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A DYNAMIC MODEL FOR DO-BOD RELATIONSHIPS IN A NON-TIDAL STREAM.

M.B. Beck † and P.C. Young *

ABSTRACT.

Dynamic models for dissolved oxygen (DO)-biochemical oxygen demand (BOD) interaction in a reach of river are not easily verified owing to the difficulties in obtaining data. In this paper a simple lumped-parameter model is presented and verified by deterministic simulation against field measurements collected over an eighty-day period from the River Cam in eastern England. An important feature of the model is a pseudo-empirical term introduced to predict explicitly the significant effects of algal growth and decay; this is achieved by manipulating data on the hours of sunlight incident on the system each day in a low-pass filter manner.

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NOTATION.

C_s = saturation concentration of DO in the reach (mg/l)

D_B = net rate of addition of DO to the reach by photosynthetic/respiratory activity of plants and algae and the decomposition of mud deposits ((mg/l)/day).

$I(t_k)$ = sustained sunlight effect at day t_k.

$\bar{I}$ = a threshold level for the sustained sunlight effect.

K_1 = reaeration rate constant for DO (day^{-1})

K_2 = BOD decay rate constant (day^{-1})

K_3 = rate constant for the sedimentation of BOD (day^{-1})

K_4 = coefficient for the sunlight effect in the DO equation

K_5 = coefficient for the sunlight effect in the BOD equation

L_A = rate of addition of BOD to the reach by local surface runoff ((mg/l)/day)

Q = volumetric flow-rate in the reach (cuft/day)

t = independent variable of time (days)

T_d = transportation delay (days)

u_1 = concentration of DO at the upstream end (input) of the reach (mg/l)

u_2 = concentration of BOD at the upstream end (input) of the reach (mg/l)

$u_3(t_k)$ = sunlight incident on the system during day t_k (hrs/day)

u_1' = hypothetically delayed DO input to the CSTR component (mg/l)

u_2' = hypothetically delayed BOD input to the CSTR component (mg/l)

V = mean volumetric hold-up in the reach (cuft)

x_1 = concentration of DO at the downstream end (output) of the reach (mg/l)

x_2 = concentration of BOD at the downstream end (output) of the reach (mg/l)

θ = river water temperature (^{o}C)

$\bar{\theta}$ = a mean river water temperature (^{o}C)

τ = a time constant for the discrete low-pass filter of the sunlight effect (days)

1. INTRODUCTION.

Although the interaction between dissolved oxygen (DO) and biochemical oxygen demand (BOD) has received much attention since the classical study of STREETER and PHELPS (1925), few investigators have used their models in a truly dynamic context or verified them against field data. This is understandable since the intensity and duration of sampling and monitoring required is often prohibitive, especially in the case of 5-day BOD tests. However, for any forecasting, parameter estimation, or control applications two factors are of considerable importance. Firstly, a river system is unlikely to be in a steady state since this implies that the BOD and DO inputs and outputs, as well as the volumetric flow-rate, are time-invariant; therefore a dynamic model should be assumed. Secondly, it is not constructive to use the model unless it has been shown to be a reasonable description of the true nature of the system.

The results described in this paper are part of a systems analysis project aimed at the application of system identification and parameter estimation methods for the solution of certain problems of river pollution (BECK (1973)). A review of the current literature indicated the initial form of a DO-BOD model which might usefully be tested against observations from the field. With this in mind a 4.7 km stretch of the River Cam in eastern England was chosen and data on the relevant variables were collected over an eighty-day period during the summer of 1972. The inadequacy of the a priori model is demonstrated under conditions of prolonged sunny, dry weather, during which significant populations of algae are established in the river; with the onset of dull weather the DO level is severely depressed due partly to the cessation of photosynthetic activity and partly to an increased BOD caused by mass algal death. It is important, therefore, to describe explicitly the effects of algae on the DO and BOD and this is accomplished by a pseudo-empirical relationship which low-pass filters the data on the hours of sunlight incident on the system during each day.

We shall only consider in detail the use of deterministic computer simulation comparisons against field data for the verification of the proposed model. However, within the stochastic setting of the problem a more sophisticated statistical validation of the model has been completed (BECK and YOUNG) using a technique known as the extended Kalman filter (see e.g. JAZWINSKI (1970)). Models of this form have also been verified against field data collected for the Bedford-Ouse study* (WHITEHEAD and YOUNG (1974)) and employed in an initial feasibility analysis of operational DO level control in a reach of river (YOUNG and BECK (1974)). Recent work has attempted to replace the original pseudo-empirical expression for algae by a more ambitious combined DO-BOD-algae model which takes account of both living and dead algal populations; a MONOD (1942) function is hypothesised for the specific growth rate of algae (BECK (1974a), (1974b)).

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* A study whose objectives are to examine the present utilisation and future potential of the Bedford-Ouse as a source of potable supply and recreation; part of the work is concerned with the development of a stochastic-dynamic model of a 55 km stretch of the Bedford-Ouse between the proposed new city of Milton Keynes and Clapham.

2. THE EXPERIMENTAL STUDY.

Within the general area of water quality modelling, which includes wastewater treatment, river quality and potable abstraction facilities, field data is at a premium. And if the available manpower is limited only small portions of a river system can be investigated in depth.

