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## Computation of Asymptotic Least Squares Estimates

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COMPUTATION OF ASYMPTOTIC LEAST SQUARES  
ESTIMATES

T. SÖDERSTRÖM

Report 7314 (C) July 1973  
Lund Institute of Technology  
Division of Automatic Control

COMPUTATION OF ASYMPTOTIC LEAST SQUARES ESTIMATES.

T. Söderström

ABSTRACT.

In this report programs are given which computes asymptotic least squares estimates.

TABLE OF CONTENTS

Page

I. Description of the Programs

1

II. Lists of the Programs

III. Examples

## I. DESCRIPTION OF THE PROGRAMS.

Section II of the report contains complete lists of the programs. A short description of the programs are given below. The programs not described (CINT, ORDER and SYMIN) can be found in the program library of the Division of Automatic Control.

1. TLST. This is a main (test) program. Parameters are read and written and a CALL LST is done. It is possible to do repeated calls of LST in one execution.

2. LST. It is assumed that the system

$$A(q^{-1})y(t) = B(q^{-1})u(t) + C(q^{-1})e(t) \quad (1)$$

is given where

$$A(q^{-1}) = 1 + a_1q^{-1} + \dots + a_{NSA}q^{-NSA}$$

$$B(q^{-1}) = b_1q^{-1} + \dots + b_{NSB}q^{-NSB}$$

$$C(q^{-1}) = 1 + c_1q^{-1} + \dots + c_{NSC}q^{-NSC}$$

$$u(t) \text{ white noise } Eu(t) = 0 \quad Eu(t)^2 = VU$$

$$e(t) \text{ white noise } Ee(t) = 0 \quad Ee(t)^2 = VE$$

$u(t)$  and  $e(t)$  independent.

An LS identification of the system is simulated in the sense that the asymptotic values of the estimates are computed using complex integrals.

The following model is obtained

$$\hat{A}(q^{-1})y(t) = \hat{B}(q^{-1})u(t) + \varepsilon(t)$$

where

$$\hat{A}(q^{-1}) = 1 + \hat{a}_1 q^{-1} + \dots + \hat{a}_{NMA} q^{-NMA}$$

$$\hat{B}(q^{-1}) = \hat{b}_1 q^{-1} + \dots + \hat{b}_{NMB} q^{-NMB}$$

The following variables are obtained

$$TM = [\hat{a}_1 \dots \hat{a}_{NMA} \hat{b}_1 \dots \hat{b}_{NMB}]$$

$$V - \text{the loss function} = \sum \varepsilon^2(t)$$

S - estimated standard deviation of  $\varepsilon(t)$

P - estimated covariance matrix of TM

CC - estimated standard deviation of the elements of TM.

3. COVM. It is assumed that the system (1) is given and that  $N = NSA = NSB = NSC$ . The following matrix is computed.

$$COV = \begin{pmatrix} r_y(0) & r_y(1) & r_y(NC-1) & r_{yu}(0) & r_{yu}(NC-1) & r_{ye}(0) & r_{ye}(NC-1) \\ \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ r_y(NC-1) & r_y(0) & r_{yu}(1-NC) & r_{yu}(0) & r_{ye}(1-NC) & r_{ye}(0) \\ r_{yu}(0) & r_{yu}(1-NC) & r_u(0) & r_u(NC-1) & r_{ue}(0) & r_{ue}(NC-1) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ r_{yu}(NC-1) & r_{yu}(0) & r_u(NC-1) & r_u(0) & r_{ue}(1-NC) & r_{ue}(0) \\ r_{ye}(0) & r_{ye}(1-NC) & r_{ue}(0) & r_{ue}(1-NC) & r_e(0) & r_e(NC-1) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ r_{ye}(NC-1) & r_{ye}(0) & r_{ue}(NC-1) & r_{ue}(0) & r_e(NC-1) & r_e(0) \end{pmatrix}$$

