	863	2000	0.148E-05
1.6	951	2000	0.477E-06
1.7	677	1985	0.804E-06
1.8	614	1787	0.145E-04
1.9	386	1151	0.274E-04
2.0	338	1009	0.592E-05
2.1	315	923	0.200E-04
2.2	954	2000	0.369E-06
2.3	950	2000	0.369E-06
2.4	956	2001	0.674E-06
2.5	950	2000	0.640E-06
2.6	947	2000	0.302E-06
2.7	376	1086	0.533E-05
2.8	163	499	0.208E-04
2.9	154	477	0.827E-05
3.0	115	358	0.724E-05
3.1	123	383	0.438E-05
3.2	444	1313	0.275E-05
3.3	328	975	0.639E-05
3.4	279	855	0.563E-05
3.5	179	553	0.474E-05
3.6	220	654	0.585E-05
3.7	173	536	0.683E-05
3.8	139	456	0.219E-04
3.9	112	354	0.201E-05
4.0	90	304	0.299E-05

PRINT FROM NELME

MINIMIZATION OF $F(X) = 100*(X(2)-X(1))^2 + (1-X(1))^2 + 90*(X(4)-X(3))^2 + (1-X(3))^2 + 10.1*((X(2)-1))^2 + (X(4)-1))^2 + 19.8*(X(2)-1)*(X(4)-1)$
 UNTIL $F(X) < 1.E-5$ FOR DIFFERENT SIZES OF THE INITIAL POLYHEDRON.
 STARTING POINT IS $(-3, -1, -3, -1)$.

SIZE OF THE INITIAL POLYHEDRON	NUMBER OF ITERATIONS REQUIRED	NUMBER OF EVALUATIONS REQUIRED	TEST0 OF NELME
4.1	502	2000	0.817E-08
4.2	320	959	0.130E-04
4.3	125	400	0.684E-05
4.4	656	2001	0.506E-04
4.5	404	1184	0.108E-04
4.6	424	1254	0.615E-05
4.7	434	1296	0.197E-04
4.8	368	1100	0.231E-04
4.9	498	2001	0.274E-08
5.0	360	1080	0.582E-05
5.1	470	1383	0.522E-04
5.2	413	1244	0.363E-05
5.3	409	1190	0.202E-04
5.4	378	1122	0.600E-05
5.5	369	1108	0.501E-05
5.6	895	2001	0.475E-05
5.7	379	1147	0.532E-04
5.8	445	1346	0.147E-05
5.9	224	690	0.964E-05
6.0	198	609	0.499E-05
6.1	233	713	0.637E-05
6.2	310	930	0.170E-04
6.3	336	999	0.930E-05
6.4	153	478	0.448E-05
6.5	149	466	0.889E-05
6.6	370	1093	0.146E-05
6.7	360	1086	0.682E-05
6.8	317	933	0.437E-04
6.9	119	388	0.782E-05
7.0	134	430	0.322E-05
7.1	104	336	0.262E-05
7.2	96	318	0.487E-05
7.3	932	2001	0.302E-06
7.4	118	376	0.860E-05
7.5	145	454	0.242E-04
7.6	129	403	0.130E-04
7.7	95	312	0.751E-05
7.8	117	376	0.668E-05
7.9	118	375	0.139E-04
8.0	199	615	0.947E-05

PRINT FROM NELME

MINIMIZATION OF $F(X) = 100*(X(2)-X(1)**2)**2 + (1-X(1))**2 +$
 $90*(X(4)-X(3))**2 + (1-X(3))**2 + 10.1*((X(2)-1)**2) + (X(4)-1)**2$
 $19.8*(X(2)-1)*(X(4)-1)$
 UNTIL $F(X) < 1.E-7$ FOR DIFFERENT SIZES OF THE INITIAL POLYHEDRON.
 STARTING POINT IS (-3,-1,-3,-1).