The reach of river chosen for this study is a 4.7 km stretch of the River Cam (a non-tidal river) near Cambridge on the edge of the Fenland district of eastern England. Figure 1 shows the major physical characteristics of this part of the River Cam. From the point of view of obtaining measurements which give a reasonably clear picture of the dynamics for DO-BOD interaction the defined system has several advantages:

(i) The input of the sewage works effluent ensures that the system is suitably excited (i.e. variations in BOD and DO can be observed which are not small enough to be attributable to measurement error or chance disturbance of the system).

(ii) The critical conditions of DO sag often occur in reaches of rivers immediately downstream of effluent outfalls.

(iii) The weir below the effluent outfall aids the assumption of complete mixing of the effluent with the stream as it enters the defined system.

(iv) The short reach between the upper weir and the upper system boundary is a precaution against obscuring the measurements of DO by entrained bubbles and other localised fluctuations resulting from the action of the weir.

(v) The geography of the surrounding land is such that there are no major tributaries or pumped land drainage inputs along the reach*.

The duration of the experiment covers the period between June 6th and August 25th, 1972, which enabled the collection of an uninterrupted record of samples for eighty days when low-flow conditions are predominant in the river. 5-day BOD samples were taken daily from the upstream and downstream boundaries of the system; with the assistance of the Water Pollution Research Laboratory and the Great Ouse River Authority (now part of the Anglian Water Authority) continuous records of DO were received from a portable monitor and a permanent station located at these two points. Volumetric flow-rate measurements are taken from the weir at Bottisham and river water temperature was observed at both Bottisham and Bait's Bite; other available data include daily samples of the effluent flow-rate and BOD content and some important meteorological information, viz. the hours of sunlight incident on the system and local rainfall during each day.

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* This is particularly relevant in the sense that BOD addition to the reach by local runoff from the surrounding land is negligible.

3. AN INITIAL DYNAMIC MODEL FOR DO-BOD INTERACTION.

A review of the recent literature on DO-BOD modelling (BECK (1973)) shows that the major contributions (see e.g. DOBBINS (1964), CAMP (1965), HANSEL and FRANKEL (1965)) have developed from the basic model of STREETER and PHELPS (1925). Their descriptions of the various biological and physico-chemical phenomena can be employed in any of three essential model structures which effectively quantify the degree of fluid mixing in the reach (YOUNG and BECK (1974)); these include a second-order partial differential equation (see e.g. DRESNACK and DOBBINS (1968)), a first-order partial differential equation (see e.g. DI TORO (1969)), or a first-order ordinary differential equation.

Bearing in mind that as control engineers we are largely concerned with time as the single independent variable the third option appears to have attractive potential. Thus, if the reach of river is schematically defined as in figure 2(a) and then idealised as a combination of transportation delay and continuously stirred tank reactor (CSTR) components, figure 2(b), the following so-called lumped-parameter model is hypothesised for DO-BOD interaction,

$$\left.\begin{aligned}
&\text{DO:} && \dot{x}_1(t) = -\left(K_1 + Q(t)/V\right)x_1(t) - K_2x_2(t) \\
& && \qquad + \left(Q(t)/V\right)u_1'(t) + K_1C_s(t) + D_B && \text{(i)} \\
&\text{BOD:} && \dot{x}_2(t) = -\left(K_2 + K_3 + Q(t)/V\right)x_2(t) \\
& && \qquad + \left(Q(t)/V\right)u_2'(t) + L_A && \text{(ii)} \\
&\text{with} && u_1'(t) = u_1(t-T_d) && \text{(iii)} \\
& && u_2'(t) = u_2(t-T_d) && \text{(iv)}
\end{aligned}\right\} \quad \text{I}$$

where the dot notation refers to differentiation with respect to time t (days) and

x_1 = concn. of DO at the downstream end (output) of the reach (mg/l)
x_2 = concn. of BOD at the downstream end (output) of the reach (mg/l)
u_1 = concn. of DO at the upstream end (input) of the reach (mg/l)
u_2 = concn. of BOD at the upstream end (input) of the reach (mg/l)
u_1' = hypothetically delayed DO input to the CSTR component (mg/l)
u_2' = hypothetically delayed BOD input to the CSTR component (mg/l)
T_d = transportation delay (days)
K_1 = reaeration rate constant for DO (day^{-1})
K_2 = BOD decay rate constant (day^{-1})
K_3 = rate constant for the sedimentation of BOD (day^{-1})
Q = volumetric flow-rate in the reach (cuft/day)
V = mean volumetric hold-up in the reach (cuft)
C_s = saturation concentration of DO in the reach (mg/l)
L_A = rate of addition of BOD to the reach by local surface runoff [(mg/l)/day]
D_B = net rate of addition of DO to the reach by photosynthetic/respiratory activity of plants and algae and the decomposition of mud deposits [(mg/l)/day].

The saturation concentration of DO can be computed from the following second-order polynomial in θ, the river water temperature ($^{\circ}C$),

$$C_s(t) = 14.54 - 0.39\theta(t) + 0.01\left(\theta(t)\right)^2 \qquad (1)$$

The model is derived from component mass-balances across the idealised representation of the reach of river and, although it has been discussed elsewhere (YOUNG and BECK (1974)), it is worth mentioning some of its important features.

