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1. INTRODUCTION

This paper gives a brief review of the results of a research project on self-tuning regulators which has been carried out at the Lund Institute of Technology. The project is part of a larger research program on adaptive control which has followed three main lines,

A - Stochastic control

B - Self-tuning regulators

C - Analysis of adaptive regulators proposed in literature.

The approach via stochastic control which leads to dual control strategies has been very useful to provide understanding and insight. So far the results have, however, not been carried out to the stage of implementation. Self-tuning regulators are a particular version of adaptive regulators based on real-time identification. They are a special case of nondual stochastic control algorithms. The work on self-tuning regulators has progressed quite far in the sense that these regulators are reasonably well understood theoretically. They have also been tried extensively in several industrial applications. On the other hand there still remains a lot of work to be done to explore other aspects of these regulators. The third project C is needed to keep abreast of the development of other adaptive schemes. It has also resulted in a long list of problems relating to understanding the strange behaviour of some algorithms in certain circumstances.

The basic idea underlying the self-tuning regulators is the following. If a description of a system and its environment is known there are many procedures available to design a control system subject to given specifications. When trying to remove the assumption that the models for the system and its environment are known we are immediately lead to the problem of controlling a system with constant but unknown parameters. This problem can in principle be solved by using stochastic control theory at the prize of exorbitant calculations. It is then meaningful to ask if there are simple control algorithms, which do not require information about the model parameters, such that the controller will converge to the controllers that could be designed if the model parameters were known. It is an empirical fact that such controllers exist in several cases. The investigation of their properties have also lead to powerful tools that can be used to analyse many other cases.

The generation of self-tuning algorithms is partly heuristic. It turns out that many algorithms can be obtained by combining a real-time identifier with a control scheme. In our work we have so far mostly considered regulators for the LQG regulator problem. This has been motivated by the particular applications we have considered. Many of the concepts and ideas can, however, be extended to many other design methods.

2. AN EXAMPLE

The main ideas will first be demonstrated using a simple example. Consider the simple discrete time system

$$y(t+1) + a\ y(t) = b\ u(t) + e(t+1) + c\ e(t) \qquad (2.1)$$

where u is the input, y the output and $\{e(t)\}$ a sequence of independent, equally distributed, random variables. The number c is assumed to be less than one. Let the criterion be to minimize the variance of the output i.e.

$$\min V = \min Ey^2 = \min E \frac{1}{t} \sum_{k=1}^{t} y^2(k) \qquad (2.2)$$

It is easy to show that the control law

$$u(t) = \frac{a - c}{b} y(t) \qquad (2.3)$$

is a minimum variance strategy, and that the output of the system (2.1) with the feedback (2.3) becomes

$$y(t) = e(t) \qquad (2.4)$$

See e.g. Åström (1970). Notice that the control law (2.3), which represents a proportional regulator, can be characterized by one parameter only.

A self-tuning regulator for the system (2.1) can be described as follows:

ALGORITHM (Self-TUning REgulator)

Step 1 (Parameter Estimation)

At each time t, fit the parameter α in the model

$$\hat{y}(k+1) + \alpha\ y(k) = u(k), \quad k = 1,\ldots,t-1 \qquad (2.5)$$

by least squares, i.e. such that the criterion

$$\sum_{k=1}^{t} \varepsilon^2(k) \tag{2.6}$$

where

$$\varepsilon(k) = y(k) - \hat{y}(k) \tag{2.7}$$

is minimal. The estimated obtained is denoted α_t to indicate that it is a function of time.

Step 2 (Control)

At each time t, choose the control

$$u(t) = \alpha_t \, y(t) \tag{2.8}$$

where α_t is the estimate obtained in step 1.

Motivation

There are several ways to arrive at the control strategy given above. The algorithm STURE can e.g. be interpreted as the certainty equivalence control for the corresponding stochastic control problem.