SIZE OF THE INITIAL POLYHEDRON	NUMBER OF ITERATIONS REQUIRED	NUMBER OF EVALUATIONS REQUIRED	TESTO OF NELME
0.1	272	789	0.796E-07
0.2	242	729	0.127E-06
0.3	142	428	0.495E-07
0.4	149	456	0.314E-06
0.5	175	541	0.345E-07
0.6	148	467	0.135E-06
0.7	170	521	0.142E-06
0.8	169	504	0.111E-06
0.9	155	485	0.128E-06
1.0	188	358	0.288E-06
1.1	151	475	0.374E-07
1.2	120	385	0.512E-07
1.3	151	475	0.175E-06
1.4	670	2001	0.105E-05
1.5	863	2000	0.148E-05
1.6	951	2000	0.477E-06
1.7	682	2001	0.882E-06
1.8	626	1829	0.396E-07
1.9	399	1199	0.295E-07
2.0	352	1059	0.540E-07
2.1	324	953	0.306E-06
2.2	954	2000	0.369E-06
2.3	950	2000	0.369E-06
2.4	956	2001	0.674E-06
2.5	950	2000	0.640E-06
2.6	947	2000	0.302E-06
2.7	391	1133	0.685E-07
2.8	168	517	0.132E-05
2.9	170	535	0.198E-07
3.0	124	390	0.562E-07
3.1	146	453	0.784E-07
3.2	456	1352	0.105E-06
3.3	338	1011	0.841E-07
3.4	297	915	0.383E-07
3.5	192	594	0.559E-07
3.6	228	679	0.431E-06
3.7	181	567	0.126E-06
3.8	181	499	0.124E-06
3.9	146	460	0.315E-07
4.0	110	362	0.313E-06

PRINT FROM NELME

MINIMIZATION OF $F(X) = 100*(X(2)-X(1)**2)**2 + (1-X(1))**2 +$
 $90*(X(4)-X(3))**2 + (1-X(3))**2 + 10.1*((X(2)-1)**2) + (X(4)-1)**2$
 $+ 19.9*(X(2)-1)*(X(4)-1)$
 UNTIL $F(X) < 1.E-7$ FOR DIFFERENT SIZES OF THE INITIAL POLYHEDRON.
 STARTING POINT IS $(-3, -1, -3, -1)$.

SIZE OF THE INITIAL POLYHEDRON	NUMBER OF ITERATIONS REQUIRED	NUMBER OF EVALUATIONS REQUIRED	TESTO OF NELME
4.1	502	2000	0.817E-08
4.2	334	1014	0.469E-07
4.3	136	438	0.106E-06
4.4	656	2001	0.506E-04
4.5	418	1232	0.386E-07
4.6	445	1322	0.743E-07
4.7	444	1332	0.170E-06
4.8	376	1132	0.133E-06
4.9	498	2001	0.274E-08
5.0	373	1126	0.388E-07
5.1	485	1434	0.116E-06
5.2	429	1295	0.126E-06
5.3	420	1226	0.240E-06
5.4	391	1165	0.134E-06
5.5	377	1139	0.717E-07
5.6	895	2001	0.475E-05
5.7	388	1182	0.113E-06
5.8	477	1438	0.874E-07
5.9	237	735	0.957E-07
6.0	224	694	0.590E-07
6.1	247	758	0.116E-06
6.2	337	1014	0.348E-07
6.3	342	1021	0.680E-06
6.4	172	539	0.180E-06
6.5	163	516	0.732E-07
6.6	386	1143	0.785E-07
6.7	374	1140	0.526E-07
6.8	324	966	0.253E-06
6.9	132	429	0.102E-06
7.0	143	459	0.327E-06
7.1	118	382	0.894E-07
7.2	110	369	0.670E-07
7.3	932	2001	0.302E-06
7.4	128	414	0.124E-06
7.5	166	522	0.675E-07
7.6	142	450	0.813E-07
7.7	104	348	0.820E-07
7.8	122	396	0.323E-06
7.9	133	423	0.792E-07
8.0	207	644	0.134E-06

PRINT FROM NELME

MINIMIZATION OF $F(X) = 100*(X(3) - 10*\text{ATAN2}(X(2), X(1)) / (2*\text{PI}))**2 +$
 $(\text{SQRT}(X(1)**2 + X(2)**2) - 1)**2 + X(3)**2$
 UNTIL $F(X) < 1.E-1$ FOR DIFFERENT SIZES OF THE INITIAL POLYHEDRON,
 STARTING POINT IS $(-1, 0, 0)$.

SIZE OF THE INITIAL POLYHEDRON	NUMBER OF ITERATIONS REQUIRED	NUMBER OF EVALUATIONS REQUIRED	TEST0 OF NELME
0.1	203	630	0.819E-01
0.2	198	609	0.278E-01
0.3	142	428	0.144E-01
0.4	153	471	0.553E-01
0.5	141	438	0.107E-01
0.6	114	364	0.983E-01
0.7	866	2001	0.119E-06
0.8	797	2001	0.000
0.9	146	436	0.681E-01
1.0	150	488	0.603E-02
1.1	184	566	0.143
1.2	148	441	0.147E-01
1.3	90	286	0.486E-02
1.4	81	258	0.471E-01
1.5	88	277	0.102
1.6	205	640	0.520E-01
1.7	117	361	0.348E-01
1.8	23	80	0.353E-01
1.9	23	90	0.443E-01
2.0	12	48	0.465
2.1	12	48	0.279
2.2	10	38	2.45
2.3	10	38	2.58
2.4	24	83	0.707E-01
2.5	98	314	0.307E-01
2.6	31	105	0.693E-02
2.7	8	30	4.04
2.8	8	30	4.26
2.9	18	69	0.619E-02
3.0	21	77	0.127E-01
3.1	26	92	0.122E-01
3.2	42	136	0.204E-01
3.3	31	112	0.574E-02
3.4	47	158	0.913E-03
3.5	29	103	0.396E-02
3.6	31	112	0.250E-01
3.7	31	107	0.407E-01
3.8	48	136	0.557E-02
3.9	48	160	0.254E-01
4.0	54	169	0.135E-01

