A Survey of Adaptive Control Methods

Wittenmark, Björn

1971

Document Version:
Publisher's PDF, also known as Version of record

Link to publication

Citation for published version (APA):
Wittenmark, B. (1971). *A Survey of Adaptive Control Methods*. (Technical Reports TFRT-7013). Department of Automatic Control, Lund Institute of Technology (LTH).

Total number of authors:
1

LUND UNIVERSITY

PO Box 117
221 00 Lund
+46 46-222 00 00

A SURVEY OF ADAPTIVE CONTROL METHODS.

BJÖRN WITTENMARK

REPORT 7110 SEPTEMBER 1971
LUND INSTITUTE OF TECHNOLOGY
DIVISION OF AUTOMATIC CONTROL

A SURVEY OF ADAPTIVE CONTROL METHODS.†

B. Wittenmark

ABSTRACT.

This report will give the background to motivate why adaptive control may be necessary and will sketch the common features of different adaptive controllers. A classification of adaptive controllers is given and a few typical systems in each class are described in further details.

† This work has been supported by the Swedish Board of Technical Development under Contract 69-631/U489

TABLE OF CONTENTS Page

1. INTRODUCTION.

In the classic servomechanism theory, which was created during World War II, only linear time invariant systems were considered. Systems with a few nonlinearities could be handled with describing function and phase plane methods. Concerning time varying parameters the philosophy was to make the systems as insensitive as possible to variations in system dynamics by means of fixed regulators. In the 1950's one became aware of the large difficulties of controlling systems with widely varying parameters. During the development of the supersonic aeroplanes one found that the behaviour of the aeroplanes could not be satisfactorily controlled with conventional time independent feedback and compensation. This leads to programmed controllers, i.e. the controllers were tuned depending on measured variables as altitude, speed, dynamic pressure etc. This type of tuning is open loop and there is no feedback from the behaviour of the airframe to the setting of the controller. It is very costly and time consuming to determine the controller for all possible flight conditions. This leads to the idea of <u>adaptive</u> control. By this is meant automatic adjustment of the system in order to adapt to changing environment. This idea was used as early as in 1939 (see [31] p 57) in an anti-aircraft fire control system. When the target was far away the gain in the system was set to give a smooth tracking. When the target came closer it was necessary to have a faster tracking and this was obtained by increasing the gain in the system.

"Adaptive" became a catch-word and great expectations became associated with it. One hoped that the adaptive systems could be the solution to all problems with time dependence and nonlinearities. Even conventional feedback was sometimes called "passive adaption". One definition of adaptive control originating from Truxal [31] is:

Definition 1:

An adaptive system is one which is designed from an adaptive point of view.

A more precise definition which was suggested early and which now has been a part of the rules of the IEEE Group of System Science and Cybernetic [1] is:

Definition 2:

An adaptive system is provided with a mean of continuously monitoring its own performance in relation to a given index of performance or optimum condition and a means of modifying its own parameters by closed loop action so as to approach this optimum.

The important part of this definition is the facility for automatic adjustment of the controller in closed loop. In connection with adaptive systems learning systems are often mentioned. The difference between adaptive and learning systems will be pointed out by giving:

Definition 3 [1]:

A learning system, in addition to having the capabilities of an adaptive system, must be able to recognize previously occuring control situations and recall the appropriate control actions, learned previously by adaptation.

The difference between adaptive and learning systems is very vague and it is difficult to draw the boundary between the two classes of systems.

Sometimes the term adaptive is given a different meaning than used in this report [3], [11]. In those cases the term adaptive control is used when controlling systems, which are influenced by disturbances, but where the statistical properties of the noise are unknown and have to be estimated and updated while controlling the system. In this report adaptive systems as defined by Definition 2 will be discussed.

The first system to be called an adaptive system was suggested by Draper and Li (1951) [19]. They designed a controller for an internal combustion engine. The task of the control was automatic adjustment of spark timing and fuel mixture to minimize manifold pressure despite changing conditions.

An old and common place adaptive controller is the automatic gain control in radio receivers, where the gain is compensating for changes in the input signal strength.

This report will try to give the background of to why adaptive control may be necessary and will give examples of processes which can be suitably regulated with adaptive controllers. This is discussed in Section 2. Section 3 includes a scheme for classification. The basic features of adaptive controllers is sketched in Section 4. Sections 5 to 7 contain brief descriptions of different controllers found in the literature. As the flora of literature on adaptive control is very large it is impossible to give a complete cover of the whole field. The author of this report surely has missed many good references, but as a first guide for the reader a commented reference list is given in Section 8.

2. WHY ADAPTIVE CONTROL?

In order to motivate further studies in the area of adaptive control we will discuss a couple of processes, which undergo large variations in their dynamics. We can find such processes in many different fields.

Aeroplane

The aeroplane is one of the first systems for which adaptive control was used. Many adaptive controllers which now can be used for different kinds of processes were from the beginning specially designed to solve the regulation problems for high performance aeroplanes.

As an example we will discuss pitchrate control. The pitchrate loop transfer function from elevator deflection to pitchrate can be approximated by:

$$G(s) = \frac{K(s+b)}{s^2 + 2\xi\omega s + \omega^2}$$

For a modern supersonic aeroplane K and ω^2 may vary by a factor 8 in one minute b and $2\xi\omega$ may vary with a factor 2-3 during the same time [47].

For the US fighter F-101B the parameters are given for five different flight conditions in the following table [42]:

Flight condition			Parameters			
No.	Altitude ft.	Speed Mach.	K	ξ	ω	b
1	0	0.2	3	0.39	1.35	0.36
2	0	1.0	44	0.30	8.59	1.67
3	20000	1.0	26	0.23	6.18	0.94
4	45000	1.0	10	0.15	3.67	0.39
5	45000	1.8	18	0.09	5.63	0.38

Table 2.1.

The purpose with the control is to maintain a desired dynamic performance despite a changing environment. It is desirable that the pilot obtains the same response from the plane for all velocities and altitudes. Biomedical investigations have shown that from the pilots point of view the system has a good behavior if the transfer function from desired pitchrate, $\dot{\theta}_r$, to pitchrate, $\dot{\theta}$, has a damping ratio $\xi = 0.7$ and a natural frequency $\omega = 3$ rad/sec.
In order to get a good performance for all flight conditions the parameters in the regulator have to be changed. Some of the flight conditions in Table 2.1 have been simulated using the regulator shown in Fig. 2.1. This regulator has been suggested to the author after discussions with engineers at the aircraft industry SAAB AB, Linköping, Sweden.

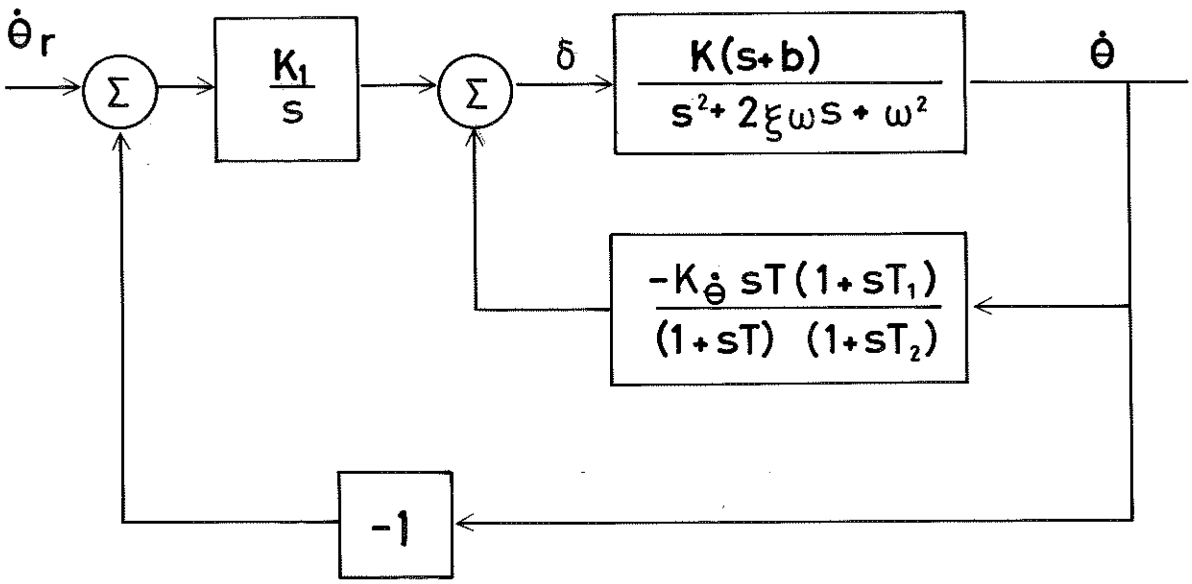

Fig. 2.1 - Pitchrate control system.
$\dot{\theta}$ - pitchrate
$\dot{\theta}_r$ - desired pitchrate
δ - elevator deflection
The parameters in the controller were set to $T=0.5$, $T_1=0.25$, $T_2=0.1$, $K_{\dot{\theta}}=1.25$, $K_1=2$.

The parameters in the controller were tuned to give a good response for flight condition 2 in Table 2.1. The step response for the flight conditions 1, 2 and 5 is given in Fig. 2.2. From the figure is seen that a very good response is obtained for condition No. 2, but when the altitude and the speed is increased then the system becomes too sluggish (No. 5) and when the speed is decreased the response becomes too oscillative (No. 1).

Nuclear reactor

Among other processes suitable for use of adaptive controllers is the nuclear reactor. The dynamic characteristics of the reactor may change, for example as a result of changes in coolant flow rate, control rod configuration, power level or pressure. Other possible causes for changes are isotope build-up, fuel depletion and fuel loading.

The feedback from the nuclear power is nonlinear in such variables as fuel and moderator temperature, coolant flow, void or fission products. In order to achieve a linear description of the dynamics, the nonlinear partial differential equations must be integrated and linearized over the core for every operational condition. It may be sufficient to describe the dynamics with some five state variables, e.g.:

- fuel temperature,
- coolant temperature,
- moderator temperature,
- nuclear power,
- pressure in heat exchange system.

The heavy water boiling reactor in Halden, Norway, is modelled by a fifteen state model. Four of these states are directly measurable. Three or four states are introduced as the difference in valve positions

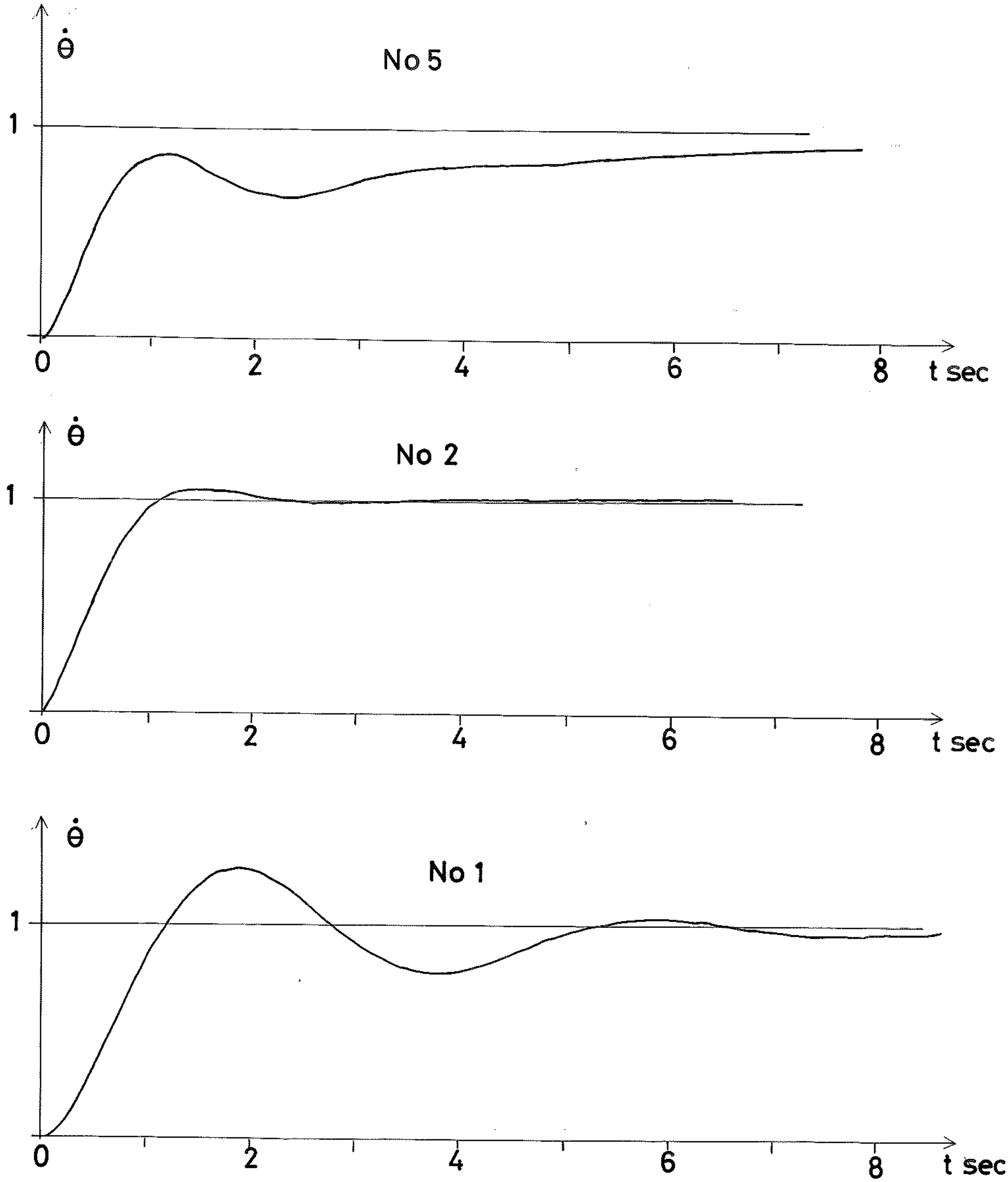

Fig. 2.2 - Step response for flight conditions Nos. 5, 2 and 1 from Table 2.1 when the regulator from Fig. 2.1 has been used.

between two intervals of time. The rest of the states are variables which are not directly measurable such as fuel temperature etc. Identification experiments show that a model of order five or six may be sufficient for the control. Simulations have been carried out using different controllers for different working conditions and to switch between the controllers depending on the actual loading. The change in the dynamics may be very large for different levels of nuclear power. Controllers which give good behaviour at low power can give rise to unstable operation if used at full power. The changes in dynamics may not be continuous but can change drastically e.g. when the water starts boiling [32].

Paper machine

A process of a different kind is paper-making. The final phase in a kraft paper mill is the drying of the paper sheet. After forming the sheet on the wire it is dried by pressing between steamheated reels. The parameters in this process can be drifting. This can be explained by the condensation of water inside the drying reels. The condensate will change the heat transfer constant and thus the grade of drying is altered.
The transferfunction can also change if the speed of the wire is altered e.g. when the paper quality is changed.

Non-linear process

Another example which illustrates the limitations of fixed linear controllers is the simple but yet realistic process in Fig. 2.3. The regulator is a motor which controls a valve with a nonlinear characteristic. For simplicity we assume that the nonlinearity is quadratic, $f(x) = x^2$.

The gain is tuned to give a good step response when x is varying around some working point x_0. If the working point is changed but the value of K is the same as before then the behaviour of the system will be changed significantly. In Fig. 2.4 the gain is determined to give a good response when $x_0 = 5$. In that case K has been chosen to 0.25. If $x_0 = 2$ this gain will give a too sluggish system, and if $x_0 = 20$ the step response will be too oscillating.

