A Design Example of a Sampled Data System

Wittenmark, Björn

1976

Document Version:
Publisher's PDF, also known as Version of record

Link to publication

Citation for published version (APA):
Wittenmark, B. (1976). *A Design Example of a Sampled Data System*. (Research Reports TFRT-3130). Department of Automatic Control, Lund Institute of Technology (LTH).

Total number of authors:
1

LUND UNIVERSITY

PO Box 117
221 00 Lund
+46 46-222 00 00

3130

A DESIGN EXAMPLE OF A SAMPLED DATA SYSTEM

B.WITTENMARK

Report 7613(C) March 1976
Department of Automatic Control
Lund Institute of Technology

A DESIGN EXAMPLE OF A SAMPLED DATA SYSTEM

B. Wittenmark

ABSTRACT

This report illustrates some design methods for sampled data systems. A second order continuous time system with a time delay is considered. The discussed design methods are based on pole placement in different ways and results from simulations are shown. The pole placement methods together with simulations are shown to be a successful approach to the design of sampled data systems.

TABLE OF CONTENTS

1. FORMULATION OF THE PROBLEM

The problem of designing digital regulators will be illustrated on a simple example. The process is a very simplified model of a control system on a turbojet motor. This process has been discussed in a master thesis, [4], done at Volvo Flygmotor, Trollhättan, in cooperation with the Department of Automatic Control, Lund.

The Process.

Consider the block diagram given in Figure 1.1. The process is a simplified model of the system for the control of the outlet area of an after-burner of a turbojet motor. The outlet area is controlled in order to get a specified pressure in the after-burner.

The numerical values used in this report are

$K \;= 60$

$T_1 = 0.015s$

$\tau \;= 0.02s$

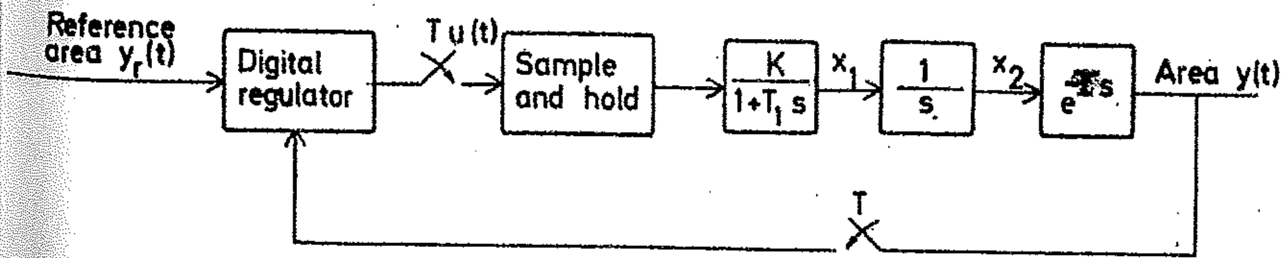


Figure 1.1 - A simple model of the area control loop of an after-burner of a turbojet motor.

Introduce the state variables $x_1(t)$ and $x_2(t)$ as in Figure 1.1. Neglecting the time delay a state space representation is given by

$$\dot{x}(t) = \begin{bmatrix} -\frac{1}{T_1} & 0 \\ 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} \frac{K}{T_1} \\ 0 \end{bmatrix} u(t)$$

The discrete time representation when using the sampling interval T is

$$x(t+T) = \begin{bmatrix} e^{-T/T_1} & 0 \\ T_1(1 - e^{-T/T_1}) & 1 \end{bmatrix} x(t) + K \begin{bmatrix} 1 - e^{-T/T_1} \\ T - T_1(1 - e^{-T/T_1}) \end{bmatrix} u(t)$$

The sampling time used in this study is T = 0.01s. The output from the system is equal to $x_2(t)$ delayed 0.02s = 2T. Introducing two state variables $x_3(t) = x_2(t-T)$ and $x_4(t) = x_3(t-T)$ gives the discrete time representation of the system.

$$x(t+0.01) = \begin{bmatrix} 0.513 & 0 & 0 & 0 \\ 0.007 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 29.196 \\ 0.162 \\ 0 \\ 0 \end{bmatrix} u(t) \qquad (1.1)$$

The pulse transfer function from the input to the output is given by

$$y(t) = \frac{0.162q^{-3}(1+0.801q^{-1})}{(1-q^{-1})(1-0.513q^{-1})} u(t) \qquad (1.2)$$

where q^{-1} is the backward shift operator.

Specifications.

The problem is to find a digital regulator which gives a smooth control of the outlet area of the after-burner. The following specification can be used:

a. The output should reach its steady state value 0.15 - 0.30s (i.e. 15 - 30 sampling intervals) after a change in the reference signal or after a disturbance.

b. The steady state value should be reached without or with a small overshoot (less than 5%).

Design Methods.

The design methods used in the following sections are based on pole placement and time domain analysis. There also exist design methods based on frequency domain analysis, see e.g. Tou [6].

The aim of this report is to show that pole placement methods together with a good simulation package can be very useful for the design of sampled data regulators.

The methods discussed in the report are:

- Pole placement using state feedback.
- Pole placement using output feedback.
- Combined feedback and feedforward control.

For a controllable system it is always possible to place the poles of the closed loop system at desired locations. This is illustrated in Section 2 where state feedback control laws are discussed.

In Section 3 a cascade controller is discussed which only uses the error between the reference value and the output signal. With a proper cascade controller it is possible to get a desired characteristic polynomial of the closed loop system.

The controller in Section 4 is also based on output feedback but the control scheme is more flexible. It is for instance possible to make a separation between the servoproblem (follow a reference signal) and the regulator problem (eliminate a disturbance).

A still more complex controller is discussed in Section 5. This controller also separates the servo and the regulator problems. The controller can be regarded as a combination of feedforward from the reference signal and feedback from the output signal.

2. POLE PLACEMENT USING STATE FEEDBACK.

If a system is controllable it is always possible to place the poles at desired locations by using a state feedback control law

$$u(t) = y_r(t) - Lx(t) \tag{2.1}$$

This requires that all state variables are measurable. In the discussed physical system, eq. (1.1), it is not possible to measure all the states. The state feedback method will, however, be discussed for the sake of completeness.

The control law (2.1) gives in general a steady state error if $y_r(t)$ is the desired reference value. The process in Figure 1.1 contains an integrator and it should be possible to follow a constant reference value by using a proper feedback law. One way to eliminate the steady state error is to use the control scheme in Figure 2.1.

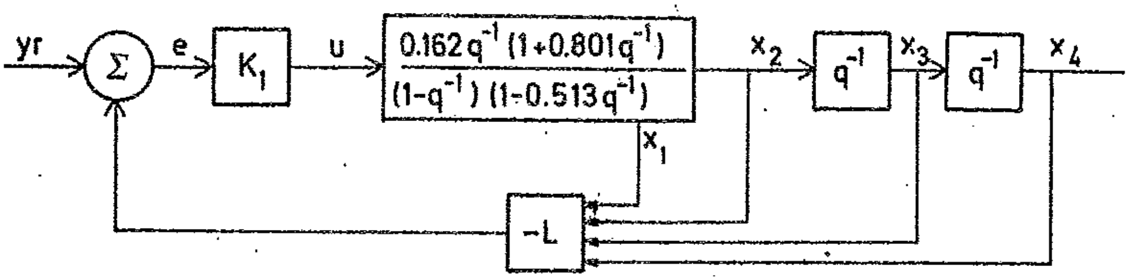


Figure 2.1 - Modified state feedback control law

$$u(t) = K_1\left(y_r(t) - Lx(t)\right)$$

The control law is now

$$u(t) = K_1 e(t) = K_1 \left(y_r(t) - Lx(t)\right) \tag{2.2}$$

A new parameter, the gain K_1, is introduced. This gives an extra degree of freedom for the choice of L. From (1.1) it is found that if steady state is reached, then $x_1 = 0$ and $x_2 = x_3 = x_4 = y$. If L is chosen such that

$$\ell_2 + \ell_3 + \ell_4 = 1$$

where the ℓ_i:s are the elements of L, then e = 0 in steady state. The characteristic polynomial can, however, still be chosen arbitrarily thanks to the extra parameter K_1.

The system (1.1) is controllable and using the control law (2.2) gives the characteristic polynomial

$$\begin{aligned}\lambda^4 &+ (29.196K_1\ell_1 + 0.162K_1\ell_2 - 1.513)\lambda^3 + \\ &+ (-29.196K_1\ell_1 + 0.130K_1\ell_2 + 0.162K_1\ell_3 + 0.513)\lambda^2 + \\ &+ (0.130K_1\ell_3 + 0.162K_1\ell_4)\lambda + 0.130K_1\ell_4 = 0\end{aligned}$$

If the desired characteristic polynomial is

$$F(\lambda) = \lambda^4 + p_1\lambda^3 + p_2\lambda^2 + p_3\lambda + p_4 = 0 \tag{2.3}$$

then by straightforward calculations it is found that the parameters in (2.2) are given by:

$$\begin{cases} K_1 = (1 + p_1 + p_2 + p_3 + p_4)/0.292 \\ K_1\ell_4 = p_4/0.130 \\ K_1\ell_3 = (p_3 - 0.162K_1\ell_4)/0.130 \\ K_1\ell_2 = (p_1 + p_2 + 1 - 0.162K_1\ell_2)/0.292 \\ K_1\ell_1 = (p_1 + 1.513 - 0.162K_1\ell_2)/29.196 \end{cases}$$

Where to place the poles?