Firstly it is a dynamic model in the sense that it relates time-varying inputs to time-varying outputs at fixed spatial locations. It is unfortunate that Streeter and Phelps named their independent variable the "time of flow" since their model concerns time-invariant quantities with variations in space and in our terminology this is a steady-state model.

Secondly, the description of T_d requires some thought since it is time-varying according to the flow-rate Q. Normally the transportation delay in a reach of river might be described by the retention-time properties; one solution is to represent eqns. I(iii) and I(iv) by a combination of discrete-time delays and first-order continuous-time exponential lags (see e.g. BECK (1973)). However, this must be treated with caution since such a representation, together with the inherent lag properties of eqns. I(i) and I(ii), can over-estimate the true transportation delay characteristics of the reach. On the other hand a correct combination of delay components has been shown to describe closely both the transportation and dispersive fluid properties of a reach of river when compared against experimental iodide tracer measurements (WHITEHEAD and YOUNG (1974)). For longer reaches this type of delay description may be important and T_d would eventually described by

$$T_d \leqslant V/Q$$

although in the present examination of a relatively short reach the inaccuracy introduced by the simplifying assumptions

$u_1'(t) = u_1(t)$

$u_2'(t) = u_2(t)$

is negligible.

Thirdly, model I does not include a description of any diurnal variations in DO, despite the availability of adequate data; they are not essential to the present discussion, but it is indicated that they prove difficult to model accurately owing to a variable phase lag observed in their equivalent sinusoidal representation (BECK (1973)).

4. A NEW MODEL FOR DO-BOD INTERACTION WITH THE INCLUSION OF ALGAL POPULATION EFFECTS.

The experimental input data for the model are shown in figure 3. An immediately observable feature is the peaking effect in the flow-rate and input BOD records which results from a thunderstorm on day t_{56}. If the output BOD data are superimposed upon those of the input BOD in figure 4 it is possible to deduce that there are apparent discrepancies between the normally predicted process of BOD decay across the reach given by eqn. I. Over the so-called "critical" periods $t_{36} \rightarrow t_{48}$ and from t_{60} onwards (approximately) the downstream BOD is persistently higher than the upstream BOD*. This would not be expected to be a function of local surface runoff since there is little to suggest that L_A would be significantly positive over the corresponding periods of dry and sunny weather (figure 3(c)).

Hence the simulated responses of model I, shown as a dashed line in figures 5(a) and 5(b), fail to adequately describe the observed effects of the critical periods.

The hypothesis that large algal populations are responsible for the inadequacy of the model, eqn. I, rest upon the experimental evidence. That the 5-day BOD test itself could obscure the true causes of the higher downstream BOD's is a possibility; however, the profiles for

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* Other (usually) isolated instances of a higher downstream BOD are attributable to a transportation delay within the reach of approximately one day, such that a rapidly falling BOD concentration at Bait's Bite is not observed until a day later at Bottisham.

the rate of oxygen uptake in BOD samples from both boundaries of the reach are similar and there is no evidence for nitrification (for example) to be occurring at only one point. On the other hand the output DO measurements show a characteristic net addition of oxygen and increased diurnal oscillations which would result from the photosynthetic/respiratory activity of a well-established algal population; moreover, with the onset of dull weather these conditions abruptly cease and DO values are notably low. Indeed, similar reports of the effects of algae are well documented in the literature (e.g. CLAY (1944), GOODMAN and TUCKER (1971)).

Thus, to summarise, the following appears to be occurring in the experimental reach of river. During periods of prolonged dry and sunny weather large, active populations of algae are established <u>within</u> the reach and their interaction with the DO and BOD levels is observed at Bottisham, the downstream boundary of the system. Such effects are well-known in the case of DO-algae relationships (see e.g. BAIN (1968), BAILEY (1970), O'CONNOR and DI TORO (1970)) but it is less widely observed that algae, either as dead or decaying material, may, in some manner, exert a biochemical oxygen demand. Secondly, according to the limitations of measurement accuracy on the DO and BOD it is not possible to detect the corresponding entry of a significant algal population into the reach at Bait's Bite. However, the presence of a sewage effluent, at relatively low levels of dilution with the stream, imbues the reach with a nutrient-rich environment for the rapid stimulation of small "seed" populations of algae passing the upstream system boundary.

4.1. A pseudo-empirical term for the prediction of algal growth and decay.

It is necessary now to establish a means whereby a quantitative measure of these effects can be predicted. As a first approximation, the only available information which might be used as a predictor of algal growth is the meteorological data on the hours of incident sunlight per day. It is known that light intensity is a factor influencing algal growth kinetics and CHEN (1970) gives an expression for the growth rate of various species of algae in terms of the light intensity and nitrogen and phosphorus stream concentrations. Unfortunately no data are available for the latter two variables, although it might be fairly argued that they would be in sufficient supply downstream of an effluent discharge and therefore not rate-limiting for the expansion of an algal population. Firstly, as a consequence of this, it is required that the prediction of algal growth be dependent only upon the light intensity and hence upon the duration of sunlight over each day. And secondly, the form of the chosen expression should be capable of discrimination between an isolated event and a sustained sunny weather period.