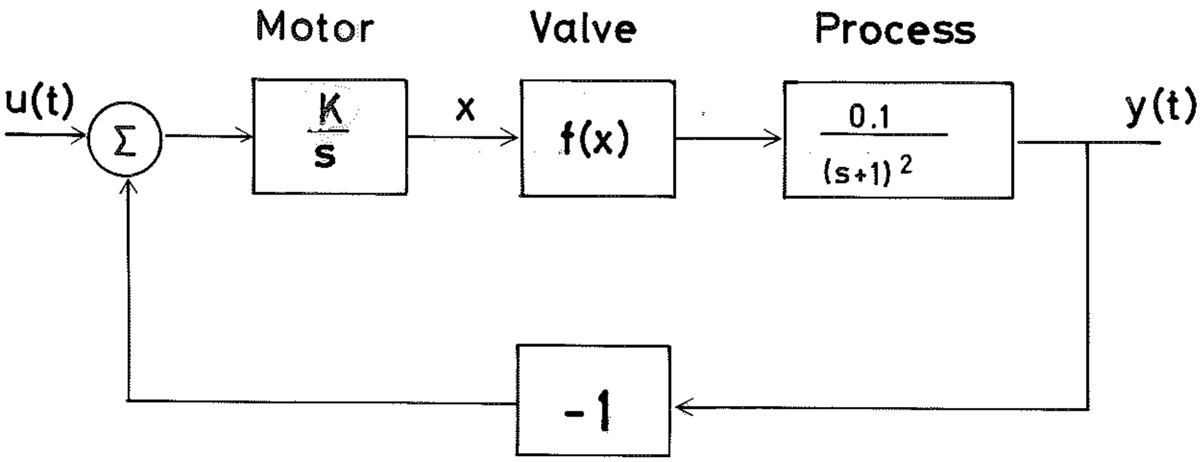

Fig. 2.3

The example clearly shows that a regulator which is good at one working point can yield a poor behaviour if the working conditions are changed. In order to circumvent this an adaptive controller can be used which measures the behaviour of the system in some way and changes the regulator in such a way that the system has an optimal behaviour all the time.

The above discussion gives a few examples of processes for which it may be fruitful to use adaptive controllers. A natural question is the following: Is it possible to obtain the same result using elaborate fixed linear controllers as might possibly be obtained using adaptive control systems?

between two intervals of time. The rest of the states are variables which are not directly measurable such as fuel temperature etc. Identification experiments show that a model of order five or six may be sufficient for the control. Simulations have been carried out using different controllers for different working conditions and to switch between the controllers depending on the actual loading. The change in the dynamics may be very large for different levels of nuclear power. Controllers which give good behaviour at low power can give rise to unstable operation if used at full power. The changes in dynamics may not be continuous but can change drastically e.g. when the water starts boiling [32].

Paper machine

A process of a different kind is paper-making. The final phase in a kraft paper mill is the drying of the paper sheet. After forming the sheet on the wire it is dried by pressing between steamheated reels. The parameters in this process can be drifting. This can be explained by the condensation of water inside the drying reels. The condensate will change the heat transfer constant and thus the grade of drying is altered.
The transferfunction can also change if the speed of the wire is altered e.g. when the paper quality is changed.

Non-linear process

Another example which illustrates the limitations of fixed linear controllers is the simple but yet realistic process in Fig. 2.3. The regulator is a motor which controls a valve with a nonlinear characteristic. For simplicity we assume that the nonlinearity is quadratic, $f(x) = x^2$.

The gain is tuned to give a good step response when x is varying around some working point x_0. If the working point is changed but the value of K is the

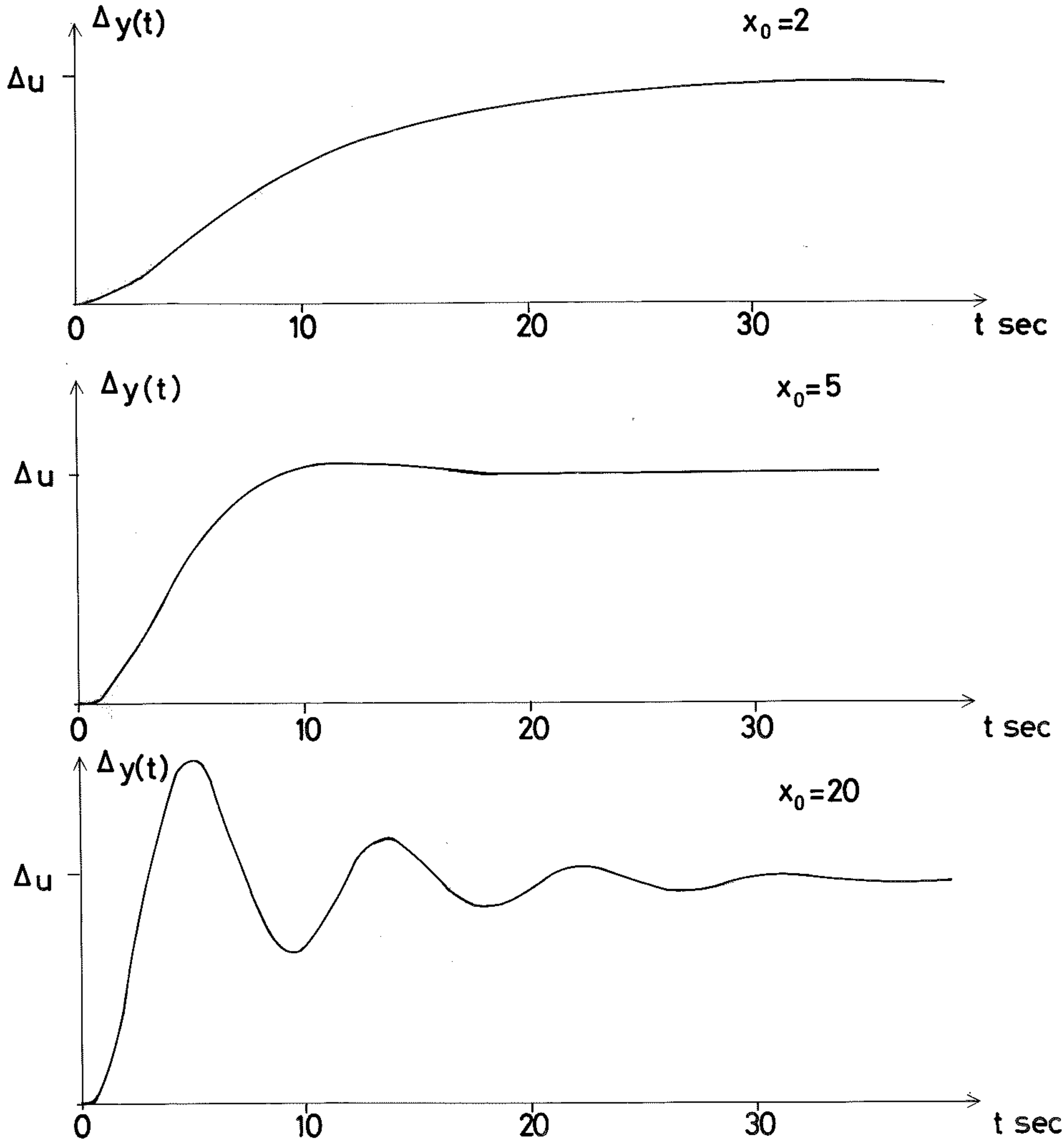

Fig. 2.4 - The change in the output $\Delta y = y(t) - 0.1x_0^2$ from the steady state value is shown for a linearized model of the system in Fig. 2.3. The change in the input is a step of magnitude Δu and the working point x_0 is 2, 5 and 20 respectively.

There has been some criticism of the arguments for introducing adaptive controllers. Horowitz [33], [34], discusses this problem and states that the reasons for using adaptive controllers are not warranted in many cases. The same things can be accomplished using ordinary feedback control. There may be many opinions about this, but there is no doubt that the angel of attack used in adaptive control has given many fruitful results. But on the other hand adaptive control is not yet developed as an easy solution for the control of systems with changing parameters. For example the installation and tuning of the adaptive controller can be difficult. In order to adapt to changes in one parameter in the process, perhaps three or more parameters in the adaptive controller have to be determined. It may on the other hand be very comfortable to install an adaptive controller which can follow parameter variations and thus making it unnecessary to determine the controller for all possible working conditions.

An alternative to adaptive control is a good feedback controller which reduces the influence of parameter changes. Another alternative is to use measurements of the environment and to use these measurements, in open loop, to tune the controller. Examples of such control are the airdata computers used for aeroplanes.

3. CLASSIFICATION OF ADAPTIVE CONTROLLERS.

There are many ways to classify adaptive controllers. We will first refer to two classifications and discuss their relevance. An early survey paper on adaptive control was written by Aseltine, Mancini and Sarture [4]. They used the following five classes:

- Passive adaptation: Systems which achieve adaptation without system parameter changes, but rather through design for operation over wide variations in environment.
- Input signal adaptation: Systems which adjust their parameters in accordance with input signal characteristics.
- Extremum adaptation: Systems which self-adjust for the maximum or minimum of some system variables.
- System variable adaptation: Systems which base self-adjustment on measurements of system variables.
- System characteristic adaptation: Systems which make self-adjustment of transfer function characteristics.

Using Definition 2 in the introduction it is seen that systems of the first class cannot be regarded as adaptive systems. This is because there is no modification of controller parameters. Further, the last two classes are very adjacent and can for simplicity be reduced to one class. With these comments the classification has been reduced to the one given by Levin [43], who suggests separation into input sensing, plant sensing and performance criterion sensing systems.

Since these two classifications were made many things have happened and a more up to date classification is needed. The number of input sensing methods are small. One of the few systems using this method is the above mentioned automatic gain control in radio receivers. We will thus overlook this class. Most of the early adaptive methods can be classified as plant sensing and a few of them as extremum seeking. The main part of "modern" adaptive methods use optimal and stochastic control theory. This will make it necessary to introduce a new class which we will call stochastic adaptive control methods. The following scheme can thus be used for classification of adaptive control systems:

- Plant sensing methods,
- Extremum seeking methods,
- Stochastic adaptive methods.

All these three classes can be divided into subclasses, but we will not elaborate our classification scheme in that direction.

Plant sensing will be classified as methods using some identification scheme to estimate characteristics of the plant, for example, estimation of gain, damping ratio, roots or other parameters in the transfer function. The estimated parameters are then used to tune coefficients in the controller. The task of this tuning can be to hold the overall gain constant or to give the transfer function other desirable qualities.

The extremum seeking methods use a performance index to evaluate the behaviour of the system. The purpose of the control is to reach the minimum or maximum points of the criterion. There are some methods classified as plant sensing which in fact use extremum seeking in one part of the controller. But in those cases the performance index is used only for identification of parameters and is not as a measure of overall behaviour.

The stochastic adaptive methods also use a criterion function in the same way as the extremum seeking methods, but here the system is given in a statistic framework. This makes it possible to regard noise as an integrated part of the system. The use of stochastic optimal control has given new ideas how to treat the adaptive control problem.

4. GENERAL PROPERTIES OF ADAPTIVE CONTROLLERS.

Before discussing different adaptive controllers in detail we will point out some features which are in common for most adaptive controllers. Three major functions can be distinguished:

- Identification of unknown process parameters,
- Decision of control strategy,
- Modification of controller parameters.

An illustration of this is Fig. 4.1, which gives a schematic picture of the philosophy behind adaptive control. The figure can serve as an illustration for simple heuristic methods as well as an illustration for methods based on stochastic optimal control.

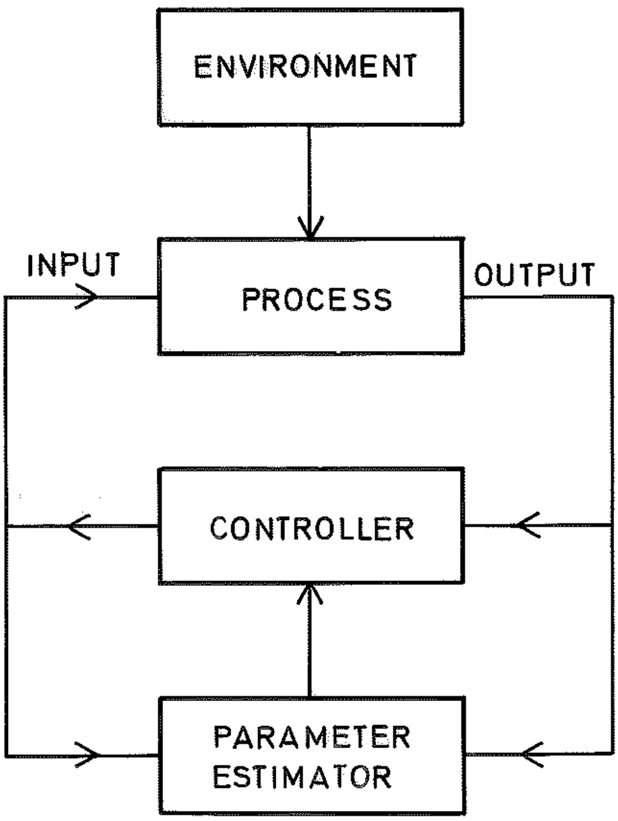

Fig. 4.1 - Schematic block diagram for adaptive controllers.

Separation of identification and control.

A very important question for the structure of adaptive controllers is if there exists a separation theorem. By this is meant that identification can be separated from decision and modification. Many adaptive controllers are constructed from the following heuristic point of view: Make an estimation of one or several parameters and design the controller as if the estimates are equal to the true values. Is the controller thus obtained optimal? The answer is in most cases: No. The reason is that the parameter estimates alone do not form an "information state" or a "sufficient statistic". By this is meant that a state vector exists, that contains all the information about the past that is needed in future steps. Separation is discussed in many papers, e.g. [8], [11], [15].

Identification.

The identification is a problem on two levels. First to determine the structure of the process. By this is meant to determine, for example, if the system is linear or nonlinear, the number of inputs and outputs or the order of the system. Second to determine the unknown parameters in the model given by the structure. The identification used in adaptive control works on the second level. The structure has to be obtained either using prior knowledge of the physical properties of the process or through real-time identification experiments. The identification experiments have to be done several times until a suitable structure of the model is obtained. Real-time identification algorithms are discussed e.g. in [67].

There is a conflict between real time identification and the control. When identifying it is desirable to have rather large output and input signals in order to obtain a good signal to noise ratio, but from the view point of control the task may be to have as small variations as possible in the output signal. This conflict may give rise to a couple of phenomena when using adaptive controllers. One of these phenomena is "burst" [69]. By this it is meant that the controller suddenly can start an oscillation in the system, but after a short while the control returns to normal conditions again. This can be explained in the following way: Assume that the parameters in the process start drifting and that the gain in the adaptive loop is small. The estimator may not notice the parameter drift until the system starts oscillating after which the estimator can rapidly improve the parameter estimates and again it is possible to achieve good control.

Another phenomenon is "turn-off" [8], [14], [70]. Here the control unintentionally may be turned off for longer or shorter periods of time. Also this phenomenon can be explained by the fact that the estimator does not receive sufficient information about the changes in the process parameters.

Performance criterion.

The performance criterion has been mentioned as an essential part both in identification and decision. We will now further discuss this concept and give examples of criteria which can be used. For a more thorough penetration of different performance criteria we refer to Eveleigh [23].

There are many expressions used as synonyms to performance criterion (PC). Examples are: performance index (PI), index of performance (IP), figure of merit (FM), quality measure, loss function.

As a definition of IP we can use:

> A <u>performance criterion</u> is a measure of system characteristics which can be used to determine optimal control operations.

A simple IP might be to count the zero crossings in the impulse response. This number can be used to determine if the gain is too high or too low. A more complex criterion is to minimize the expectation of the output variance.

Notice that the control becomes optimal only for the particular performance index used. This makes it very difficult to choose the performance index, because one cannot be sure that another choice of criterion would not give the system a more satisfactory behaviour. This makes the choice of IP more an art than a science.

The performance criterion can be classified as odd or even. By even is at this point meant that the function is unimodal. The two types are sketched in Fig. 4.2 where the performance index J is shown as function of a single control variable x. The optimal value of x is denoted x_0.

Using an odd criterion has several advantages. To determine in which direction the control variable shall be changed it is sufficient to make a single measurement. This makes it possible to use a conventional control loop to adjust the variable x. Odd criteria is difficult to use when two or more parameters shall be adjusted because the parameters can in most cases not be adjusted independently.

