A Program Package for GLS Identification of Dynamic Systems

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1973

TFRT-7048

A PROGRAM PACKAGE FOR GLS IDENTIFICATION OF DYNAMIC SYSTEMS

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Report 7311 (C) July 1973
Lund Institute of Technology
Division of Automatic Control

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OF DYNAMIC SYSTEMS.

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ABSTRACT.

In this report some programs are given, which perform identification of a linear dynamic system using the GLS method.

TABLE OF CONTENTS

I. DESCRIPTION OF THE PROGRAMS.

The program package for GLS identification consists of two parts, one for simulation and identification of data (the main program is TGLS) and the other one for identification of real data (the main program IGLS or the subroutine IDGLS is to be used). The part containing TGLS includes the subroutines SIMUL, PRBSTA, PRB, NODI, LS, LSQ, EIGS and DSYMIN from the program library of the Division of Automatic Control. The part containing IDGLS includes the subroutines LS, LSQ, EIGS, DSYMIN, STDAT POST, VMEAN, VLIM, REST, DESYM, SOLVS, CSGFT, FINO and ROT from the same program library.

The method and the programs are earlier described in Clarke (1967) and Söderström (1972).

Note the possibility to use LS + CFAC which will increase the accuracy and decrease the computing time (see Söderström (1973 a) and Söderström (1973 b)).

1. TGLS. This is a main program for identification of simulated data. It consists of the following parts:

   1. Parameters are read and printed.
   2. Generation of simulated data (the subroutine SIMUL is used).
   3. Identification (a CALL GLS(..) is done).

   There is possibility to do several simulations and identifications in one execution.

2. IDGLS and IGLS. IDGLS is a subroutine for identification of real data. If IDGLS performs all wanted operations it is suitable to use the main program IGLS. The subroutine IDGLS performs the following operations on the data:

1. The data are read into core using the subroutine STDAT.
2. Parameters for the identification are read and printed.
3. Identification (a CALL GLS(...) is done).
4. Computation of the poles, the zeros and the static gain.
5. Test of the residuals of the model (the subroutine REST is used).

It is possible to repeat from Step 2 several times. This is suitable if it is desired to compute models of different orders of the same data.

Note that the execution of IDGLS is ended if ITER = 0.

3. GLS. This is the major subroutine for the identification, where all administration takes part.

Special parameters

IFILT = 0 gives the model

$$\hat{A}(q^{-1})y(t) = \hat{B}(q^{-1})u(t) + \frac{1}{\hat{C}(q^{-1})}\varepsilon(t) \tag{1}$$

$$\hat{A}(q^{-1}) = 1 + \hat{a}_1 q^{-1} + \ldots + \hat{a}_{NA} q^{-NA}$$

$$\hat{B}(q^{-1}) = \hat{b}_1 q^{-1} + \ldots + \hat{b}_{NB} q^{-NB}$$

$$\hat{C}(q^{-1}) = 1 + \hat{c}_1 q^{-1} + \ldots + \hat{c}_{NC} q^{-NC}$$

IFILT = 1 corresponds to the model

$$\hat{A}(q^{-1})y(t) = \hat{B}(q^{-1})u(t) + \frac{1}{\prod_i \hat{C}_i(q^{-1})} \varepsilon(t) \tag{2}$$

However, the product $\prod_i \hat{C}_i$ is not computed and when the algorithm has converged a new $\hat{C}$-polynomial is estimated and a model of the form (1) is obtained as a final result.

ITER - maximum number of iterations.

EPST - the algorithm is considered to have converged if

$$\max_{i, |\hat{\theta}_i| \geq 0.01} \left| \frac{\Delta \hat{\theta}_i}{\hat{\theta}_i} \right| \leq \text{EPST}$$

where $\hat{\theta}$ is the actual estimate and $\Delta\hat{\theta}$ the difference between the actual estimate and the estimate one iteration ago.

A small value of EPST will in general cause many iterations. Under idealized assumptions the number of necessary iterations n_i is given by

$$\lambda^{n_i} = K \cdot \text{EPST}$$

where $0 \leq \lambda < 1$ and λ as well as K depends on the system.

ITER1 - maximum number of approximate Newton-Raphson iterations in VGLS.

Rough rule of thumb. Take ITER = 10 - 20 (more for bad conditioned systems). EPST = 10^{-3} and ITER1 = 3 as a first choice.

4. FILT. This routine performs filtering of a signal.

5. RESID1. This program computes residuals.

6. VGLS. This program evaluates the loss function and its first order and second order derivatives. Also a diagonalization of the matrix of second order derivatives is performed as well as an inversion to get an estimated covariance matrix. Some approximate Newton-Raphson iterations are performed in order to improve the minimization.

7. FACT. This subroutine computes and prints the zeros of the $\hat{A}$-, $\hat{B}$- and $\hat{C}$-polynomials of the model (1), the poles of the corresponding continuous system and the static gain.

Possible extensions.

The package may be improved by a more compact storage of the data.

The package may be modified (especially GLS) to estimate the parameters of the model

$$y(t) = \frac{\hat{B}(q^{-1})}{\hat{A}(q^{-1})} u(t) + \frac{1}{\hat{C}(q^{-1})} \varepsilon(t) \tag{3}$$

See Clarke (1973).

II. REFERENCES.

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On the Convergence Properties of the Generalized Least Squares Identification Method. Report 7228, Division of Automatic Control, Lund Institute of Technology.

Söderström, T. (1973 a).
A Program for Test of Common Factors of Two Polynomials. Report 7313(C), Division of Automatic Control, Lund Institute of Technology.

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On the Simplification of Dynamic Models Obtained by Least Squares Identification (Preliminary title). Forthcoming report, Division of Automatic Control, Lund Institute of Technology.