One problem is now to determine the desired characteristic polynomial (2.3). This problem arises since the specifications are not given in terms of poles but as specifications on the step response.

Figure 2.2 can be useful in order to get a feeling for the relations between the poles of a continuous and a sampled data system. The figure shows the transformation of curves with constant damping ratio and constant real part when sampling a system with the sampling time T.

The transient of the sampled data system can now be determined by chosing an appropriate damping factor and a radius in the z-plane.

To determine the duration of the transient a rule of thumb can be given. Consider the first order system

$$z(t+T) = \alpha z(t) \qquad z(0) = a$$

The solution is given by

$$z(nT) = a\alpha^n$$

The transient is reduced to 0.1 of its initial value when $\alpha^n = 0.1$ which gives:

$$n = \frac{\ln 0.1}{\ln \alpha} \approx \frac{2}{1 - \alpha} \tag{2.4}$$

Using Figure 2.2 and equation (2.4) it is thus possible to get a good starting point for the choice of the characteristic polynomial. The final choice must, however, be done by using the rules of thumb as well as simulations. This means that the synthesis is highly facilitated if a good simulation package is available.

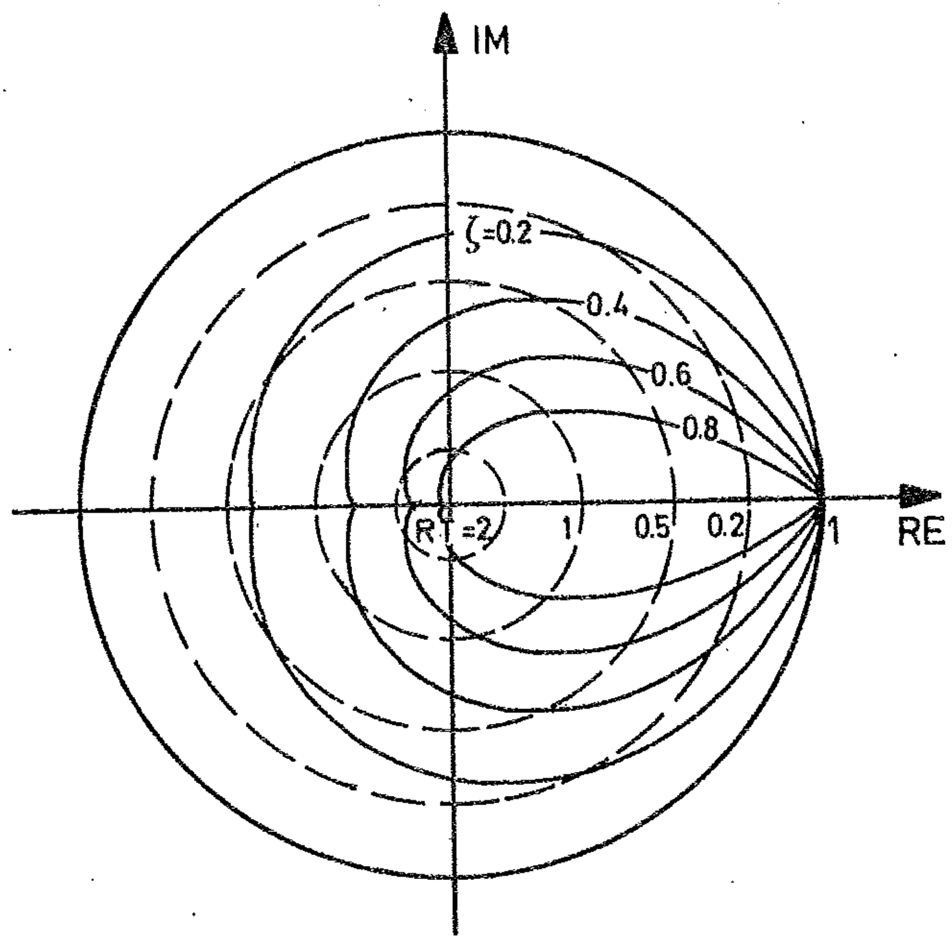


Figure 2.2 - The transformation of curves with constant damping, ξ, and real part, $R = \xi\omega$, when sampling with a sampling time of T seconds.

Simulations.

The specifications given in Section 1 implies that the transient should vanish after 15 - 30 samples. The poles of the characteristic polynomial should thus be placed within the area in Figure 2.2 bounded by $\xi = 0.8$ and a circle with radius less than $\approx 0.8 - 0.9$.

The system has been simulated using a simulations program, SIMNON, available at the Department of Automatic Control [5]. A listing of the system descriptions are given in Appendix. For the state feedback case the simulation is defined by specifying the subsystems:

PROC - continuous system describing the process
STFB - discrete system for the control law (2.2)
CON1 - a connecting system which connects the subsystems into a closed loop system.

Figure 2.3 shows the step responses for three different characteristic polynomials:

$$F_1(q^{-1}) = (1 - 0.75q^{-1})^2(1 - 0.5q^{-1})$$

$$F_2(q^{-1}) = (1 - 1.5q^{-1} + 0.5727q^{-2})(1 - 0.5q^{-1})$$

$$F_3(q^{-1}) = (1 - 1.5q^{-1} + 0.64q^{-2})(1 - 0.5q^{-1})$$

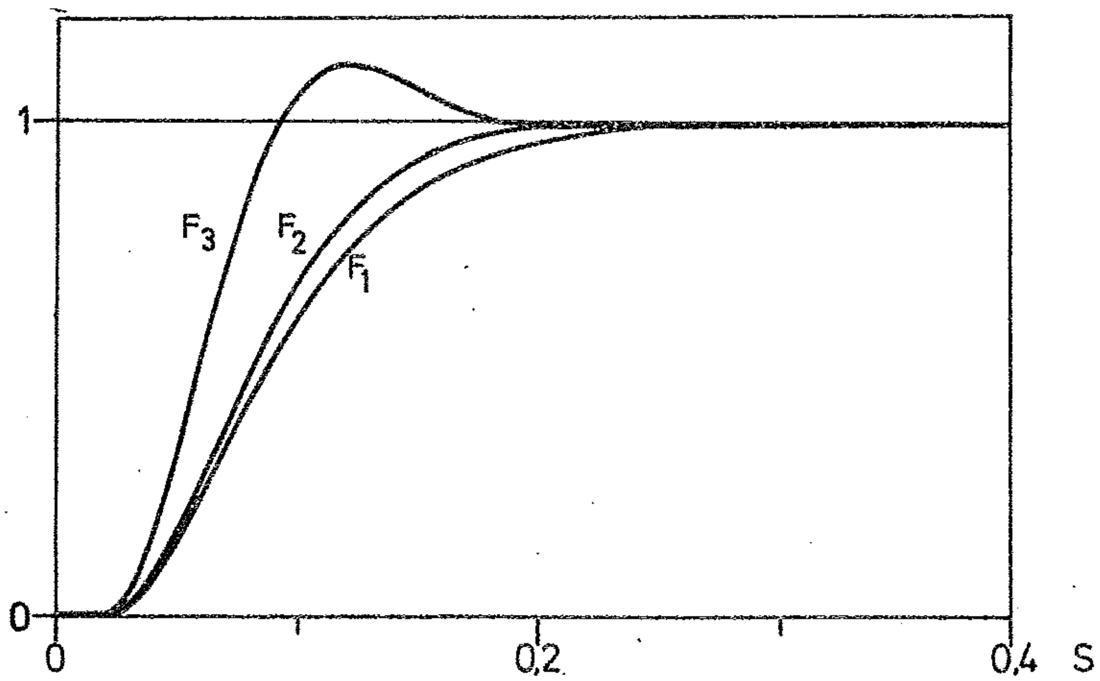


Figure 2.3 - Step responses using state feedback when the desired characteristic polynomial has been chosen to be F_1, F_2 and F_3 respectively.

The real parts of the poles are the same in the three cases but the imaginary parts are varied. F_1 gives a fairly good response but the solution time might be somewhat too long. F_3 gives a too high overshoot, since the imaginary parts of the complex poles are too large. The second polynomial, F_2, gives a good response which satisfies the specifications in Section 1.

Figure 2.4 again shows the step response for the characteristic polynomial F_2 this time together with the control signal. Figure 2.5 shows the transient after an initial value disturbance $x_2(0) = x_3(0) = x_4(0) = 0.25$.

Using the rules of thumb given above together with a good simulation package makes it easy to determine a characteristic polynomial which makes the system fulfil the specifications.