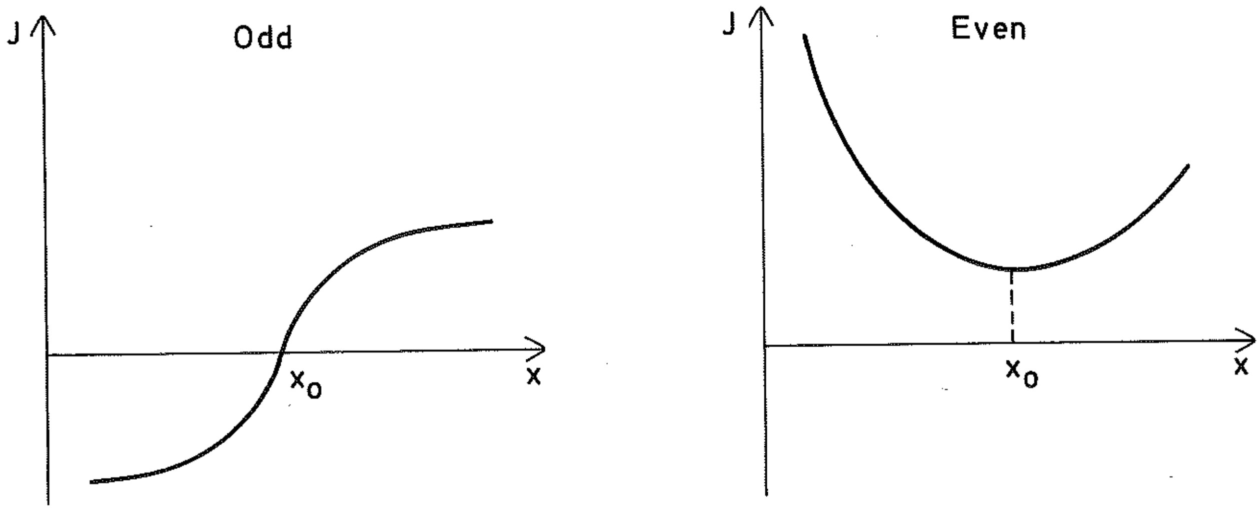

Fig. 4.2 - One dimensional odd and even performance criteria, J, as function of one control variable, x.

There are essentially two ways to determine the direction of changes in the control variable when using an even criterion. First using a gradient method. Second to make two or more measurements and use the differences for direction determination.

As an example of an odd criterion we can choose the impulse response area ratio which is used in the Aeronutronic autopilot [31, p. 349]. The areas A^+ and A^- are defined in Fig. 4.3.

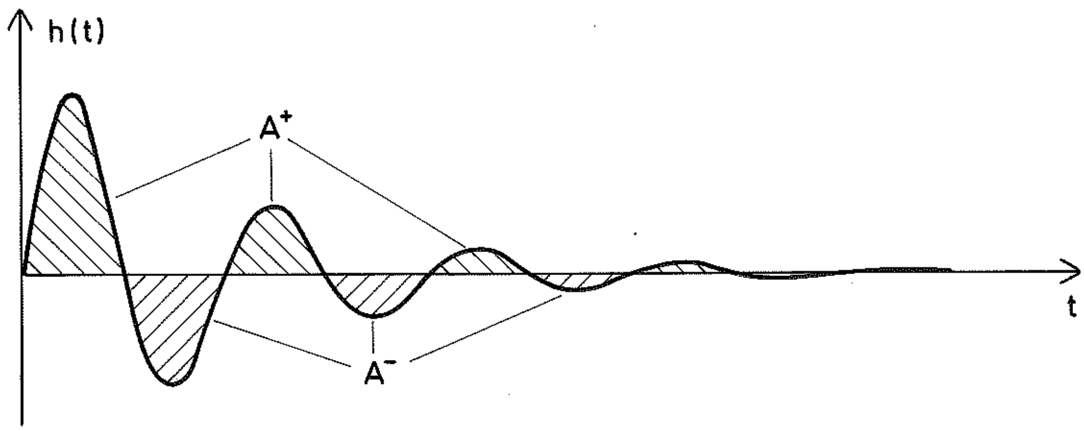

Fig. 4.3 - Typical impulse response for the second order system

$$G(s) = \frac{\omega^2}{s^2 + 2\xi\omega s + \omega^2}$$

For a second order system it can be shown that

$$\frac{A^+}{A^-} = \exp \frac{\pi\xi}{\sqrt{1-\xi^2}}$$

The criterion can be chosen as

$$J = A^+ - kA^-$$

where k is a positive constant.
The integral square error

$$\int e(t)^2 dt \tag{4.1}$$

and the integral absolute error

$$\int |e(t)| dt \tag{4.2}$$

are examples of even performance criteria. The integral square error is the most frequently used even performance index. This because the analysis is much easier than when using other more complex functions. When handling processes with noise the minimization of the mean square error is almost the only criterion used. It should also be pointed out that the value of the performance index depends on the initially stored energy in the system and on the applied command input. The influence of the command input can be decreased by normalizing the performance index with a factor that depends on the energy exciting the system [56]. For example the performance index can be chosen to:

$$J = \frac{1}{N} \int_0^T e(t)^2 dt$$

$$N = \int_0^T u_r(t)^2 dt$$

where $u_r(t)$ is the command input to the system.

To see the difference between different performance criteria we will use the system in Fig. 4.4 and the criteria (4.1) and (4.2) to compute the optimal step response by changing the location of the pole -a.

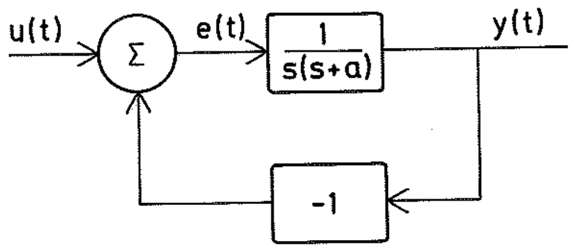

Fig. 4.4

For the integral square error criterion the minimum is obtained for a = 1 and for the integral absolute error criterion for a = 1.4. But the criteria do not have sharp minima. When using the integral square error criterion an increase of a from 1.0 to 1.4 will only increase the value of the integral with 6%.

The step response for the two cases is shown in Fig. 4.5. When using integral square error the response will be too lightly damped while the integral absolute error will give a "better" response. By better is meant that it corresponds more closely to what is called a good step response.

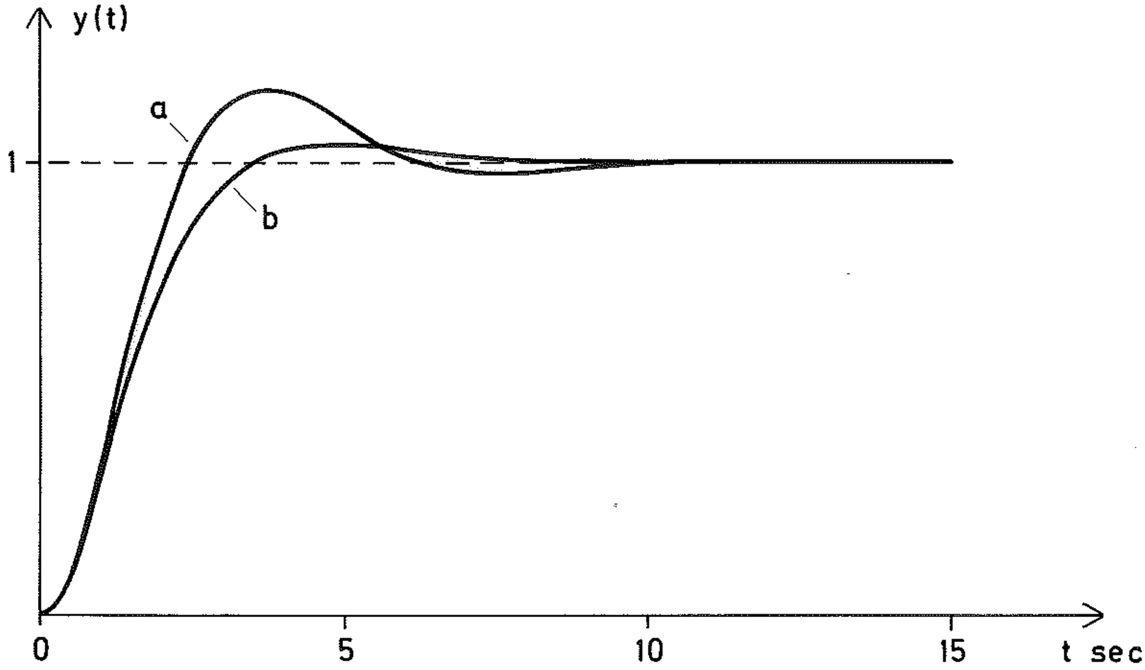

Fig. 4.5 - Optimal step response for the system in Fig. 4.4 when using

a. Integral square error criterion
b. Integral absolute error criterion.

Stability.

Most adaptive controllers have a disadvantage in common: It is very hard and almost impossible to analyse the stability properties of a system regulated by an adaptive controller. This is because nonlinearities are involved partly in the identification and partly in the modification parts of the controller. Thus most regulator schemata are derived in a heuristic way and stability is investigated for systems with constant parameters. The results then are generalized to slowly varying parameters.

Stability can be investigated for simple regulators when a few parameters are constant but unknown and the input to the process is simple, e.g. step, ramp or sinusoidal. This type of analysis is sometimes called "adaptive step analysis", i.e. there is an initial offset in the parameters and the path of adaption is investigated. But in most cases simulations are the only way to test the stability of the system. This has serious limitations because one cannot be sure that the worst cases have been treated.

One of the few adaptive systems which can be designed to be stable is the model reference method [66]. The stability of this system can be ensured using Liapunov theory. A brief resumé of the Liapunov theory for continuous as well as discrete systems is given by Kalman and Bertram in [37], [38]. An analysis of the model reference method has been given e.g. by Parks [51]. This will further be discussed in connection with the model reference method in Section 5.

In recent years there has been much research in the area of stability. Methods other than Liapunov-like methods have been developed, see e.g. Popov [58]. These methods treat in most cases a linear system with a nonlinear feedback. Criteria on the nonlinearity are given which ensure stability of the closed loop system. These results have been used by Landau [41] to analyze the model reference method, see Section 5.2.

5. PLANT SENSING METHODS.

In this section some plant sensing methods will be described in more detail. The plant sensing methods are here separated into the three classes:

- Indirect methods,
- Model reference methods,
- Learning model methods.

These three classes correspond to three common ways of designing adaptive controllers.

5.1 Indirect Methods.

One of the first designed and used adaptive controller is the Minneapolis-Honeywell (M-H) adaptive regulator [31, p. 123]. This controller was designed for high performance aeroplanes and has been proven in test flights on the US aeroplane F-94C and a more elaborate version has been used on F-101 and X-15. This regulator is chosen to examplify the indirect methods. A simplified block scheme for this type of controller is given in Fig. 5.1.

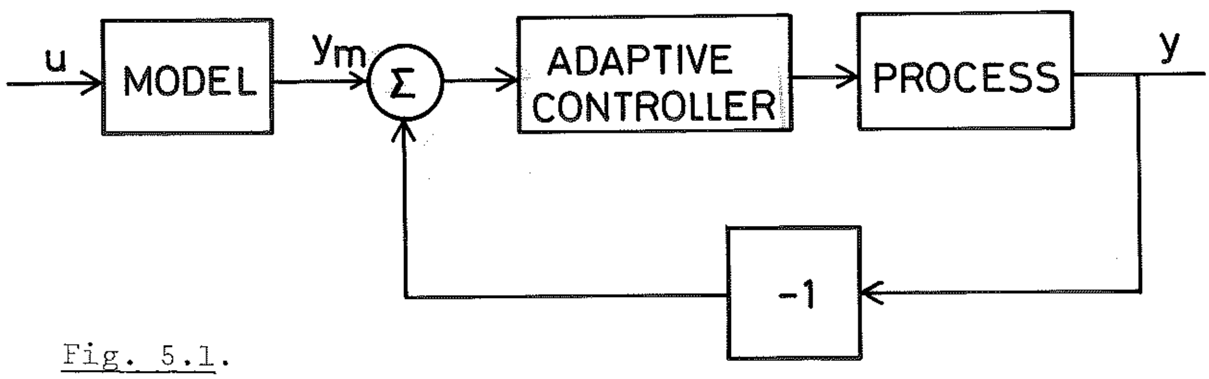

Fig. 5.1.

The input u is fed through a model with desired characteristics. The desired output is thus the signal y_m. The closed loop part is now designed to give as quick response as possible to changes in y_m. The block "adaptive controller" is used to give as high gain as possible in the forward loop. The objective is to make the transfer function from y_m to y as equal to one as possible. A way to increase the gain and still ensure stability is to use a relay in the adaptive controller. The nonlinearity will introduce a limit cycle into the system. But the limit cycle can be neglected in the over all behaviour of the system if the amplitude is small and the frequency is high. In the M-H system this is ensured by controlling the amplitude of the limiter. A more detailed drawing of the M-H adaptive controller is given in Fig. 5.2.

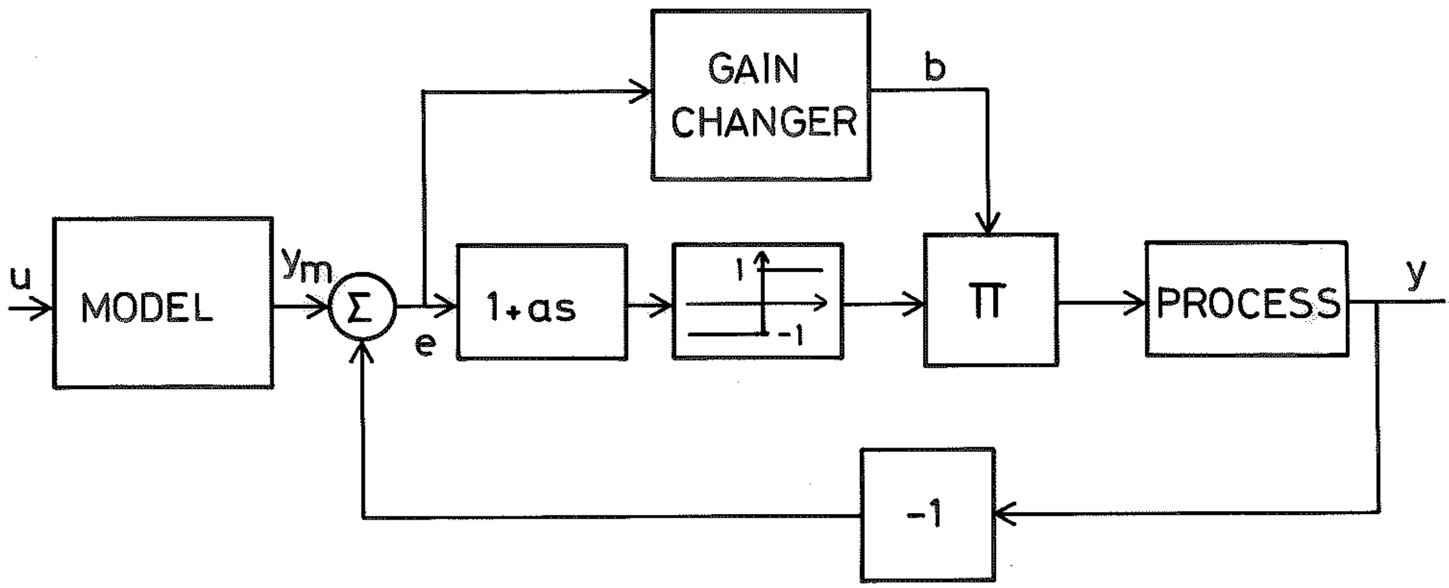

Fig. 5.2 - Block diagram of the Minneapolis-Honeywell adaptive controller

The signal $e + a \frac{de}{dt}$ is used to determine the sign of the output from the relay. The function of the gain changer is to change the absolute value, b, of the input to the process. As even a small error can give a large input signal to the process the forward loop gain will be high and thus the signal y can be almost equal to y_m.