III. LISTS OF THE PROGRAMS

```
PROGRAM TGLS

MAIN PROGRAM FOR GENERALIZED LEAST SQUARES IDENTIFICATION
OF SIMULATED DATA
AUTHOR TORSTEN SODERSTROM 1971-10-01
REFERENCE REPORT 7311(C)

THE FOLLOWING DATA ARE READ FROM CARDS
1- M,ISYST,IMOD,INF - 4I10
M - NUMBER OF SAMPLES (MAX 1000)
IF M=0 THE PROGRAM STOPS
ISYST=10000*NA+100*NB+NC  ORDER OF TRUE OPERATORS
IMOD=10000*MNA+100*MNB+MNC  ORDER OF ESTIMATED OPERATORS
INF=10000*ITER+1000*IFILT+100*INIT+10*IPRINT+ISIM
ITER - MAX NUMBER OF ITERATIONS
IFILT  =0-FILTER ORIGINAL DATA   =1-FILTER FILTERED DATA
INIT  =0 START WITH LS ESTIMATE OF A AND B
      =1 START WITH VALUES OF A AND B FROM CARD
      =2 START WITH VALUES OF C       FROM CARD
IPRINT  =0-LITTLE OUTPRINT  =1 GREAT OUTPRINT
  REPORT 7311(C) CONTAINS EXAMPLES
ISIM =0 U(T) IS A PRBS
     =1 U(T) IS A WHITE NOISE INDEPENDENT OF E(T)
2-(T(I),I=1,(NA+NB+NC),AL,EPST - 8F10.5/F10.5
T - PARAMETER VECTOR  (TRUE VALUES)
AL - STANDARD DEVIATION OF THE NOISE
EPST - TEST QUANTITY FOR CONVERGENCE
3- /IF INIT=1/ (T(I),I=1,(MNA+MNB)) - 8F10.5 START VALUES OF A AND B
3- /IF INIT=2/ (T(I)+MNA+MNB),I=1,MNC) -8F10.5  START VALUES OF C

AFTER THE IDENTIFICATION A NEW CARD 1 IS READ

SUBROUTINE REQUIRED
      SIMUL
      PRBSTA
      PRB
      NODI
      GLS
      LS
      LSQ
      VGLS
      DSYMIN
      FILT
      RESID
      RESID1
      EIGS

  DIMENSION U(1000),Y(1000),DAT(3000),AB(1000,11)
  DIMENSION TSYST(30),TMOD(30)

  EPST=0.01
1 READ 100,M,ISYST,IMOD,INF
100 FORMAT(4I10)
  IF(M.EQ.0) GO TO 99
  IS=ISYST
  IM=IMOD
  NA=(IS-MOD(IS,10000))/10000
  MNA=(IM-MOD(IM,10000))/10000
  IS=MOD(IS,10000)
  IM=MOD(IM,10000)
  NB=(IS-MOD(IS,100))/100
  MNB=(IM-MOD(IM,100))/100
  NC=MOD(IS,100)
```

```
      MNC=MOD(IM,100)
      ITER=(INF-MOD(INF,10000))/10000
      INF=MOD(INF,10000)
      IFILT=(INF-MOD(INF,1000))/1000
      INF=MOD(INF,1000)
      INIT=(INF-MOD(INF,100))/100
      INF=MOD(INF,100)
      IPRINT=(INF-MOD(INF,10))/10
      IPRINT=IPRINT+1
      ISIM=MOD(INF,10)
      N2=NA+NB
      N3=N2+NC
      MN2=MNA+MNB
      MN3=MN2+MNC

      IF(N3.GT.0) READ 101,(TSYST(I),I=1,N3),AL
      IF(N3.EQ.0) READ 101,AL
      READ 101,EPST
101   FORMAT(8F10.5)

      PRINT 300,ISIM,M
300   FORMAT(1H1,9X,5HISIM=,I5,10X,17HNUMBER OF SAMPLES,I5)
      PRINT 301,NA,NB,NC
301   FORMAT(10X,25HORDER OF SYSTEM OPERATORS,3I5)
     F10X,17HSYSTEM OPERATORS)
      IF(N3.GT.0) PRINT 201,(TSYST(I),I=1,N3),AL
201   FORMAT(8E15.7)

      IF(MN3.EQ.0) GO TO 1
      DO 2 I=1,MN3
    2 TMOD(I)=0.
      IF(INIT.EQ.1.AND.MN2.NE.0) READ 101,(TMOD(I),I=1,MN2)
      IF(INIT.EQ.2.AND.MNC.NE.0) READ 101,(TMOD(I+MN2),I=1,MNC)

      PRINT 302,MNA,MNB,MNC
302   FORMAT(10X,24HORDER OF MODEL OPERATORS,3I5/
     F10X,37HINITIAL ESTIMATES OF MODEL PARAMETERS)
      PRINT 201,(TMOD(I),I=1,MN3)
      PRINT 303,INIT,IFILT,IPRINT,ITER
303   FORMAT(10X,5HINIT=,I7/10X,6HIFILT=,I6/10X,7HIPRINT=,I5/
     F10X,24HMAX NUMBER OF ITERATIONS,I5)

      SIMULATION

      JS=1
      IF(ISIM.EQ.1) JS=4
      IE=3323
      CALL SIMUL(U,Y,TSYST,1.0,AL,M,NA,NB,NC,JS,7,IE)
      DO 3 I=1,M
      DAT(3*I-1)=U(I)
    3 DAT(3*I)=Y(I)

      IDENTIFICATION

      IA=1000
      IB=11
      K1=MSLEFT(DUM)
      CALL GLS(DAT,TMOD,AB,M,MNA,MNB,MNC,ITER,5,IFILT,INIT,IPRINT,EPST,I
     *A)
      K2=MSLEFT(DUM)
      K=K1-K2
      PRINT 400,K
400   FORMAT(//10X,'TIME FOR IDENTIFICATION WAS',I5,' MS')
      GO TO 1
```

```
99 CONTINUE
   STOP
   END
```

```
SUBROUTINE IDGLS(AB,DAT,U,Y,RES,RESF,IA)

MAIN ROUTINE FOR IDENTIFICATION WITH THE GLS METHOD

THE MODEL
A(Q)*C(Q)*Y(T)=B(Q)*C(Q)*U(T)+RESF(T)
IS COMPUTED
A(Q)=1+A(1)*Q**(-1)+...A(NA)*Q**(-NA)
B(Q)=  B(1)*Q**(-1)+...B(NB)*Q**(-NB)
C(Q)=1+C(1)*Q**(-1)+...C(NC)*Q**(-NC)

AUTHOR TORSTEN SODERSTROM 1971-12-24
REVISED TORSTEN SODERSTROM 1973-04-12
REFERENCE REPORTS 7228,7311(C)

THE FOLLOWING DATA ARE READ FROM CARDS
1-M,JPRINT
  FORMAT(2I5)
M-NUMBER OF SAMPLES (MAX 1000)
JPRINT -CONTROLS THE OUTPRINT OF STDAT

A CALL OF STDAT IS PERFORMED
SEE DESCRIPTION OF THIS ROUTINE

2-NA,NB,NC,ITMAX,ITER,IFILT,IPRINT,INIT
  FORMAT(8I5)
NA,NB,NC ORDER OF OPERATORS (MIN 0,MAX 10)
ITMAX -MAX NUMBER OF ITERATIONS IN GLS (MIN 0,NO MAX)
ITER - MAX NUMBER OF NEWTON RAPHSON ITERATIONS (NO MIN,NO MAX)
  IT ITER.EQ.0 THE ROUTINE IS ENDED
  IT ITER.LT.0 THE LOSS FUNCTION AND ITS DERIVATIVES ARE NEVER EVALUATED
IFILT=0 IS VERSION 1,IFILT=1 IS VERSION 2  OF REPORT 7228
IPRINT CONTROLS THE OUTPRINT
  IF IPRINT=0 SMALL AMOUNT OF OUTPRINT
  IF IPRINT=1 LARGE AMOUNT OF OUTPRINT
  REPORT 7311(C) CONTAINS EXAMPLES
INIT - IF INIT=0 THE ITERATION IS STARTED WITH THE LS ESTIMATE OF A AND B
       IF INIT=1 THE ITERATION IS STARTED WITH GIVEN VALUES OF A AND B
       IF INIT=2 THE ITERATION IS STARTED WITH GIVEN VALUES OF C
3-EPST,TSAMP
  FORMAT(2F10.5)
EPST -TEST QUANTITY FOR STOP OF THE ITERATIONS
TSAMP - SAMPLING PERIOD IN SECONDS
/4-IF INIT=1 T(I),I=1,(NA+NB)
/4-IF INIT=2 T(I),I= (NA+NB+1)...(NA+NB+NC)
T-VECTOR OF ORDER (NA+NB+NC) CONTAINING THE PARAMETERS
FOR IDENTIFICATIONS AFTER THE FIRST ONE
ONLY CARDS 2-3(4) ARE NEEDED FOR FURTHER IDENTIFICATIONS ON THE SAME DATA

AB-MATRIX OF ORDER M*MAX(NA+NB+1,NC+1) USED AS WORKING AREA
DAT-VECTOR OF ORDER 3*M CONTAINING THE DATA (DAT IS CREATED IN STDAT)
U-VECTOR OF ORDER M,AT RETURN CONTAINING THE INPUT
Y-VECTOR OF ORDER M,AT RETURN CONTAINING THE OUTPUT
RES-VECTOR OF ORDER M,AT RETURN CONTAINING THE SIGNALS A(Q)*Y(T)-B(Q)*U(T)
RESF-VECTOR OF ORDER M,AT RETURN CONTAINING THE RESIDUALS
IA-DIMENSION PARAMETER

SUBROUTINE REQUIRED
   STDAT
      POST
      VLIM
      VMEAN
   GLS
```

```
      LS
        LSQ
      RESID1
      FILT
      VGLS
        DSYMIN
        EIGS
    REST

      DIMENSION AB(IA,1),DAT(1),U(1),Y(1),RES(1),RESF(1)
      DIMENSION T(30),DMIN(3),DMEAN(3),DMAX(3),MSD(3),TQ(15),RRES(201),
     FRRESU(201)

C     CREATE THE DAT-VECTOR

      READ 100,M,JPRINT
  100 FORMAT(16I5)
      CALL STDAT(DAT,DMIN,DMEAN,DMAX,MSD,U,M,1,1,JPRINT,IE)
      IF(IE.EQ.1) GO TO 99

C     READ AND PRINT PARAMETERS

    1 READ 100,NA,NB,NC,ITMAX,ITER,IFILT,IPRINT,INIT
      IF(ITER.EQ.0) GO TO 99
      READ 101,EPST,TSAMP
  101 FORMAT(8F10.5)
      PRINT 200
  200 FORMAT(1H1,'RESULT FROM IDENTIFICATION WITH GLS'
     F/1X,35(1H*))
      PRINT 201,NA,NB,NC
  201 FORMAT(/10X,'ORDER OF OPERATORS ',3I5)
      PRINT 202,IFILT,IPRINT,ITMAX,ITER
  202 FORMAT(/10X,'IFILT=',I5/10X,'IPRINT=',I4/10X,'ITMAX=',I5/
     F10X,'ITER=',I6)
      PRINT 204,INIT,M,EPST,TSAMP
  204 FORMAT(10X,'INIT=',I6/10X,I4,' SAMPLES ARE USED'/10X,'EPST=',G12.5
     F/10X,'SAMPLING PERIOD=',G12.5,' SECONDS')

      N2=NA+NB
      N3=N2+NC
      DO 5 I=1,N3
    5 T(I)=0.
      IF(INIT.EQ.1) READ 101,(T(I),I=1,N2)
      IF(INIT.EQ.2) READ 101,(T(I+N2),I=1,NC)
      PRINT 211
  211 FORMAT(/10X,'INITIAL VALUES')
      PRINT 101,(T(I),I=1,N3)

      IPRINT=IPRINT+1
      CALL GLS(DAT,T,AB,M,NA,NB,NC,ITMAX,ITER,IFILT,INIT,IPRINT,EPST,IA)

C     FACTORIZATION

      CALL FACT(T,NA,NB,NC,TSAMP)

C     TEST OF RESIDULAS

      DO 22 I=1,M
      U(I)=DAT(3*I-1)
   22 Y(I)=DAT(3*I)
      CALL RESID1(U,Y,RES,T,M,NA,NB)
      IF(NC.EQ.0) GO TO 24
      DO 23 I=1,NC
```

```
   23 TQ(I)=T(I+NA+NB)
   24 CALL FILT(RES,RESF,TQ,M,NC)
      CALL REST(RESF,U,TQ,RRES,RRESU,M,50,10,5,1,2)
C
      PRINT 207
  207 FORMAT(//10X,'10 FIRST VALUES OF THE RESIDUALS')
      PRINT 208,(RESF(I),I=1,10)
  208 FORMAT(10G12.5)
C
      GO TO 1
C
   99 CONTINUE
      STOP
      END
```

```
C       PROGRAM IGLS
C       THIS MAIN PROGRAM IS SUITABLE TO USE FOR SUBROUTINE IDGLS
C
        PARAMETER M=1000,N=11
        PARAMETER M3=M*3
        DIMENSION AB(M,N),DAT(M3),U(M),Y(M),RES(M),RESF(M)
        CALL IDGLS(AB,DAT,U,Y,RES,RESF,M,N,M3)
        STOP
        END
```

```
      SUBROUTINE GLS(DAT,T,AB,M,NA,NB,NC,ITER,ITER1,IFILT,INIT,IPRINT,
     *EPST,IA)
C
C     COMPUTES THE GENERALIZED LEAST SQUARES ESTIMATE
C     A(Q)*C(Q) Y(T) = B(Q)*C(Q) U(T) + E(T)
C     A(Q)=1 + A(1)*Q**(-1) +...+ A(NA)*Q**(-NA)
C     B(Q)=    B(1)*Q**(-1) +...+ B(NB)*Q**(-NB)
C     C(Q)=1 + C(1)*Q**(-1) +...+ C(NC)*Q**(-NC)
C
C     AUTHOR  TORSTEN SODERSTROM  1971-10-01
C     REFERENCE REPORT 7311(C)
C
C     DAT - VECTOR OF ORDER 3*M, CONTAINING THE DATA IN THE FOLLOWING FORM
C     TIME(1),U(1),Y(1),TIME(2),... Y(M)
C     T - VECTOR OF ORDER (NA+NB+NC) AT RETURN CONTAINING THE PARAMETER
C     ESTIMATES