Analysis

The properties of a closed loop system controlled by a self-tuning regulator will now be discussed. Since the closed loop system is nonlinear, timevarying and stochastic, the analysis is not trivial.

It is fairly obvious that the regulator will perform well if it is applied to a system (2.1) with $b = 1$ and $c = 0$, because in this case the least squares estimate α_t will be an unbiased estimate of a. The regulator (2.8) will thus converge to a minimum variance regulator if the parameter estimate α_t converges. It is surprising, however, that the regulator will also converge to the minimum variance regulator if $c \neq 0$ as will be demonstrated below.

There may also be some difficulties because the control law is of the certainty equivalence type. Because of the special model structure (2.5) the feedback gain will however be bounded if the estimate α_t is bounded.

The least squares estimate is given by the normal equation

$$\frac{1}{t}\sum_{k=1}^{t} y(k+1)y(k) + \alpha_{t+1}\frac{1}{t}\sum_{k=1}^{t} y^2(k) = \frac{1}{t}\sum_{k=1}^{t} y(k)u(k) .$$

Assuming that the estimate α_t converges towards a value which gives a stable closed loop system, then it is straightforward to show that

$$\frac{1}{t}\sum_{k=1}^{t} (\alpha_{t+1} - \alpha_k)y^2(k) \rightarrow 0$$

Thus the closed loop system has the property

$$\lim_{t\to\infty} \frac{1}{t}\sum_{k=1}^{t} y(t+1)y(t) = 0 \qquad (2.9)$$

Furthermore, assuming that the system to be controlled is governed by (2.1), the output of the closed loop system obtained in the limit is given by

$$y(t) + [a - \alpha b]\, y(t) = e(t) + c\, e(t-1) \qquad (2.10)$$

The covariance of $\{y(t)\}$ at lag 1 is then given by

$$E\, y(t+1)y(t) = -f(\alpha) = \frac{(c-a+\alpha b)(1-ac+\alpha bc)}{1 - (a-\alpha b)^2} \qquad (2.11)$$

The condition (2.9) gives

$$f(\alpha) = 0$$

This is a second order equation for α which has the solutions

$$\alpha = \alpha_1 = \frac{a - c}{b}$$

$$\alpha = \alpha_2 = \frac{a - 1/c}{b}$$

The corresponding poles of the closed loop system are $\lambda_1 = c$ and $\lambda_2 = 1/c$ respectively. Since c was assumed less than one, only the value α_1 corresponds to a stable closed loop system. Notice that α_1 corresponds to the gain of the minimum variance regulator (2.3).

Hence, if the parameter estimate α_t converges to a value which gives a stable closed loop system, then the closed loop system obtained must be such that (2.7) holds. This means that the algorithm can be thought of as a regulator which attempts to bring the covariance of the output at lag one, i.e. $r_y(1)$, to zero in the same way as an integrating regulator brings the integral of the control error to zero.

If the system to be controlled is actually governed by (2.1), then the self-tuning regulator will converge to a minimum variance regulator if it converges at all.

Slide 1 shows the results of a simulation of the algorithm. It is clear from this simulation that the algorithm converges in the particular case. The least squares estimate will be a biased estimate of the model parameter $a = -0.5$ because of the correlation between the model errors. As can be expected from the previous analysis the bias is, however, such that the limiting regulator corresponds to the minimum variance regulator.

The lower part of slide 1 shows the asymptotic value of the loss function obtained if the regulator gain is fixed to the current value. It is clear from this slide that the loss function is very close to the minimum loss for the case of known parameter after 50 steps.