As an application of the M-H regulator the following system will be used [13], [16].

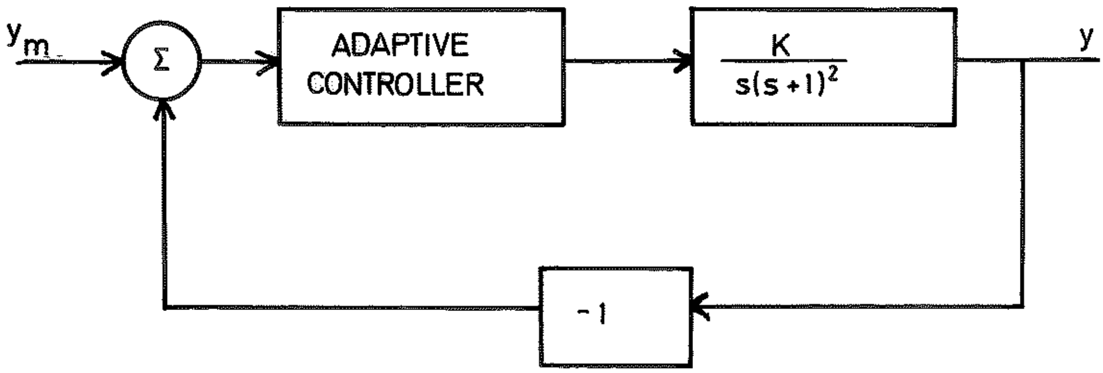

Fig. 5.3 - Adaptive control of the system

$\frac{K}{s(s+1)^2}$ where K is time varying.

The adaptive controller is the Minneapolis-Honeywell regulator, compare fig 5.2.

Without the adaptive controller the closed loop system is stable for $0 \leq K \leq 2$. Let the reference signal y_m be sinusoidal and let the gain, K, vary in the interval (0.5,10). Result from simulation of this system is shown in Fig. 5.4. There is a good tracking of y_m despite the gain is changing by a factor 20.

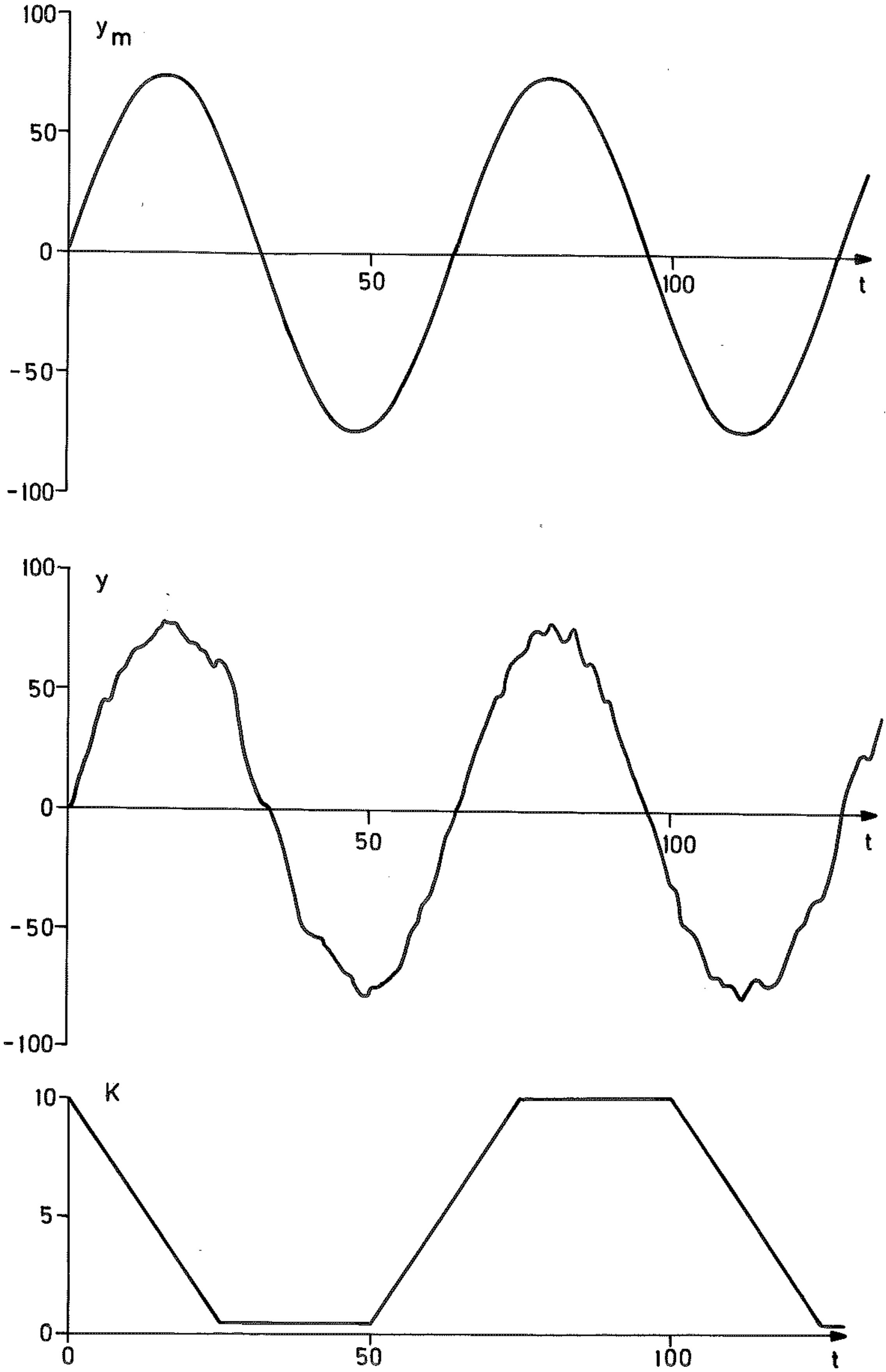

Fig. 5.4 - Desired, y_m, and actual, y, outputs for the system in Fig. 5.3 when the gain, K, is varying (from [13]).

The Minneapolis-Honeywell adaptive controller is reported to give good performance of the over all system. Application to aeroplane and to roll autopilot for a missile are discussed in [31, p. 123] and [62] respectively. The controller is suitable for systems which will be unstable for high gain and for which the influence of the limit cycle can be neglected. The response to changing parameters is quick. Another advantage is that the mechanization is simple and reliable. A disadvantage is the limit cycle which can cause wear.

5.2. Model Reference Methods.

The model reference method [22], [46], [55], [66], is often called the MIT adaptive controller or Whitaker model reference system.

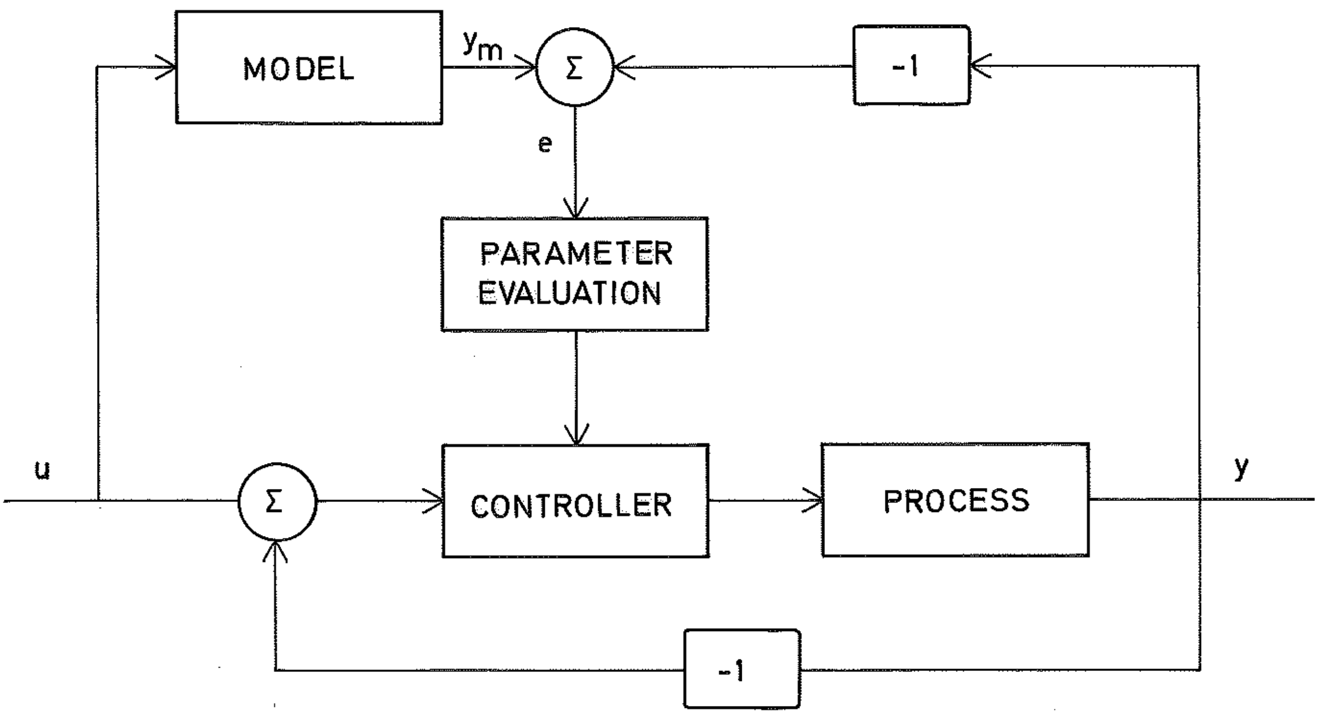

Fig. 5.5 - Block diagram of the model reference method.

In order to keep the over all characteristics of the system as near a desired optimum as possible the input signal u is fed into a model as well as into the system. The model has the desired properties and an error signal, e(t), is obtained by comparing the output from the model y_m with the output from the process, y. The error signal is used to regulate the parameters in the controller in such a way that the behaviour of the model and the system will be the same. The process can as in Fig. 5.5 be in closed loop or in open loop. There are many ways to use the information in the error signal to alter the parameters in the controller. One way, called the "MIT rule", is to minimize the integral squared error:

$$J = \int e(t)^2 dt$$

As an example consider a second order system with time varying gain [51]:

$$G_p(s) = \frac{K_v(t)}{s^2 + a_1 s + a_2}$$

The parameters a_1 and a_2 are assumed to be known and K_v is unknown. Let the purpose be to keep the over all gain constant equal to K. The model will thus be

$$G_m(s) = \frac{K}{s^2 + a_1 s + a_2}$$

and the controller is simply a gain constant K_c. Minimization of the integral squared error yields that the gain in the controller, K_c, shall be changed according to

$$\dot{K}_c = - Be \frac{\partial e}{\partial K_c}$$

where B is a positive constant which has to be chosen.

The partial derivative of e with respect to K_c is proportional to $-y_m$ and we get

$$\dot{K}_c = B'ey_m$$

The process and regulator will thus be as in Fig. 5.6.

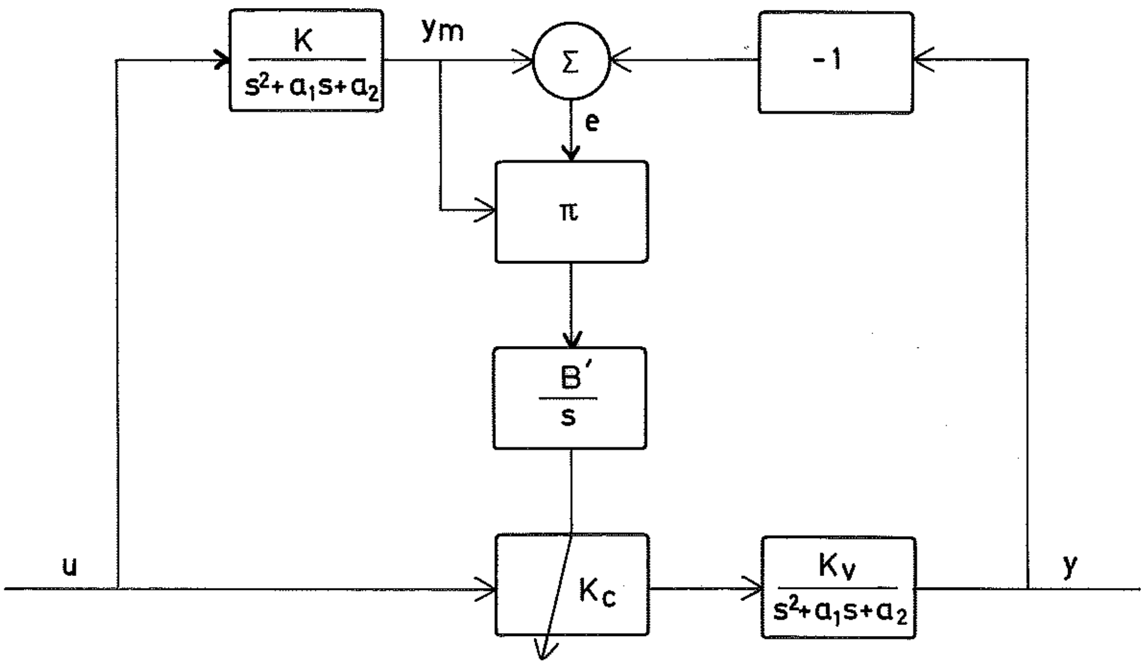

Fig. 5.6 - The model reference method using the MIT rule applied to a second order system with time varying gain.

As for many adaptive regulators it is difficult to say anything about the stability of the regulated process when using the MIT rule. For simple processes and simple inputs it is possible to decide which values of the parameter B lead to a stable system. In order to overcome this Liapunov theory has been used to derive regulators which will give stable systems [29], [49], [51].

The technique of Parks [51] will be used on the example above when K_v is constant but initially $K_v K_c \neq K$.

The system is of second order and the Liapunov function, V, is chosen to be quadratic in e, $\dot{e}$ and $x = K - K_c K_v$:

$$V = a_1 a_2 e^2 + a_1 \dot{e}^2 + \lambda x^2$$

Then

$$\frac{dV}{dt} = -2a_1^2 \dot{e}^2 + 2a_1 \dot{e} x u + 2\lambda x \dot{x}$$

The time derivative of the Liapunov function will be negative if

$$\dot{x} = -K_v \dot{K}_c = -\frac{a_1}{\lambda} \dot{e} u$$

or

$$\dot{K}_c = \frac{a_1}{\lambda K_v} \dot{e} u$$

This will give an asymptotically stable system, which was not guaranteed when using the MIT rule. But the equation now obtained has some drawbacks. First it contains the unknown gain K_v which thus has to be estimated. Second it contains the derivative of the error signal. By using a lemma of Kalman conditions can be given when it is possible to avoid the use of derivatives [51]. The Liapunov technique used by Parks can be extended to processes with time varying gain (see paper by Monopoli et al [49]). The convergence can be improved by using more elaborate Liapunov functions [29].

The model reference method is also possible to use when more than one parameter is varying, but the interaction between the different loops is a problem as it slows down the speed of adaptation. Multi-variable model reference systems are discussed in [59], where a Liapunov approach similar to Parks' is used.

An alternative way to adjust the parameters than those discussed above is taken in [20] and [40]. In these references sensitivity analysis is used to derive the equations for the parameter adjustment. More general results on the stability of model reference methods are recently given by Landau [41]. These new results are obtained by using Popov's results in the field of hyperstable systems [58]. In [41] a theorem is given which includes Parks' results with Liapunov functions as well as Dressler's results [20] with sensitivity analysis.

5.3. Learning Model Methods.

This section will treat systems using a mathematical model with adjustable parameters. The model is used to identify the parameters of the process real time. The main part of the suggested methods uses a hill-climbing method to adjust the model. The estimated

parameters are used to set parameters in a controller. The purpose of the control is often to give the response from the process certain qualities as a given damping ratio, raise time etc. These methods are clearly based upon the hypothesis of a separation of identification and control.
The method is examplified by a system, given by Margolis and Leondes [44], [45].