C     T = (A(1)...A(NA),B(1)...B(NB),C(1)...C(NC))
C     AB - MATRIX OF ORDER M * MAX(NA+NB+1,NC+1) USED AS WORKING AREA
C     M - ORDER OF U AND Y (NUMBER OF SAMPLES) (MIN 31,MAX 1000)
C     NA,NB,NC - ORDER OF A,B,C RESP.
C     (NA+NB+NC)  (MIN 0,MAX 30)
C     ITER - MAX NUMBER OF ITERATIONS (MIN 0,NO MAX)
C     ITER1 - MAX NUMBER OF VGLS-CALLS (MIN 1,NO MAX)
C     IFILT - IFILT=0 THE FILTER C(Q) IS APPLIED TO ORIGINAL DATA
C           - IFILT=1 THE FILTER C(Q) IS APPLIED TO FILTERED DATA
C     INIT - INIT=0 THE ITERATION IS STARTED WITH THE LS-ESTIMATES OF A AND B
C            INIT=1 THE ITERATION IS STARTED WITH GIVEN VALUES OF A AND B
C            INIT=2 THE ITERATION IS STARTED WITH GIVEN VALUES OF C
C     IPRINT - IPRINT =0 MINIMAL RESULTS ARE PRINTED
C              IPRINT =1 MEDIUM  RESULTS ARE PRINTED
C              IPRINT =2 MUCH    RESULTS ARE PRINTED
C     EPST - TEST QUANTITY FOR STOP OF ITERATIONS
C     IA - DIMENSION PARAMETER
C
C     THE VECTOR DAT IS NOT DESTROYED
C
C     SUBROUTINE REQUIRED
C        LS
C           LSQ
C        RESID1
C        FILT
C        VGLS
C           DSYMIN
C           EIGS
C
      DIMENSION DAT(1),T(1),AB(IA,1)
      DIMENSION U(1000),UF(1000),Y(1000),YF(1000),RES(1000),DATA(3000)
      DIMENSION T1(30),T2(30),TT(30),NNB(1)
      COMMON/LSCOM/ V,SS,P(50,50),C(50),Q(50)
C
      JPRINT=0
      IF(IPRINT.EQ.2) JPRINT=2
      EPS=1.0E-02
      N2=NA+NB
      N3=N2+NC
      K=0
      NNB(1)=NB
      PRINT 200
  200 FORMAT(//10X,15HRESULT FROM GLS)
      IF(N3.EQ.0) GO TO 99
      DO 1 I=1,N3
    1 TT(I)=0.
      DO 2 I=1,M
```

```
      U(I)=DAT(3*I-1)
    2 Y(I)=DAT(3*I)
C
C     INITIAL VALUES OF THE ESTIMATES.
C
      IF(INIT-1) 11,12,18
   11 CALL LS(DAT,T,AB,M,1,NA,NNB,IA,IB,JPRINT)
      IF(IPRINT.EQ.2) PRINT 201
  201 FORMAT(1X,50(2H -))
   12 IF(N2.EQ.0) GO TO 14
      DO 13 I=1,N2
   13 T1(I)=T(I)
   14 CALL RESID1(U,Y,RES,T1,M,NA,NB)
      S1=0.
      S2=0.
      DO 15 I=1,M
      S1=S1+RES(I)*RES(I)
   15 S2=S2+Y(I)*Y(I)
      S=S1/S2
      IF(S.GT.1.0-05) GO TO 16
      PRINT 101,S
  101 FORMAT(/10X,23HRESIDUALS TOO SMALL  S=,G12.5)
      IF(IPRINT.EQ.2.OR.N2.EQ.0) GO TO 61
      PRINT 100,(T1(I),I=1,N2)
  100 FORMAT(10G12.5)
      GO TO 81
   16 IF(NC.EQ.0) GO TO 24
      DO 17 I=1,M
   17 DATA(2*I)=RES(I)
      CALL LS(DATA,T2,AB,M,0,NC,NNB,IA,IB,JPRINT)
      IF(IPRINT.EQ.2) PRINT 202
  202 FORMAT(1X,100(1H-))
      IF(IFILT.EQ.0) GO TO 22
      SM=0.
      DO 27 I=1,NC
   27 SM=AMAX1(SM,ABS(T2(I)/C(I)))
      IF(SM.LT.1.0) GO TO 61
      GO TO 22
C
   18 IF(NC.EQ.0) GO TO 20
      DO 19 I=1,NC
   19 T2(I)=T(N2+I)
   20 IF(N2.EQ.0) GO TO 22
      DO 21 I=1,N2
   21 T1(I)=0.
C
   22 CALL FILT(U,UF,T2,M,NC)
      CALL FILT(Y,YF,T2,M,NC)
      IF(NC.EQ.0) GO TO 24
      DO 23 I=1,NC
   23 T(I+N2)=T2(I)
   24 IF(N2.EQ.0) GO TO 26
      DO 25 I=1,N2
   25 T(I)=T1(I)
   26 PRINT 102
  102 FORMAT(/10X,14HINITIAL VALUES)
      PRINT 100,(T(I),I=1,N3)
      IF(IPRINT.EQ.2) PRINT 201
C
C     START ITERATION.
C
      IF(ITER.EQ.0) GO TO 64
      IF(IPRINT.LT.2) PRINT 107
  107 FORMAT(5X,44HITERATION NUMBER AND CORRESPONDING ESTIMATES)
```

```
      DO 50 K=1,ITER
      IF(IPRINT.LT.2) GO TO 31
      PRINT 103,K
  103 FORMAT(/10X,16HITERATION NUMBER,I5)

      ESTIMATE A AND B PARAMETERS

      PRINT 104
  104 FORMAT(/10X,32HESTIMATION OF A AND B-PARAMETERS)
   31 DO 32 I=1,M
      DATA(3*I-1)=UF(I)
   32 DATA(3*I)=YF(I)
      CALL LS(DATA,T1,AB,M,1,NA,NNB,IA,IB,JPRINT)
      IF(IPRINT.EQ.2) PRINT 201
      IF(N2.EQ.0) GO TO 34
      DO 33 I=1,N2
      TT(I)=T(I)
   33 T(I)=T1(I)
   34 IF(IFILT.EQ.0) CALL RESID1(U,Y,RES,T1,M,NA,NB)
      IF(IFILT.EQ.1) CALL RESID1(UF,YF,RES,T1,M,NA,NB)

      ESTIMATION OF C-PARAMETERS

      IF(IPRINT.EQ.2) PRINT 105
  105 FORMAT(/10X,26HESTIMATION OF C-PARAMETERS)
      S1=0.
      S2=0.
      DO 41 I=1,M
      S1=S1+RES(I)*RES(I)
   41 S2=S2+YF(I)*YF(I)
      S=S1/S2
      IF(S.GT.1.0-05) GO TO 42
      PRINT 101,S
      IF(IPRINT.EQ.2.OR.N2.EQ.0) GO TO 42
      PRINT 100,(T1(I),I=1,N2)
      GO TO 61
   42 IF(NC.EQ.0) GO TO 47
      DO 43 I=1,M
   43 DATA(2*I)=RES(I)
      CALL LS(DATA,T2,AB,M,0,NC,NNB,IA,IB,JPRINT)
      DO 44 I=1,NC
      TT(I+N2)=T(I+N2)
   44 T(I+N2)=T2(I)
      IF(IFILT.EQ.0) GO TO 45
      SM=0.
      DO 51 I=1,NC
   51 SM=AMAX1(SM,T2(I)/C(I))
      IF(SM.LT.1.0) GO TO 61
   45 IF(IPRINT.EQ.2) PRINT 202
      IF(IFILT.EQ.1) GO TO 46
      CALL FILT(U,UF,T2,M,NC)
      CALL FILT(Y,YF,T2,M,NC)
      GO TO 47
   46 CALL FILT(UF,UF,T2,M,NC)
      CALL FILT(YF,YF,T2,M,NC)
   47 IF(IPRINT.LT.2) PRINT 106,K,(T(I),I=1,N3)
  106 FORMAT(5X,I5,9G12.5/(10X,9G12.5))
      TA=0.
      DO 48 I=1,N3
      IF(ABS(T(I)).LT.EPS) GO TO 48
      TA=AMAX1(TA,ABS(1.0-TT(I)/T(I)))
   48 CONTINUE
      IF(TA.LT.EPST) GO TO 61
   50 CONTINUE
```

```
C
C      FINAL ESTIMATION OF C IF IFILT=1
C
       PRINT 108,K
  108 FORMAT(/10X,33HCONVERGENCE HAS NOT OCCURED AFTER,I3,11H ITERATIONS
     F)
       GO TO 64
   61 PRINT 109,K
  109 FORMAT(/10X,29HCONVERGENCE HAS OCCURED AFTER,I3,11H ITERATIONS)
   64 PRINT 110,EPST
  110 FORMAT(/10X,60HTEST QUANTITY FOR RELATIVE CHANGE IN THE PARAMETER
     FESTIMATES/10X,G12.5)
       IF(IFILT.EQ.0.OR.NC.EQ.0) GO TO 71
       CALL RESID1(U,Y,RES,T1,M,NA,NB)
       DO 62 I=1,M
   62 DATA(2*I)=RES(I)
       CALL LS(DATA,T2,AB,M,0,NC,NB,IA,IB,JPRINT)
       CALL FILT(U,UF,T2,M,NC)
       CALL FILT(Y,YF,T2,M,NC)
       DO 63 I=1,NC
       T1(I+N2)=0.
   63 T(I+N2)=T2(I)
C
C      COMPUTATION OF LOSS FUNCTION
C
   71 IF(IPRINT.LT.2.AND.IFILT.EQ.1) PRINT 106,K,(T(I),I=1,N3)
       PRINT 202
       IF(ITER.GT.0.OR.NC.GT.0) GO TO 72
       CALL FILT(U,UF,T,M,NC)
       CALL FILT(Y,YF,T,M,NC)
   72 CONTINUE
       IF(ITER1.LT.0) GO TO 99
       CALL VGLS(U,UF,Y,YF,RES,T,M,NA,NB,NC,IFILT,IPRINT,ITER1)
       GO TO 99
C
C      RESIDUALS ARE TOO SMALL
C
   81 IF(N2.EQ.0) GO TO 99
       CALL LS(DAT,T1,AB,M,1,NA,NNB,IA,IB,2)
       DO 82 I=1,N2
   82 T(I)=T1(I)
       IF(NC.EQ.0) GO TO 99
       DO 83 I=1,N3
   83 T(I+N2)=0.
C
C
   99 CONTINUE
       RETURN
       END
```

```
      SUBROUTINE FILT(U,UF,X,M,N)
C
C     COMPUTES THE FILTERED SIGNAL
C     UF(T)=U(T)+X(1)*U(T-1)+...+X(N)*U(T-N)
C     STARTVALUES OF U(T) ARE ASSUMED TO BE ZERO
C
C     AUTHOR, TORSTEN SODERSTROM 1971-10-15
C
C     U - VECTOR OF ORDER M, CONTAINING THE SIGNAL TO BE FILTERED
C     UF- VECTOR OF ORDER M, CONTAINING THE FILTERED SIGNAL
C     X - VECTOR OF ORDER N, CONTAINING THE FILTER
C     M - ORDER OF U (MIN 1,NO MAX)
C     N - ORDER OF X (MIN 0,MAX 20)
C     N.LE.M
C
C     SUBROUTINE REQUIRED
C            NONE
C
      DIMENSION U(1),UF(1),X(1)
      DIMENSION FI(20)
C
      IF(N.EQ.0) GO TO 98
      DO 1 I=1,N
    1 FI(I)=U(M-I)
C
      DO 6 I=M,1,-1
      UF(I)=U(I)
      DO 2 J=1,N
    2 UF(I)=UF(I)+FI(J)*X(J)
      IF(N-1) 99,5,3
    3 N1=N-1
      DO 4 J=1,N1
    4 FI(J)=FI(J+1)
    5 FI(N)=0.
    6 IF(I-N-1.GT.0) FI(N)=U(I-N-1)
      GO TO 99
C
   98 DO 7 I=1,M
    7 UF(I)=U(I)
C
   99 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE RESID1(U,Y,RES,X,M,NA,NB)
C
C     COMPUTES THE RESIDULAS
C     RES(T)=Y(T)+A(1)*Y(T-1)+...A(NA)*Y(T-NA)-
C     -B(1)*U(T-1)-...-B(NB)*U(Y-NB)
C     RES(T)=0   T=1,... MAX(NA,NB)
C
C     AUTHOR  TORSTEN SODERSTROM 1971-10-15
C
C     U - VECTOR OF ORDER M, CONTAINING THE INPUT SIGNAL
C     Y - VECTOR OF ORDER M, CONTAINING THE OUTPUT SIGNAL
C     RES - VECTOR OF ORDER M ,CONTAINING THE RESIDUALS
C     X - VECTOR OF ORDER (NA+NB)
C     X=(A(1),...,A(NA),B(1),...B(NB))
C     M- NUMBER OF SAMPLES  (MIN 1,NO MAX)
C     NA,NB - ORDER OF A RESP B
C     (NA+NB) (MIN 0,MAX 20)
C     MAX(NA,NB) .LT. M
C
C     SUBROUTINE REQUIRED
C            NONE
C
      DIMENSION U(1),Y(1),RES(1),X(1)
      DIMENSION FI(21)
C
      N2=NA+NB
      IF(N2.EQ.0) GO TO 99
      N=MAX0(NA,NB)
      DO 1 I=1,N
    1 RES(I)=0.
      DO 12 I=1,NA
   12 FI(I)= Y(N+1-I)
      DO 13 I=1,NB
   13 FI(I+NA)=-U(N+1-I)
C
      NN=N+1
      DO 10 I=NN,M
      RES(I)=Y(I)
      DO 2 J=1,N2
    2 RES(I)=RES(I)+FI(J)*X(J)
      IF(NA-1) 5,4,3
    3 DO 6 JJ=2,NA
      J=NA+2-JJ
    6 FI(J)=FI(J-1)
    4 FI(1)=Y(I)
    5 IF(NB-1) 10,8,7
    7 DO 11 JJ=2,NB
      J=N2+2-JJ
   11 FI(J)=FI(J-1)
    8 FI(NA+1)=-U(I)
   10 CONTINUE
C
   99 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE VGLS(U,UF,Y,YF,RES,T,M,NA,NB,NC,IFILT,IPRINT,ITMAX)