3. GENERALISATIONS

A regulator which generalizes the simple self-tuner of the previous section is shown in slide 2. The regulator can be thought of as being composed of three parts: a parameter estimator (block 1), a controller (block 3) and a third part (block 2), which relates the controller parameters to the estimated parameters. The parameter estimator acts on the process inputs and outputs and produces estimates of certain process parameters. The controller is simply a linear filter characterized by the coefficients of its transfer function. These coefficients are in general a non-linear function of the estimated parameters. This function is frequently not one to one. This way of describing the regulator is convenient from the point of view of explaining how it works. The subdivision is, however, largely arbitrary, and the regulator can equally well be regarded simply as one non-linear regulator. The functions of the blocks 1, 2 and 3 are also simple, but the interconnection of these blocks represents a system with a rather complex input-output relation. The partitioning of the regulator as indicated in slide 2 is also convenient from the point of view of implementation, because the parameter estimator and the controller parameter calculation are often conveniently time shared between several loops.

There are many different ways to estimate the parameters θ and to calculate the regulator parameters, ϑ. Some possibilities are shown in slide 3.

The complexity of the algebraic equation which relates the controller parameters to the estimated parameters can vary significantly. From a simple variable substitution for minimum variance regulators to solution of an algebraic Riccati equation for the general LQG case.

Analysis

A brief statement of some properties of the self-tuning regulators will now be given. The results are fairly technical and only a few main points will be given here. A review of available results are given in Åström, Borisson, Ljung and Wittenmark (1975). The major results were proven in Åström and Wittenmark (1973), Ljung (1974) and Ljung and Wittenmark (1974b).

For the analysis it is assumed that the process to be controlled is governed by

$$A(q^{-1})\ y(t) = B(q^{-1})\ u(t-h) + v(t) \qquad (3.1)$$

where $A(q^{-1})$ and $B(q^{-1})$ are polynomials in the backward shift operator q^{-1} and $\{v(t)\}$ is a sequence of random variables with bounded fourth moment. The analysis will basically cover the case $\mu(t) \to 0$ as $t \to \infty$ which corresponds to the case when the parameters are constant.

The following problems can partially be resolved by analysis.

- Overall stability of the closed loop system
- Convergence of the regulator
- The properties of the possible limiting regulators

The analysis is far from trivial because the closed loop system is a nonlinear time variable stochastic system. Even if the recursive identification schemes used are well known, their convergence properties are largely unknown except for the least squares case. The input is also generated by a time varying feedback, which introduces additional difficulties. If the process noise $\{v(t)\}$ is correlated, the least squares estimates will be biased and the bias will depend on the feedback used.

A global stability result was proven by Ljung and Wittenmark (1974). See slide 4. This result applies to a regulator composed of a least squares identifier and a minimum variance controller. The result requires that the system (3.1) is minimum phase and that the timedelay k and the parameter β_o are known.

A key result in the analysis is the observation made by Ljung (1974) that the paths of the estimates are closely related to the trajectories of the differential equation

$$\begin{cases} \frac{d\theta}{d\tau} = S\ f(\theta) \\ \frac{dS^{-1}}{d\tau} = G(\theta) - S^{-1} \end{cases} \tag{3.2}$$

where

$$\begin{cases} f(\theta) = E\ [\Psi^T(t,\theta)\ \varepsilon(t,\theta)] \\ G(\theta) = E\ [\Psi^T(t,\theta)\ \Psi(t,\theta)] \end{cases} \tag{3.3}$$

In the particular case of the regulator LS+MV the control law is chosen in such a way that $\hat{y}(t,\theta) = 0$ and the stationary points are then given by

$$\theta = f(\theta) = E\left[y(t+1)\ \phi(t)\right] = 0$$

The regulator LS+MV thus attempts to zero the autocovariance of the output and the crosscovariance of the input and the output for certain lags. This result which generalizes the simple example discussed in section 2 was shown in Åström and Wittenmark (1972).
In this paper it was also shown that the equation

$$f(\theta) = 0 \tag{3.4}$$

has only one stationary solution for the regulator of LS+MV if the orders of the system and the model are compatible.

The differential equations (3.3) can be used in several different ways. Ljung has exploited them to construct both convergence proofs and examples which shows that the parameter estimates are not converge. The differential equations have also been very useful in simulations. See e.g. Wittenmark (1973).