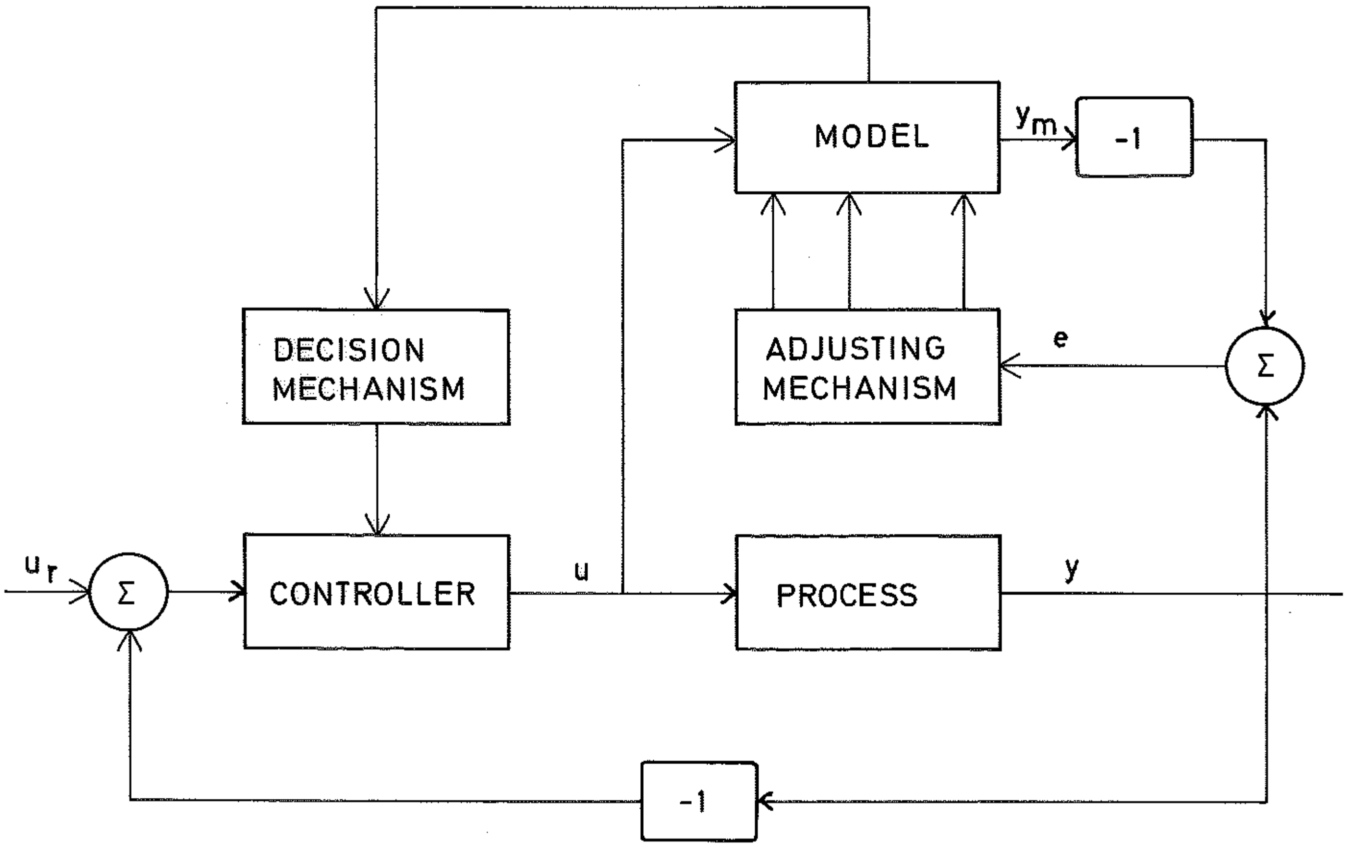

Fig. 5.7 - Block diagram of the learning model method.

By comparing the output from the process, y, with the output from the model, y_m, an error signal, e, is obtained. The error signal is used to adjust the parameters in the model.

Let the process be described by a linear differential equation with time varying coefficients

$$\sum_{i=0}^{n} a_i(t) \frac{d^i y}{dt^i} = \sum_{i=0}^{n} b_i(t) \frac{d^i u}{dt^i}$$

and the model

$$\sum_{i=0}^{n'} \hat{a}_i(t) \frac{d^i y}{dt^i} = \sum_{i=0}^{n'} \hat{b}_i(t) \frac{d^i u}{dt^i}$$

where

u(t) - input to the process and model,
y(t) - output from process
$y_m(t)$ - output from model

If the model is synthesized in a digital computer the equations above are substituted by linear difference equations.

As always in identification there is a problem: How to choose the order of the model. We overlook this difficulty and use n = n'.

The error

$$e(t) = y(t) - y_m(t)$$

is used to determine how the parameters shall be adjusted. An appropriate loss function which has a minimum when $\hat{a}_i = a_i$ and $\hat{b}_i = b_i$ is chosen. An example of such a loss function is

$$\ell(e,\dot{e},\ldots) = (e + c_1\dot{e} + c_2\ddot{e} + \ldots)^2$$

Using a gradient method to minimize the loss we get the following equations

$$\frac{d\hat{a}_i}{dt} = - k_i \frac{\partial \ell}{\partial \hat{a}_i}$$

$$\frac{d\hat{b}_i}{dt} = - k_i' \frac{\partial \ell}{\partial b_i}$$

This requires evaluation of quantities like $\frac{\partial \dot{e}}{\partial a_i}$ which is conveniently done through the filters for the sensitivity derivatives [44], [45], [71].

There is one degree of freedom in the choice of the coefficients k_i and k_i'. These constants have a great influence on the stability and the speed of the parameter tracking. Too small values will give a too sluggish tracking. Too large values will make the tracking unstable.

To illustrate how to choose the gain parameters in the tracking algorithm a discrete time system with constant but unknown parameters will be discussed. Let the system be

$$\begin{cases} x(t+1) = Ax(t) + Bu(t) \\ y(t) = x(t) \end{cases}$$

where

x(t) - state vector of order n×1

A - unknown matrix of order n×n

B - unknown vector of order n×1

u(t) - input of order 1×1

y(t) - output vector of order n×1

Here it is assumed that the whole A and B matrices are unknown. In practice it is possible to introduce same structure of the matrices in order to decrease the number of unknown parameters.

The model used is

$$\hat{x}(t+1) = \hat{A}x(t) + \hat{B}u(t)$$

where $\hat{x}$, $\hat{A}$ and $\hat{B}$ are of the same order as x, A and B respectively.

Use the loss function

$$\ell = \frac{1}{2}\tilde{x}^T\tilde{x} = \frac{1}{2}(x-\hat{x})^T(x-\hat{x})$$

The equation for the error, $\tilde{x}$, will be

$$\tilde{x}(t+1) = x(t+1) - \hat{A}x(t) - \hat{B}u(t) =$$

$$= \left(A - \hat{A}(t)\right)x(t) + \left(B - \hat{B}(t)\right)u(t)$$

Partial derivation of ℓ with respect to the coefficients $\hat{a}_{ij}$ and $\hat{b}_i$ give

$$\frac{\partial \ell(t+1)}{\partial \hat{a}_{ij}(t)} = - \tilde{x}_i(t+1)x_j(t)$$

$$\frac{\partial \ell(t+1)}{\partial \hat{b}_i(t)} = - \tilde{x}_i(t+1)u(t)$$

To update the estimates of A and B the following equations are used

$$\hat{a}_{ij}(t+1) = \hat{a}_{ij}(t) + k\tilde{x}_i(t+1)x_j(t)$$

$$\hat{b}_i(t+1) = \hat{b}_i(t) + k\tilde{x}_i(t+1)u(t)$$

Notice that the same gain k is used for all parameters. The question is how to choose k. The residuals $\tilde{x}(t+1)$ are due to non-exact or incorrect estimates, but if the updated values of A and B are used the new residuals would be zero by an appropriate choice of k, i.e.

$$\tilde{x}'(t+1) = x(t+1) - \hat{A}(t+1)x(t) - \hat{B}(t+1)u(t)$$

$$= x(t+1) - \hat{A}(t)x(t) - \hat{B}(t)u(t) -$$

$$- k\left(\tilde{x}(t+1)x(t)^T x(t) + \tilde{x}(t+1)u(t)^2\right)$$

$$= \tilde{x}(t+1)\left[1 - k\left(x(t)^T x(t) + u(t)^2\right)\right]$$

The updated residuals $\tilde{x}'$ are made zero choosing

$$k = \frac{1}{x(t)^T x(t) + u(t)^2}$$

It can be shown that this choice of k will give convergence of $\hat{A}$ and $\hat{B}$ to the true values A and B. Denoting the i:th row in the A and $\hat{A}$ matrices by A_i and $\hat{A}_i$ respectively we have

$$\left(\hat{A}_i(t+1) - A_i\right)\left(\hat{A}_i(t+1) - A_i\right)^T + \left(\hat{b}_i(t+1) - b_i\right)^2 =$$

$$= \left(\hat{A}_i(t) - A_i + k\tilde{x}_i(t+1)x(t)^T\right) \cdot$$

$$\cdot \left(\hat{A}_i(t) - A_i + k\tilde{x}_i(t+1)x(t)^T\right)^T +$$

$$+ \left(\hat{b}_i(t) - b_i + k\tilde{x}_i(t+1)u(t)\right)^2 =$$

$$= \left(\hat{A}_i(t) - A_i\right)\left(\hat{A}_i(t) - A_i\right)^T +$$

$$+ \left(b_i(t) - b_i\right)^2 - k\tilde{x}_i(t)^2$$

The left hand side is always greater than or equal to zero. The last term on the right hand side is always nonnegative hence if the system is persistently excited

$$\hat{A}_i \rightarrow A_i$$

$$\hat{b}_i \rightarrow b_i$$

Using this special choice of k will thus give convergence to the true values in the case with constant but unknown parameters. This way of choosing k originated from Kaczmarz who used this type of algorithm to solve large systems of linear equations. For further details we refer to [10 pp. 450 - 455].

For nonconstant matrices it is very hard to say anything about the stability or convergence. For the case with specific inputs, e.g. steps, sinusoidal and few parameters the stability problem is examined in [45].

In [52] the learning model identification for continuous time systems is studied using Liapunov theory. Through an appropriate choice of Liapunov function the stability of the parameter identification can be ensured.

A rule of thumb is given in [44] for the relation between process characteristics and the ability to follow changes in parameters. For the adjustment of one parameter a period of two time constants of the process is required. For two parameters the time is increased by four to nine time constants. An advantage with the learning model technique is the relative easy mechanization of the parameter tracking servo. Further, the identification does not need any perturbation signal applied to the system, but in that case the natural input to the system must excite the system all the time, otherwise all parameters cannot be tracked. A drawback is the difficulty to follow rapid changes and even slow changes if more than one parameter is adjusted. Concerning the stability of the closed loop adaptive system very little can be said. Even if it can be shown that the identification algorithm converges it is not necessary that the closed loop system will be stable. This is for example reported in [60] where a system with time varying gain is investigated. In that case it was impossible to get the closed loop system stable. For some special cases it is possible to prove this instability.

6. EXTREMUM SEEKING METHODS.

In earlier sections some adaptive controllers have been discussed which used an even performance index for the identification part, but the even criterion has not been used to control the over all behaviour of the system. Adaptive controllers which use an even performance index to optimize the over all behaviour of the system will be called extremum seeking. The extremum seeking methods will be separated into two classes. First controllers using an intentional perturbation signal to determine how to change the input to the system or the parameters in the controller. Second controllers using gradient or hillclimbing methods to drive the system to the optimum.

6.1. Perturbation Methods.

Systems using an intentional perturbation signal are discussed in many papers. See e.g. [18], [19], [39]. An even performance index is used to change a parameter or a control signal. The control variable is given a small periodic perturbation and the performance index is used to drive the system to its optimum. The perturbation is for example a sinusoidal or a square wave.

Let the performance index be $J(x)$ with minimal value for $x = x_0$, where x is the controlled variable. The minimum point, x_0, may be time varying. The actual value of the parameter x is x_a. The parameter is now perturbed around $x = x_a$

$$x = x_a + A \sin \omega t$$

Expand $J(x)$ around $x = x_a$

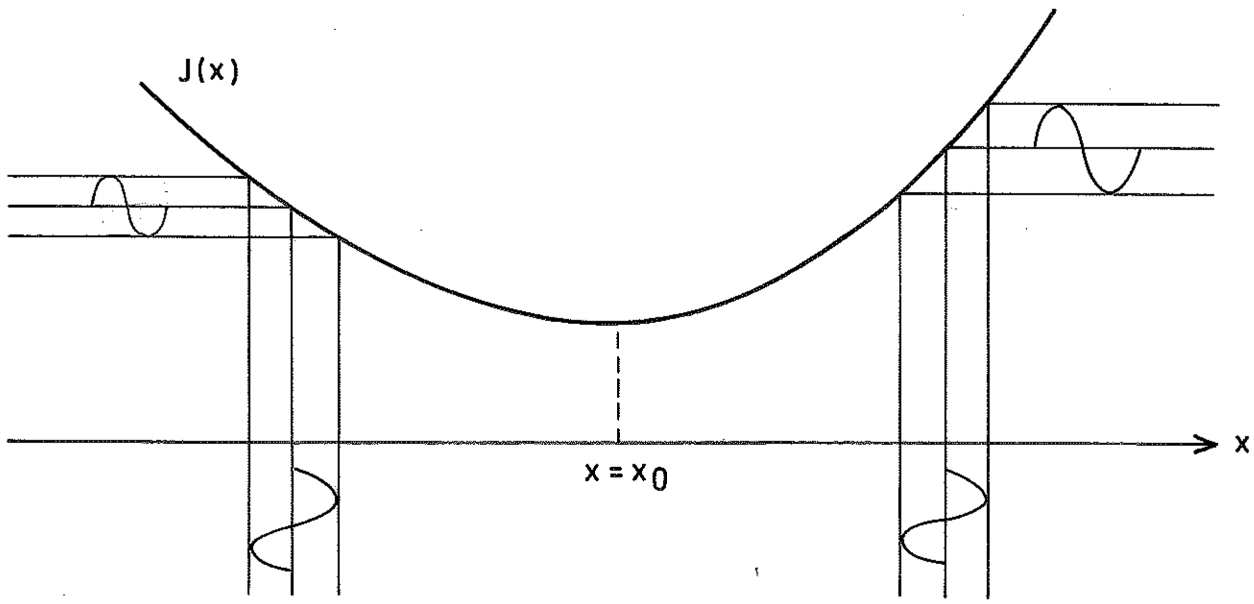

Fig. 6.1 - Perturbation of x will give different phase in the output J(x) depending on if x is less or greater than x_0.

$$J(x) = J(x_a) + \left.\frac{\partial J}{\partial x}\right|_{x=x_a} \cdot A \sin \omega t + \left.\frac{\partial^2 J}{\partial x^2}\right|_{x=x_a} \cdot A^2 \sin^2 \omega t$$

The output from the performance computer is fed through a bandpass filter with center frequency $\omega_c = \omega$. The signal out from the filter will thus have an amplitude proportional to the derivative of the performance index at the point $x = x_a$. A demodulator will give a signal proportional to $\partial J/\partial x$. The sign of this signal indicates if x_a is greater or smaller than x_0. The absolute value gives a measure of how far the actual value is from the optimum. The parameter x is now changed until $x_a = x_0$. If the optimum point is varying the parameter will be adjusted in order to follow the moving optimum point.

A block scheme for adaptive regulation of one parameter is shown in Fig. 6.2.