COV is the covariance matrix of the vector

$$[y(t-1) \dots y(t-NC) \ u(t-1) \dots u(t-NC) \ e(t-1) \dots e(t-NC)]$$

5. PMPY. This program performs polynomial multiplication,  
see the comments in the list.

## II. LISTS OF THE PROGRAMS

```

1 C PROGRAM TLST
2 C
3 C TEST OF SUBROUTINE LST
4 C AUTHOR TORSTEN SODERSTROM 1971-10-05
5 C
6 C SUBROUTINE REQUIRED
7 C LST
8 C COVM
9 C CINT
10 C ORDER
11 C PMPY
12 C GURV
13 C
14 C DIMENSION TS(30),TM(20),P(20,20),C(20)
15 C
16 C 1 READ 101,M,NA,NB,NC
17 101 FORMAT(1U15)
18 IF(M.EQ.0) GO TO 99
19 N=NA+NB+NC
20 IF(N.GT.0) READ 100,(TS(I),I=1,N),VU,VE
21 IF(N.EQ.0) READ 100,VU,VE
22 100 FORMAT(8F10.5)
23 C 2 READ 101,NMA,NMB
24 IF(NMA*NMB.EQ.0) GO TO 1
25 C
26 PRINT 200
27 200 FORMAT(1H1,09X,'SYSTEM'/10X,6(1H*))
28 PRINT 201,NA,NB,NC,M
29 201 FORMAT(/10X,'NUMBER OF A-PARAMETERS',I5
30 */10X,'NUMBER OF B-PARAMETERS',I5
31 */10X,'NUMBER OF C-PARAMETERS',I5
32 */10X,'NUMBER OF SAMPLING INTERVALS',I8)
33
34 PRINT 202
35 202 FORMAT(/10X,'PARAMETERS')
36 IF(N.GT.0) PRINT 100,(TS(I),I=1,N)
37 PRINT 204,VU,VE
38 204 FORMAT(/10X,'VARIANCE OF THE INPUT SIGNAL',G12.5
39 */10X,'VARIANCE OF THE NOISE',7X,G12.5)
40 PRINT 205
41 205 FORMAT(///10X,'MODEL'/10X,5(1H*))
42 PRINT 206,NMA,NMB
43 206 FORMAT(/10X,'NUMBER OF A-PARAMETERS',I5
44 */10X,'NUMBER OF B-PARAMETERS',I5)
45 C
46 CALL LST(TS,VU,VE,TM,V,S,P,C,NA,NB,NC,NMA,NMB,M,1,20)
47 C
48 GO TO 2
49 C
50 99 CONTINUE
51 STOP
52 END