The simulations shown in slides 5, 6 and 7 illustrate the behaviour of different versions of the self-tuner.

4. THE SERVOPROBLEM

So far the self-tuning regulator has only been discussed in the framework of the regulator problem. It is straightforward to apply self-tuning to the servo-problem too. Clarke and Gawthrop (1974) proposes to do so by posing a linear quadratic problem for a servo problem.

Another approach is simply to introduce the reference values by the standard procedure using feedforward and an inverse model. In the case of known parameters the problem is handled as follows. Assume that the process is described by (3.1) and introduce the reference values $u^r(t)$ and $y^r(t)$ which satisfies the same dynamics as the process i.e.

$$A(q^{-1})\ y^r(t) = B(q^{-1})\ u^r(t-k) \tag{4.1}$$

Hence

$$A(q^{-1})\ [y(t) - y^r(t)] = B(q^{-1})[u(t-k) - u^r(t-k)] + v(t).$$

A design procedure for the regulator then gives the feedback

$$u(t) - u^r(t) = \frac{G(q^{-1})}{F(q^{-1})}\ [y(t) - y^r(t)]$$

If the comand signal $y^r(t)$ is specified this gives

$$u(t) = \frac{A(q^{-1})}{B(q^{-1})}\ y^r(t+k) + \frac{G(q^{-1})}{F(q^{-1})}\ [y(t) - y^r(t)] \tag{4.2}$$

This system can not be realized unless the change in reference value is known or can be predicted k steps ahead. If this is not the case a timedelay in the response to of k units must be accepted.

Observe that the control law (4.2) can be written as

$$B(q^{-1})\ F(q^{-1})\ u(t) =$$

$$= G\ y(t) + \left(A(q^{-1})\ F(q^{-1}) - q^{-k}\ G(q^{-1})\right)\ y^r(t+k) \tag{4.2}$$

The servo-problem can be incorporated into the self-tuning regulator simply by changing the model in the parameter estimation step to

$$M: \quad y(t) = - A(q^{-1})y(t-1) + B(q^{-1})u(t-k) + C(q^{-1})\varepsilon(t-1) + $$
$$+ D(q^{-1})y^{r}(t-1)$$

and making the modification (4.2) in the control step.

5. APPLICATIONS

The self-tuning regulators are conveniently implemented using a digital computer. The simple regulator LS+MV requires no more than 30 lines of FORTRAN code while the regulator RML+LQ requires an order of magnitude more code because of the necessity of solving the algebraic Riccati equation in each iteration. The regulators have been applied to a number of industrial processes. Among the applications that are currently known to me I can mention

- o paper machine [Cegrell-Hedqvist (1973) and Borisson-Wittenmark (1974)]
- o digester [Cegrell-Hedqvist (1974)]
- o ore crusher [Borisson-Syding (1974)]
- o enthalpy exchanger [Jensen-Hänsel (1974)]
- o supertanker [Källström (1974)]

Several of these applications have been in operation for a long time. A self-tuning regulator has for example been running as an adaptive autopilot for a supertanker for more than a year.

Even if the regulators discussed automatically tune its parameters, it is necessary to determine some parameters in advance. These are for instance:

- o The number of parameters in the prediction model (p, r and s).
- o The initial values of the parameter estimates.
- o Value of any fixed parameters in the model.
- o Rate of exponential forgetting of past data in the estimation algorithm.
- o The sampling rate.

Experience has shown that it is fairly easy to make the proper choice in practice. These parameters are also much easier to choose than to directly determine the coefficients of a complex control law. It is our experience that system engineers without previous exposure to this type of algorithms have been able to learn how to use them after a short training period only.

There have also been several misapplications. The most common mistake is to attempt a self-tuner for a control design that will not work even if the parameters are known.