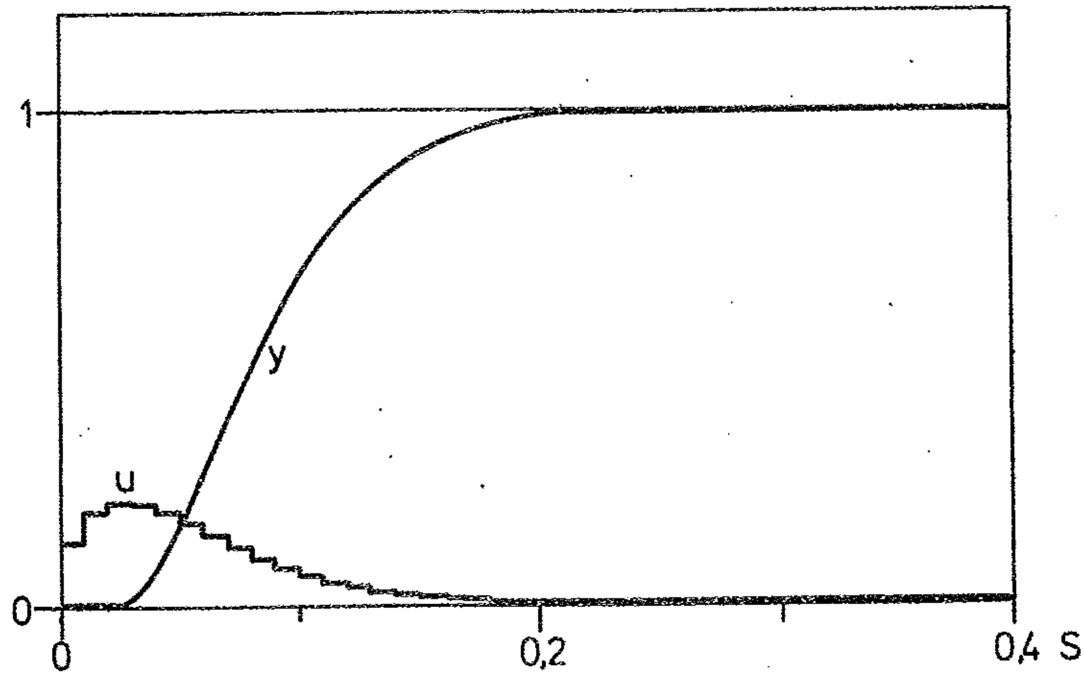


Figure 2.4 - Step response and control signal when state feedback is used to get the characteristic polynomial F_2.

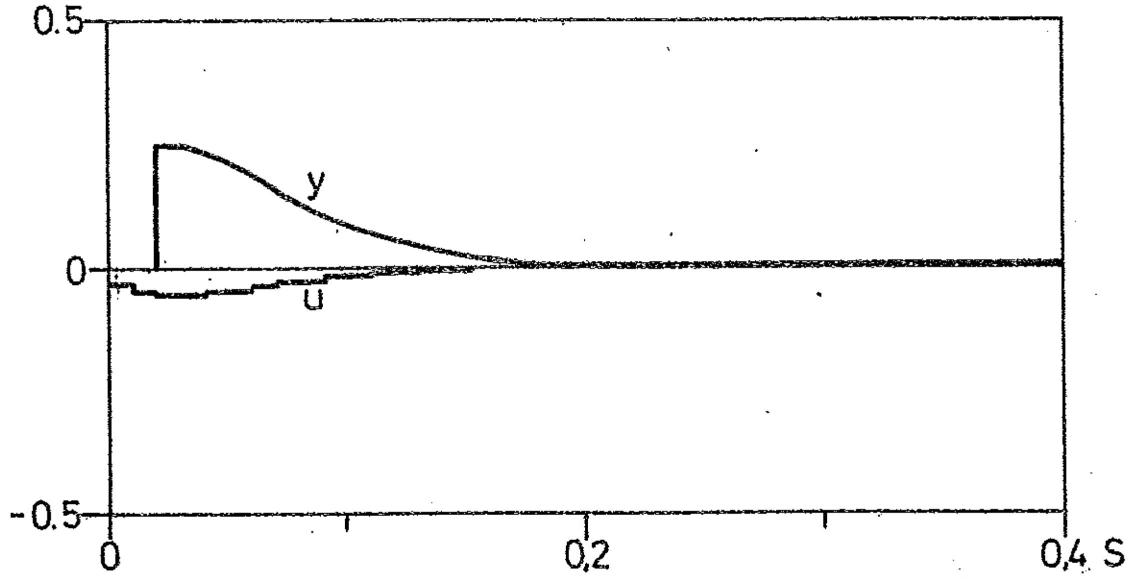


Figure 2.5 - Output and control signal after an initial value disturbance when the characteristic polynomial is F_2.

3. POLE PLACEMENT USING OUTPUT FEEDBACK 1.

This section will show how it is possible to make pole placement by just using the output from the process. This method can be suitable if it is not necessary to make a reconstruction of the state.

Consider the control scheme in Figure 3.1. The closed loop system has the pulse transfer function

$$y(t) = \frac{q^{-k}BR}{AS + q^{-k}BR} y_r(t) \tag{3.1}$$

Assume that it is desired that the closed loop system has the characteristic polynomial $F(q^{-1}) = 0$. The problem of pole placement is now reduced to find the polynomials $R(q^{-1})$ and $S(q^{-1})$ which satisfies the algebraic equation

$$F(q^{-1}) = A(q^{-1})S(q^{-1}) + q^{-k}B(q^{-1})R(q^{-1}) \tag{3.2}$$

This equation always has a solution if R and S have sufficiently high orders and if A and B have no common factor.

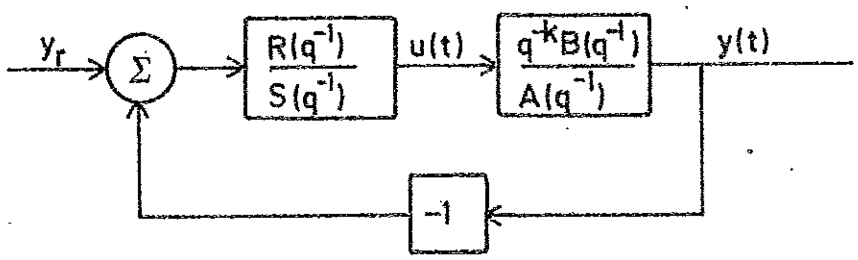


Figure 3.1 - Control scheme using only output feedback.

For the discussed system the order of A and B are two and one respectively and k = 3. If R and S are of order one and three respectively then equation (3.2) will give five equations which can be used to determine the five unknown parameters in R and S.

Assume that

$$A(q^{-1}) = 1 + a_1 q^{-1} + a_2 q^{-2}$$

$$B(q^{-1}) = b_0 + b_1 q^{-1}$$

$$R(q^{-1}) = r_0 + r_1 q^{-1}$$

$$S(q^{-1}) = 1 + s_1 q^{-1} + s_2 q^{-2} + s_3 q^{-3}$$

$$F(q^{-1}) = 1 + f_1 q^{-1} + f_2 q^{-2} + f_3 q^{-3} + f_4 q^{-4} + f_5 q^{-5}$$

The unknown parameters are now obtained from the system of equations.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ a_1 & 1 & 0 & 0 & 0 \\ a_2 & a_1 & 1 & b_0 & 0 \\ 0 & a_2 & a_1 & b_1 & b_0 \\ 0 & 0 & a_2 & 0 & b_1 \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ r_0 \\ r_1 \end{bmatrix} = \begin{bmatrix} f_1 - a_1 \\ f_2 - a_2 \\ f_3 \\ f_4 \\ f_5 \end{bmatrix}$$

The 5×5 matrix on the left hand side is invertable if there is no common factor in the A and B polynomials [3]. The output feedback has been simulated using the subsystem OFB1 and CON2, which are listed in Appendix.

Subsystem OFB1 accepts the coefficients of the desired characteristic polynomial and determines the parameters in the polynomials R and S.

By making a couple of simulations it was found that the same characteristic polynomial as in Section 2 gives the system a satisfactory behaviour, i.e.

$$F(q^{-1}) = 1 - 2q^{-1} + 1.3225q^{-2} - 0.28625q^{-3} \qquad (3.3)$$

The closed loop transfer functions are, however, not the same. One extra zero is introduced in this case due to the R polynomial, see equation (3.1).

Figure 3.2 shows the output and the control signal at a set point change.
Figure 3.3 shows the output and the control signal after an initial value disturbance in x_2.