```



```

1      SUBROUTINE LST(TS,VU,VE,TM,V,S,P,CC,NSA,NSB,NSC,NMA,NMB,M,IPR,IA)
2      C
3      C
4      C   GIVEN THE SYSTEM
5      C    $A(Q)*Y(T)=B(Q)*U(T)+C(Q)*E(T)$ 
6      C    $A(Q)=1+A(1)*Q**(-1)+...+A(NSA)*Q**(-NSA)$ 
7      C    $B(Q)= B(1)*Q**(-1)+...+B(NSB)*Q**(-NSB)$ 
8      C    $C(Q)=1+C(1)*Q**(-1)+...+C(NSC)*Q**(-NSC)$ 
9      C   U(T) AND E(T) INDEPENDENT WHITE NOISES WITH VARIANCES VU RESP VE
10     C
11     C   COMPUTED THE THEORETICAL, ASYMPTOTIC LEAST SQUARES MODEL
12     C    $AM(Q)*Y(T) = BM(Q)*U(T) + EPS(T)$ 
13     C   AND RELATED VARIABLES
14     C    $AM(Q)=1+AM(1)*Q**(-1)+...+AM(MNA)*Q**(-MNA)$ 
15     C    $BM(Q)= BM(1)*Q**(-1)+...+BM(MNB)*Q**(-MNB)$ 
16     C
17     C   AUTHOR TORSTEN SODERSTROM 1971-10-05
18     C   REFERENCE REPORT 731+(C)
19     C
20     C   TS-VECTOR OF ORDER (NSA+NSB+NSC)
21     C   TS=(A(1),...,A(NSA),B(1),...,B(NSB),C(1),...,C(NSC))
22     C
23     C   VU-VARIANCE OF U(T)
24     C   VE-VARIANCE OF E(T)
25     C
26     C   TM-VECTOR OF ORDER (NMA+NMB)
27     C   TM=(AM(1),...,AM(NMA),BM(1),...,BM(NMB))
28     C
29     C   V - THE LOSS FUNCTION (=SUM OF SQUARED RESIDUALS)
30     C
31     C   S-ESTIMATED STANDARD DEVIATION OF THE NOISE
32     C
33     C   P-ESTIMATED COVARIANCE MATRIX OF TM
34     C
35     C   CC-ESTIMATED STANDARD DEVIATIONS OF THE PARAMETERS
36     C
37     C   NSA,NSB,NSC -ORDERS OF A,B,C (MIN0,MAX10)
38     C
39     C   NMA,NMB-ORDERS OF AM,BM (MIN 0,MAX 9)
40     C
41     C   M-NUMBER OF SAMPLES
42     C
43     C   IPR-PRINT PARAMETER
44     C   IPR=0 NOTHING PRINTED
45     C   1 OUTPUT GENERATED (T,CC,P,V,S ARE PRINTED)
46     C
47     C   IA-DIMENSION PARAMETER OF P
48     C
49     C   SUBROUTINE REQUIRED
50     C       COVM
51     C       CINT
52     C       ORDER
53     C       PMPY
54     C       SYMIN
55     C
56     C   DIMENSION TS(1),TM(1),P(IA,IA),CC(1)
57     C   DIMENSION COV(30,30),R(20),A(10),B(10),C(10)
58     C
59     C   N=MAX0(NSA,NSB,NSC)
60     C   MN=NMA+NMB
61     C   IF(N*MN.EQ.0) GO TO 99
62     C   NC=MAX0(NMA,NMB)+1
63     C
64     C   DO 1 I=1,N
65     C   A(I)=0.
66     C   B(I)=0.
67     C   1 C(I)=0.
68     C   IF(NSA.EQ.0) GO TO 3
69     C   DO 2 I=1,NSA
70     C   2 A(I)=TS(I)
71     C   3 IF(NSB.EQ.0) GO TO 5
72     C   DO 4 I=1,NSB
73     C   4 B(I)=TS(1+NSA)
74     C   5 IF(NSC.EQ.0) GO TO 7
75     C   DO 6 I=1,NSC

```

```

64      6 C(I)=TS(1+NSA+NSB)
65      C
66      7 CALL COVR(A,B,C,COV,VU,VE,N,NC,30)
67      C
68      C      COMPUTE P=LIM FI(T)*FI/M AND R=LIM FI(T)*Y/M
69      C
70      IF(NMA.EQ.0) GO TO 11
71      DO 10 I=1,NMA
72      DO 8 J=1,NMA
73      8 P(I,J)=COV(I,J)
74      R(I)=-COV(1,I+1)
75      IF(NMB.EQ.0) GO TO 10
76      DO 9 J=1,NMB
77      9 P(I,J+NMA)=-COV(I,J+NC)
78      10 CONTINUE
79      11 IF(NMB.EQ.0) GO TO 13
80      DO 12 I=1,NMB
81      R(I+NMA)=COV(1,I+NC+1)
82      DO 12 J=1,NMB
83      12 P(I+NMA,J+NMA)=COV(I+NC,J+NC)
84      DO 14 I=1,NN
85      DO 14 J=1,I
86      14 P(I,J)=P(J,I)
87      13 CONTINUE
88      C
89      C      COMPUTE THE SOLUTION
90      C
91      CALL SYMIN(NN,IA,IFAIL,P)
92      IF(IFAIL.EQ.0) GO TO 31
93      PRINT 116
94      116 FORMAT(/10X,'THE SYSTEM IS SINGULAR')
95      GO TO 99
96      31 CONTINUE
97      C
98      DO 15 I=1,NN
99      TM(I)=0.
100     DO 15 J=1,NN
101     15 TM(I)=TM(I)+P(I,J)*R(J)
102     V=COV(1,1)
103     DO 16 I=1,NN
104     16 V=V-R(I)*TM(I)
105     IF(V.LT.0.0) S=SQRT(-V)
106     IF(V.GE.0.0) S=SQRT(V)
107     V=V*M
108     DO 17 I=1,NN
109     DO 17 J=1,NN
110     17 F(I,J)=P(I,J)*S*S/M
111     DO 18 I=1,NN
112     IF(P(I,I).LT.0.0) CC(I)=SQRT(-P(I,I))
113     18 IF(P(I,I).GE.0.0) CC(I)=SQRT(P(I,I))
114     C
115     C      PRINT RESULTS
116     C
117     IF (IPR) 99,99,19
118     19 PRINT 101
119     101 FORMAT(/10X,'THEORETICAL LS ESTIMATE'/10X,23(1H-
120     */10X,'PARAMETER ESTIMATES')
121     PRINT 100,(TM(I),I=1,NN)
122     100 FORMAT(10G12.5)
123     PRINT 102
124     102 FORMAT(/10X,'STANDARD DEVIATIONS')
125     PRINT 100,(CC(I),I=1,NN)
126     PRINT 103
127     103 FORMAT(/10X,'COVARIANCE MATRIX')