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SIMULATIONS

EXAMPLE 1

$$y(t) + a\,y(t-1) = b\,u(t-1) + e(t) + c\,e(t-1)$$

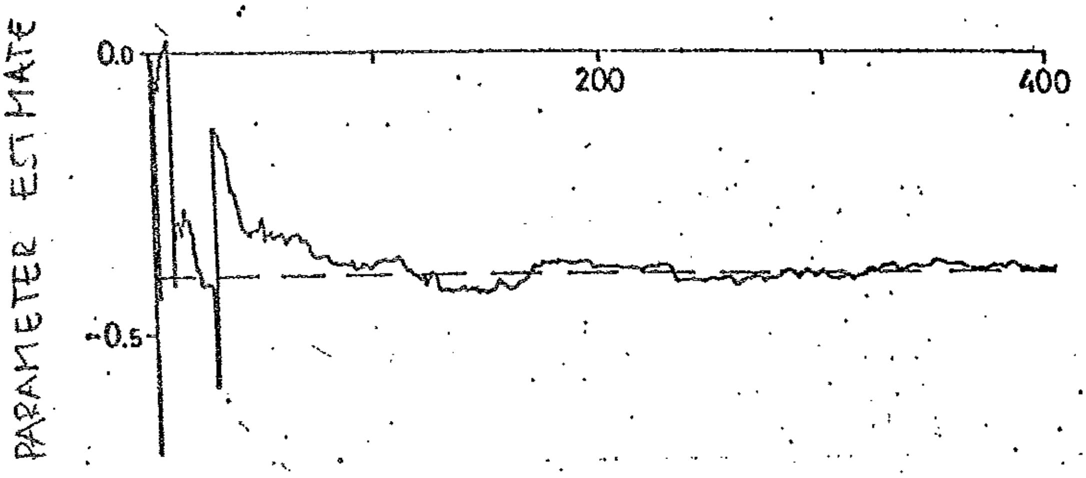
$$a = -0.5, \quad b = 3, \quad c = 0.7$$