C
C     COMPUTES THE LOSS FUNCTION ETC FOR THE GLS PROBLEM
C
C     AUTHOR  TORSTEN SODERSTROM 1971-10-01
C     REFERENCE REPORT 7311(C).
C
C     U - VECTOR OF ORDER M CONTAINING THE INPUT
C     UF- VECTOR OF ORDER M CONTAINING THE FILTERED INPUT
C     Y - VECTOR OF ORDER M CONTAINING THE OUTPUT
C     YF- VECTOR OF ORDER M CONTAINING THE FILTERED OUTPUT
C     RES-VECTOR OF ORDER M CONTAINING THE RESIDUALS  RES(T)=A(Q)*Y(T)-B(Q)*U
C     T - VECTOR OF ORDER (NA+NB+NC) CONTAINING THE ACTUAL PARAMETER VALUES
C     M-ORDER OF U AND Y (NUMBER OF SAMPLES) (MIN 31,MAX 1000)
C     NA,NB,NC - NUMBER OF A,B,C PARAMETERS RESP
C     (NA+NB+NC) (MIN 0,MAX 30)
C     IFILT - IFILT=0 THE FILTER C(Q) IS APPLIED TO ORIGINAL DATA
C           - IFILT=1 THE FILTER C(Q) IS APPLIED TO FILTERED DATA
C     IPRINT -PRINT PARAMETER
C     THE FOLLOWING VARIABLES ARE PRINTED
C     IPRINT=0 THE LOSS FUNCTION AND THE GRADIENT
C              STANDARD DEVIATIONS OF THE PARAMETERS AND THE NOISE
C              EXTRAPOLATED PARAMETER ESTIMATES BASED ON NEWTON-RAPHSON
C     ELSE     AS IPRINT=0 +
C              THE MATRIX OF SECOND ORDER DERIVATIVES
C              ITS EIGENVALUES AND EIGENVECTORS
C              THE ESTIMATED COVARIANCE FUNCTION
C              THE ESTIMATED COVARIANCE MATRIX OF THE PARAMETER ESTIMATES
C     ITMAX - MAX NUMBER OF NEWTON RAPHSON STEPS.
C
C     SUBROUTINE REQUIRED
C            FILT
C            RESID1
C            DSYMIN
C            EIGS
C
      DIMENSION U(1),UF(1),Y(1),YF(1),RES(1),T(1)
      DIMENSION RESF(1000),VT(30),VTT(30,30),P(30,30),DT(30),
     FT2(30),R(30,30),EV(30),C(20)
      DOUBLE PRECISION P
C
      IA=30
      NA1=NA+1
      NB1=NB+1
      NC1=NC+1
      N2=NA+NB
      N3=N2+NC
      IF(N3.EQ.0) GO TO 99
      IF(NC.EQ.0) GO TO 2
      DO 1 I=1,NC
    1 T2(I)=T(I+N2)
    2 CALL FILT(RES,RESF,T2,M,NC)
      V=0.
      DO 3 I=1,M
    3 V=V+RESF(I)*RESF(I)/M/2.
      S=SQRT(2.*V)
      DS=S*SQRT(0.5/M)
C
C     COMPUTE FIRST ORDER DERIVATIVES
C
      IF(NA.EQ.0) GO TO 12
      DO 11 I=1,NA
      VT(I)=0.
```

```
      II=I+1
      DO 11 J=II,M
   11 VT(I)=VT(I)+RESF(J)*YF(J-I)/M
   12 IF(NB.EQ.0) GO TO 14
      DO 13 I=1,NB
      VT(I+NA)=0.
      II=I+1
      DO 13 J=II,M
   13 VT(I+NA)=VT(I+NA)-RESF(J)*UF(J-I)/M
   14 IF(NC.EQ.0) GO TO 21
      DO 15 I=1,NC
      VT(I+N2)=0.
      II=I+1
      DO 15 J=II,M
   15 VT(I+N2)=VT(I+N2)+RESF(J)*RES(J-I)/M
C
C     COMPUTE SECOND ORDER DERIVATIVES
C
   21 DO 22 I=1,N3
      DO 22 J=1,N3
   22 VTT(I,J)=0.
      IF(NA.EQ.0) GO TO 28
      DO 37 I=1,NA
      DO 23 J=I,NA
      II=J+1
      DO 23 K=II,M
   23 VTT(I,J)=VTT(I,J)+YF(K-I)*YF(K-J)/M
      IF(NB.EQ.0) GO TO 25
      DO 24 J=1,NB
      K1=MAX0(I,J)+1
      DO 24 K=K1,M
   24 VTT(I,NA+J)=VTT(I,NA+J)-UF(K-J)*YF(K-I)/M
   25 IF(NC.EQ.0) GO TO 37
      DO 27 J=1,NC
      K1=MAX0(I,J)+1
      K2=I+J
      K3=K2+1
      DO 26 K=K1,M
   26 VTT(I,N2+J)=VTT(I,N2+J)+RES(K-J)*YF(K-I)/M
      DO 27 K=K3,M
   27 VTT(I,N2+J)=VTT(I,N2+J)+RESF(K)*Y(K-I-J)/M
   37 CONTINUE
   28 IF(NB.EQ.0) GO TO 32
      DO 38 I=1,NB
      DO 29 J=I,NB
      K1=J+1
      DO 29 K=K1,M
   29 VTT(I+NA,J+NA)=VTT(I+NA,J+NA)+UF(K-I)*UF(K-J)/M
      IF(NC.EQ.0) GO TO 38
      DO 31 J=1,NC
      K1=MAX0(I,J)+1
      K2=I+J
      K3=K2+1
      DO 30 K=K1,M
   30 VTT(I+NA,J+N2)=VTT(I+NA,J+N2)-RES(K-J)*UF(K-I)/M
      DO 31 K=K3,M
   31 VTT(I+NA,J+N2)=VTT(I+NA,J+N2)-RESF(K)*U(K-I-J)/M
   38 CONTINUE
   32 IF(NC.EQ.0) GO TO 34
      DO 33 I=1,NC
      DO 33 J=I,NC
      K1=J+1
      DO 33 K=K1,M
   33 VTT(I+N2,J+N2)=VTT(I+N2,J+N2)+RES(K-I)*RES(K-J)/M
```

```
   34 DO 35 I=1,N3
      DO 35 J=1,I
   35 VTT(I,J)=VTT(J,I)
      DO 36 I=1,N3
      DO 36 J=1,N3
   36 P(I,J)=VTT(I,J)
C
C     PRINT RESULTS
C
      PRINT 101
  101 FORMAT(//10X,16HRESULT FROM VGLS)
      PRINT 111
  111 FORMAT(/10X,28HVALUE OF PARAMETER ESTIMATES)
       PRINT 100,(T(I),I=1,N3)
      PRINT 102
  102 FORMAT(/10X,13HLOSS FUNCTION)
      PRINT 100,V
  100 FORMAT(10G12.5)
      PRINT 103
  103 FORMAT(/10X,31HSTANDARD DEVIATION OF THE NOISE)
      PRINT 100,S,DS
      PRINT 112
  112 FORMAT(/10X,15HDERIVATIVE OF V)
      PRINT 100,(VT(I),I=1,N3)
      IF(IPRINT.EQ.0) GO TO 51
      PRINT 104
  104 FORMAT(/10X,34HMATRIX OF SECOND ORDER DERIVATIVES)
      DO 41 I=1,N3
   41 PRINT 100,(VTT(I,J),J=1,N3)
C
C     COMPUTE THE INVERSE OF VTT ETC.
C
   51 CALL DSYMIN(N3,IA,IERR,P)
      IF(IERR.EQ.0) GO TO 52
      PRINT 105
  105 FORMAT(/10X,22HMATRIX VTT IS SINGULAR)
      GO TO 56
   52 CONTINUE
      DO 54 I=1,N3
      DO 54 J=1,N3
   54 P(I,J)=P(I,J)*V/M*2.
      DO 55 I=1,N3
      IF(P(I,I).LT.0.0) DT(I)=0.0
   55 IF(P(I,I).GE.0.0) DT(I)=SQRT(P(I,I))
   56 CALL EIGS(VTT,R,EV,N3,IA,0)
      COND=EV(1)/EV(N3)
C
C     COMPUTE NEWTON RAPHSON STEPS
C
      IF(IERR.NE.0.OR.ITMAX.EQ.0) GO TO 68
      DO 67 K=1,ITMAX
      DO 61 I=1,N3
      T2(I)=T(I)
      DO 61 J=1,N3
   61 T2(I)=T2(I)-P(I,J)*VT(J)/2.*M/V
      VV=V
      DO 62 I=1,N3
   62 VV=VV+VT(I)*(T2(I)-T(I))/2.
      IF(VV.GT.0.) SS=SQRT(2.0*VV)
      IF(VV.LE.0.0) SS=-SQRT(-2.0*VV)
C
      CALL RESID1(U,Y,RES,T2,M,NA,NB)
      IF(NC.EQ.0) GO TO 64
      DO 63 I=1,NC
```

```
63 C(I)=T2(I+N2)
64 CALL FILT(RES,RESF,C,M,NC)
   W=0.
   DO 65 I=1,M
65 W=W+RESF(I)**2/M/2.
   S=SQRT(2.*W)

   PRINT 107
107 FORMAT(/10X,53HEXTRAPOLATED PARAMETER ESTIMATES USING NEWTON-RAPHS
  FON)
   PRINT 100,(T2(I),I=1,N3)
   PRINT 114,VV,SS
114 FORMAT(/10X,'ASSOCIATED LOSS FUNCTION AND STANDARD DEVIATION OF TH
  FE NOISE'/10X,2G12.5)
   PRINT 115,W,S
115 FORMAT(/10X,'TRUE VALUES OF V AND LAMBDA IN THIS POINT'/10X,2G12.5
  F)
   IF(W.GE.V) GO TO 68
   DO 66 I=1,N3
66 T(I)=T2(I)
67 CONTINUE
68 CONTINUE

   PRINT RESULTS

   PRINT 113,COND
113 FORMAT(/10X,22HCONDITON NUMBER OF VTT,G12.5)
   IF(IERR.EQ.-1) GO TO 73
   PRINT 106
106 FORMAT(/10X,47HESTIMATED STANDARD DEVIATIONS OF THE PARAMETERS)
   PRINT 100,(DT(I),I=1,N3)
   IF(IPRINT.EQ.0) GO TO 99
   PRINT 108
108 FORMAT(/10X,45HESTIMATED COVARIANCE MATRIX OF THE PARAMETERS)
   DO 72 I=1,N3
72 PRINT 100,(P(I,J),J=1,N3)
73 IF(IPRINT.EQ.0) GO TO 99
   PRINT 109
109 FORMAT(/10X,18HEIGENVALUES OF VTT)
   PRINT 100,(EV(I),I=1,N3)
   PRINT 110
110 FORMAT(/10X,19HEIGENVECTORS OF VTT)
   DO 71 I=1,N3
71 PRINT 100,(R(I ,J),J=1,N3)

99 CONTINUE
   RETURN
   END
```

```
      SUBROUTINE FACT(T,NA,NB,NC,TSAMP)
C
C     SUBROUTINE FOR THE GLS PACKAGE
C
C     COMPUTES THE ZEROS OF THE A-,B- AND C-POLYNOMIALS, THE STATIC GAIN
C     AND THE POLES OF THE CONTINOUS MODEL
C
C     AUTHOR TORSTEN SODERSTROM 1971-12-24
C
C     T-VECTOR OF ORDER (NA+NB+NC)
C     T = ( A(1)...A(NA) B(1)...B(NB) C(1)...C(NC) )
C     NA,NB,NC - DEGREES OF THE A-,B-,C- POLYNOMIALS RESP. (MIN 0,MAX 10)
C     TSAMP - THE SAMPLING INTERVAL
C
C     SUBROUTINE REQUIRED
C             ROT
C
      DIMENSION T(1),PR(10),PI(10),A(10)
C
      EPS=1.0E-10
      PRINT 101
  101 FORMAT(///10X,'RESULT OF FACTORIZATION'/10X,23(1H*))
C
      IF(NA.LE.0) GO TO 10
      CALL ROT(NA,1,T,T,PR,PI,NE)
      PRINT 102
  102 FORMAT(//10X,'POLES OF DISCRETE MODEL')
      DO 1 I=1,NA
    1 PRINT 100,PR(I),PI(I)
  100 FORMAT(2G12.5)
      PRINT 103
  103 FORMAT(//10X,'POLES OF CONTINOUS MODEL')
      DO 2 I=1,NA
      IF(PR(I).LT.EPS.AND.ABS(PI(I)).LT.EPS) GO TO 2
      PRE=ALOG(PR(I)**2+PI(I)**2)/2./TSAMP
      PIM=ATAN2(PI(I),PR(I))/TSAMP
      PRINT 100,PRE,PIM
    2 CONTINUE
C
   10 IF(NB.LE.1) GO TO 20
      NB1=NB-1
      DO 11 I=1,NB1
   11 A(I)=T(NA+1+I)/T(NA+1)
      CALL ROT(NB1,1,A,A,PR,PI,NE)
      PRINT 104
  104 FORMAT(//10X,'ZEROS OF DISCRETE MODEL')
      DO 12 I=1,NB1
   12 PRINT 100,PR(I),PI(I)
C
   20 IF(NC.LE.0) GO TO 30
      DO 21 I=1,NC
   21 A(I)=T(NA+NB+I)
      CALL ROT(NC,1,A,A,PR,PI,NE)
      PRINT 105
  105 FORMAT(//10X,'ZEROS OF THE C-POLYNOMIAL')
      DO 22 I=1,NC
   22 PRINT 100,PR(I),PI(I)
      PRINT 106
  106 FORMAT(/10X,'CONTINOUS EQUIVALENT')
      DO 23 I=1,NC
      IF(PR(I).LT.EPS.AND.ABS(PI(I)).LT.EPS) GO TO 23
      PRE=ALOG(PR(I)**2+PI(I)**2)/2./TSAMP
      PIM=ATAN2(PI(I),PR(I))/TSAMP
```

```
      PRINT 100,PRE,PIM
   23 CONTINUE
C
   30 IF(NB.LE.0) GO TO 40
      S2=0.
      DO 31 I=1,NB
   31 S2=S2+T(NA+I)
      S1=1.
      IF(NA.EQ.0) GO TO 33
      DO 32 I=1,NA
   32 S1=S1+T(I)
   33 AK=S2/S1
      PRINT 107
  107 FORMAT(//10X,'STATIC GAIN')
      PRINT 100,AK
C
   40 RETURN
      END
```