```

```
128      DO 20 I=1,NN
129          20 PRINT 100,(P(I,J),J=1,NN)
130              PRINT 104
131          104 FORMAT(/10X,'LOSS FUNCTION')
132              PRINT 100,V
133              PRINT 105
134          105 FORMAT(/10X,'STANDARD DEVIATION OF THE NOISE')
135              PRINT 100,S
136      C
137          99 CONTINUE
138              RETURN
139              END
```

```

1      SUBROUTINE COVM(A,B,C,COV,VU,VE,N,NC,IA)
2      C
3      C      GIVEN THE SYSTEM
4      C       $A(Q)*Y(T)=B(Q)*U(T)+C(Q)*E(T)$ 
5      C
6      C       $A(Q)=1+A(1)*Q**(-1)+...+A(N)*Q**(-N)$ 
7      C       $B(Q)= B(1)*Q**(-1)+...+B(N)*Q**(-N)$ 
8      C       $C(Q)=1+C(1)*Q**(-1)+...+C(N)*Q**(-N)$ 
9      C
10     C      U(T) WHITE NOISE
11     C      E(T) WHITE NOISE
12     C      U(T) AND E(T) INDEPENDENT
13     C
14     C      COMPUTED THE EXTENDED SYSTEM COVARIANCE MATRIX
15     C           RY   RYU   RYE
16     C      COV = ( RUY   RU   RUE )
17     C           REY   REU   RE
18     C
19     C      =COVARIANCE MATRIX OF THE VECTOR
20     C      ( Y(T-1) Y(T-2)...Y(T-NC) U(T-1)...U(T-NC) E(T-1)...E(T-NC) )
21     C
22     C      AUTHOR TORSTEN SÖDERSTRÖM 1971-12-24
23     C      REFERENCE REPORT 7314(C)
24     C
25     C      A VECTOR OF ORDER N
26     C      B VECTOR OF ORDER N
27     C      C VECTOR OF ORDER N
28     C      COV-MATRIX OF ORDER (3*NC)*(3*NC), AT RETURN CONTAINING THE
29     C      EXTENDED SYSTEM COVARIANCE MATRIX
30     C      VU-VARIANCE OF U(T)
31     C      VE-VARIANCE OF E(T)
32     C      N-ORDER OF THE SYSTEM (MIN1,MAX10)
33     C      J*NC-ORDER IF THE MATRIX COV
34     C      NC (MIN 1,MAX 10)
35     C      IA-DIMENSION PARAMETER OF COV
36     C
37     C      SUBROUTINE REQUIRED
38     C           CINT
39     C           ORDER
40     C           PMPY
41     C
42     C      DIMENSION A(1),B(1),C(1),COV(IA,1)
43     C      DIMENSION RYE(20),RYU(20),RYYE(10),RYYU(10)
44     C      DIMENSION AI(20),BI(20),CIU(30),CIE(30)
45     C      DIMENSION C1E(30),C1U(30),C1(11)
46     C
47     C      NC3=NC*3
48     C      N1=N+1
49     C      N2=N*2
50     C
51     C      DO 1 I=1,NC
52     C      DO 1 J=1,NC
53     C      1 COV(NC+I,2*NC+J)=0.
54     C      DO 2 J=1,NC
55     C      DO 3 I=1,J
56     C      COV(NC+I,NC+J)=0.
57     C      3 COV(NC*2+I,NC*2+J)=0.
58     C      COV(NC+J,NC+J)=VJ
59     C      2 COV(2*NC+J,2*NC+J)=VE
60     C      DO 5 I=1,NC
61     C      DO 5 J=1,I
62     C      COV(I,NC+J)=0.
63     C      5 COV(I,2*NC+J)=0.