These considerations lead to the following discrete, low-pass filter mechanism for a "sustained sunlight effect", $I(t_k)$,

$$I(t_k) = I(t_{k-1}) + \frac{1}{\tau}\left[u_3(t_k)\left\{\frac{\theta(t_k) - \bar{\theta}}{\bar{\theta}}\right\} - I(t_{k-1})\right] \qquad (2)$$

where u_3 is the sunlight incident on the system during day t_k (hrs/day), $\bar{\theta}$ is an arbitrary mean river water temperature (^{o}C) and τ is the time constant of the low-pass filter (days)*. Figure 5(c) shows the pattern of $I(t_k)$ for $\bar{\theta} = 8^{o}C$ and $\tau = 4$ days; the correlation between the critical periods and peaks in $I(t_k)$ is remarkably precise between t_{36} and t_{48}.

In order to introduce this sunlight-temperature effect into the model, additive terms $K_4(I(t_k)-\bar{I})$ and $K_5(I(t_k)-\bar{I})$ are inserted into equations I(i) and I(ii), respectively, so that the DO-BOD interaction is now described by

$$\left.\begin{aligned}
\dot{x}_1(t) &= -\left(K_1 + Q(t)/V\right)x_1(t) - K_2x_2(t) \\
&\quad + \left(Q(t)/V\right)u_1(t) + K_1C_s(t) \\
&\quad + K_4\left(I(t_k)-\bar{I}\right) + D_B \qquad \text{(i)} \\
\dot{x}_2(t) &= -\left(K_2 + K_3 + Q(t)/V\right)x_2(t) + \left(Q(t)/V\right)u_2(t) \\
&\quad + K_5\left(I(t_k)-\bar{I}\right) + L_A \qquad \text{(ii)} \\
I(t_k) &= I(t_{k-1}) + \frac{1}{\tau}\left[u_3(t_k)\left\{\frac{\theta(t_k) - \bar{\theta}}{\bar{\theta}}\right\} - I(t_{k-1})\right] \qquad \text{(iii)} \\
\left(I(t_k)-\bar{I}\right) &= 0 \text{ for } I(t_k) < \bar{I} \qquad \text{(iv)}
\end{aligned}\right\} \text{II}$$

Here $\bar{I}$ is a "threshold" level of the sustained sunlight effect; in other words, the presence of $\bar{I}$ indicates the requirement for a certain minimum level of $I(t_k)$, i.e. eqn. II(iv), before algal populations can become established. Similarly the exponential weighting into the

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* The temperature coefficient {...} is included for completeness and, although its effects are marginal, it had been hoped that it might explain the features of the second critical period.

past introduced by the low-pass filter mechanism embodies the concept that prolonged sunny and warm weather is required to produce algal populations of sufficient magnitude to have any discernible effect on the DO-BOD balance in the river system. K_4 and K_5 are simple proportionality constants in this context and they relate the pseudo-empirical sustained sunlight effect to the DO and BOD dynamics.

Figures 5(a) and 5(b) show the improved prediction in output DO and BOD given by the new lumped-parameter differential-difference equation II. Particularly over the first critical period the model competently describes the net daily addition of DO in the reach by the photosynthesis/ /respiration cycle of algae and also the production of a BOD from dead or decaying algal material. The parameter values estimated for the model are given in table 1*. D_B, which for model II refers only to the withdrawal of oxygen by decomposing mud deposits, is notably time-dependent; this could be a consequence of measurement error on the DO and BOD (BECK (1973)), although recent results suggest that flow conditions may be accountable for such variations (YOUNG and WHITEHEAD (1974)). On the other hand, L_A is suitably zero and this agrees with the nature of the physical system where local surface runoff is collected in drainage dykes and pumped into the river downstream of Bottisham weir.

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* Most of the parameter estimates are taken from a more complete statistical verification study of the model (BECK (1973), BECK and YOUNG).

Table 1 - Parameter values for model II.

K_1	0.17	K_5 †	0.32	V	$5.4(10^6)$
K_2	0.32	$\bar{I}$ †	6.0	L_A	0.0
K_3	0.0	$\bar{\theta}$	8.0	D_B	-2.7 for $0 \leq t \leq t_{19}$
K_4 †	0.31	τ	4		-0.4 for $t > t_{19}$

† No specific units are assigned to these quantities owing to the dimensional anomoly of eqn. II(iii).

5. FURTHER COMMENTS ON DO-BOD-ALGAE MODELS.

Over the past two years the data collected from the River Cam have been the subject of considerable analysis. Only the deterministic simulation aspects are reported here but clearly more sophisticated statistical approaches are warranted if the model is to be properly examined in an environment where the system is subject to chance disturbances and random errors of measurement. Such an approach is the extended Kalman filter and it is particularly useful in identifying and validating the structure of the model (BECK and YOUNG); once the model structure is defined the analysis can proceed to further "refinement" of the parameter estimates using techniques of recursive time-series analysis (see e.g. YOUNG (1974), YOUNG and WHITEHEAD (1974)).

Indeed, other identification and parameter estimation studies have yielded more information on the nature of DO-BOD-algae interaction. The model of eqn. II has been verified against data from the Bedford-Ouse river (WHITEHEAD and YOUNG (1974)); in this case the basic structure is improved by including a term relating to the chlorophyll-A content of the stream for the prediction of DO production by algae.