IV. EXAMPLES

Example 1.

```
ISIM=       0                NUMBER OF SAMPLES  500
ORDER OF SYSTEM OPERATORS        2       2        0
-.1000000+00   -.5600000+00    .1000000+01    .7000000+00    .1000000+01
ORDER OF MODEL OPERATORS         1       1        1
INITIAL ESTIMATES OF MODEL PARAMETERS
.0000000        .0000000        .0000000
INIT=        0
IFILT=       0
IPRINT=      1
MAX NUMBER OF ITERATIONS    20
```

true standard deviation of the noise, λ

```
RESULT FROM GLS

INITIAL VALUES
.57276       .99262        .46616
ITERATION NUMBER AND CORRESPONDING ESTIMATES
   1   -.77398      1.0179        .68462
   2   -.80323      1.0106        .69682
   3   -.80420      1.0103        .69714
   4   -.80422      1.0103        .69715

CONVERGENCE HAS OCCURED AFTER  4 ITERATIONS

TEST QUANTITY FOR RELATIVE CHANGE IN THE PARAMETER ESTIMATES
   .10000-02
```

```
RESULT FROM VGLS

VALUE OF PARAMETER ESTIMATES
-.80422     1.0103       .69715

LOSS FUNCTION
.52025

STANDARD DEVIATION OF THE NOISE
1.0200      .32257-01
```

← estimation of λ ($\hat{\lambda}$ and $\hat{\sigma}_{\hat{\lambda}}$ resp.)

```
DERIVATIVE OF V
-.28201-03    .63116-03    .69343-06

MATRIX OF SECOND ORDER DERIVATIVES
```

(called VTT)

```
8.8256      -.35944       .67053
-.35944      1.4653       .98743-02
.67053       .98743-02    2.0244

EXTRAPOLATED PARAMETER ESTIMATES USING NEWTON-RAPHSON
-.80421      1.0099       .69715

ASSOCIATED LOSS FUNCTION AND STANDARD DEVIATION OF THE NOISE
   .52025        1.0200

TRUE VALUES OF V AND LAMBDA IN THIS POINT
   .52025        1.0200

EXTRAPOLATED PARAMETER ESTIMATES USING NEWTON-RAPHSON
-.80419      1.0095       .69714

ASSOCIATED LOSS FUNCTION AND STANDARD DEVIATION OF THE NOISE
   .52025        1.0200

TRUE VALUES OF V AND LAMBDA IN THIS POINT
```

```
         .52025        1.0200

        EXTRAPOLATED PARAMETER ESTIMATES USING NEWTON-RAPHSON
-.80418        1.0090         .69714

        ASSOCIATED LOSS FUNCTION AND STANDARD DEVIATION OF THE NOISE
         .52025        1.0200

        TRUE VALUES OF V AND LAMBDA IN THIS POINT
         .52025        1.0200

        CONDITON NUMBER OF VTT    6.1676

        ESTIMATED STANDARD DEVIATIONS OF THE PARAMETERS
 .15634-01    .37884-01    .32481-01

        ESTIMATED COVARIANCE MATRIX OF THE PARAMETERS
 .24443-003   .60504-004 -.81257-004
 .60504-004   .14352-002 -.27041-004
-.81257-004  -.27041-004  .10550-002

        EIGENVALUES OF VTT
 8.9082        1.9628         1.4444

        EIGENVECTORS OF VTT
 .99415      -.92391-01    .55864-01
-.47883-01    .86453-01    .99510
 .96769-01    .99196      -.81523-01

        TIME FOR IDENTIFICATION WAS 1488 MS
```

Example 2. (The same as example 1 with the excetption IPRINT=2)

```
ISIM=     0                NUMBER OF SAMPLES  500
ORDER OF SYSTEM OPERATORS     2     2     0
-.1000000+00   -.5600000+00    .1000000+01    .7000000+00    .1000000+01
ORDER OF MODEL OPERATORS      1     1     1
INITIAL ESTIMATES OF MODEL PARAMETERS
.0000000        .0000000        .0000000
INIT=        0
IFILT=       0
IPRINT=      2
MAX NUMBER OF ITERATIONS    20


RESULT FROM GLS

RESULT FROM IDENTIFICATION, MODEL OF ORDER  1
  A( 1)=   -.57276                   STAND DEV=    .30495-01
B 1( 1)=    .99262                   STAND DEV=    .60603-01

VALUE OF LOSS FUNCTION     906.98
ESTIMATED STANDARD DEVIATION S=    1.3509

SINGULAR VALUES
44.425         22.275

COVARIANCE MATRIX OF THE PARAMETER ESTIMATES
.92994-03   -.12065-03
-.12065-03   .36728-02
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
```

Estimation of initial C-parameters

```
RESULT FROM IDENTIFICATION, MODEL OF ORDER  1
  A( 1)=    .46616                   STAND DEV=    .39644-01

VALUE OF LOSS FUNCTION     709.89
ESTIMATED STANDARD DEVIATION S=    1.1939

SINGULAR VALUES
30.116

COVARIANCE MATRIX OF THE PARAMETER ESTIMATES
.15717-02
-----------------------------------------------------------------------------

INITIAL VALUES
-.57276         .99262          .46616
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -

ITERATION NUMBER     1

ESTIMATION OF A AND B-PARAMETERS

RESULT FROM IDENTIFICATION, MODEL OF ORDER  1
  A( 1)=   -.77398                   STAND DEV=    .18452-01
B 1( 1)=    1.0179                   STAND DEV=    .43811-01

VALUE OF LOSS FUNCTION     570.76
ESTIMATED STANDARD DEVIATION S=    1.0716

SINGULAR VALUES
58.271         24.446

COVARIANCE MATRIX OF THE PARAMETER ESTIMATES
.34048-03    .59820-04
.59820-04    .19194-02
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
```

```
ESTIMATION OF C-PARAMETERS

RESULT FROM IDENTIFICATION, MODEL OF ORDER  1
  A( 1)=    .68462                     STAND DEV=    .32663-01

VALUE OF LOSS FUNCTION      524.08
ESTIMATED STANDARD DEVIATION S=    1.0259

SINGULAR VALUES
31.407

COVARIANCE MATRIX OF THE PARAMETER ESTIMATES
.10669-02
------------------------------------------------------------------------

ITERATION NUMBER     2

ESTIMATION OF A AND B-PARAMETERS

RESULT FROM IDENTIFICATION, MODEL OF ORDER  1
  A( 1)=   -.80323                     STAND DEV=    .15579-01
B 1( 1)=    1.0106                     STAND DEV=    .38192-01

VALUE OF LOSS FUNCTION      519.84
ESTIMATED STANDARD DEVIATION S=    1.0227

SINGULAR VALUES
66.034      26.752

COVARIANCE MATRIX OF THE PARAMETER ESTIMATES
.24272-03    .58948-04
.58948-04    .14586-02
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -

ESTIMATION OF C-PARAMETERS

RESULT FROM IDENTIFICATION, MODEL OF ORDER  1
  A( 1)=    .69682                     STAND DEV=    .32141-01

VALUE OF LOSS FUNCTION      520.25
ESTIMATED STANDARD DEVIATION S=    1.0221

SINGULAR VALUES
31.801

COVARIANCE MATRIX OF THE PARAMETER ESTIMATES
.10330-02
------------------------------------------------------------------------

ITERATION NUMBER     3

ESTIMATION OF A AND B-PARAMETERS

RESULT FROM IDENTIFICATION, MODEL OF ORDER  1
  A( 1)=   -.80420                     STAND DEV=    .15474-01
B 1( 1)=    1.0103                     STAND DEV=    .37973-01

VALUE OF LOSS FUNCTION      519.67
ESTIMATED STANDARD DEVIATION S=    1.0226

SINGULAR VALUES
66.482      26.902
```

```
 COVARIANCE MATRIX OF THE PARAMETER ESTIMATES
.23943-03    .58716-04
.58716-04    .14420-02
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -

        ESTIMATION OF C-PARAMETERS

        RESULT FROM IDENTIFICATION, MODEL OF ORDER  1
          A( 1)=    .69714                STAND DEV=    .32127-01

        VALUE OF LOSS FUNCTION    520.25
        ESTIMATED STANDARD DEVIATION S=    1.0221

        SINGULAR VALUES
31.814

        COVARIANCE MATRIX OF THE PARAMETER ESTIMATES
.10321-02
------------------------------------------------------------------------

        ITERATION NUMBER     4

        ESTIMATION OF A AND B-PARAMETERS

        RESULT FROM IDENTIFICATION, MODEL OF ORDER  1
          A( 1)=   -.80422                STAND DEV=    .15471-01
        B 1( 1)=    1.0103                STAND DEV=    .37968-01

        VALUE OF LOSS FUNCTION    519.67
        ESTIMATED STANDARD DEVIATION S=    1.0226

        SINGULAR VALUES
66.494      26.906

        COVARIANCE MATRIX OF THE PARAMETER ESTIMATES
.23935-03    .58710-04
.58710-04    .14416-02
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -

        ESTIMATION OF C-PARAMETERS

        RESULT FROM IDENTIFICATION, MODEL OF ORDER  1
          A( 1)=    .69715                STAND DEV=    .32126-01

        VALUE OF LOSS FUNCTION    520.25
        ESTIMATED STANDARD DEVIATION S=    1.0221

        SINGULAR VALUES
31.815

        COVARIANCE MATRIX OF THE PARAMETER ESTIMATES
.10321-02
------------------------------------------------------------------------

        CONVERGENCE HAS OCCURED AFTER  4 ITERATIONS

        TEST QUANTITY FOR RELATIVE CHANGE IN THE PARAMETER ESTIMATES
          .10000-02
------------------------------------------------------------------------

        RESULT FROM VGLS

        VALUE OF PARAMETER ESTIMATES
```

```
.80422       1.0103       .69715

 LOSS FUNCTION
.52025

 STANDARD DEVIATION OF THE NOISE
1.0200    .32257-01

 DERIVATIVE OF V
-.28201-03    .63116-03    .69343-06

 MATRIX OF SECOND ORDER DERIVATIVES
 8.8256      -.35944       .67053
-.35944      1.4653        .98743-02
 .67053      .98743-02    2.0244

 EXTRAPOLATED PARAMETER ESTIMATES USING NEWTON-RAPHSON
-.80421      1.0099        .69715

 ASSOCIATED LOSS FUNCTION AND STANDARD DEVIATION OF THE NOISE
  .52025      1.0200

 TRUE VALUES OF V AND LAMBDA IN THIS POINT
  .52025      1.0200

 EXTRAPOLATED PARAMETER ESTIMATES USING NEWTON-RAPHSON
-.80419      1.0095        .69714

 ASSOCIATED LOSS FUNCTION AND STANDARD DEVIATION OF THE NOISE
  .52025      1.0200

 TRUE VALUES OF V AND LAMBDA IN THIS POINT
  .52025      1.0200

 EXTRAPOLATED PARAMETER ESTIMATES USING NEWTON-RAPHSON
-.80418      1.0090        .69714

 ASSOCIATED LOSS FUNCTION AND STANDARD DEVIATION OF THE NOISE
  .52025      1.0200

 TRUE VALUES OF V AND LAMBDA IN THIS POINT
  .52025      1.0200

 CONDITON NUMBER OF VTT    6.1676

 ESTIMATED STANDARD DEVIATIONS OF THE PARAMETERS
 .15634-01    .37884-01    .32481-01

 ESTIMATED COVARIANCE MATRIX OF THE PARAMETERS
 .24443-003   .60504-004  -.81257-004
 .60504-004   .14352-002  -.27041-004
-.81257-004  -.27041-004   .10550-002

 EIGENVALUES OF VTT
 8.9082       1.9628       1.4444

 EIGENVECTORS OF VTT
 .99415      -.92391-01    .55864-01
-.47883-01    .86453-01    .99510
 .96769-01    .99196      -.81523-01

 TIME FOR IDENTIFICATION WAS 1950 MS
```

```
PRINTOUT FROM STDAT
*******************
NUMBER OF SAMPLING EVENTS =        240
NUMBER OF INPUT   SIGNALS =          1
NUMBER OF OUTPUT  SIGNALS =          1

CONSTANT SAMPLING RATE WITH

INITIAL TIME =        .00000
SAMPLING INTERVAL =  1.00000

DATA FROM

UNIT NUMBER =  5
FILE NUMBER =  0
NUMBER OF SHIFTS =          0
SCALE FACTOR =    .10000000-01

INPUT SIGNAL NR =  1
 .5000     .5000     .5000    -.5000    -.5000    -.5000     .1028-05  .5000     .5000    -.5000
 .5000    -.5000     .1028-05  .1028-05  .1028-05 -.5000     .1028-05  .1028-05 -.5000    -.5000
 .1028-05 -.5000     .5000    -.5000    -.5000     .1028-05  .1028-05  .5000    -.5000     .1028-05
 .5000     .1028-05 -.5000     .5000     .5000    -.5000     .1028-05 -.5000     .1028-05 -.5000
-.5000    -.5000    -.5000     .5000     .5000     .5000     .1028-05 -.5000    -.5000     .5000
-.5000     .5000     .1028-05  .1028-05  .1028-05  .5000     .1028-05  .1028-05  .5000     .5000
 .1028-05  .5000    -.5000     .5000     .5000     .1028-05  .1028-05 -.5000     .5000     .1028-05
-.5000     .1028-05  .5000    -.5000    -.5000     .5000     .1028-05  .5000     .1028-05  .5000
 .5000     .5000     .5000    -.5000    -.5000    -.5000     .1028-05  .5000     .5000    -.5000
 .5000    -.5000     .1028-05  .1028-05  .1028-05 -.5000     .1028-05  .1028-05 -.5000    -.5000
 .1028-05 -.5000     .5000    -.5000    -.5000     .1028-05  .1028-05  .5000    -.5000     .1028-05
 .5000     .1028-05 -.5000     .5000     .5000    -.5000     .1028-05 -.5000     .1028-05 -.5000
-.5000    -.5000    -.5000     .5000     .5000     .5000     .1028-05 -.5000    -.5000     .5000
-.5000     .5000     .1028-05  .1028-05  .1028-05  .5000     .1028-05  .1028-05  .5000     .5000
 .1028-05  .5000    -.5000     .5000     .5000     .1028-05  .1028-05 -.5000     .5000     .1028-05
-.5000     .1028-05  .5000    -.5000    -.5000     .5000     .1028-05  .5000     .1028-05  .5000
 .5000     .5000     .5000    -.5000    -.5000    -.5000     .1028-05  .5000     .5000    -.5000
 .5000    -.5000     .1028-05  .1028-05  .1028-05 -.5000     .1028-05  .1028-05 -.5000    -.5000
 .1028-05 -.5000     .5000    -.5000    -.5000     .1028-05  .1028-05  .5000    -.5000     .1028-05
 .5000     .1028-05 -.5000     .5000     .5000    -.5000     .1028-05 -.5000     .1028-05 -.5000
-.5000    -.5000    -.5000     .5000     .5000     .5000     .1028-05 -.5000    -.5000     .5000
-.5000     .5000     .1028-05  .1028-05  .1028-05  .5000     .1028-05  .1028-05  .5000     .5000
 .1028-05  .5000    -.5000     .5000     .5000     .1028-05  .1028-05 -.5000     .5000     .1028-05
-.5000     .1028-05  .5000    -.5000    -.5000     .5000     .1028-05  .5000     .1028-05  .5000

DATA FROM

UNIT NUMBER =  5
FILE NUMBER =  0
NUMBER OF SHIFTS =          0
SCALE FACTOR =    .10000000-01

OUTPUT SIGNAL NR =  1
-.6412    -.5812    -.9012    -1.291    -1.671    -1.901    -1.871    -1.381    -.6512    -.5812
-1.051    -.9312    -1.151    -1.101    -.8612    -.8112    -.9012    -.6512    -.3212    -.3612
-.3212     .7883-01  .2188     .5388     .4988     .4788     .8088     .8688    1.259      .9088
 .8988    1.199      .8888     .5788     .9088     .7688     .3288     .1788     .2188     .3488
 .5188     .6588     .7888    1.079     1.859     1.839     1.479     1.089      .9088     .7788
1.379     1.159     1.399     1.159      .9688     .8788    1.049      .7288     .4688     .6088
 .4888     .1883-01 -.6117-01 -.3412    -.3312    -.3312    -.6412    -1.071    -1.091    -.7212
-.8612    -1.051    -.9112    -.5012    -.7612    -.7712    -.3112    -.4212    -.2112    -.5712
-.6912    -.9912    -1.201    -1.631    -1.971    -2.251    -2.151    -1.561    -.9812    -1.031
-1.291    -1.321    -1.511    -1.381    -1.171    -1.071    -1.001    -.7712    -.6112    -.6612
-.6212    -.1612    -.1117-01  .5388     .4688     .3888     .6488     .8488    1.079      .8388
 .7988     .9788     .8088     .6188     .9688     .8288     .4888     .3188     .3188     .4288
 .5788     .6188     .9188    1.129     1.899     2.019     1.689     1.299     1.149     1.019
1.599     1.349     1.629     1.459     1.119     1.159     1.279      .9088     .6288     .6888
 .5588     .2388     .8883-01 -.2512    -.2912    -.2112    -.6112    -.8112    -.9012    -.6812
-.6412    -.8912    -.8312    -.4012    -.6612    -.8412    -.1712    -.2112    -.1712    -.3912
```