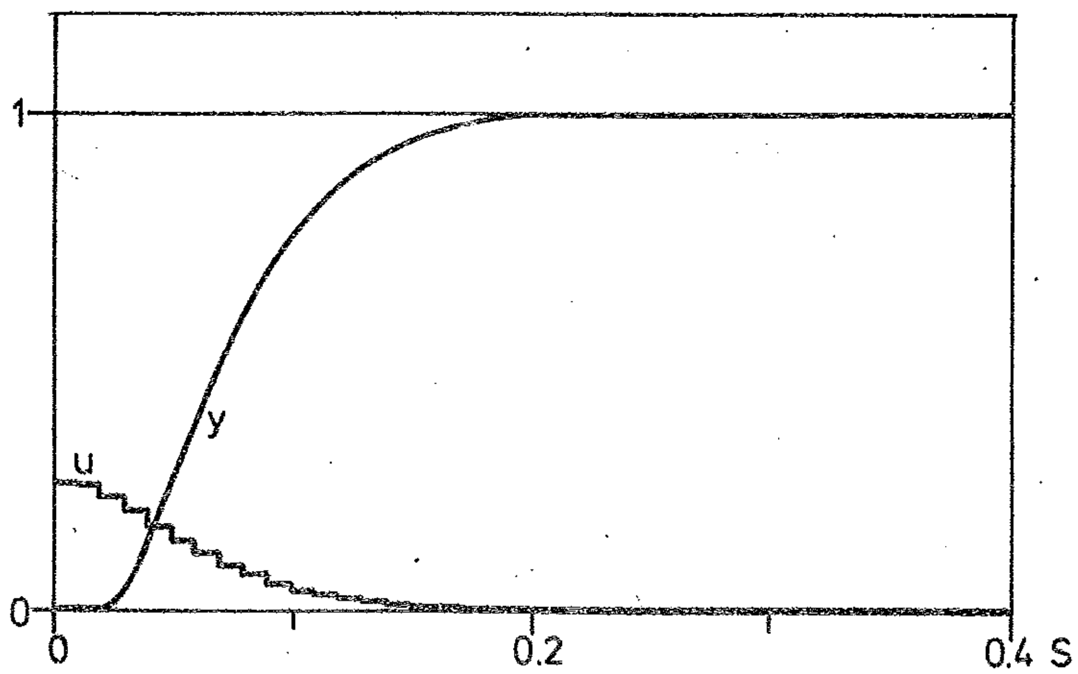


Figure 3.2 - Output and control signal when using output feedback to give the closed loop system the characterized polynomial (3.3).

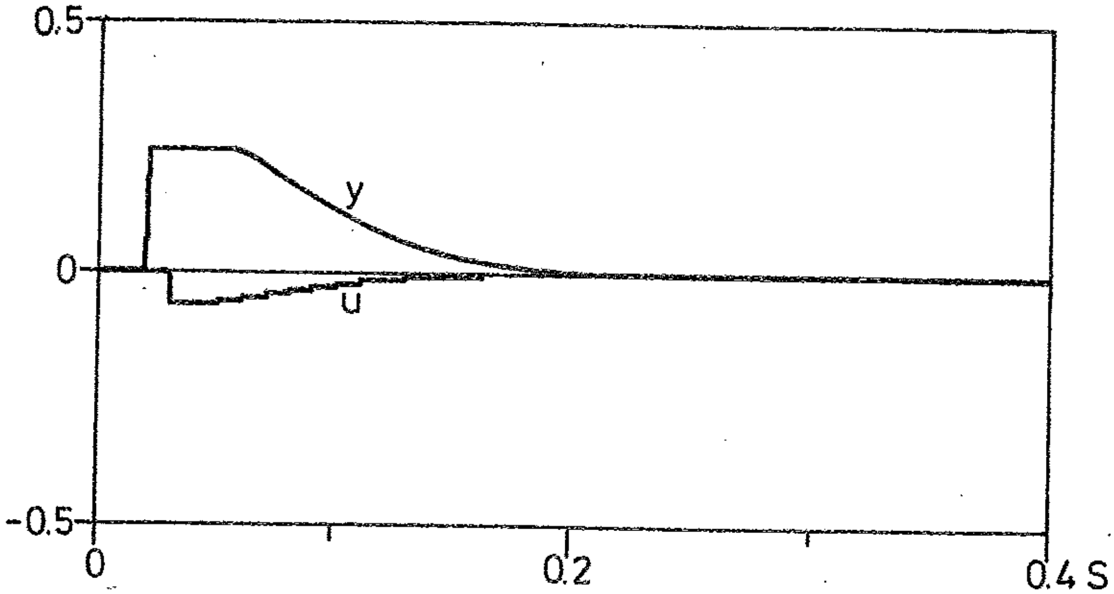


Figure 3.3 - Output and control signal after an initial value disturbance, $x_2(0) = 0.25$ when using output feedback.

4. POLE PLACEMENT USING OUTPUT FEEDBACK 2.

With the design method in the previous section it was possible to get desired locations of the poles of the close loop system. One drawback is, however, that extra zeroes must be introduced, see eq (3.1). These zeroes are depending on the location of the desired poles and on the system and is not in the hands of the designer. In this section a modification will be done which makes it possible to get control of the introduced zeroes. Further this method can be used to separate the servoproblem (the problem of following a reference signal) and the regulator problem (the problem of eliminating a disturbance).

Consider the control scheme in Figure 4.1. P, R, S and C are polynomials in the backward shift operator, q^{-1}. A disturbance, e(t), which can be stochastic or deterministic, is also introduced.

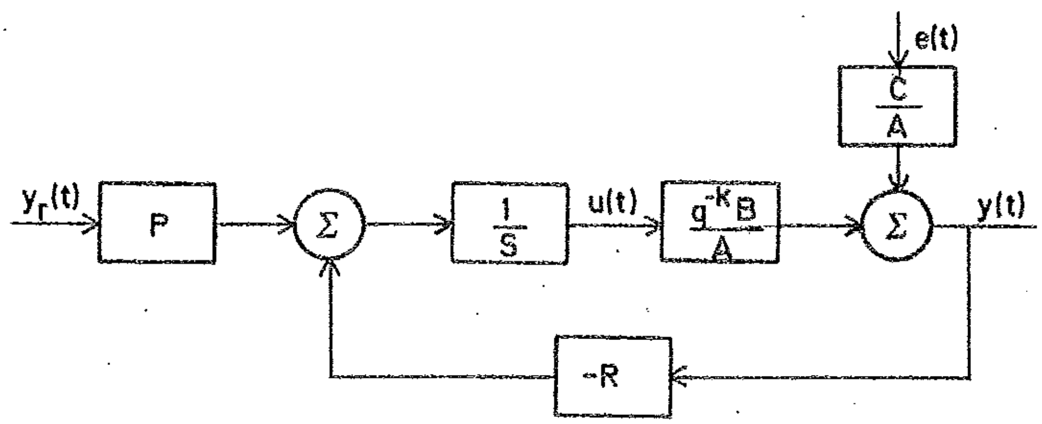


Figure 4.1 - Control scheme for the design method based on output feedback.

The control law is thus

$$u(t) = \left(Py_r(t) - Ry(t)\right)/S$$

The closed loop system is given by

$$y(t) = \frac{q^{-k}PB}{AS + q^{-k}BR} y_r(t) + \frac{CS}{AS + q^{-k}BR} e(t) \qquad (4.1)$$

Compare eq. (3.1). As in the previous section it is possible to get a desired characteristic polynomial, F, by solving the algebraic equation

$$F = AS + q^{-k}BR$$

In contrast to the method in Section 3 it is now possible to arbitrarily specify where the introduced zeroes should be located. For instance it might not be desired to introduce any extra zeroes.

In order to get a correct steady state value the following equality must be satisfied

$$P(1)B(1) = A(1)S(1) + B(1)R(1)$$

If the open loop system contains an integrator then $A(1)S(1) = 0$ and a correct steady state value will be obtained if

$$P(1) = R(1)$$

When using the design method scetched in this section it is thus possible to obtain the same transfer function from y_r to y as when state feedback is used. The order of the closed loop system is, however, increased by the order of the S-polynomial. This design method uses only the output which means that it can be used instead of state feedback when all states are not measurable. It can in fact be shown that the structure in Figure 4.1 is equivalent to using a minimum order observer combined with a state feedback.

The Servo-Problem.

If only the servo-problem is considered, i.e. $e(t) = 0$, then the system (4.1) can be given almost an arbitrarily pulse transfer function. It is only the time delay and the zeroes originating from the open loop system that cannot be influenced. However, if the system is minimum phase then F can be chosen in such a way that it cancels the undesired zeroes from the B-polynomial. Extra zeroes can then be introduced by using the P-polynomial. It is, as in the continuous time case, more difficult to determine the effect of a zero than that of a pole. Figure 4.2 can be of some help to determine if a zero will give a lead or lag effect.