Now, while it is possible to extend the original pseudo-empirical expression, what is really required is a more fundamental growth and decay model for algae. Recent work using maximum likelihood methods of identification (see ÅSTRÖM and BOHLIN (1965)) applied to the Cam data indicate that it is feasible to describe algal growth in terms of a MONOD (1942) function, where sunlight is taken to be the rate-limiting factor. A combined DO-BOD-algae model is proposed in which the living and dead algal populations are considered explicitly by separate component mass balances across the reach of river. In particular, such a model is able to eliminate the error in BOD prediction over the latter twenty days of the experimental period (see figure 5(b)), where the effects of algae* appear to be very sensitive to the retention-time conditions existing in the reach (BECK (1974a), (1974b)).

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* This is not to ignore that such effects could also be a consequence of either decreased turbidity conditions or increased growths of attached plants over the summer period.

6. CONCLUSIONS.

A lumped-parameter differential-difference equation is presented for the description of the dynamic interaction between DO and BOD in a non-tidal stream. A significant improvement included in the model is a pseudo-empirical term which describes the effects of algal populations on the DO and BOD and when tested against field data the model is shown to simulate adequately the major variations observed in practice.

The model is simple in the sense that it requires only the solution of ordinary differential equations as opposed to partial differential equations; this means that many of the techniques of control system synthesis can be readily applied for analyses of in-stream DO level maintenance (YOUNG and BECK (1974)). At the same time the transportation delay and fluid dispersion properties of a reach of river can be simulated where necessary.

The basic structure of the model appears to be applicable outside the specific system chosen for the experimental study reported here; however, parameter values are different for different systems as evidenced by the results from the River Cam and the Bedford-Ouse. But no model can claim to be universally applicable and if it is to be used wisely then some form of prior experiment, within the obvious limitations of manpower and instrumentation, is always necessary for model validation.

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LEGENDS FOR ILLUSTRATIONS

Figure 1 The experimental reach of river: a part of the River Cam outside Cambridge.

Figure 2 A simple model for DO-BOD interaction in a reach of river; all variables are as defined for eqn I.
(a) single reach of river
(b) CSTR representation

Figure 3 Experimental data from the River Cam.

Figure 4 The comparison between the upstream, u_2, and downstream, x_2, BOD data.

Figure 5 Deterministic simulation responses for DO-BOD interaction given by model I (dashed line) and model II (continuous line); experimental observations are indicated by dots.