PRINT FROM NELME

MINIMIZATION OF $F(X)=100*(X(3)-10*ATAN2(X(2),X(1)))/(2*PI)**2+$
 $(SQRT(X(1)**2+X(2)**2)-1)**2+X(3)**2$
 UNTIL $F(X) < 1.E-1$ FOR DIFFERENT SIZES OF THE INITIAL POLYHEDRON.
 STARTING POINT IS $(-1,0,0)$.

SIZE OF THE INITIAL POLYHEDRON	NUMBER OF ITERATIONS REQUIRED	NUMBER OF EVALUATIONS REQUIRED	TESTQ OF NELME
4.1	42	137	0.267E-01
4.2	50	159	0.351E-01
4.3	26	89	0.101
4.4	33	107	0.364E-01
4.5	49	163	0.130E-01
4.6	30	103	0.373E-01
4.7	52	167	0.908E-01
4.8	48	158	0.497E-01
4.9	38	122	0.189E-01
5.0	82	260	0.184E-01
5.1	38	129	0.694E-02
5.2	70	231	0.198E-01
5.3	32	114	0.442E-01
5.4	48	161	0.205E-01
5.5	42	138	0.142E-01
5.6	23	86	0.167
5.7	31	109	0.571E-01
5.8	38	133	0.616E-01
5.9	109	333	0.116E-01
6.0	36	118	0.556E-01
6.1	35	121	0.589E-01
6.2	64	202	0.342E-01
6.3	37	134	0.208E-01
6.4	44	150	0.203E-01
6.5	53	176	0.169E-01
6.6	51	171	0.149E-01
6.7	62	199	0.324E-01
6.8	84	262	0.665E-01
6.9	111	355	0.656E-01
7.0	68	224	0.449E-01
7.1	96	300	0.885E-02
7.2	65	212	0.102
7.3	56	192	0.733E-02
7.4	85	267	0.486E-01
7.5	40	141	0.229E-01
7.6	56	179	0.367E-01
7.7	50	166	0.126E-01
7.8	99	313	0.231E-01
7.9	59	187	0.351E-01
8.0	81	253	0.329E-01

PRINT FROM NELME

MINIMIZATION OF F(X)=T08*(X(3)-T0KATN02*(X(2),X(1)))/(2*(PI))**2+
(S0RT(X(1)**2+X(2)**2)-1)**2+X(3)**2
UNTIL F(X) < T'E-3 FOR DIFFERENT SIZES OF THE INITIAL POLYHEDRON.
STARTING POINT IS (-1,0,0)

SIZE OF THE INITIAL POLYHEDRON	NUMBER OF ITERATIONS REQUIRED	NUMBER OF EVALUATIONS REQUIRED	TESTS OF NELME
--------------------------------------	-------------------------------------	--------------------------------------	-------------------

0.1	228	708	0.455E-03
0.2	235	719	0.686E-03
0.3	186	561	0.496E-03
0.4	194	591	0.948E-03
0.5	194	593	0.128E-03
0.7	866	601	0.138E-03
0.8	797	2001	0.119E-06
0.9	194	579	0.754E-03
1.0	185	576	0.553E-03
1.1	248	758	0.648E-03
1.2	174	525	0.302E-03
1.3	148	467	0.288E-03
1.4	98	287	0.184E-03
1.5	124	387	0.134E-03
1.6	251	784	0.673E-03
1.7	142	442	0.188E-03
1.8	38	103	0.343E-03
1.9	42	139	0.117E-03
2.0	56	185	0.646E-03
2.1	77	251	0.603E-03
2.2	75	243	0.199E-03
2.3	84	270	0.118E-03
2.4	68	212	0.171E-03
2.5	134	425	0.797E-03
2.6	59	195	0.428E-03
2.7	49	165	0.183E-03
2.8	64	204	0.948E-03
2.9	51	164	0.192E-03
3.0	51	162	0.110E-03
3.1	57	179	0.117E-03
3.2	79	246	0.569E-03
3.3	74	236	0.451E-03
3.4	88	261	0.346E-03
3.5	88	285	0.264E-03
3.6	68	222	0.281E-03
3.7	88	274	0.164E-03
3.8	88	282	0.278E-03
3.9	88	267	0.444E-03
4.0	85	296	0.197E-03