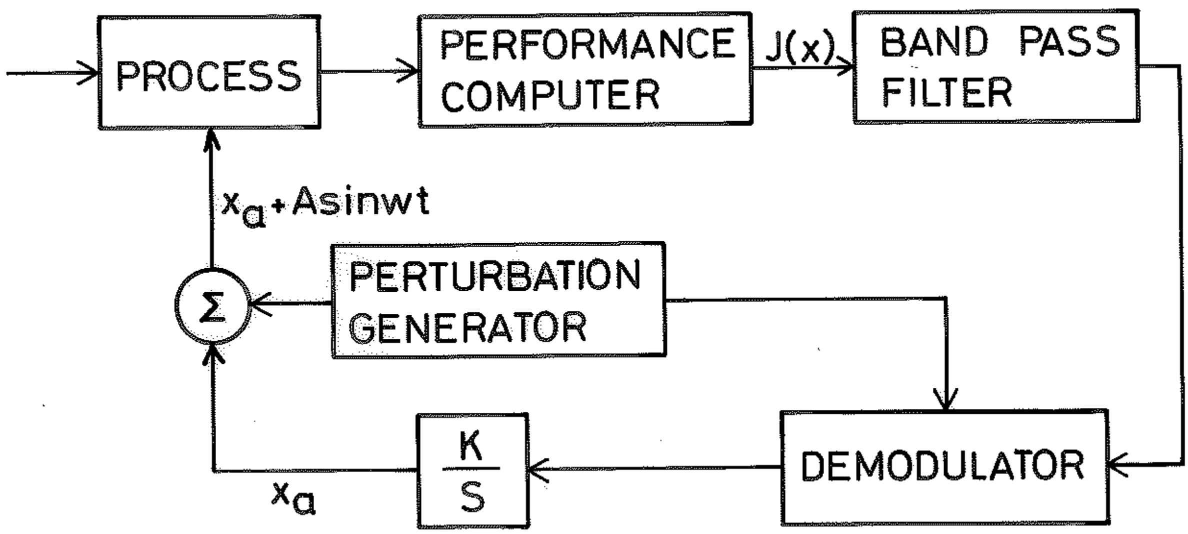

Fig. 6.2 - Adaptive controller with intentional perturbation signal.

The perturbation signal adaptive method can be used for a wide range of processes, e.g. internal combustion engine [19], roll autopilot [63], chemical reactor [39].

The system of Draper and Li [19] is one of the first reported adaptive systems. This system was designed for the optimum control of an aeroplane engine. The controlled variables are fuel-air ratio and ignition timing. The purpose of the control is to maximize the break mean effective pressure. This output as function of fuel-air ratio and spark timing form a two-dimensional surface with one extreme point. The optimum point can move when the environment changes. In order to follow the moving optimum point the input signals are perturbated and the reference values are changed until the extremum point is reached. This will ensure that the process works under optimal conditions all the time.

If more than one parameter or input signal shall be adjusted one has to assure that the interaction between the different channels can be neglected. Because of the interaction this type of controller is mostly discussed for one or two parameters, but a six-channel adaptive controller is discussed in [17]. In the sinusoidal case the signals are separated in frequency. Another way to decouple the different channels is to use independent noise. E.g. a PRBS signal can be used [18]. If the period is sufficiently large and if the PRBS signal is shifted half a period one gets two signals which can be regarded as uncorrelated and can thus be used to control two parameters.

6.2. Gradient Methods.

A way to find the extremum point of an even performance index is to use gradient or steepest descent methods. In this case an algorithm for finding the minimum can be:

1. Give an initial value to the parameter vector, x.
2. Measure and store the performance index value J(x) and its gradient $\frac{\partial J}{\partial x}$.
3. Change the parameters with a vector along the opposite direction of the gradient, $x \rightarrow x - k \frac{\partial J}{\partial x}$.
4. Repeat from 2 as long as the performance index is significantly reduced.

The difference between the various gradient methods are in the way of changing the parameter vector. For example one parameter can be changed at each time or the whole parameter vector can be changed at the same time. Other differences can be that the change in the parameter vector has a fixed step length or the gradient is used to determine the length of the step. The convergence and stability of the gradient methods depend critically upon the step length. Too small a step length will give slow convergence rate and may give an unsatisfactory behaviour of the system if the optimum point is time varying. On the other hand too large a step length can cause instability.

There are two commercially available controllers using this type of adaptive optimization. The Opcon controller [28] is built by Westinghouse Electric Corporation and the Varitrack controller [2] by Motorola Control Systems Ltd. The algorithms used are in principle the same: Measure the performance of the system over an interval of time and alter the set point values of the control variables. In the Opcon the stepsize is constant until an optimum is reached, then the stepsize is reduced. The Varitrack works in a similar way and adjusts the control variables to follow the path of steepest descent or ascent within the limits given by constraints on the set point values, for example. In both controllers the control variables are changed in a cyclic manner even after the optimum is reached. This facilitates following a drifting optimum point.

Several papers discuss gradient method controllers. Feldbaum [24] and Stakhovskii [61] measure the gradient in discrete times and change the parameters with a step length which depends on the norm of the gradient vector. In [50] a self-optimizing system is discussed which continuously changes the parameters in order to minimize a mean square criterion. The parameters are changed using an error gradient which is computed by cross correlation methods.

We will further discuss one gradient method, given in [53], [56]. The control system is assumed to be represented by the equations:

$$\begin{cases} \dot{x} = f(x,u,\gamma,\omega,t) \\ y = Cx \end{cases}$$

f is a known vector-valued function which depends on the state of the system, x, the control signal, u, the adjustable parameters in the controller, γ, and a collection of unknown process parameters, ω. Even if the function f is known the behaviour of the system is not known as f depends on the unknown parameters, ω. The parameter vector, γ, is changed in discrete times in order to minimize an integral performance index

$$P = \int_{t_i}^{t_{i+1}} G\big(y(t),t\big)dt$$

Now γ is changed as

$$\gamma_{i+1} = \gamma_i - \mu S_i(\gamma_i)$$

where $S_i(\gamma_i)$ is an approximation of the true gradient of P and μ is a positive scalar. With a proper choice of the adaptive loop gain, μ, and the length of (t_i,t_{i+1}) this algorithm can be used for adaptive control of linear as well as nonlinear systems. The time intervals (t_i,t_{i+1}) should be chosen small enough to achieve a fast response time for the adaptive loop, yet long enough to give a proper detection of the effect of the last change in the parameters, γ. The problem is here formulated to guarantee that P has metric properties. By doing this it can be shown that if $\gamma_1, \gamma_2, \ldots$ converges it will converge to the true optimum independent of the applied command input. In [56] the method is applied to the adaptive control of the pitch rate loop of an aeroplane.

7. STOCHASTIC ADAPTIVE METHODS.

During the first decade of development of adaptive controllers the design was in most cases not based on a statistical description of the noisy environment in which the processes always work.
The noise can originate from measurement errors, drift in calibration of instruments, wind gusts, changes in quality of incoming products etc. This approach was taken because one did not have the tools to handle the noise in the systems. The earlier analysis was in most cases carried out to achieve a performance of the systems in servomechanism theory terms. If noise was considered it was mostly approximated by steps, ramps or other test signals. The systems then were designed to be as insensitive to these signals as possible, but around 1960 much work was done in the area of stochastic optimal control and to formulate the control problems in a statistical framework. This and the immense development in the field of computers have given the control engineers new and sharper tools to work with.

This section will handle adaptive systems where the problems have been formulated as stochastic control problems. The philosophy of stochastic adaptive control and ways to attack these problems are discussed by Bellman [11] and Bellman and Kalaba [12]. Many stochastic adaptive methods are formulated as optimal Bayesian control problems. Aoki has given a thorough penetration of this kind of problems [3].

In most cases the derivation of optimal control laws for stochastic systems will lead to a functional equation which has to be solved using dynamic programming. This will limit the class of problems that can be solved. A stochastic variational problem solved by Åström [6], [7] clearly shows the difficulties that arise when solving multistep decision problems. It is much easier to solve single step decision problems. Minimizing over several steps of time will give dual control laws. Minimizing over just one step will give non-dual control laws. The term dual control was first introduced by Feldbaum [25]. By dual control is meant that the control law has a twofold action. First it minimizes the loss and second, which is very important, it makes control actions in order to get better estimates of the unknown parameters of the process. The non-dual control laws just minimize the loss at each step of time. Heuristically it can be said that non-dual control laws just make the best out of a given situation. The dual control laws try to reach a better situation from which better control can be achieved. The stochastic adaptive controllers will be classified as non-dual or dual controllers.

7.1. Non-Dual Control Methods.

Two ways of obtaining non-dual control laws will be discussed. First one can use the equations for the optimization over several steps of time and make approximation or introduce some restrictions. Second one can from the beginning look at the problem as a single step decision problem and minimize the loss for just one step. The first approach is used e.g. in [3], [9], [21], [27], [64]. As an example we will follow [9]. Represent the system as

$$\begin{cases} x(t+1) = A(t)x(t) + B(t)u(t) \\ y(t) = C(t)x(t) + e(t) \end{cases}$$

where e(t) is white noise and the matrices A(t) and B(t) have random elements. Further the matrix C(t) is assumed known. The purpose of the control is to minimize a loss function over N steps

$$V_N = \sum_{t=0}^{N-1} [x(t+1)^T Q(t+1)x(t+1) + \lambda u(t)^2]$$

i.e. find the control sequence u(0), u(1), ..., u(N-1) which minimizes the expected value of V_N.

Three different types of controllers are discussed. First optimal open loop controllers where only the a priori information of initial states and probability distributions are used to determine the control sequence over the whole interval. Second optimal closed loop controller which gives a dual control law. Third a suboptimal closed loop controller. When using the first type of controller one does not use any measurements during the control period. In contrast to the optimal closed loop control law it is possible in this case to get an analytic solution to the open loop problem. This can now be used to form a suboptimal control law which at each step uses the last estimate of the parameters to compute the open loop control sequence for the remaining control period. This control sequence is used only for one step. When a new measurement is taken this is used to make a new estimate and a new open loop control sequence is computed. This means that at each instant of time the control signal is computed on basis of the last measurement and under the assumption that no further measurements will be taken. This is often called open loop feedback control [21]. Properties of open loop feedback controllers are further discussed in [64].

Other ways to derive non-dual control laws are discussed in [57] and [68]. We will follow the way outlined in [68]:

Assume that the system can be described by the equation

$$y(t) + a_1(t)y(t-1) + \dots + a_n(t)y(t-n) =$$

$$= b_1(t)u(t-1) + \dots + b_n(t)u(t-n) + e(t)$$

where $a_i(t)$ and $b_i(t)$ are unknown time varying parameters and e(t) is white noise. The control problem is to choose u(t) as function of old outputs and inputs in such a way that the expected variance of the output in the next instant of time is minimized, i.e. min $Ey(t)^2$. In the general case unknown parameters of a process can be estimated by augmenting the state vector with the unknown parameters. In general this will lead to nonlinear estimation problems that has to be solved using dynamic programming, but because of the special structure of the model and the criterion chosen here the problem is linear in the unknown parameters and the least squares method can be used to estimate the parameters. Further the problem formulation will make it possible to separate estimation and control. The parameters are estimated using a real time identification algorithm based on Kalman theory.

The system equation can be written as

$$y(t) = [-y(t-1) \dots -y(t-n)u(t-1) \dots u(t-n)] \begin{bmatrix} a_1(t) \\ \vdots \\ a_n(t) \\ b_1(t) \\ \vdots \\ b_n(t) \end{bmatrix} + e(t)$$

or

$$y(t) = \varphi(t-1)\theta(t) + e(t) \tag{7.1}$$

The vector $\theta(t)$ which contains all the unknown parameters is assumed to be a stochastic process described by the equation

$$\theta(t+1) = \theta(t) + v(t) \tag{7.2}$$

where $v(t)$ is a sequence of independent normal random vectors. It is further assumed that the statistical properties of the noises $e(t)$ and $v(t)$ are known. The equations (7.1) and (7.2) can now be regarded as measurement and state equation of a dynamic system. The state of the system, $\theta(t)$, can be estimated using Kalman theory. The filter equations do not only give the parameter estimates but also the variance of the estimation error. This variance matrix is a measure of the quality of the estimates. Due to the formulation of the problem the optimal solution can be given analytically. The control signal will be a function of inputs, outputs, parameter estimates and further of the accuracy of the parameter estimates. If in the obtained control law all the variances are put equal to zero the control law will be reduced to the deterministic minimum variance strategy [5], but with the true parameter values substituted by estimates.

To illustrate the behaviour of this type of adaptive controllers we use the system

$$y(t) - 2.6y(t-1) + 1.2y(t-2) =$$

$$= u(t-1) + b(t)u(t-2) + e(t)$$

where $b(t)$ is varying as

$b(t) = -0.5 + 0.0005 \cdot t$

The control signal at time t is allowed to be a function of u(t-1), u(t-2), ..., y(t-1), y(t-2), ... Fig. 7.1 shows how b(t) is tracked by the real time estimator. The accumulated loss $\Sigma y(t)^2$ is shown in Fig. 7.2. It is seen that the adaptive controller in this case only gives a slight increase in the loss compared with the minimal expected loss (see [5]). A minimal variance regulator is derived for the system based upon the true parameter values at time t = 0. The accumulated loss when using this regulator on the time varying system is also seen in Fig. 7.2. The used system is evidently sensitive for the changes in b(t), but with the adaptive controller it is possible to follow the changes in b(t) and thus achieve good control all the time.

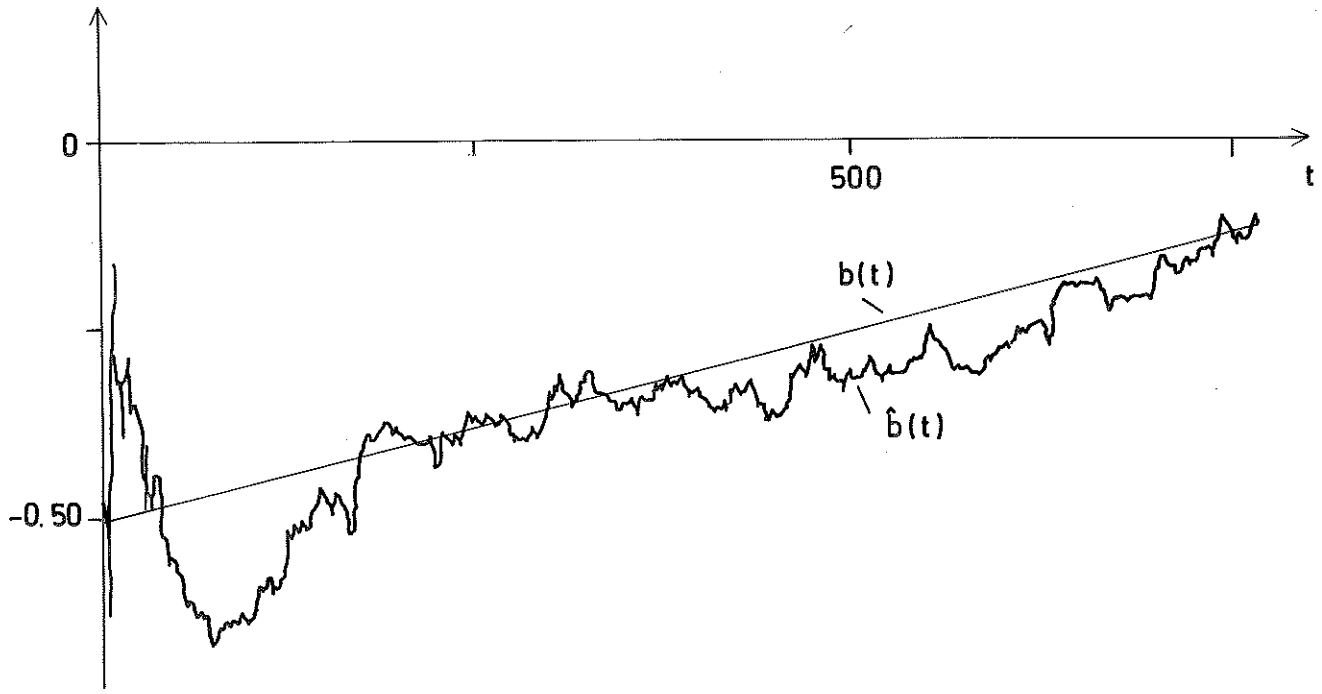

Fig. 7.1 - True and estimated values of b(t).