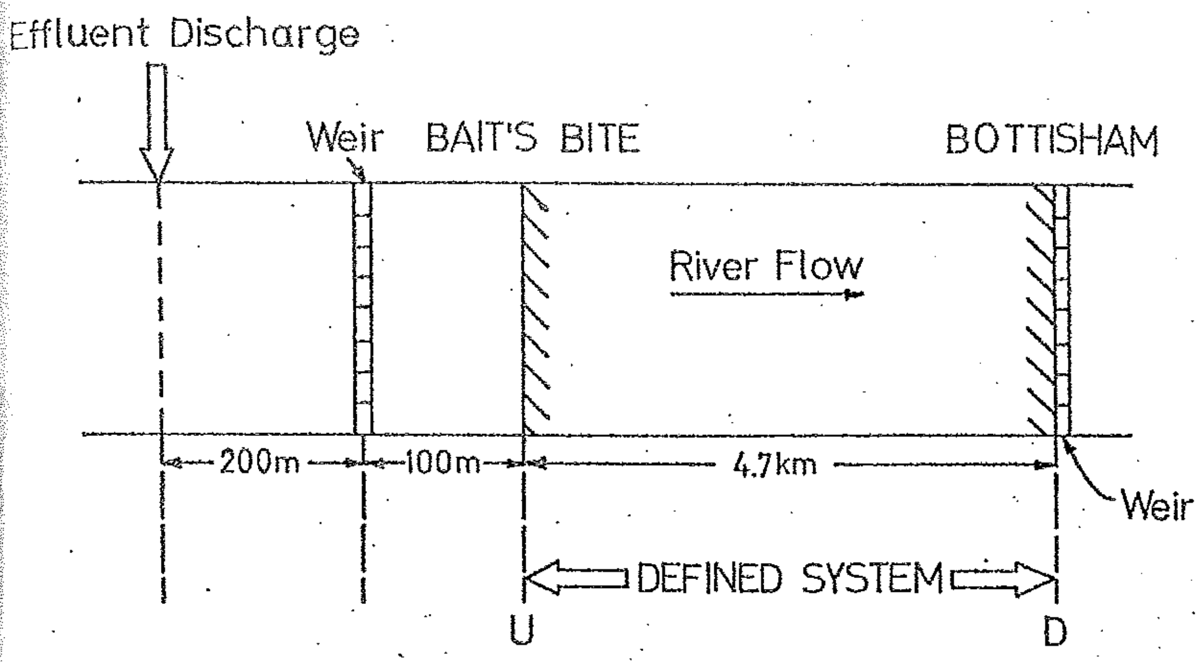
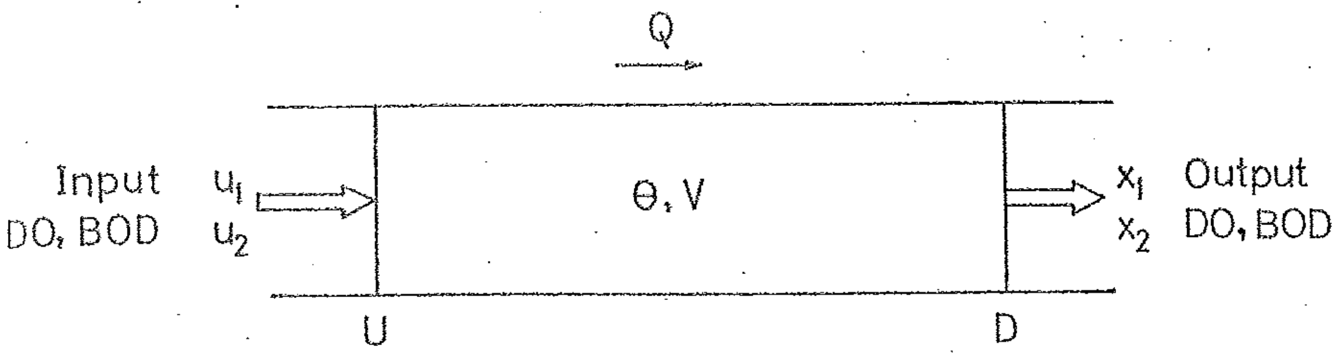
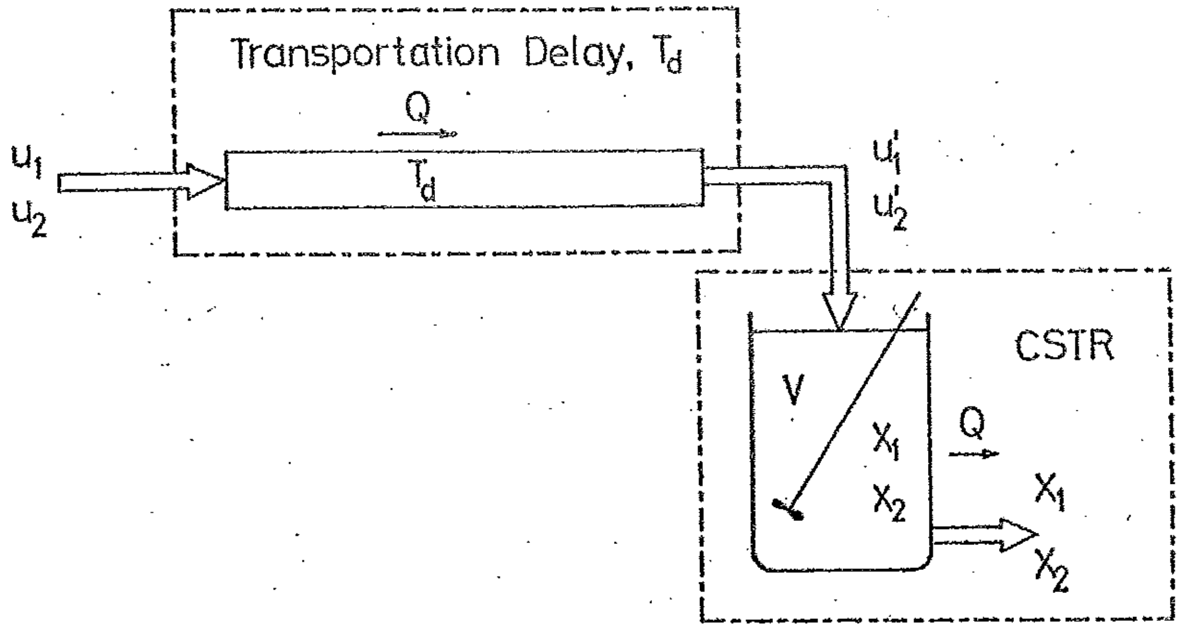

Effluent Discharge
Weir
BAIT'S BITE
BOTTISHAM
River Flow
200m
100m
4.7km
Weir
DEFINED SYSTEM
U
D

Figure 1.