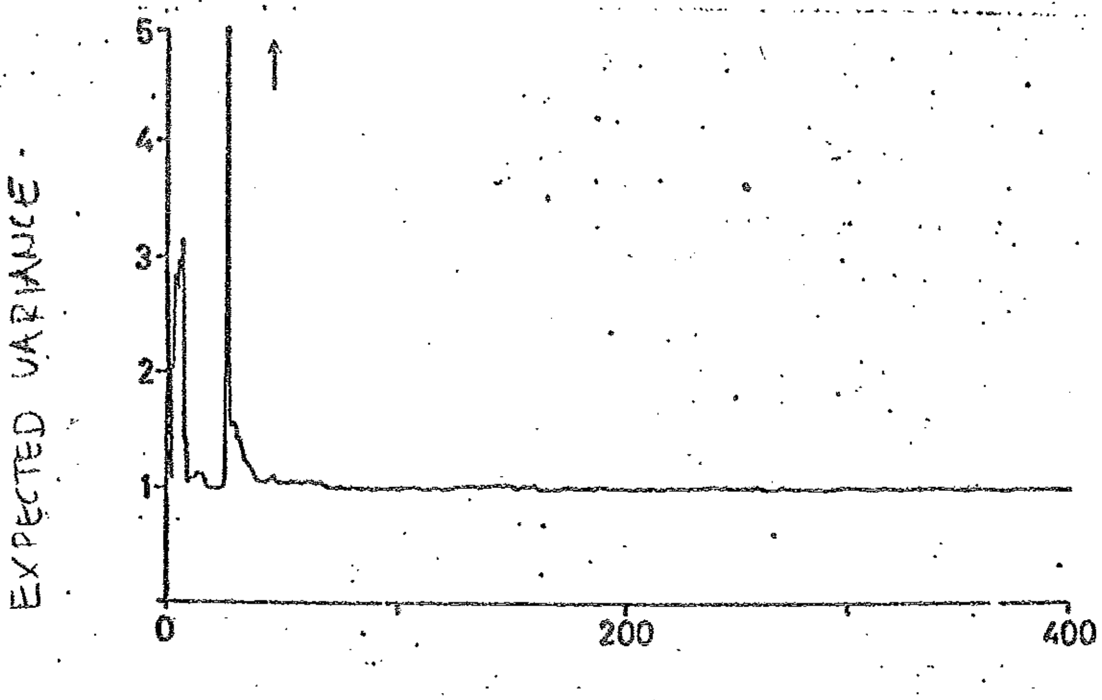
MIN VARIANCE REGULATOR

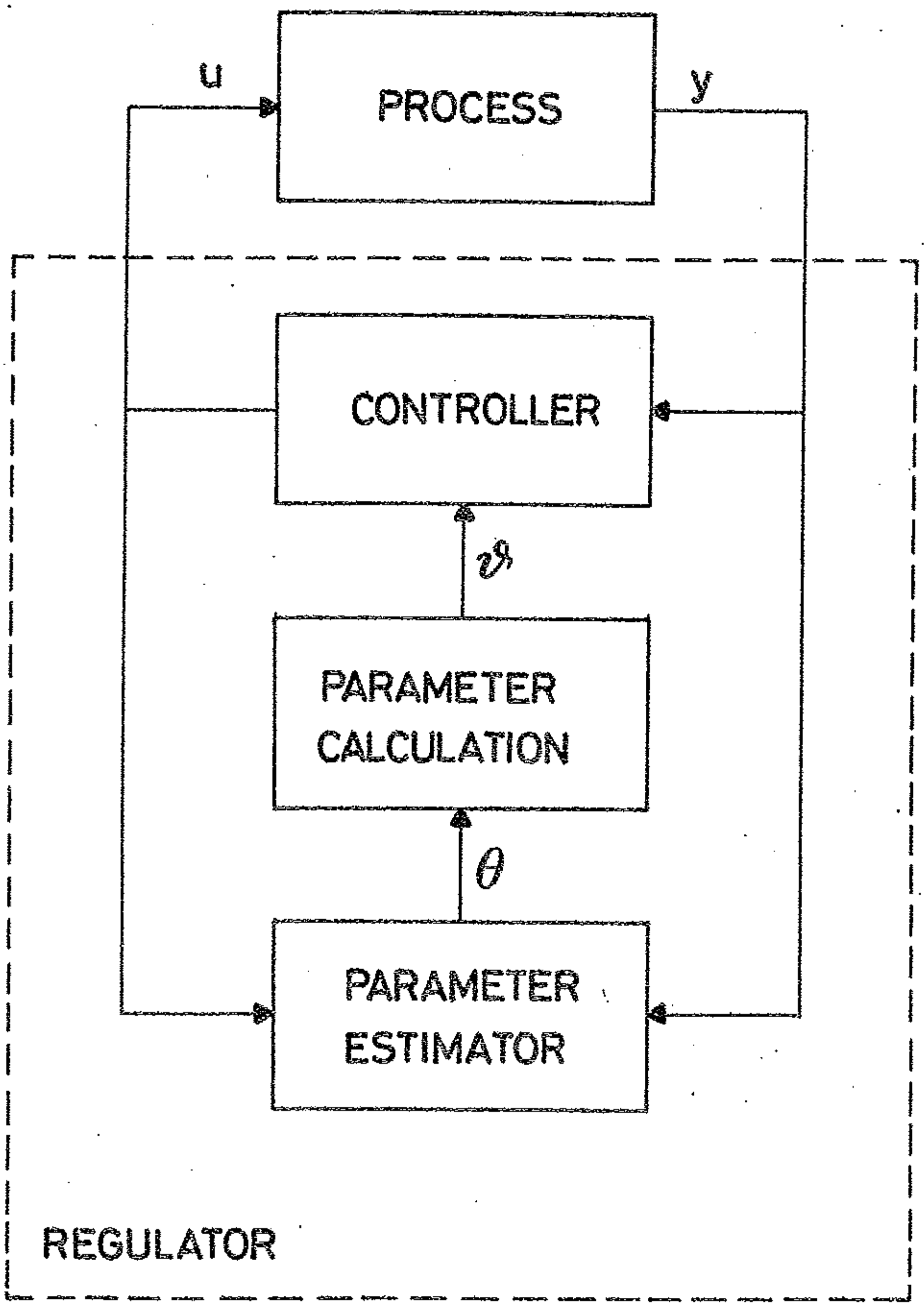
$$u(t) = \frac{a-c}{b}\,y(t) = -0.4\,y(t)$$