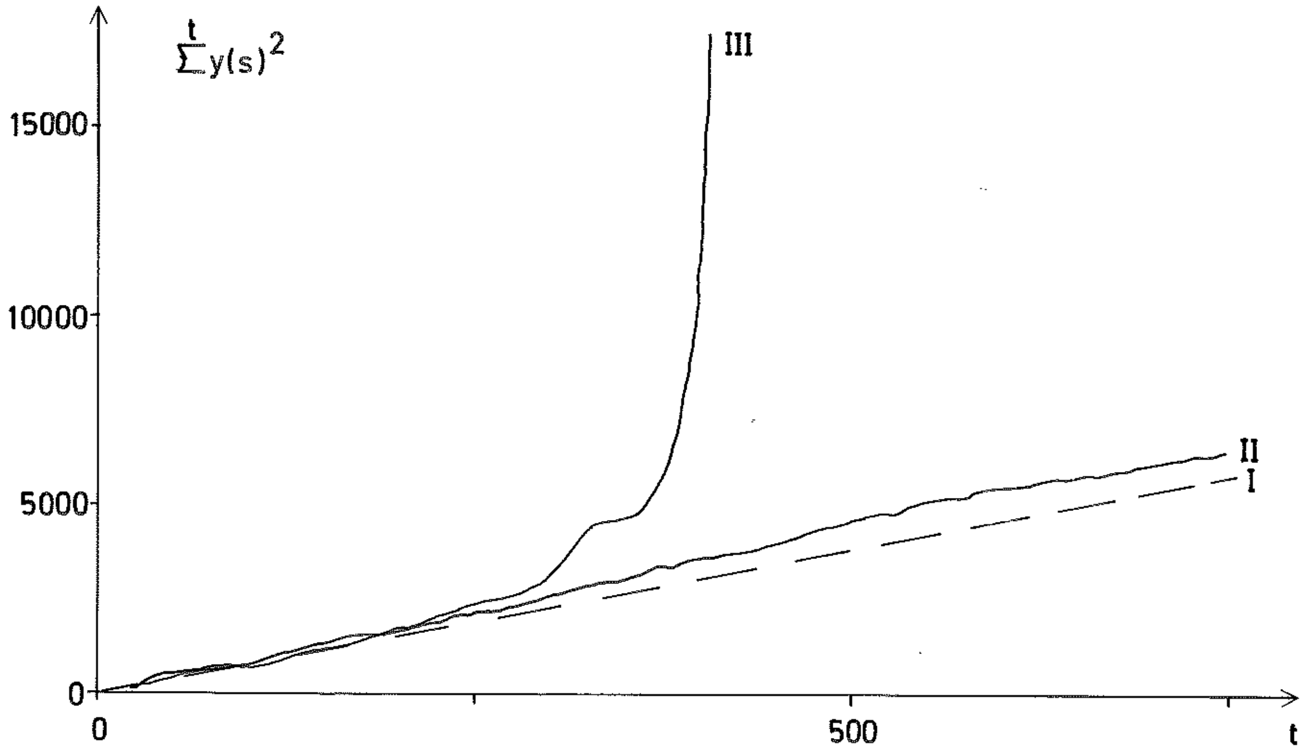

Fig. 7.2 - I Expected minimal loss if the parameters were known.

II Accumulated loss, $\Sigma y(t)^2$, when using adaptive controller.

III $\Sigma y(t)^2$ when using optimal controller based upon the true parameter values at time t = 0.

7.2. Dual Control Methods.

The dual control laws are given as solutions to functional equations which have to be solved using dynamic programming. This has the consequence that it is only possible, because of computational reasons, to solve problems of low order.

Problems leading to dual control are formulated in many papers, but only a few complete solutions are reported.

Systems with fixed but unknown gain parameters are discussed in [30] for continuous systems and in [26] for discrete time systems. In the first paper different ways are discussed to get approximate solutions to the dynamic programming equations. In the second paper a one dimensional problem is discussed. Let the system be described by the equation

$$x(t+1) = x(t) + bu(t) + v(t)$$

where b is fixed but unknown, with a gaussian a priori distribution. v(t) is white noise with known distribution. The purpose with the control is to minimize

$$E \sum_{t=1}^{N} [x(t-1)^2 + u(t)^2]$$

The problem is solved by stepping backwards from the final time N. After three steps it is reported that a stationary control law is obtained as a function of the state variable, x(t), estimate of b and the variance of the estimation error. The control policy is then approximated by an analytic expression which can be used as control laws, but there is no comparison between the behaviour of non-dual control laws and the obtained dual control law.

We will now discuss three fully solved problems which give dual control laws. In all three papers only first order systems are used.

The first problem is given by Jacobs and Langdon [35], [36], and treats the extremum control problem described by Fig. 7.3. The observed output c is a quadratic function of the variable x:

$$c(t) = Ax(t)^2 \tag{7.3}$$

A is assumed to be known.

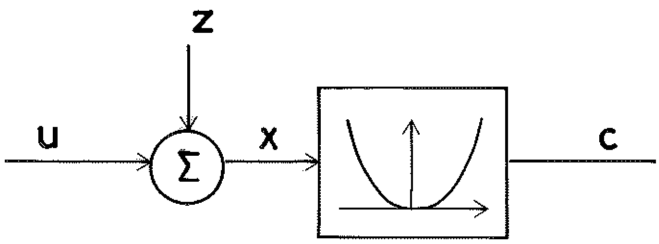

Fig. 7.3 - Simplified extremum control problem.

z(t) is a stochastic process given by the equation

$$z(t+1) = z(t) + r(t) \tag{7.4}$$

where $r(t) \in N(0,\sigma)$.

The problem is to minimize the expected time average of the output c.

$$\bar{c} = \lim_{N\to\infty} \frac{1}{N} \sum_{t=1}^{N} c(t) \tag{7.5}$$

Introducing $v(t) = u(t+1) - u(t)$ the system can be written as

$$x(t+1) = x(t) + v(t) + r(t) \qquad (7.6)$$

The system (7.6) and the criterion (7.5) formulates a linear quadratic stochastic control problem which has the solution $v(t) = -x(t)$, but the variable $x(t)$ is only measurable through the variable $c(t)$ and thus only the absolute value is known and not the sign. Introducing the probability for $x(t)$ to be positive it is possible to derive a functional equation for choosing the control variable. Using a digital computer for solving the recursive functional equation a steady state control table is obtained, i.e. the control signal is given in discrete points as a function of the probability for positive $x(t)$ and the absolute value of x.

Bohlin [14] and Wittenmark [69] have discussed a problem with an unknown time varying gain.

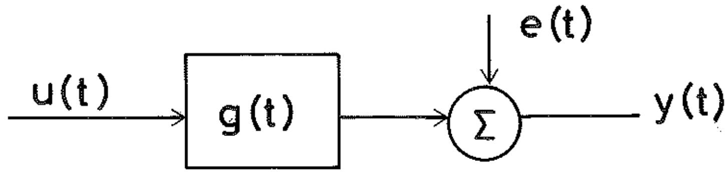

Fig. 7.4 - The system used in [14] and [69].

In [14] the time varying gain is modelled as

$$g(t) = g_0 + d[g(t-1) - g_0] + v(t)$$

and in [69] as

$$g(t) = ag(t-1) + v(t-1) \tag{7.7}$$

In both cases the distributions for e(t) and v(t) are normal with zero mean and known variances λ^2 and σ^2 respectively. Also the constants g_0, d and a are assumed to be known.

The loss function in [14] is

$$\ell_N' = E \sum_{t=1}^{N} (y(t) - g_0)^2$$

and in [69]

$$\ell_N = E \sum_{t=1}^{N} (1 + g(t)u(t))^2 \tag{7.8}$$

As the two problems and solutions are very much the same we will only discuss the solution given in [69] at this point.

Consider the system

$$y(t) = g(t)u(t) + e(t) \tag{7.9}$$

where g(t) is varying according to (7.7) and use the loss function (7.8). If the gain was known the optimal solution would be

$$u(t) = - \frac{1}{g(t)}$$

The unknown gain can be estimated using a Kalman filter.

$$\begin{cases} \hat{g}(t+1|t) = a\hat{g}(t|t-1) + K(t)\big(y(t) - u(t)\hat{g}(t|t-1)\big) \\ K(t) = \dfrac{ap(t)}{p(t)u(t)^2 + \lambda^2} \\ p(t+1) = \dfrac{\lambda^2 a^2 p(t)}{p(t)u(t)^2 + \lambda^2} + \sigma^2 \end{cases} \tag{7.10}$$

Consider the situation at time $t = N$. Then it only remains to find the control signal $u(N)$ as a function of old inputs and outputs. The optimal $u(N)$ is given by

$$u(N) = - \frac{\hat{g}(N|N-1)}{\hat{g}(N|N-1)^2 + p(N)}$$

Define

$$V\big(\hat{g}(t|t-1),p(t),t\big) =$$

$$= \min_{u(t)\ldots u(N)} E\left\{\sum_{s=t}^{N} \big(1 + u(s)g(s)\big)^2 \,\Big|\, \mathcal{Y}_{t-1}\right\}$$

where

$$\mathcal{Y}_{t-1} = [y(t-1),\ y(t-2),\ \ldots,\ u(t-1),\ u(t-2),\ \ldots]$$

is the sequence of old outputs and inputs.

This will now give the functional equation

$$V\big(\hat{g}(t|t-1),p(t),t\big) =$$

$$= \min_{u(t)} \Big[\big(1 + u(t)\hat{g}(t|t-1)\big)^2 + u(t)^2 p(t) +$$

$$+ E\Big\{V\big(\hat{g}(t+1|t),p(t+1),t+1\big) \,\Big|\, \mathcal{Y}_{t-1}\Big\}\Big] \tag{7.11}$$

For t = N

$$V(\hat{g}(N|N-1),p(N),N) = \frac{p(N)}{\hat{g}(N|N-1)^2 + p(N)}$$

The equations (7.11) and (7.10) define a dynamic programming problem which is solved backwards from t = N. The equations are iterated backwards until the steady state solution is obtained. The control signal, u(t), is now given in discrete points as function of the estimated gain, $\hat{g}(t|t-1)$, and the estimation error variance, p(t). To simulate the system the control table is stored and the actual control signal is given through interpolation in the control table.

There are two things concerning dual control laws which are pointed out in all three papers discussed above. First we will discuss a property of the optimal dual control law and second discuss how to derive suboptimal control laws which preserve the dual action.

The optimal dual control law has a property which at the first glance may seem strange. When the uncertainty of the parameter estimate is increased the control signal as function of the uncertainty has a discontinuity. This can be explained by the twofold action of the optimal control. When the estimates are good there is no reason to use any control to improve the estimates and the strategy just minimizes the expected loss, but when the estimates are poor they have to be improved. This is necessary in order to be able to make better control in future steps. The strategy is thus changed and the control is achieved partly to minimize the expected loss and partly to get better estimates. The function V(g,p,t) has two local minima corresponding to these two strategies and depending on the variance p(t) it is the one or the other that will give the absolute minimum.

The twofold property of the optimal dual control law can be used to form suboptimal control laws which are much easier to derive and thus can be used on higher order systems. When the accuracy of the estimates are good it is a good strategy to use the non-dual control law obtained by minimizing over just one step of time, but when the uncertainty in the parameter estimates is too large the estimates have to be improved. This can be done by superimposing a perturbation signal on the non-dual control law. The perturbation signal can e.g. be a small amplitude square wave [69] or a random signal [68]. The random signal can e.g. be a pseudo random binary sequence (PRBS). The perturbation signal can be used all the time or only when the estimates are poor. A way to judge the quality of the estimates can be to examine the variance of the estimates or perhaps better to use the relative accuracy.

In [69] an example is given comparing the one step controller and the optimal dual controller:

The system is

$$\begin{cases} g(t+1) = 0.9g(t) + v(t) \\ y(t) = g(t)u(t) + e(t) \end{cases}$$

where

$$Ev(t) = Ee(t) = 0$$

$$Ev(t)^2 = 1$$

$$Ee(t)^2 = 0.25$$

$$Ev(t)e(t) = 0$$

The loss function is chosen to minimize

$$E \sum_{1}^{N} \left(1 + g(s)u(t)\right)^2$$

The accumulated loss

$$\sum_{1}^{t} \left(1 + g(s)u(s)\right)^2$$

is shown in Fig. 7.5 for the different control laws.

It is possible to compute the expected loss when using the optimal dual control law, line e in Fig. 7.5. From the figure it is seen that the perturbated one step controller gives a rather large reduction of the loss. The same result when using suboptimal dual control laws is reported in [36]. This gives a hint as to how suboptimal dual controllers can be derived, i.e. use the one step non-dual controller as long as the accuracy of the parameter estimates are good and superimpose a perturbation signal when the estimates are too poor. Since the dimension of the dynamic programming problem increases with the square of the order of the system [8] it is not realistic to think that optimal solutions can be obtained for higher order systems. Thus the control laws sketched above might be a way to make suboptimal dual control on higher order systems.

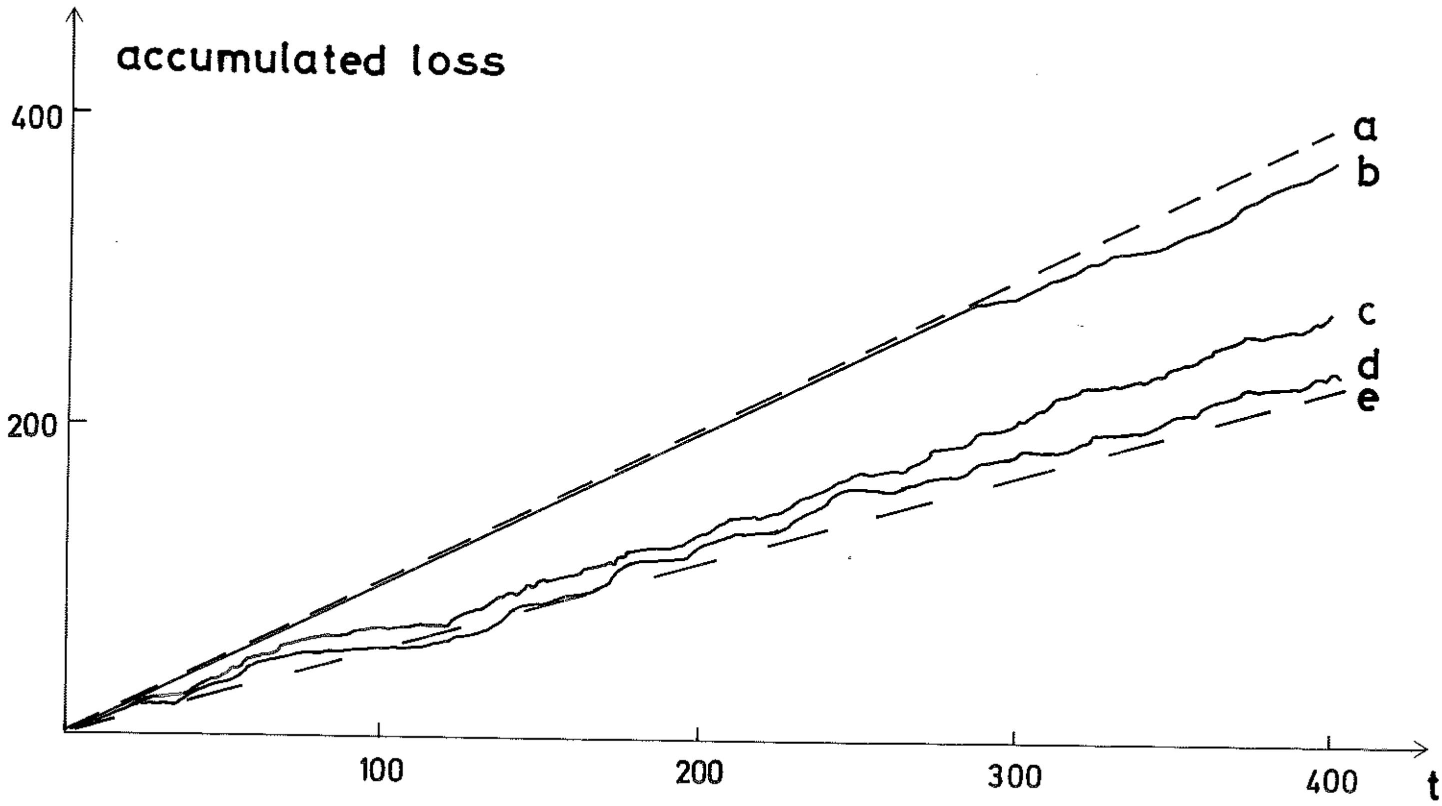

Fig. 7.5 - Accumulated loss

$$\sum_{1}^{t}\left(1 + g(s)u(s)\right)^2$$

for different cases.

a. Expected value for $u(t) \equiv 0$

b. Non-dual control

$$u(t) = - \frac{\hat{g}(t)}{\hat{g}(t)^2 + p(t)}$$

c. Suboptimal dual control

$$u(t) = - \frac{\hat{g}(t)}{\hat{g}(t)^2 + p(t)} + (-1)^t \cdot 0.15$$

d. Dual control

e. Expected value when using dual control

8. REFERENCES.

The flora of literature on adaptive control is very large and in this report only a part of all suggested adaptive controllers has been discussed. Even if this report is a survey it can be hard to find the right way through the jungle of given references. The intention is therefore to point out a few books and the main references to the different methods. This smaller subset can be used as a first guide into the fascinating field of adaptive control.