Q
Input
DO, BOD
u_1
u_2
Θ, V
x_1
x_2
Output
DO, BOD
U
D

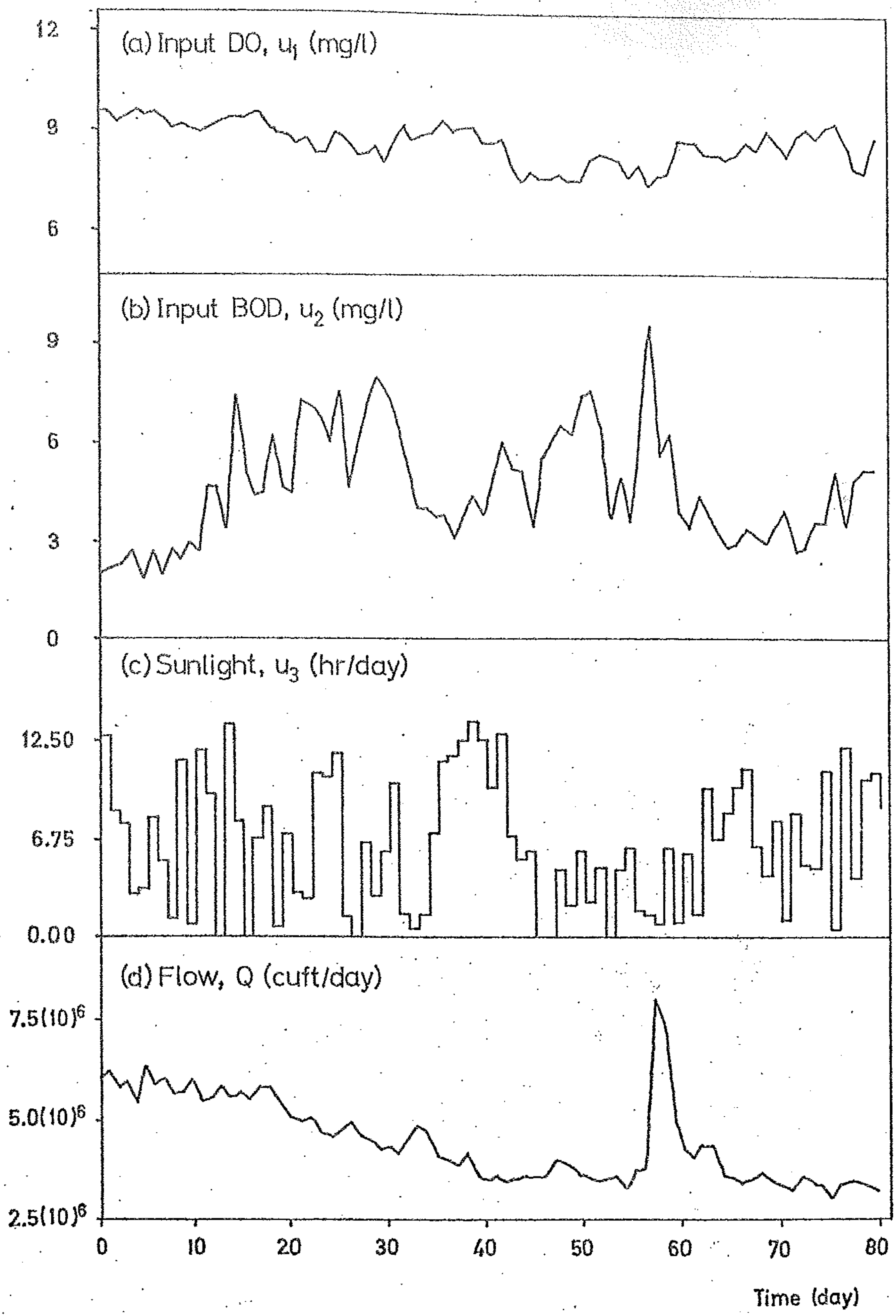
(a)



Transportation Delay, T_d
Q
T_d
u_1
u_2
u_1'
u_2'
CSTR
V
X_1
X_2
Q
X_1
X_2

(b)

Figure 2.



(a) Input DO, u_1 (mg/l)
12
9
6
(b) Input BOD, u_2 (mg/l)
9
6
3
0
(c) Sunlight, u_3 (hr/day)
12.50
6.75
0.00
(d) Flow, Q (cuft/day)
$7.5(10)^6$
$5.0(10)^6$
$2.5(10)^6$
0
10
20
30
40
50
60
70
80
Time (day)

Figure 3.