Example 3. Real data, the subroutine IDGLS is used with IPRINT = 0 first and then IPRINT = 1.

-.3912	-.7212	-1.141	-1.621	-1.891	-2.031	-2.021	-1.601	-.6812	-.9712
-1.321	-1.291	-1.421	-1.451	-1.361	-1.131	-1.891	-.8212	-.8412	-.3712
-.5012	-.6117-01	.1088	.4888	.4288	.4488	.6488	.9088	1.189	.8888
.7788	.9788	.8288	.4788	.8088	.8488	.4888	.2488	.3188	.4388
.5388	.6188	.8088	1.169	1.769	1.889	1.599	1.199	.9888	.8588
1.419	1.189	1.459	1.219	.9388	.9488	.9388	.7088	.6788	.6688
.3688	-.4117-01	-.1812	-.4912	-.3912	-.6012	-.9712	-1.131	-1.181	-.8912
-.9212	-1.091	-1.031	-.4912	-.8112	-.8612	-.3312	-.3912	-.2512	-.4812

RESULT FROM IDENTIFICATION WITH GLS

```
ORDER OF OPERATORS     1     1     1

IFILT=     0
IPRINT=    0
ITMAX=    10
ITER=      3
INIT=      0
 240 SAMPLES ARE USED
EPST=    .10000-02
SAMPLING PERIOD=   1.0000     SECONDS

        INITIAL VALUES
.00000       .00000      .00000


        RESULT FROM GLS

        INITIAL VALUES
-.95229      .22805      -.49007
   ITERATION NUMBER AND CORRESPONDING ESTIMATES
     1   -.87505      .38005      -.67141
     2   -.76646      .39359      -.79264
     3   -.62159      .38509      -.89535
     4   -.46779      .36996      -.94381
     5   -.41415      .36415      -.95288
     6   -.40722      .36339      -.95386
     7   -.40653      .36331      -.95396
     8   -.40647      .36330      -.95397

        CONVERGENCE HAS OCCURED AFTER  8 ITERATIONS

        TEST QUANTITY FOR RELATIVE CHANGE IN THE PARAMETER ESTIMATES
          .10000-02
---------------------------------------------------------------------------------


        RESULT FROM VGLS

        VALUE OF PARAMETER ESTIMATES
-.40647      .36330      -.95397

        LOSS FUNCTION
.15934-01

        STANDARD DEVIATION OF THE NOISE
.17852       .81482-02

        DERIVATIVE OF V
.77317-03    .87681-03    .17342-07

        MATRIX OF SECOND ORDER DERIVATIVES
.75421-01    .36264-01    .48179-01
.36264-01    .32232      -.43914-03
.48179-01   -.43914-03    .34763

        EXTRAPOLATED PARAMETER ESTIMATES USING NEWTON-RAPHSON
-.41690      .36176      -.95252

        ASSOCIATED LOSS FUNCTION AND STANDARD DEVIATION OF THE NOISE
          .15930-01   .17849
```

```
        TRUE VALUES OF V AND LAMBDA IN THIS POINT
         .15930-01    .17849

        EXTRAPOLATED PARAMETER ESTIMATES USING NEWTON-RAPHSON
-.42733      .36021      -.95108

        ASSOCIATED LOSS FUNCTION AND STANDARD DEVIATION OF THE NOISE
         .15930-01    .17849

        TRUE VALUES OF V AND LAMBDA IN THIS POINT
         .15934-01    .17852

        EXTRAPOLATED PARAMETER ESTIMATES USING NEWTON-RAPHSON
-.43776      .35867      -.94963

        ASSOCIATED LOSS FUNCTION AND STANDARD DEVIATION OF THE NOISE
         .15930-01    .17849

        TRUE VALUES OF V AND LAMBDA IN THIS POINT
         .15948-01    .17859

        CONDITON NUMBER OF VTT    5.7382

        ESTIMATED STANDARD DEVIATIONS OF THE PARAMETERS
 .45320-01    .20930-01    .20531-01

        ESTIMATED COVARIANCE MATRIX OF THE PARAMETERS
 .20539-002 -.23148-003 -.28495-003
-.23148-003  .43806-003  .32634-004
-.28495-003  .32634-004  .42150-003

        EIGENVALUES OF VTT
 .35698      .32618      .62212-01

        EIGENVECTORS OF VTT
 .18888      .10099      .97679
 .18538      .97315     -.13646
 .96434     -.20685     -.16509


        RESULT OF FACTORIZATION
        ***********************


        POLES OF DISCRETE MODEL
 .42733      .00000


        POLES OF CONTINOUS MODEL
-.85020      .00000


        ZEROS OF THE C-POLYNOMIAL
 .95108      .00000

        CONTINOUS EQUIVALENT
-.50159-01   .00000

        STATIC GAIN
 .62901
```

OUTPUT FROM SUBROUTINE CSEFT

MEANVALUE = -.1396-02 STANDARD DEVIATION = .1789+00

DEGREES OF FREEDOM: 21 TEST QUANTITY: 18.9 CORRECTION FROM THE TAILS: 1.7

SKEWNESS: -.21+00 KURTOSIS: .11-01

ABSOLUTE FREQUENCIES:

2	0	1	6	10	10	8	14	19	26
25	16	27	18	20	11	8	10	6	3
0	0								

RELATIVE CUMULATIVE FREQUENCIES:

.008	.008	.012	.037	.079	.121	.154	.212	.292	.400
.504	.571	.683	.758	.842	.887	.921	.962	.987	1.000
1.000	1.000								