MODEL

$$y(t) + \alpha y(t-1) = \beta_0\,u(t-1) + e(t)$$







u
PROCESS
y
CONTROLLER
ϑ
PARAMETER
CALCULATION
θ
PARAMETER
ESTIMATOR
REGULATOR

PARAMETER ESTIMATION

$$\hat{y}(t) = -\mathcal{A}(q^{-1})y(t-1) + \mathcal{B}(q^{-1})u(t-1) + \mathcal{C}(q^{-1})\varepsilon(t-1)$$

$$= \varphi(t)\theta \quad \text{(MODEL } \mathcal{M}\text{)}$$

$$\varepsilon(t,\theta) = y(t) - \varphi(t)\theta =$$

$$\theta(t+1) = \theta(t) + \mu(t)S(t+1)\,\psi^T(t)\,\varepsilon(t, \theta(t))$$

$$S^{-1}(t+1) = S^{-1}(t) + \mu(t+1)\left[\psi^T(t+1)\psi(t+1) - S^{-1}(t)\right]$$

ELS: $\varphi(t) = \psi(t)$ LS: $\mathcal{C} = 0$

RML: $\psi(t) = -\text{grad}_{\theta}\ \varepsilon(t, \theta)$

CONTROL STRATEGIES

$$u(t) = \frac{G(q^{-1})}{F(q^{-1})}\ y(t)$$

REGULATOR PARAMETERS

$\vartheta = \text{col}\left[g_1\ g_2 \ldots g_m\ f_1\ f_2 \ldots\ f_1\right]$

MINIMUM VARIANCE (DEAD BEAT)

LINEAR QUADRATIC $\sum\left[y^2(t) + \ u^2(t)\right]$

THEOREM

LET THE SYSTEM BE CONTROLLED BY REGI (LS+MV)
IF TIME DELAY k AND β_0 ARE KNOWN, IF THE ORDER IS NOT UNDERESTIMATED AND IF