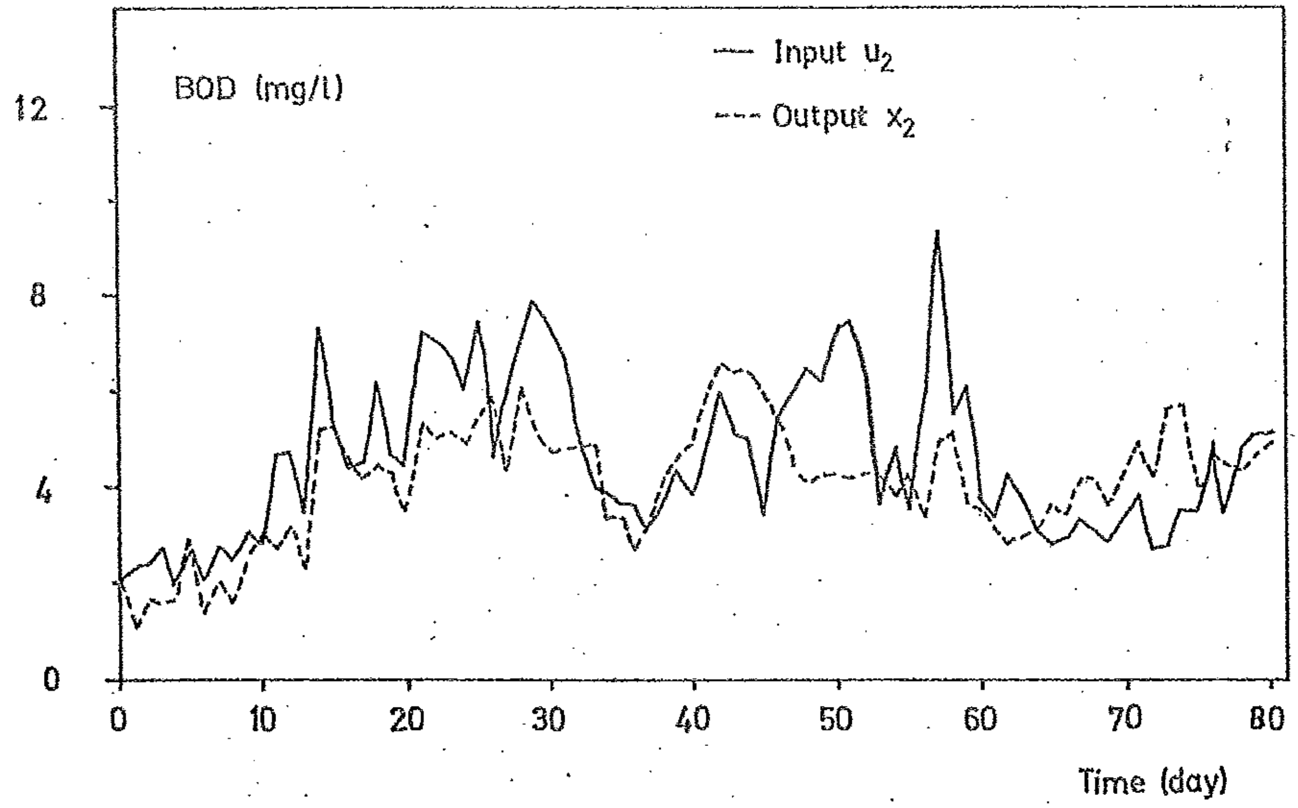

BOD (mg/l)
Input u_2
Output x_2
12
8
4
0
0
10
20
30
40
50
60
70
80
Time (day)

Figure 4.

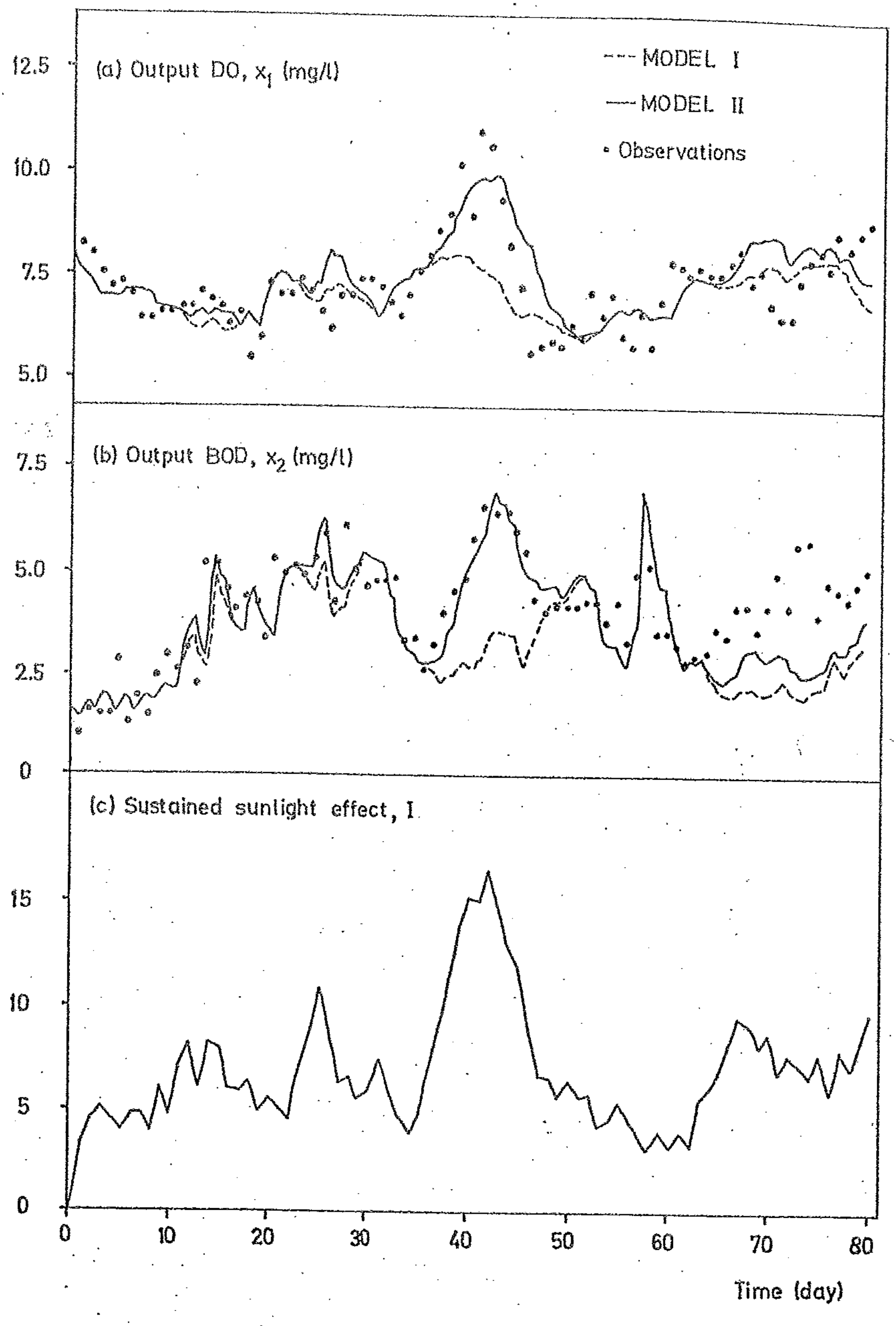

12.5
10.0
7.5
5.0
(a) Output DO, x_1 (mg/l)
--- MODEL I
— MODEL II
• Observations
7.5
5.0
2.5
0
(b) Output BOD, x_2 (mg/l)
(c) Sustained sunlight effect, I
15
10
5
0
0
10
20
30
40
50
60
70
80
Time (day)

Figure 5.