```

```

64 C
65 C COMPUTE RYU AND RYE.
66 C
67 DO 5 I=1,N
68 CIU(I)=B(N+1-I)
69 CIE(I)=C(N+1-I)
70 6 AI(I+1)=A(N+1-I)
71 AI(1)=0.
72 AI(N+2)=1.
73 CIE(N+1)=1.
74 BI(1)=1.
75 DO 6 K=1,NC
76 NCU=N+K-2
77 NCE=N+K-1
78 IF(K.EQ.1) GO TO 7
79 DO 17 I=1,N
80 CIU(NCU+2-I)=CIU(NCU+1-I)
81 17 CIE(NCE+2-I)=CIE(NCE+1-I)
82 CIU(K-1)=0.
83 CIE(K)=C(N)
84 CIE(K-1)=0.
85 7 RYU(K)=VU*CINT(AI,CIU,BI,N1,NCU,0)
86 8 RYE(K)=VL*CINT(AI,CIE,BI,N1,NCE,0)
87 C
88 C COMPUTE RYYU AND RYYE.
89 C
90 DO 4 I=1,N
91 4 BI(I+1)=A(I)
92 BI(1)=1.
93 DO 9 I=1,N
94 9 AI(I+1)=A(N+1-I)
95 AI(1)=0.
96 AI(N+2)=1.
97 DO 10 I=1,N
98 CIU(I)=B(N+1-I)
99 CIE(I)=C(N+1-I)
100 10 C1(I+1)=C(I)
101 C1(1)=1.
102 CIU(N1)=0.
103 CIE(N1)=1.
104 N2=N2+1
105 CALL PMPY(CIU,NCU,B,N,CIU,N1)
106 NS=NCU-1
107 DO 18 I=1,NS
108 18 CIU(NCU+1-I)=CIU(NCU-I)
109 CIU(1)=0.
110 CALL PMPY(CIE,NCE,C1,N1,CIE,N1)
111 C
112 NCU=NCU-2
113 NCE=NCE-2
114 DO 12 K=1,NC
115 IF(K.EQ.1) GO TO 13
116 NK=K-1
117 DO 11 I=1,N2
118 KI=NK+N2+1-I
119 CIU(KI)=CIU(KI-1)
120 11 CIE(KI+1)=CIE(KI)
121 CIU(KK)=0.
122 CIE(KK+1)=CIE(KK)
123 CIE(KK)=0.
124 13 NCU=NCU+1
125 NCE=NCE+1
126 RYYU(K)=VU*CINT(AI,CIU,BI,N1,NCU,N)
127 12 RYYE(K)=VE*CINT(AI,CIE,BI,N1,NCE,N)