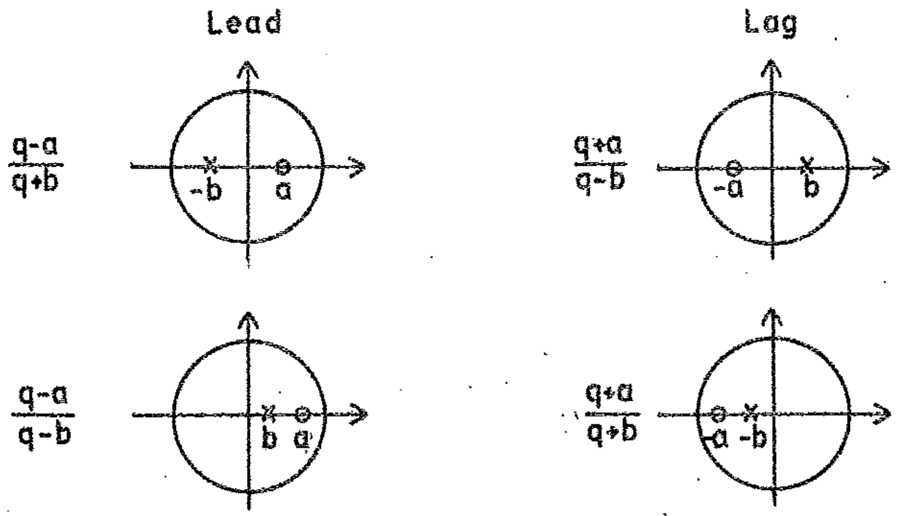


Figure 4.2 - Pole-zero constellations for first order lead and lag networks, from Tou [6].

The Combined Servo and Regulator Problem.

Using the control scheme in Figure 4.1 it is possible to separate the servo and the regulator problems. The regulator problem can first be solved by choosing the R and S polynomials. The P-polynomial can then be used to give the system a desired response at set point changes.

If e(t) is white noise it can be desired to use a minimum variance regulator [2]. In that case

$$S = BF$$
$$R = G$$

where the F-and G-polynomials are determined from the identity

$$C = AF + q^{-k}G$$

The closed loop system will then be

$$y(t) = \frac{q^{-k}P}{C} y_r(t) + Fe(t)$$

When e(t) is a stochastic process it can be assumed that the C-polynomial has all its zeroes inside the unit circle [2]. The P-polynomial can now be used to give the system a good response from set point changes. If P is chosen as

$$P(q^{-1}) = C(q^{-1})P'(q^{-1})$$

then the system will have a finite memory for reference value changes. In some cases it can be advantageous to replace P by $P(q^{-1})/Q(q^{-1})$ in order to get a smoother step response. This will be further discussed in Section 5.

Simulations.

The regulator structure in Figure 4.1 is simulated using the SIMNON system OFB2 and the total system is defined by CON3, see Appendix. By choosing the same characteristic polynomial as in Section 2, i.e.

$$F(q^{-1}) = 1 - 2q^{-1} + 1.3225q^{-2} - 0.28625q^{-3}$$

it is verified that the same transfer function from y_r to y can be obtained, see Figure 4.3. P is chosen to be a constant equal to R(1) in order to get a correct steady state value. The response for initial value disturbances is not the same as can be seen in Figure 4.4. The response is, however, satisfactory.

In order to illustrate the influence of the polynomial P a simulation has been done when the characteristic polynomial is

$$F(q^{-1}) = 1 - 0.8q^{-1}$$

The result is shown in Figure 4.5 when two different P polynomials are used

$$P_1(q^{-1}) = 0.681 \tag{4.2}$$

$$P_2(q^{-1}) = 0.081 + 0.15q^{-1} + 0.20q^{-2} + 0.25q^{-3} \tag{4.3}$$

In order to get the correct steady state $P_1(1) = P_2(1) = R(1) = 0.681$. The step response is more smooth when P_2 is used compared to the case when P_1 is used. By using a P-polynomial of higher order or by replacing P by $P(q^{-1})/Q(q^{-1})$ it is possible to get a smooth response at reference value changes and still having a system which quickly eliminates disturbances.

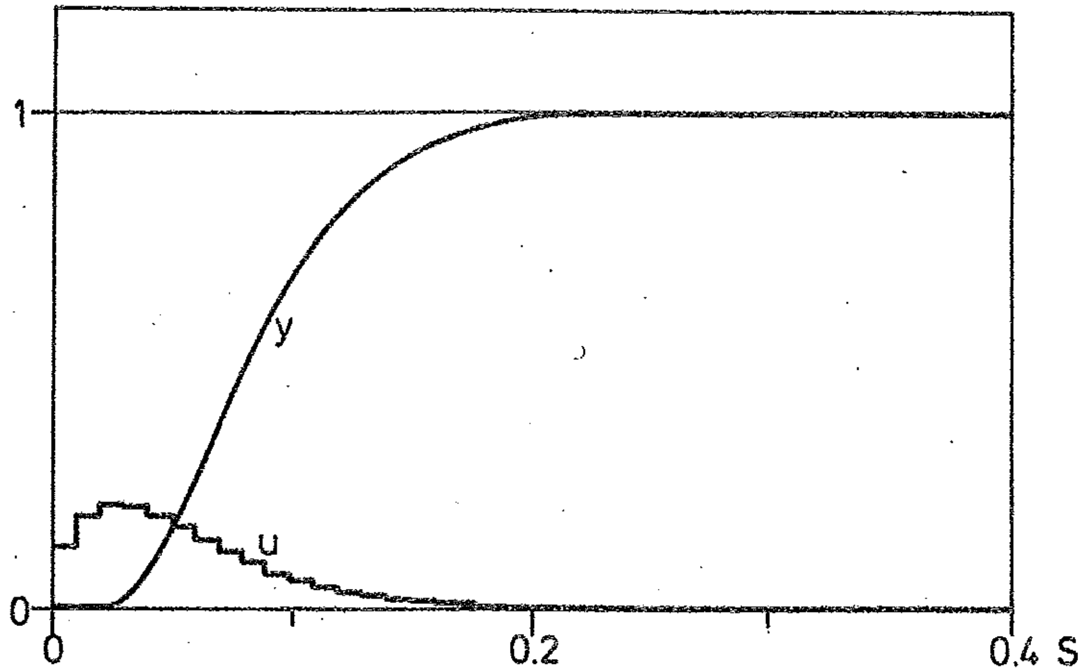


Figure 4.3 - Step response when the control scheme in Figure 4.1 is used.

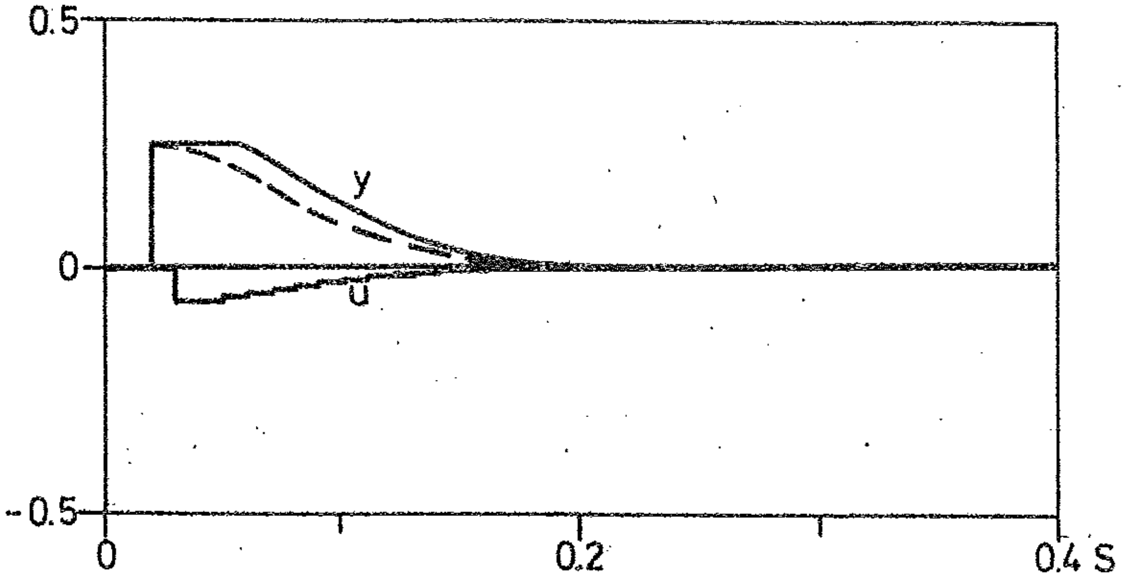


Figure 4.4 - The transient after an initial value disturbance when the control scheme in Figure 4.1 is used. The dashed line is the output when the state feedback law (2.2) is used, compare Figure 2.5.