RESULT FROM TEST OF RESIDUALS

```
NUMBER OF SAMPLES  240

MAX LAG AT THE CALCULATION OF THE COVARIANCE FUNCTIONS   50

MEAN AND STANDARD DEVIATION OF THE RESIDUALS  -.13957-02   .17888

NUMBER OF CHANGES OF SIGN 130
5 PERCENT TOLERANCE LIMITS   103   134

MAX ABS(RES(T))/SIGMA(RES(T))   3.4129
  .83 PERCENT OF THE RESIDUALS OUTSIDE 0.26 PERCENT LIMITS

MAX ABS(RRES(T)) WHEN T.NE.0   .38894
  6.00 PERCENT OF RRES(T) OUTSIDE 5 PERCENT LIMITS
5 PERCENT LIMIT   .12652

TEST OF NORMALITY
TEST QUANTITY   18.938
DEGREES OF FREEDOM 21

TEST OF INDEPENDENCE OF THE RESIDUALS
E RES(T)*RES(T+TAU) ARE USED FOR TAU=1,  5
TEST QUANTITY   7.1197
DEGREES OF FREEDOM  5

MEAN AND STANDARD DEVIATION OF THE INPUT   .10233-05   .41165

MAX ABS(RRESU(T)   .44376
  8.91 PERCENT OF RRESU(T) OUTSIDE 5 PERCENT LIMITS
5 PERCENT LIMIT   .12652

TEST OF INDEPENDENCE OF RESIDUALS AND INPUT
E (RES(T) * U(T-TAU)) ARE USED FOR TAU= 11, 15
TEST QUANTITY   3.2111
DEGREES OF FREEDOM  5

TEST OF INDEPENDENCE OF RESIDUALS AND INPUT
E (RES(T) * U(T-TAU)) ARE USED FOR TAU=  0, -4
TEST QUANTITY   1.6797
DEGREES OF FREEDOM  5
```

```
RRES(TAU) =( E(RES(T)*RES(T+TAU)) -E(RES(T))*E(RES(T)) ) /VAR(RES(T))
RRESU(TAU)=( E(RES(T)*  U(T+TAU)) -E(RES(T))*E(  U(T)) ) /SIGMA(RES(T))/SIGMA(U(T))
```

TAU	RRES	RRESU
0	1.0000	.37872-02
1	-.10296-01	-.31431-01
2	.15854	.41822-01
3	-.69009-02	.62176-01
4	.32472-01	-.19052-01
5	-.57640-01	-.19701-01
6	.26522-01	.40934-01
7	-.16908-01	-.72203-01
8	.10897	-.67391-02
9	-.20367-01	.57368-01
10	.65220-01	-.79578-03
11	-.44027-01	.45439-02
12	.68279-01	-.15153-02
13	-.10821	.12992-01
14	.35096-01	.14805-01
15	-.10139-01	.33961-01
16	-.30540-01	-.16277-01
17	.62192-02	.52957-01
18	-.70176-01	-.63211-02
19	.58946-01	.15254-02
20	-.37371-01	.42436-01
21	.16862-01	-.60642-03
22	-.56509-01	.24937-01
23	-.21508-01	.20752-01
24	.14556-02	-.47307-01
25	.14090-01	.22988-01
26	.31460-01	-.42562-01
27	-.80026-01	.76497-01
28	.10002	-.46900-02
29	-.36952-01	-.15654-02
30	-.36070-01	.35612-01
31	-.49892-01	-.21513-02
32	.77954-01	.42946-01
33	-.11880	-.74745-01
34	.68408-01	.44211-01
35	-.32009-01	.12040-01
36	.86226-01	.96387-02
37	-.84126-01	.41533
38	-.99254-01	.28786
39	-.38894	.27807
40	-.10263	-.22353-01
41	-.38248	.53013-01
42	-.61294-01	-.41411-01
43	-.72290-01	-.63368-01
44	.12563	.10956-01
45	-.70955-01	.25000-01
46	.32860-01	-.36266-01
47	-.87569-01	.59545-01
48	.29246-01	.65444-02
49	-.92129-01	-.36596-01
50	-.33066-01	-.19680-01

TAU	RRESU
0	.37872-02
-1	-.32007
-2	-.34903
-3	-.44376
-4	.32248-01
-5	.84505-02
-6	-.27686-01
-7	.63193-01
-8	-.89828-01
-9	-.18752-01
-10	-.39980-01
-11	-.44148-01
-12	.40605-01
-13	-.64298-01
-14	.67561-01
-15	-.34260-01
-16	-.24272-02
-17	.94449-02
-18	.24538-01
-19	-.48245-02
-20	-.42850-01
-21	.36038-02
-22	.14358-01
-23	-.54718-01
-24	.46128-01
-25	-.21826-01
-26	.96604-03
-27	-.13686-01
-28	.58466-02
-29	-.29991-01
-30	.30859-01
-31	-.67714-01
-32	.50466-01
-33	.24579-01
-34	-.46945-01
-35	-.37750-02
-36	.26820-02
-37	-.18992-01
-38	-.77210-02
-39	.84238-02
-40	.60499-02
-41	.25957
-42	.29915
-43	.36131
-44	-.30550-01
-45	-.44935-01
-46	.57637-01
-47	-.52445-01
-48	.81809-01
-49	.15818-01
-50	.25397-01

10 FIRST VALUES OF THE RESIDUALS

.00000 -.37275 -.26049 -.29400 .93735-01 -.11357 .79026-01 .23409 .31203 -.25574

```
RESULT FROM IDENTIFICATION WITH GLS
***********************************

        ORDER OF OPERATORS      1     1     1

        ISILT=     0
        IPRINT=    1
        ITMAX=    10
        ITER=      3
        INIT=      0
         240 SAMPLES ARE USED
        EPST=   .10000-02
        SAMPLING PERIOD=   1.0000    SECONDS

        INITIAL VALUES
 .00000      .00000      .00000


        RESULT FROM GLS

        RESULT FROM IDENTIFICATION, MODEL OF ORDER  1
          A( 1)=   -.95229               STAND DEV=   .17174-01
        B 1( 1)=    .22805               STAND DEV=   .40930-01

        VALUE OF LOSS FUNCTION    15.877
        ESTIMATED STANDARD DEVIATION S=    .25882

        SINGULAR VALUES
 15.130      6.3192

        COVARIANCE MATRIX OF THE PARAMETER ESTIMATES
 .29496-03   .56688-04
 .56688-04   .16752-02
-------------------------------------------------------------------------

        RESULT FROM IDENTIFICATION, MODEL OF ORDER  1
          A( 1)=   -.49007               STAND DEV=   .56640-01

        VALUE OF LOSS FUNCTION    12.078
        ESTIMATED STANDARD DEVIATION S=    .22527

        SINGULAR VALUES
 3.9772

        COVARIANCE MATRIX OF THE PARAMETER ESTIMATES
 .32081-02
-------------------------------------------------------------------------

        INITIAL VALUES
-.95229      .22805     -.49007
-------------------------------------------------------------------------

        ITERATION NUMBER    1

        ESTIMATION OF A AND B-PARAMETERS

        RESULT FROM IDENTIFICATION, MODEL OF ORDER  1
          A( 1)=   -.87505               STAND DEV=   .25307-01
        B 1( 1)=    .38005               STAND DEV=   .29575-01

        VALUE OF LOSS FUNCTION    10.380
        ESTIMATED STANDARD DEVIATION S=    .20928

        SINGULAR VALUES
```

```
8.2966      7.0594

        COVARIANCE MATRIX OF THE PARAMETER ESTIMATES
 .64044-03  -.31511-04
-.31511-04   .87466-03
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -

        ESTIMATION OF C-PARAMETERS

        RESULT FROM IDENTIFICATION, MODEL OF ORDER  1
          A( 1)=  -.67141              STAND DEV=   .48204-01

        VALUE OF LOSS FUNCTION    9.9104
        ESTIMATED STANDARD DEVIATION S=    .20406

        SINGULAR VALUES
 4.2332

        COVARIANCE MATRIX OF THE PARAMETER ESTIMATES
 .23236-02
--------------------------------------------------------------------------------

        ITERATION NUMBER     2

        ESTIMATION OF A AND B-PARAMETERS

        RESULT FROM IDENTIFICATION, MODEL OF ORDER  1
          A( 1)=  -.76646              STAND DEV=   .32748-01
        B 1( 1)=   .39359              STAND DEV=   .26026-01

        VALUE OF LOSS FUNCTION    9.3062
        ESTIMATED STANDARD DEVIATION S=    .19816

        SINGULAR VALUES
 5.9886     7.7433

        COVARIANCE MATRIX OF THE PARAMETER ESTIMATES
 .10724-02  -.96814-04
-.96814-04   .67734-03
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -

        ESTIMATION OF C-PARAMETERS

        RESULT FROM IDENTIFICATION, MODEL OF ORDER  1
          A( 1)=  -.79264              STAND DEV=   .39702-01

        VALUE OF LOSS FUNCTION    9.1217
        ESTIMATED STANDARD DEVIATION S=    .19577

        SINGULAR VALUES
 4.9310

        COVARIANCE MATRIX OF THE PARAMETER ESTIMATES
 .15763-02
--------------------------------------------------------------------------------

        ITERATION NUMBER     3

        ESTIMATION OF A AND B-PARAMETERS

        RESULT FROM IDENTIFICATION, MODEL OF ORDER  1
          A( 1)=  -.82159              STAND DEV=   .38711-01
        B 1( 1)=   .38509              STAND DEV=   .23585-01
```