PRINT FROM NELME

MINIMIZATION OF $F(X) = 100*(X(3) - 10*ATAN2(X(2), X(1)))/(2*PI)**2 +$
 $(SQRT(X(1)**2 + X(2)**2) - 1)**2 + X(3)**2$
 UNTIL $F(X) < 1.E-3$ FOR DIFFERENT SIZES OF THE INITIAL POLYHEDRON.
 STARTING POINT IS (-1,0,0).

SIZE OF THE INITIAL POLYHEDRON	NUMBER OF ITERATIONS REQUIRED	NUMBER OF EVALUATIONS REQUIRED	TESTO OF NELME
4.1	54	176	0.432E-02
4.2	92	287	0.199E-02
4.3	68	216	0.252E-02
4.4	101	313	0.534E-03
4.5	90	288	0.694E-03
4.6	56	189	0.516E-03
4.7	90	282	0.197E-02
4.8	84	273	0.409E-04
4.9	73	233	0.443E-03
5.0	126	391	0.108E-02
5.1	69	219	0.749E-03
5.2	127	403	0.624E-03
5.3	64	217	0.586E-03
5.4	126	397	0.396E-03
5.5	118	380	0.114E-03
5.6	79	255	0.429E-03
5.7	88	272	0.292E-02
5.8	118	369	0.115E-02
5.9	121	371	0.645E-03
6.0	74	228	0.150E-02
6.1	58	197	0.812E-03
6.2	109	344	0.655E-03
6.3	80	266	0.191E-02
6.4	111	353	0.633E-03
6.5	110	346	0.638E-03
6.6	113	358	0.370E-03
6.7	111	351	0.401E-03
6.8	114	355	0.784E-04
6.9	124	401	0.340E-03
7.0	130	404	0.352E-03
7.1	156	478	0.729E-03
7.2	113	363	0.299E-02
7.3	87	284	0.100E-02
7.4	136	421	0.377E-02
7.5	109	353	0.187E-02
7.6	84	272	0.499E-02
7.7	102	329	0.312E-03
7.8	110	349	0.330E-02
7.9	135	417	0.325E-03
8.0	115	352	0.757E-03

PRINT FROM NELME

MINIMIZATION OF $F(X)=100*(X(3)-10*ATAN2(X(2),X(1)))/(2*PI))**2+$
 $(SQRT(X(1)**2+X(2)**2)-1)**2+X(3)**2$
 UNTIL $F(X) < 1.E-5$ FOR DIFFERENT SIZES OF THE INITIAL POLYHEDRON.
 STARTING POINT IS $(-1,0,0)$.

SIZE OF THE INITIAL POLYHEDRON	NUMBER OF ITERATIONS REQUIRED	NUMBER OF EVALUATIONS REQUIRED	TESTO OF NELME
0.1	235	732	0.623E-04
0.2	254	774	0.213E-04
0.3	201	604	0.140E-04
0.4	206	632	0.908E-05
0.5	203	622	0.148E-04
0.6	209	654	0.176E-04
0.7	866	2001	0.119E-06
0.8	797	2001	0.000
0.9	201	604	0.289E-04
1.0	193	606	0.672E-05
1.1	269	822	0.181E-04
1.2	186	560	0.620E-05
1.3	166	522	0.132E-04
1.4	161	498	0.469E-05
1.5	140	433	0.173E-04
1.6	260	815	0.880E-05
1.7	150	473	0.648E-05
1.8	60	196	0.138E-04
1.9	75	240	0.154E-04
2.0	79	261	0.719E-05
2.1	99	319	0.202E-04
2.2	87	280	0.315E-04
2.3	93	299	0.116E-04
2.4	89	255	0.366E-05
2.5	147	467	0.636E-05
2.6	67	221	0.733E-04
2.7	57	196	0.811E-05
2.8	71	229	0.146E-04
2.9	60	195	0.533E-05
3.0	60	191	0.109E-04
3.1	90	280	0.225E-04
3.2	96	303	0.400E-05
3.3	116	367	0.276E-05
3.4	112	356	0.402E-05
3.5	107	346	0.778E-05
3.6	76	251	0.620E-05
3.7	100	315	0.143E-04
3.8	103	327	0.128E-04
3.9	100	313	0.468E-05
4.0	107	335	0.876E-05