$\limsup 1/N \sum v^2(t) < \infty$

THEN

$\limsup 1/N \sum y^2(t) < \infty$

IF THE SYSTEM IS MINIMUM PHASE THEN ALSO

$\limsup 1/N \sum u^2(t) < \infty$

EXAMPLE 1

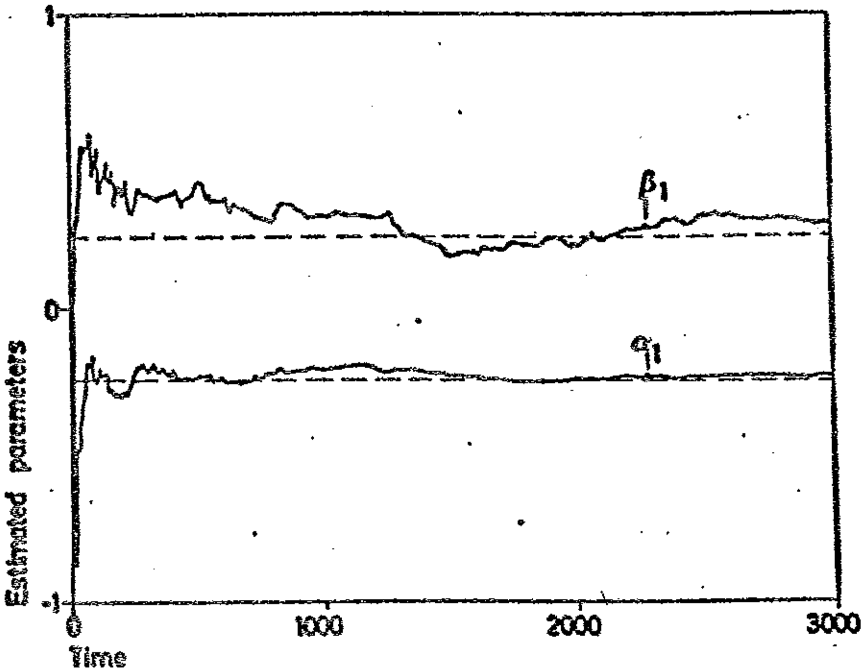
SYSTEM

$y(t) - 0.95\,y(t-1) = u(t-2) + e(t) - 0.7\,e(t-1)$

MODEL

$y(t) = -\alpha\, y(t-2) + u(t-2) + \beta\, u(t-3)$

ALGORITHMS : LS + MV



EXAMPLE 2

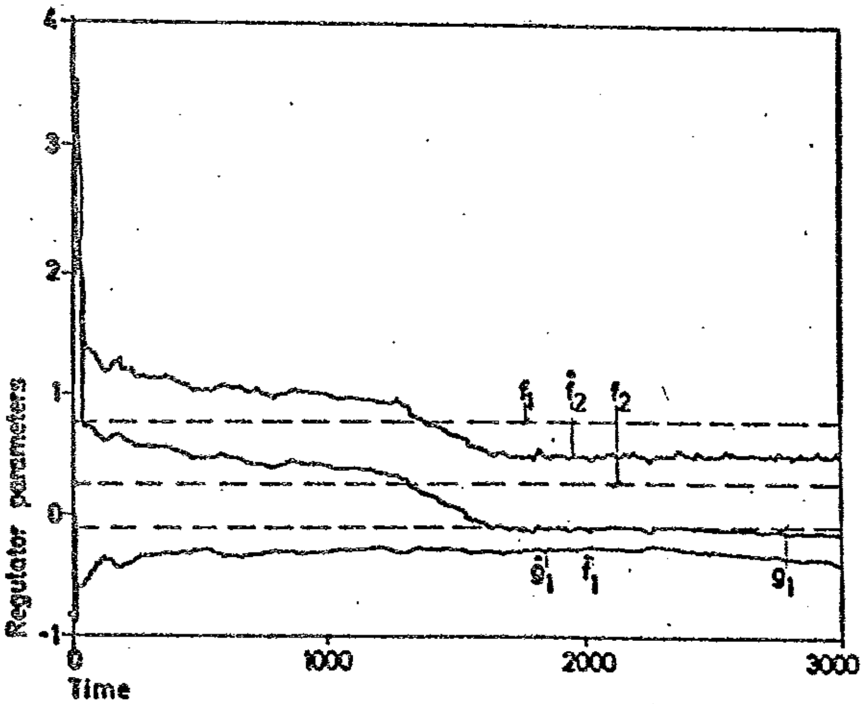
SYSTEM

$$y(t) - 0.95y(t-1) = u(t-2)+2u(t-3)+e(t)-0.7e(t-1)$$

MODEL

$$\hat{y}(t) = -\alpha y(t-1) + \beta_1 u(t-2)+ \beta_2 u(t-3)$$

ALGORITHMS: LS+LQ (RICCATI)



EXAMPLE 3

SYSTEM

$$y(t)-0.95y(t-1)=u(t-2)+2u(t-3)+e(t)-0.7e(t-1)$$

MODEL

$$y(t) = -\alpha y(t-1)+\beta_1 u(t-2)+\beta_2 u(t-3)+\gamma\varepsilon(t-1)$$

ALGORITHMS: ELS + LQ (RICCATI)