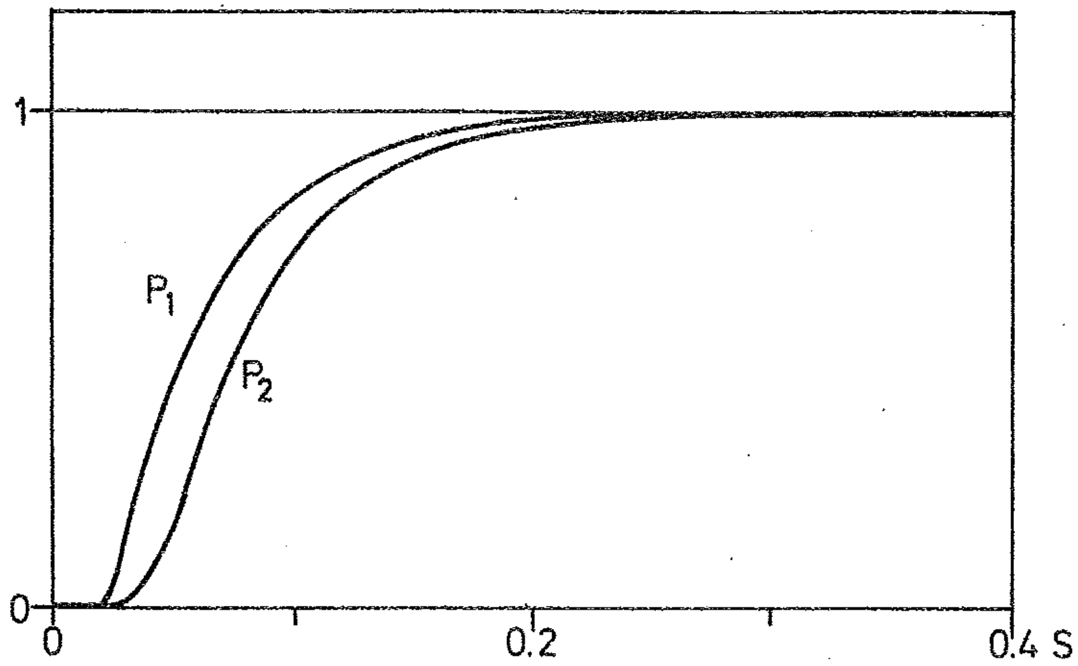


Figure 4.5 - Step responses when the control law (4.1) is used for two different P-polynomials, (4.2) and (4.3), when $F = 1 - 0.8q^{-1}$.

5. COMBINED FEEDBACK AND FEEDFORWARD CONTROL.

There are several ways to solve the combined servo and regulator problem. One way was shown in Section 4, in this section an other way will be discussed.

Consider the control scheme in Figure 5.1.

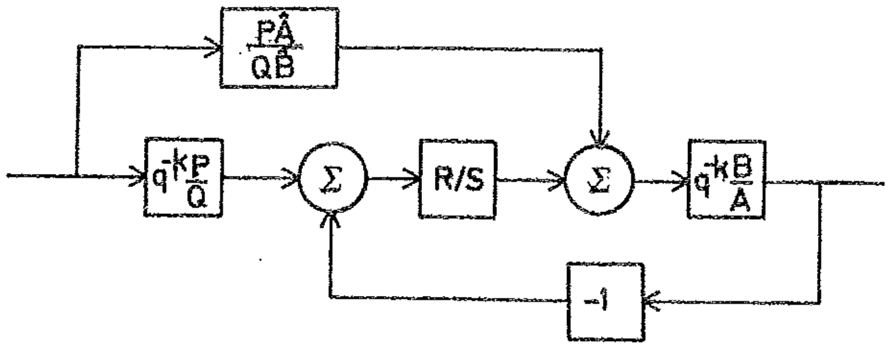


Figure 5.1 - A general feedback and feedforward scheme.

For this structure the design problem is separated into two parts:

- o Feedforward from the reference signal. The block $P\hat{A}/G\hat{B}$ will take care of reference value changes.
- o A feedback regulator R/S which takes care of the disturbances acting on the process.

The pulse transfer function from y_r to y is given by

$$y(t) = \frac{q^{-k}P}{Q} \cdot \frac{\left(\hat{A}S\,\frac{B}{\hat{B}} + q^{-k}BR\right)}{AS + q^{-k}BR}\, y_r(t)$$

If $\hat{A} = A$ and $\hat{B} = B$ then

$$y(t) = \frac{q^{-k}P}{Q} y_r(t)$$

The model $M = q^{-k}P/Q$ thus gives the desired behaviour at set point changes and the regulator R/S can be designed in order to eliminate disturbances and also to take care of errors due to incomplete knowledge of the process, i.e. if $\hat{A} \neq A$ and $\hat{B} \neq B$. The feedforward block $P\hat{A}/Q\hat{B}$ can be interpreted as a combination of a model and an estimated inverse of the process.

Simulations.

The model M was first determined. Models of different orders and with different locations of the poles were used. Figure 5.2 shows the step responses for the models.

$$M_1(q^{-1}) = \frac{0.25}{1 - 0.75q^{-1}}$$

$$M_2(q^{-1}) = \frac{0.0725}{1 - 1.5q^{-1} + 0.5725q^{-2}}$$

$$M_3(q^{-1}) = \frac{0.03625}{1 - 2q^{-1} + 1.3225q^{-2} - 0.28625q^{-3}}$$

When using the model M_1 the control signals were large and the output had a tendency to ripple between the sampling points. By increasing the order of the model it was possible to get a good step response. Also when the order of the model was increased the amplitude of the control signals was decreased.

The regulator R/S can be determined in different ways. One special type of regulators is the so-called dead-beat regulators. These will bring the output back to zero in as few steps as possible after a disturbance. For the discussed process the dead-

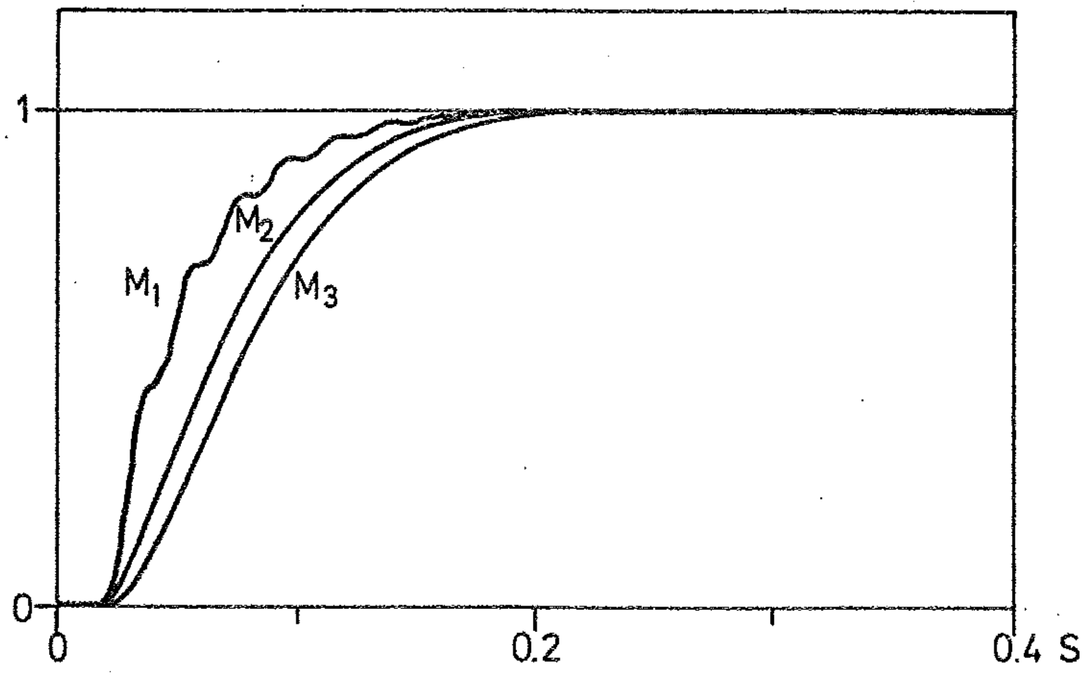


Figure 5.2 - The step response for three different models M_1, M_2 and M_3 when the control scheme in Figure 5.1 was used.

beat regulator is

$$\frac{R}{S} = \frac{11.804 - 5.625q^{-1}}{1 + 2.3q^{-1} + 2.988q^{-2} + 1.423q^{-3}} \tag{5.1}$$

Figure 5.3 shows the output when the dead-beat regulator is used to eliminate an initial value disturbance.

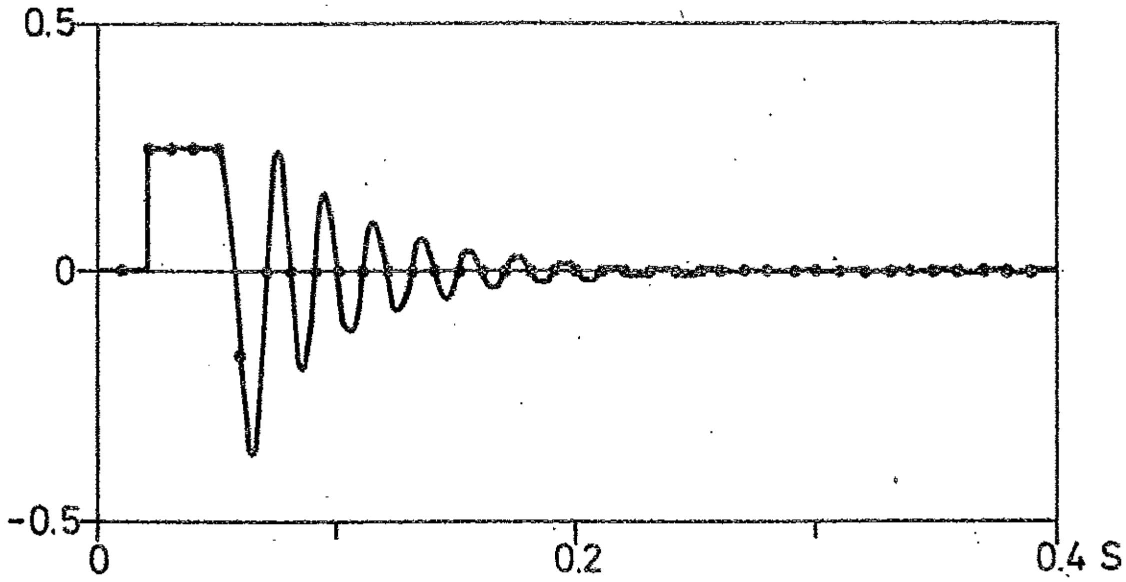


Figure 5.3 - The output after an initial value disturbance, $x_2(0) = 0.25$, when using the dead-beat regulator (5.1). The sampling times are indicated by points.