```

```
128 C
129 C
130 C STORE RESULT IN COV.
131 C
132 DO 14 J=1,NC
133 DO 14 I=1,J
134 COV(I,NC+J)=RYU(J-I+1)
135 COV(I,2*NC+J)=RYE(J-I+1)
136 14 COV(I,J)=RYYE(J-I+1)+RYYU(J-I+1)
137 DO 15 I=1,NC3
138 DO 15 J=1,I
139 15 COV(I,J)=COV(J,I)
140 C
141 RETURN
142 END
```

```

1      SUBROUTINE PMPY(Z, IDIMZ, X, IDIMX, Y, IDIMY)
2      C
3      C      POLYNOMIAL MULTIPLICATION
4      C      GIVEN
5      C      X, IDIMX, Y, IDIMY
6      C       $X(T) = X(1)*T**(IDIMX-1) + \dots + X(IDIMX)$ 
7      C       $Y(T) = Y(1)*T**(IDIMY-1) + \dots + Y(IDIMY)$ 
8      C      COMPUTED
9      C      Z, IDIMZ
10     C       $Z(T) = X(T)*Y(T)$ 
11     C       $IDIMZ = IDIMX + IDIMY - 1$ 
12     C       $Z(T) = Z(1)*T**(IDIMZ-1) + \dots + Z(IDIMZ)$ 
13     C
14     C      AUTHORS IBM+I GUSTAVSSON+E BURSTRÖM +T SÖDERSTRÖM, ANNO DAZUMAL
15     C      SUBROUTINE REQUIRED
16     C      NONE
17     C
18     C      DIMENSION Z(1), X(1), Y(1)
19     C      IF(IDIMX*IDIMY) 10, 10, 20
20     10 IDIMZ=0
21     C      GO TO 50
22     20 IDIMZ=IDIMX+IDIMY-1
23     C      DO 30 I=1, IDIMZ
24     30 Z(I)=0.
25     C      DO 40 I=1, IDIMX
26     C      DO 40 J=1, IDIMY
27     C      K=I+J-1
28     40 Z(K)=X(I)*Y(J)+Z(K)
29     50 RETURN
30     C      END

```

### III. EXAMPLES

SYSTEM  
\*\*\*\*\*

NUMBER OF A-PARAMETERS 1  
 NUMBER OF B-PARAMETERS 1  
 NUMBER OF C-PARAMETERS 1  
 NUMBER OF SAMPLING INTERVALS 500

PARAMETERS  
 --.80000 1.00000 .70000

VARIANCE OF THE INPUT SIGNAL 1.0000  
 VARIANCE OF THE NOISE 1.0000

MODEL  
 \*\*\*\*\*

NUMBER OF A-PARAMETERS 2  
 NUMBER OF B-PARAMETERS 2

THEORETICAL LS ESTIMATE  
 -----

PARAMETER ESTIMATES  
 -1.2857 .42249 1.0000 --.48573

STANDARD DEVIATIONS  
 .39949-01 .37905-01 .47958-01 .62417-01

COVARIANCE MATRIX  
 .15959-02 -.13082-02 .00000 .15959-02  
 -.13882-02 .14368-02 .00000 -.13882-02  
 .00000 .00000 .23000-02 .00000  
 .15959-02 -.13082-02 .00000 .38959-02

LOSS FUNCTION  
 575.00

STANDARD DEVIATION OF THE NOISE  
 1.0724



SYSTEM

\*\*\*\*\*

NUMBER OF A-PARAMETERS 2  
NUMBER OF B-PARAMETERS 2  
NUMBER OF C-PARAMETERS 0  
NUMBER OF SAMPLING INTERVALS 500

PARAMETERS

-.10000    -.56000    1.00000    .70000

VARIANCE OF THE INPUT SIGNAL 1.0000  
VARIANCE OF THE NOISE 1.0000

MODEL

\*\*\*\*\*

NUMBER OF A-PARAMETERS 2  
NUMBER OF B-PARAMETERS 2

THEORETICAL LS ESTIMATE

PARAMETER ESTIMATES

-.10000    -.56000    1.0000    .70000

STANDARD DEVIATIONS

.33507-01    .29368-01    .44721-01    .55882-01

COVARIANCE MATRIX

.11227-02    -.66922-03    .00000    .11227-02  
-.66922-03    .86249-03    .00000    -.66922-03  
.00000    .90000    .20000-02    .90000  
.11227-02    -.66922-03    .00000    .31227-02

LOSS FUNCTION

500.00

STANDARD DEVIATION OF THE NOISE

1.0000