VALUE OF LOSS FUNCTION 8.4349
ESTIMATED STANDARD DEVIATION S= .18865

SINGULAR VALUES
4.8325 8.1888

COVARIANCE MATRIX OF THE PARAMETER ESTIMATES
.14985-02 -.15710-03
-.15710-03 .55626-03

- -

ESTIMATION OF C-PARAMETERS

RESULT FROM IDENTIFICATION, MODEL OF ORDER 1
A(1)= -.89535 STAND DEV= .29055-01

VALUE OF LOSS FUNCTION 8.2298
ESTIMATED STANDARD DEVIATION S= .18595

SINGULAR VALUES
6.4000

COVARIANCE MATRIX OF THE PARAMETER ESTIMATES
.84422-03

ITERATION NUMBER 4

ESTIMATION OF A AND B-PARAMETERS

RESULT FROM IDENTIFICATION, MODEL OF ORDER 1
A(1)= -.46779 STAND DEV= .41937-01
B 1(1)= .36990 STAND DEV= .21423-01

VALUE OF LOSS FUNCTION 7.5519
ESTIMATED STANDARD DEVIATION S= .17862

SINGULAR VALUES
4.2250 8.6113

COVARIANCE MATRIX OF THE PARAMETER ESTIMATES
.17587-02 -.19523-03
-.19523-03 .45896-03

- -

ESTIMATION OF C-PARAMETERS

RESULT FROM IDENTIFICATION, MODEL OF ORDER 1
A(1)= -.94381 STAND DEV= .21610-01

VALUE OF LOSS FUNCTION 7.6857
ESTIMATED STANDARD DEVIATION S= .17970

SINGULAR VALUES
8.3158

COVARIANCE MATRIX OF THE PARAMETER ESTIMATES
.46699-03

ITERATION NUMBER 5

ESTIMATION OF A AND B-PARAMETERS

```
RESULT FROM IDENTIFICATION, MODEL OF ORDER  1
  A( 1)=  -.41415                 STAND DEV=   .42462-01
B 1( 1)=   .36415                 STAND DEV=   .20673-01

VALUE OF LOSS FUNCTION   7.3435
ESTIMATED STANDARD DEVIATION S=   .17603

SINGULAR VALUES
4.1123    8.8213

COVARIANCE MATRIX OF THE PARAMETER ESTIMATES
 .18030-02  -.20255-03
-.20255-03   .42739-03
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -

ESTIMATION OF C-PARAMETERS

RESULT FROM IDENTIFICATION, MODEL OF ORDER  1
  A( 1)=  -.95288                 STAND DEV=   .19849-01

VALUE OF LOSS FUNCTION   7.6468
ESTIMATED STANDARD DEVIATION S=   .17925

SINGULAR VALUES
9.0304

COVARIANCE MATRIX OF THE PARAMETER ESTIMATES
.39399-03
------------------------------------------------------------------------------------

ITERATION NUMBER    6

ESTIMATION OF A AND B-PARAMETERS

RESULT FROM IDENTIFICATION, MODEL OF ORDER  1
  A( 1)=  -.40722                 STAND DEV=   .42509-01
B 1( 1)=   .36339                 STAND DEV=   .20576-01

VALUE OF LOSS FUNCTION   7.3346
ESTIMATED STANDARD DEVIATION S=   .17592

SINGULAR VALUES
4.1053    8.8615

COVARIANCE MATRIX OF THE PARAMETER ESTIMATES
 .18070-02  -.20328-03
-.20328-03   .42337-03
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -

ESTIMATION OF C-PARAMETERS

RESULT FROM IDENTIFICATION, MODEL OF ORDER  1
  A( 1)=  -.95386                 STAND DEV=   .19647-01

VALUE OF LOSS FUNCTION   7.6482
ESTIMATED STANDARD DEVIATION S=   .17926

SINGULAR VALUES
9.1260

COVARIANCE MATRIX OF THE PARAMETER ESTIMATES
.38602-03
------------------------------------------------------------------------------------
```

```
ITERATION NUMBER     7

ESTIMATION OF A AND B-PARAMETERS

RESULT FROM IDENTIFICATION, MODEL OF ORDER  1
  A( 1)=  -.40653                 STAND DEV=   .42514-01
B 1( 1)=   .36331                 STAND DEV=   .20566-01

VALUE OF LOSS FUNCTION    7.3364
ESTIMATED STANDARD DEVIATION S=    .17592

         SINGULAR VALUES
4.1048        8.8657

         COVARIANCE MATRIX OF THE PARAMETER ESTIMATES
 .18074-02  -.20335-03
-.20335-03   .42298-03
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -

ESTIMATION OF C-PARAMETERS

RESULT FROM IDENTIFICATION, MODEL OF ORDER  1
  A( 1)=  -.95396                 STAND DEV=   .19628-01

VALUE OF LOSS FUNCTION    7.6484
ESTIMATED STANDARD DEVIATION S=    .17927

         SINGULAR VALUES
9.1332

         COVARIANCE MATRIX OF THE PARAMETER ESTIMATES
 .38525-03
-----------------------------------------------------------------------------------------------------------------------

ITERATION NUMBER     8

ESTIMATION OF A AND B-PARAMETERS

RESULT FROM IDENTIFICATION, MODEL OF ORDER  1
  A( 1)=  -.40647                 STAND DEV=   .42514-01
B 1( 1)=   .36330                 STAND DEV=   .20565-01

VALUE OF LOSS FUNCTION    7.3364
ESTIMATED STANDARD DEVIATION S=    .17592

         SINGULAR VALUES
4.1048        8.8661

         COVARIANCE MATRIX OF THE PARAMETER ESTIMATES
 .18075-02  -.20336-03
-.20336-03   .42294-03
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -

ESTIMATION OF C-PARAMETERS

RESULT FROM IDENTIFICATION, MODEL OF ORDER  1
  A( 1)=  -.95397                 STAND DEV=   .19626-01

VALUE OF LOSS FUNCTION    7.6484
ESTIMATED STANDARD DEVIATION S=    .17927

         SINGULAR VALUES
[illegible]
```

```
 .38518-03 COVARIANCE MATRIX OF THE PARAMETER ESTIMATES
------------------------------------------------------------------------------------------

          CONVERGENCE HAS OCCURED AFTER  8 ITERATIONS

          TEST QUANTITY FOR RELATIVE CHANGE IN THE PARAMETER ESTIMATES
               .10000-02
------------------------------------------------------------------------------------------


          RESULT FROM VGLS

          VALUE OF PARAMETER ESTIMATES
 -.40647       .36330       -.95397

          LOSS FUNCTION
 .15934-01

          STANDARD DEVIATION OF THE NOISE
 .17852        .81482-02

          DERIVATIVE OF V
 .77317-03     .87681-03     .17342-07

          MATRIX OF SECOND ORDER DERIVATIVES
 .75421-01     .36264-01     .48179-01
 .36264-01     .32252       -.43914-03
 .48179-01    -.43914-03     .34763

          EXTRAPOLATED PARAMETER ESTIMATES USING NEWTON-RAPHSON
 -.41690       .36176       -.95252

          ASSOCIATED LOSS FUNCTION AND STANDARD DEVIATION OF THE NOISE
               .15930-01     .17849

          TRUE VALUES OF V AND LAMBDA IN THIS POINT
               .15930-01     .17849

          EXTRAPOLATED PARAMETER ESTIMATES USING NEWTON-RAPHSON
 -.42733       .36021       -.95108

          ASSOCIATED LOSS FUNCTION AND STANDARD DEVIATION OF THE NOISE
               .15930-01     .17849

          TRUE VALUES OF V AND LAMBDA IN THIS POINT
               .15934-01     .17852

          EXTRAPOLATED PARAMETER ESTIMATES USING NEWTON-RAPHSON
 -.43776       .35867       -.94963

          ASSOCIATED LOSS FUNCTION AND STANDARD DEVIATION OF THE NOISE
               .15930-01     .17849

          TRUE VALUES OF V AND LAMBDA IN THIS POINT
               .15948-01     .17859

          CONDITON NUMBER OF VTT   5.7382

          ESTIMATED STANDARD DEVIATIONS OF THE PARAMETERS
 .45320-01     .20930-01     .20531-01

          ESTIMATED COVARIANCE MATRIX OF THE PARAMETERS
 .20539-002 -.23148-003 -.2[illegible]95-003
```

```
-.23140-003   .43806-003   .32634-004
-.28495-003   .32634-004   .42150-003

          EIGENVALUES OF VTT
 .35698        .32618         .62212-01

          EIGENVECTORS OF VTT
 .18888        .10099         .97679
 .18538        .97315        -.13646
 .96434       -.20685        -.16509


          RESULT OF FACTORIZATION
          ***********************


          POLES OF DISCRETE MODEL
 .42733         .00000


          POLES OF CONTINOUS MODEL
-.85020         .00000


          ZEROS OF THE C-POLYNOMIAL
 .95108         .00000

          CONTINOUS EQUIVALENT
-.50159-01      .00000


          STATIC GAIN
 .62901

     OUTPUT FROM SUBROUTINE CSGFT

     MEANVALUE =   -.1396-02         STANDARD DEVIATION =    .1789+00

     DEGREES OF FREEDOM: 21         TEST QUANTITY:  18.9         CORRECTION FROM THE TAILS:   1.7

     SKEWNESS: -.21+00         KURTOSIS:  .11-01

     ABSOLUTE FREQUENCIES:
        2        0        1        6       10       10        8       14       19       26
       25       16       27       18       20       11        8       10        6        3
        0        0

     RELATIVE CUMULATIVE FREQUENCIES:
     .008     .008     .012     .037     .079     .121     .154     .212     .292     .400
     .504     .571     .683     .758     .842     .887     .921     .962     .987    1.000
    1.000    1.000
```

RESULT FROM TEST OF RESIDUALS

NUMBER OF SAMPLES 240

MAX LAG AT THE CALCULATION OF THE COVARIANCE FUNCTIONS 50

MEAN AND STANDARD DEVIATION OF THE RESIDUALS -.13957-02 .17888

NUMBER OF CHANGES OF SIGN 130
5 PERCENT TOLERANCE LIMITS 103 134

MAX ABS(RES(T))/SIGMA(RES(T)) 3.4129
.83 PERCENT OF THE RESIDUALS OUTSIDE 0.26 PERCENT LIMITS

MAX ABS(RRES(T)) WHEN T.NE.0 .38894
6.00 PERCENT OF RRES(T) OUTSIDE 5 PERCENT LIMITS
5 PERCENT LIMIT .12652

TEST OF NORMALITY
TEST QUANTITY 18.938
DEGREES OF FREEDOM 21

TEST OF INDEPENDENCE OF THE RESIDUALS
E RES(T)*RES(T+TAU) ARE USED FOR TAU=1, 5
TEST QUANTITY 7.1197
DEGREES OF FREEDOM 5

MEAN AND STANDARD DEVIATION OF THE INPUT .10233-05 .41165

MAX ABS(RRESU(T)) .46376
8.91 PERCENT OF RRESU(T) OUTSIDE 5 PERCENT LIMITS
5 PERCENT LIMIT .12652

TEST OF INDEPENDENCE OF RESIDUALS AND INPUT
E (RES(T) * U(T-TAU)) ARE USED FOR TAU= 11, 15
TEST QUANTITY 3.2111
DEGREES OF FREEDOM 5

TEST OF INDEPENDENCE OF RESIDUALS AND INPUT
E (RES(T) * U(T-TAU)) ARE USED FOR TAU= 0, -4
TEST QUANTITY 1.6797
DEGREES OF FREEDOM 5

```
RRES(TAU) =( E(RES(T)*RES(T+TAU)) -E(RES(T))*E(RES(T)) ) /VAR(RES(T))
RRESU(TAU)=( E(RES(T)*  U(T+TAU)) -E(RES(T))*E(  U(T)) ) /SIGMA(RES(T))/SIGMA(U(T))
```

TAU	RRES	RRESU	TAU	RRESU
0	1.0000	.37872-02	0	.37872-02
1	-.10296-01	-.31431-01	-1	-.32007
2	.15854	.41822-01	-2	-.34903
3	-.69009-02	.62176-01	-3	-.44376
4	.32472-01	-.19052-01	-4	.32248-01
5	-.57640-01	-.19701-01	-5	.84505-02
6	.26522-01	.40934-01	-6	-.27686-01
7	-.16908-01	-.72203-01	-7	.63193-01
8	.10897	-.67391-02	-8	-.89828-01
9	-.20367-01	.57368-01	-9	-.18752-01
10	.65220-01	-.79578-03	-10	-.39980-01
11	-.44027-01	.45439-02	-11	-.44148-01
12	.68279-01	-.15153-02	-12	.40605-01
13	-.10821	.12992-01	-13	-.64298-01
14	.35096-01	.14805-01	-14	.67561-01
15	-.10139-01	.33961-01	-15	-.34260-01
16	-.30540-01	-.16277-01	-16	-.24272-02
17	.62192-02	.52957-01	-17	.94449-02
18	-.70176-01	-.63211-02	-18	.24538-01
19	.58946-01	.15254-02	-19	-.48245-02
20	-.37371-01	.42436-01	-20	-.42850-01
21	.16862-01	-.60642-03	-21	.36038-02
22	-.56509-01	.24937-01	-22	.14358-01
23	-.21508-01	.20752-01	-23	-.54718-01
24	.14556-02	-.47307-01	-24	.46128-01
25	.14090-01	.22988-01	-25	-.21826-01
26	.31460-01	-.42562-01	-26	.96604-03
27	-.80026-01	.76497-01	-27	-.13686-01
28	.10002	-.46900-02	-28	.58466-02
29	-.36952-01	-.15654-02	-29	-.29991-01
30	-.36070-01	.35612-01	-30	.30859-01
31	-.49892-01	-.21513-02	-31	-.67714-01
32	.77954-01	.42946-01	-32	.50466-01
33	-.11880	-.74745-01	-33	.24579-01
34	.68408-01	.44211-01	-34	-.46945-01
35	-.32009-01	.12040-01	-35	-.37750-02
36	.86226-01	.96387-02	-36	.26820-02
37	-.84126-01	.41533	-37	-.18992-01
38	-.99254-01	.28786	-38	-.77210-02
39	-.38894	.27807	-39	.84238-02
40	-.10263	-.22353-01	-40	.60499-02
41	-.38248	.53013-01	-41	.25957
42	-.61294-01	-.41411-01	-42	.29915
43	-.72290-01	-.63368-01	-43	.36131
44	.12563	.10956-01	-44	-.30550-01
45	-.70955-01	.25000-01	-45	-.44935-01
46	.32860-01	-.36266-01	-46	.57637-01
47	-.87569-01	.59545-01	-47	-.52445-01
48	.29244-01	.65444-02	-48	.81809-01
49	-.92129-01	-.36596-01	-49	.15818-01
50	-.33066-01	-.19680-01	-50	.25397-01

10 FIRST VALUES OF THE RESIDUALS

.00000 -.37275 -.28819 -.29400 .93735-01 -.11357 .79026-01 .25409 .31205 -.33374