At the sampling points the output will be equal to zero after just a few steps, but between the sampling points the output oscillates. This depends on the severe specification that the output at the sampling time should be equal to zero as quickly as possible. To meet this specification the input signal has to be large and this starts up the oscillation.

Another way to determine the regulator is to use a lead network. One possible net is obtained by eliminating one zero and one pole in the process i.e. to use

$$\frac{R}{S} = K \cdot \frac{1 - 0.513q^{-1}}{1 + 0.801q^{-2}} \qquad (5.2)$$

Using Figure 4.2 it can be seen that this regulator has a lead effect. Figure 5.4 shows the output after an initial value disturbance for different values of K in (5.2).

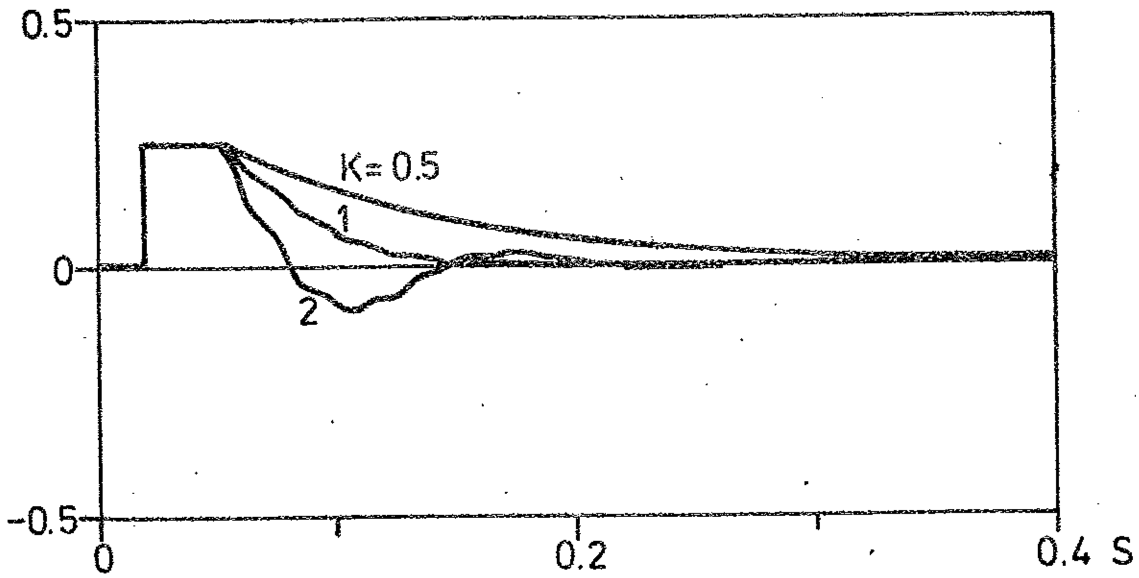


Figure 5.4 - Output signal after an initial value disturbance when using the lead network (5.2) for different values of K.

6. SUMMARY

The purpose with this report has been to illustrate some different design methods for sampled data systems. The design methods have been based on pole placement. In Section 2 rules of thumb are given which can be useful when determining the desired locations of the poles.

The use of the discussed design methods are highly facilitated if a good simulation package is available. All the simulations in this report are done using SIMNON [5].

For the discussed example it has been easy to find regulators which make it possible to fulfil the given specifications. Also by changing the numerical values of the parameters in the process (K, T_1 and τ) it was found that the discussed design schemes, at least for this example, was insensitive to parameter changes.

There are also other design methods for sampled data systems, see e.g. [1] and [6]. One important method is linear quadratic control theory (LQC). Using LQC it is possible to introduce a weighting between the allowed magnitude of the control signals and the state variables. Instead of specifying the poles of the closed loop system a loss function has to be specified. In most cases the design of regulators using LQC is also iterative in the same way as the design methods discussed in this report since different loss functions have to be tried.

7. REFERENCES

[1] Ackerman, J.: Abtastregelung, Springer Verlag, 1972.

[2] Åström, K.J.: Introduction to Stochastic Control Theory, Academic Press, 1970.

[3] Dickson, L.E.: First Course in the Theory of Equations, Wiley, 1922.

[4] Eliasson, B.: Dimensionering av digital regulator för reglering av utloppsarean till turbojetmotor, Master Thesis at the Department of Automatic Control, to appear (in Swedish).

[5] Elmquist, H.: SIMNON, User's Manual, Report 7502, Department of Automatic Control, 1975.

[6] Tou, J.T.: Digital and Sampled-Data Control Systems, McGraw-Hill, 1959.

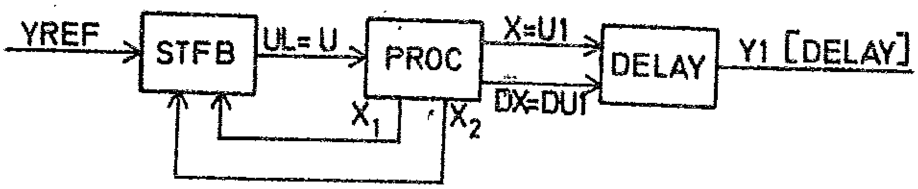
APPENDIX

The Appendix contains SIMNON listings of the systems used for the simulations. Also figures are given which specifies the notations used in the connecting systems.

The subsystem DELAY is a standard system in SIMNON for simulating time delays. The process is defined by the subsystem PROC.

```
CONTINUOUS SYSTEM PROC
STATE X1 X2
DER DX1 DX2
INPUT U
OUTPUT X DX

INITIAL
K:60
T1:0.015
XLIM:11.8

OUTPUT
X=X2
DX=X1

DYNAMICS
DX1=(-X1+K*U)/T1
DX2=IF ABS(X1)<XLIM THEN X1 ELSE SIGN(X1)*XLIM

END
```

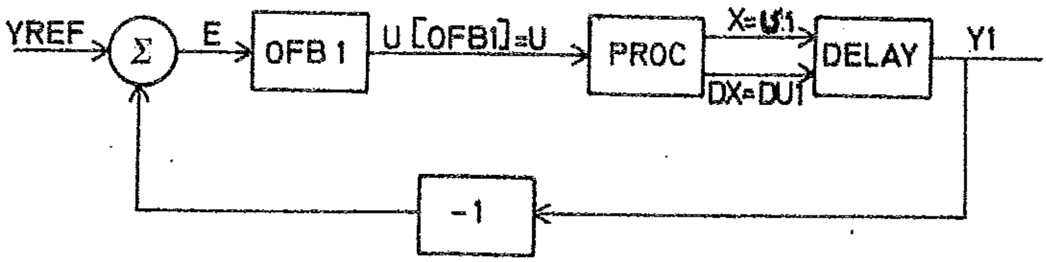
STATE FEEDBACK



Note: The states X3 and X4 are computed
in STFB

```
DISCRETE SYSTEM STFB
STATE X3 X4
NEW NX3 NX4
INPUT X1 X2
OUTPUT UL
TIME T
TSAMP TS

INITIAL
YREF:1
P1:0
P2:0
P3:0
P4:0
K1=(1+P1+P2+P3+P4)/0.292
L4=P4/0.130
L3=(P3-0.162*L4)/0.130
L2=(P1+P2+1-0.162*L3)/0.292
L1=(P1+1.513-0.162*L2)/29.196
DT:0.01

OUTPUT
UL=K1*YREF-L1*X1-L2*X2-L3*X3-L4*X4
TS=T+DT

DYNAMICS
NX3=X2
NX4=X3

END

- - - - - - - - - - - - - - - - - - - -

CONNECTING SYSTEM CON1
TIME T
TD1[DELAY]=T-TAU
TAU:0.02
X1[STFB]=X1[PROC]
X2[STFB]=X2[PROC]
U[PROC]=UL[STFB]
U1[DELAY]=X[PROC]
DU1[DELAY]=DX[PROC]
END
```

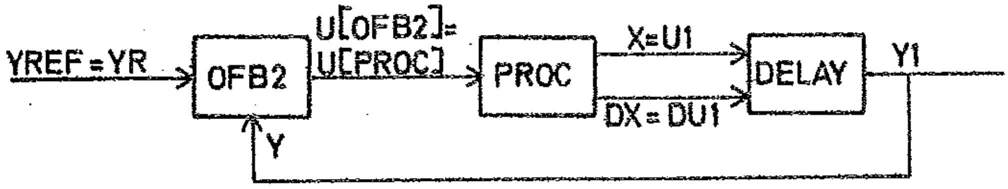
OUTPUT FEEDBACK 1




```
DISCRETE SYSTEM OFB1
STATE UM UMM UMMM EM
NEW NUM NUMM NUMMM NEM
INPUT E
OUTPUT U
TIME T
TSAMP TS

INITIAL
F1:0
F2:0
F3:0
F4:0
F5:0
BM0:0.162
BM1:0.1298
AM1:-1.513
AM2:0.513
S1=F1-AM1
S2=F2-AM2-AM1*S1
K1=BM0-AM1*BM1/AM2+BM1*BM1/BM0/AM2
SL=F4+F5*(BM1/BM0/AM2-AM1/AM2)-AM2*S2
K2=SL-F3*BM1/BM0+BM1/BM0*(AM1*S2+AM2*S1)
R1=K2/K1
R0=(F3-F5/AM2-AM1*S2-AM2*S1+BM1*R1/AM2)/BM0
S3=(F5-BM1*R1)/AM2
DT:0.01

OUTPUT
U=R0*E+R1*FM-S1*UM-S2*UMM-S3*UMMM
TS=T+DT

DYNAMICS
NUMMM=UMM
NUMM=UM
NUM=U
NEM=E

END
```

```
CONNECTING SYSTEM CON2
TIME T
TD1[DELAY]=T-TAU
TAU:0.02
YREF:1
E[OFB1]=YREF-Y1[DELAY]
U[PROC]=U[OFB1]
U1[DELAY]=X[PROC]
DU1[DELAY]=DX[PROC]
END
```

OUTPUT FEEDBACK 2



Note: If the swich ISW > 0.5 then the user has to specify the polynomials P, R and S. If ISW ≤ 0.5 then the R and S polynomials are determined in the program such that the characteristic polynomial of the closed loop system is F and such that P(1)=R(1).

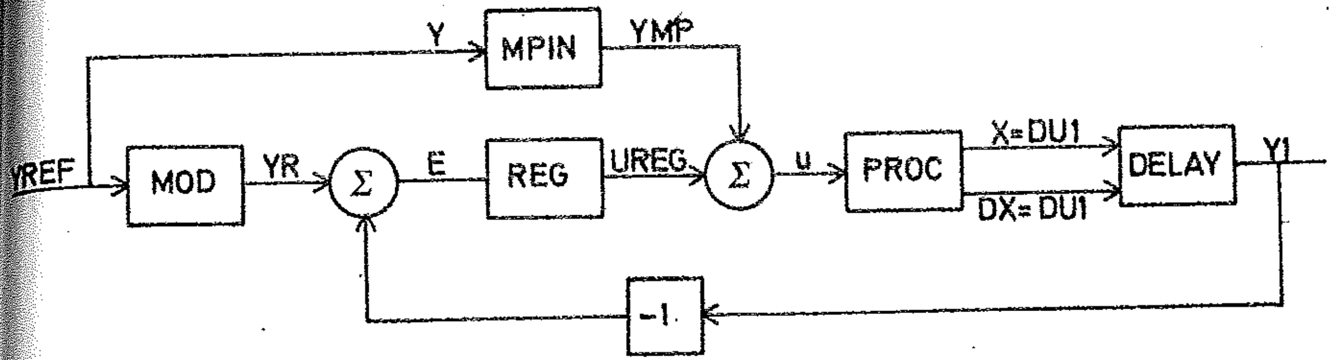
```
DISCRETE SYSTEM OFB2
STATE UM1 UM2 UM3 YRM1 YRM2 YRM3 YM1 YM2
NEW NUM1 NUM2 NUM3 NYRM1 NYRM2 NYRM3 NYM1 NYM2
INPUT YR Y
OUTPUT U
TIME T
TSAMP TS

INITIAL
S1:0
S2:0
S3:0
R1:0
R2:0
R3:0
P1:0
P2:0
P3:0
P4:0
```

```
F1:0
F2:0
F3:0
F4:0
F5:0
BM0:0.162
BM1:0.1298
AM1:-1.513
AM2:0.513
D1=F1-AM1
D2=F2-AM2-AM1*D1
K1=BM0-AM1*BM1/AM2+BM1*BM1/BM0/AM2
SS1=F4+F5*(BM1/BM0/AM2-AM1/AM2)-AM2*D2
K2=SS1-F3*BM1/BM0+BM1/BM0*(AM1*D2+AM2*D1)
C1=K2/K1
C0=(F3-F5/AM2-AM1*D2-AM2*D1+BM1*C1/AM2)/BM0
D3=(F5-BM1*C1)/AM2
DT:0.01
ISW:0
SR1= IF ISW>0.5 THEN S1 ELSE D1
SR2= IF ISW>0.5 THEN S2 ELSE D2
SR3= IF ISW>0.5 THEN S3 ELSE D3
RR1= IF ISW>0.5 THEN R1 ELSE C0
RR2= IF ISW>0.5 THEN R2 ELSE C1
RR3= IF ISW>0.5 THEN R3 ELSE 0

OUTPUT
U1=P1*YR+P2*YRM1+P3*YRM2+P4*YRM3
U2=-RR1*Y-RR2*YM1-RR3*YM2
U3=-SR1*UM1-SR2*UM2-SR3*UM3
U4=(RR1+RR2+RR3)*YR
U= IF ISW>0.5 THEN U1+U2+U3 ELSE U4+U2+U3
TS=T+DT

DYNAMICS
NUM1=U
NUM2=UM1
NUM3=UM2
NYRM1=YR
NYRM2=YRM1
NYRM3=YRM2
NYM1=Y
NYM2=YM1

END
```

- -

```
CONNECTING SYSTEM CON3
TIME T
TD1[DELAY]=T-TAU
TAU:0.02
YREF:1
YR[OFB2]=YREF
Y[OFB2]=Y1[DELAY]
U[PROC]=U[OFB2]
U1[DELAY]=X[PROC]
DU1[DELAY]=DX[PROC]
END
```

COMBINED FEEDBACK AND FEEDFORWARD




```
DISCRETE SYSTEM MOD
STATE YM1 YM2 YM3 YRM1 YRM2 YRM3
NEW NYM1 NYM2 NYM3 NYRM1 NYRM2 NYRM3

INPUT Y
OUTPUT YR
TIME T
TSAMP TS

INITIAL
AM1:-1.5
AM2:0.7
AM3:0
DT:0.01

OUTPUT
YR=(1+AM1+AM2+AM3)*YM3-AM1*YRM1-AM2*YRM2-AM3*YRM3
TS=T+DT

DYNAMICS
NYM3=YM2
NYM2=YM1
NYM1=Y
NYRM3=YRM2
NYRM2=YRM1
NYRM1=YR

END
- - - - - - - - - - - - - - - - - - - - - - - - - - -

DISCRETE SYSTEM MPIN
STATE MPM1 MPM2 YM1 YM2 MPM3 MPM4
NEW NMPM1 NMPM2 NYM1 NYM2 NMPM3 NMPM4
INPUT Y
OUTPUT YMP
TIME T
TSAMP TS
```

```
INITIAL
AM1:-1.5
AM2:0.7
AM3:0
B0=(1+AM1+AM2+AM3)/0.162
B1=-1.513
B2=0.513
A1=AM1+0.801
A2=AM1*0.801+AM2
A3=AM2*0.801+AM3
A4=0.801*AM3
DT:0.01

OUTPUT
YMP=B0*(Y+B1*YM1+B2*YM2)-A1*MPM1-A2*MPM2-A3*MPM3-A4*MPM4
TS=T+DT

DYNAMICS
NMPM4=MPM3
NMPM3=MPM2
NMPM2=MPM1
NMPM1=YMP
NYM2=YM1
NYM1=Y

END
```

- -

```
DISCRETE SYSTEM REG
STATE UM1 UM2 UM3 EM1
NEW NUM1 NUM2 NUM3 NEM1
INPUT E
OUTPUT UREG
TIME T
TSAMP TS

INITIAL
B1:11.8037
B2:-5.6247
A1:2.314
A2:2.9881
A3:1.4227
DT:0.01

OUTPUT
UREG=B1*E+B2*EM1-A1*UM1-A2*UM2-A3*UM3
TS=T+DT

DYNAMICS
NUM3=UM2
NUM2=UM1
NUM1=UREG
NEM1=E

END
```

```
CONNECTING SYSTEM CON4
TIME T
TD1[DELAY]=T-TAU
TAU:0.02
YREF:1
Y[MOD]=YREF
Y[MPIN]=YREF
E[REG]=YR[MOD]-Y1[DELAY]
U[PROC]=UREG[REG]+YMP[MPIN]
U1[DELAY]=X[PROC]
DU1[DELAY]=DX[PROC]
END
```