There is no book which treats all the different classes of adaptive controllers discussed in this report. The books

Mishkin, E., Braun, L.Jr. (Eds.): Adaptive Control Systems [48]

Eveleigh, V.W.: Adaptive Control and Optimization Techniques [23]

discuss many of the earlier adaptive controllers. A review of the state of art up to about 1959 is given in

Gregory, P.C.: Proceedings of the Self-Adaptive Flight Control Systems Symposium [31]

An excellent exposition of the philosophy behind stochastic adaptive methods is given in

Bellman, R.: Adaptive Control Processes - A Guided Tour [11]

This book does not give the detailed solutions but mainly discusses the problems arising when formulating the adaptive problems in a statistic framework.

Extremum seeking methods are discussed and an introduction to stochastic adaptive methods is found in

Westcott, J.H. (Ed.): An Exposition of Adaptive Control [65]

Apart from the books referred above the main part of the literature on adaptive control is found in journals and conference papers. The main sources of this kind which have been used for this report are

IEEE Transactions
Automation and Remote Control
International Journal of Control
Preprints from the Joint American Control Conferences (JACC)
Proceedings from IFAC Conferences and Symposia

In the end of this section an alphabetic reference list is given. To get a total view of the references they are below listed in different categories. The main references of the different classes of controllers are denoted by a star (*).

General discussion of adaptive methods:

[3] [6] [11] [12] [15] [23] [28] [31] [48] [54] [65]

Plant sensing methods:

Indirect methods:
[13] [16] [31 p. 123]* [62]

Model reference methods:
[20] [22] [29] [40] [41]* [46] [49] [51]* [55] [59] [66]*

Learning model methods:
[10] [44]* [45] [52] [60]

Extremum seeking methods:

Perturbation methods:
[17] [18] [19]* [39] [63]

Gradient methods:
[2] [24] [28]* [50] [53] [56] [61]

Stochastic adaptive methods:

Non-dual control methods:
[3]* [8] [9] [21] [27] [57] [64] [68]

Dual control methods:
[6] [8] [12] [14] [25]* [30] [35] [36] [69]

Alphabetic Reference List.

[1] Anonymous, IEEE SSG Group Newsletter, No. 18, Dec., 1969, p. 8.

[2] Anonymous, The Veritrak Performance - Optimizing Controller, Motorola Control Systems Ltd., Bulletin 03120-11.

[3] Aoki, M.: Optimization of Stochastic Systems, Academic Press, 1967.

[4] Aseltine, J.A., Mancini, A.R., Sarture, C.W.: A Survey of Adaptive Control Systems, IRE T-AC, Dec., 1958, pp. 102-108.

[5] Åström, K.J.: Introduction to Stochastic Control Theory, Academic Press, 1970.

[6] Åström, K.J.: Optimal Control of Markov Processes with Incomplete State Information, Journal of Mathematical Analysis and Applications, Vol. 10, No. 1, Feb., 1965, pp. 174-205.

[7] Åström, K.J.: Optimal Control of Markov Processes with Incomplete State Information II - The Convexity of the Loss Function, Journal of Mathematical Analysis and Applications, Vol. 26, No. 2, May, 1969, pp. 403-406.

[8] Åström, K.J., Wittenmark, B.: Problems of Identification and Control, Journal of Mathematical Analysis and Applications, Vol 34, No 1 April 1971, pp 90-113.

[9] Bar-Shalom, Y., Sivan, R.: On the Optimal Control of Discrete-Time Linear Systems with Random Parameters, IEEE T-AC, Vol. 14, No. 1, Feb. 1969, pp. 3-8.

[10] Beckenback, E.F. (Ed.): Modern Mathematics for the Engineer, McGraw Hill, 1956.

[11] Bellman, R.: Adaptive Control Processes - A Guided Tour, Princeton University Press, 1961.

[12] Bellman, R., Kalaba: Dynamic Programming and Adaptive Processes. Mathematical Foundation, IRE T-AC, Vol. 5, Jan., 1960, pp. 5-10.

[13] Bergman, J.: Undersökning av Minneapolis-Honeywells adaptiva regulator genom digital simulering (in Swedish), RE-41, Division of Automatic Control, Lund Institute of Technology.

[14] Bohlin, T.: Optimal Dual Control of a Simple Process with Unknown Gain, TP 18.196, Febr., 1969, IBM Nordic Laboratory.

[15] Bohlin, T.: Information Pattern for Linear Discrete Time Models with Stochastic Coefficients, IEEE, T-AC, Vol 15, No 1, Febr. 1970 pp 104-106.

[16] Borisson, U., Sogndal, C.: Adaptiva reglermetoder (in Swedish), Elteknik, No. 9, pp. 182-185, 1968.

[17] Douce, J.L., Ng, K.C.: Six-Channel Adaptive Computer, Proc. IEE, Vol. 111, No. 10, Oct., 1964.

[18] Douce, J.L., Ng, K.C.: The Use of Pseudo-Random Signals in Adaptive Control, Proc. of the 2nd Symposium on the Theory of Self-Adaptive Control Systems, Teddington, 1965, pp. 274-280.

[19] Draper, C.S., Li, Y.T.: Principles of Optimizing Control Systems and an Application to the Internal Combustion Engine, American Society of Mechanical Engineers, New York, 1951. A reprint of the first part is given in:
Oldenburg, R. (Ed.): Optimal and Self-Optimizing Control, The MIT Press, 1966, pp. 415-429.

[20] Dressler, R.M.: An Approach to Model-Referenced Adaptive Control Systems, IEEE T-AC, Vol. 12, No. 1, Febr., 1967, pp. 75-80.

[21] Dreyfus, S.: Some Types of Optimal Control of Stochastic Systems, SIAM J. Control, Vol. 2, No. 1, pp. 120-134, 1964.

[22] Dymock, A.J., Meredith, J.F., Hall, A., White, K.M.: Analysis of a Type of Model Reference Control Systems, Proc. IEE, No. 4, April, 1965, p. 743.

[23] Eveleigh, V.W.: Adaptive Control and Optimization Techniques, McGraw Hill, 1967.

[24] Feldbaum, A.A.: Automatic Optimalizer, Automation and Remote Control, Vol. 19, No. 8, Aug., 1958, pp. 718-728.

[25] Feldbaum, A.A.: Dual Control Theory, I-IV, Automation and Remote Control, Vol. 21, No. 9, April, 1961, pp. 874-880; Vol. 21, No. 11, May, 1961, pp. 1033-1039; Vol. 22, No. 1, Jan., 1962, pp. 1-12, Vol. 22, No. 2, Febr., 1962, pp. 109-121.

[26] Florentine, J.J.: Optimal Probing Adaptive Control of a Simple Bayesian System, Journal of Electronics and Control, Vol. 11, 1962, pp. 167-177.

[27] Frost, V.A.: Identification and Control of Linear Discrete-Time Systems with Noisy Output Measurements, Int. J. Control, Vol. 11, No. 4, pp. 571-580.

[28] Gibson, J.E.: Mechanizing the Adaptive Principle, Control Engineering, Oct., 1960, pp. 109-114.

[29] Gilbart, J.W., Monopoli, R.V., Price,C .F.: Improved Convergence and Increased Flexibility in the Design of Model Reference Adaptive Control Systems, IEEE Symposium on Adaptive Processes, 1970, p. IV 3.1-3.10.

[30] Gormar, D., Zaborszky, J.: Stochastic Optimal Control of Continuous Time Systems with Unknown Gain, IEEE, T-AC, Vol. 13, No. 6, Dec., 1968, pp. 630-638.

[31] Gregory, P.C. (Ed.): Proceedings of the Self-Adaptive Flight Control Systems Symposium, WADC Technical report 59-49, March, 1959.

[32] Grumbach, R.: Private Communication.

[33] Horowitz, I.M.: Plant Adaptive Systems Versus Ordinary Feedback Systems, IRE, T-AC, Jan., 1962, pp. 48-56.

[34] Horowitz, I.M.: Optimum Linear Adaptive Design of Dominant-Type Systems with Large Parameter Variations, IEEE, T-AC, June, 1969, pp. 261-269.

[35] Jacobs, O.L.R.: Extremum Control and Optimal Control Theory, Preprints IFAC Symposium on Identification in Automatic Control Systems, Prague, 1967, paper 5.10.

[36] Jacobs, O.L.R., Langdon, S.M.: An Optimal Extremum Control System, Preprints, 4th IFAC Congress, Warsaw, 1969, paper 69.6.

[37] Kalman, R.E., Bertram, J.E.: Control System Analysis and Design Via the "Second Method" of Lyapunov, I Continuous-Time Systems, Journal of Basic Engineering, June, 1960, pp. 371-393.

[38] Kalman, R.E., Bertram, J.E.:Control System Analysis and Design Via the "Second Method" of Lyapunov, II Discrete-Time Systems, Journal of Basic Engineering, June, 1960, pp. 394-399.

[39] Kisiel, A.J., Rippin, D.W.T.: Adaptive Optimization of a Water-Gas Shift Reactor, Proc. of the 2nd Symposium on the Theory of Self-Adaptive Control Systems, Teddington, 1965, pp. 335-346.

[40] Kokotović, P.V., Medanić, J.W., Vušković, M.I.: Sensitivity Method in the Experimental Design of Adaptive Control Systems, 3rd Congress of IFAC, London, 1966, paper 45B.

[41] Landau, I.D.: A Hyperstability Criterion for Model Reference Adaptive Control Systems, IEEE T-AC, Vol. 14, No. 5, Oct., 1969, pp. 552-555.

[42] Lee, Y.S.: A Time-Optimal Adaptive Control System via Adaptive Switching Hypersurface, IEEE T-AC, Aug., 1967, p. 367-375.

[43] Levin, M.J.: Methods for the Realization of Self-Optimizing Systems, ISA Paper No. FCS-2-58 presented at ASME-IRD Conference, Newark, Del., April 2-4, 1958.

[44] Margolis, M., Leondes, C.T.: A Parameter Tracking Servo for Adaptive Control Systems, Wescon. Conv. Rec. Inst. Radio Engrs., N.Y., Pt. IV, Aug., 1959.

[45] Margolis, M., Leondes, C.T.: On the Theory of Adaptive Control Systems; The Learning Model Approach, Proc. of the First IFAC Congress, Moscow, 1960, Vol. 2, pp. 556-563.

[46] Maršik, J.: Quick-Response Adaptive Identification, IFAC Symposium on Identification in Automatic Control Systems, Prague, 1967, paper 5-5.

[47] Meredith, J.F., Dymock, A.J.: A Self-Adaptive System Employing High Speed Parameter Identification, Proc. of the 2nd Symposium on the Theory of Self-Adaptive Control Systems, Teddington, 1965, pp. 165-174.

[48] Mishkin, E., Braun, L.Jr. (Eds.): Adaptive Control Systems, McGraw Hill, 1961.

[49] Monopoli, R.V., Gilbart, J.W., Thayer, W.D.: Model Reference Adaptive Control Based on Liapunov-like Techniques, Preprints from Second IFAC Symposium on System Sensitivity and Adaptivity, 1968, paper F-24.

[50] Narendra, K.S., McBride, L.E.: Multiparameter Self-Optimization Systems Using Correlation Techniques, IEEE, T-AC, Jan., 1964, pp. 31-38.

[51] Parks, P.C.: Liapunov Redesign of Model Reference Adaptive Control Systems, IEEE, T-AC, Vol. 11, No. 3, p. 362, July, 1966.

[52] Pazdera, J.S., Pottinger, H.J.: Linear System Identification via Liapunov Design Techniques, JACC, 1969, pp. 795-801.

[53] Pearson, A.E.: A Modified Gradient Procedure for Adaption in Nonlinear Control Systems, JACC, 1969, pp. 596-602.

[54] Pearson, A.E.: A Survey on Adaptive Control Systems, Office of Control Theory and Application, Electronics Research Center, Cambridge, Mass., Oct., 1968.

[55] Pearson, A.E., Noonan, F.: On the Model Reference Adaptive Control Problem, JACC, 1968, p. 538.

[56] Pearson, A.E., Vanguri, K-S.V.R.: A Synthesis Procedure for Parameter Control Systems, Report Brown University, Providence RI.

[57] Peterka, V.: Adaptive Digital Regulation of Noisy Systems, Preprints, 2nd Prague IFAC Symposium, 1970, paper 6.2.

[58] Popov, V.M.: The Solution of a New Stability Problem for Controlled Systems, Automation and Remote Control, Vol. 24, pp. 1-23, Jan., 1963.

[59] Porter, B., Tatnal, M.L.: Performance Characteristics of Multi-Variable Model-Reference Adaptive Systems Synthesized by Liapunov's Direct Method, Int.J.Control, Vol. 10, No. 3, pp. 241-257, Sept., 1969.

[60] Skoog, H.: Analys av Margolis Leondes adaptiva reglersystem, RE-21, Division of Automatic Control, Lund Institute of Technology, 1967 (in Swedish).

[61] Stakhovskii, R.I.: Twin-Channel Automatic Optimalizer, Automation and Remote Control, Vol. 19, No. 8, Aug., 1958, pp. 729-740.

[62] Stallard, D.V.: A Missile Adaptive Autopilot with a Small Amplitude Limit Cycle, JACC, 1965, pp. 408-422.

[63] Stallard, D.V.: A Missile Adaptive Roll Autopilot with a New Dither Principle, IEEE T-AC, Vol. 11, No. 3, July, 1966, p. 368-378.

[64] Tse, E.: Athans, M.: Adaptive Stochastic Control for Linear Systems, Part I, II, IEEE Symposium on Adaptive Control, 1970, p. IV 1.1-1.11, 2.1-2.17.

[65] Westcott, J.H. (Ed.): An Exposition of Adaptive Control, Pergamon Press, 1962.

[66] Whitaker, H.P., Yarmon, J., Kezer, A.: Design of Model Reference Adaptive Control Systems for Aircraft, MIT Instrumentation Lab, Report R-164, Sept., 1958.

[67] Wieslander, J.: Real Time Identification, Part I, Report 6908, Division of Automatic Control, Lund Institute of Technology.

[68] Wieslander J., Wittenmark, B.: An Approach to Adaptive Control Using Real Time Identification, Preprints, 2nd Prague IFAC Symposium, 1970, paper 6.3.

[69] Wittenmark, B.: On Adaptive Control of Low Order Systems, Report 6918, Division of Automatic Control, Lund Institute of Technology.

[70] Wittenmark, B.: On the Turn-Off Phenomenon in Adaptive Control, Report 7105, Division of Automatic Control, Lund Institute of Technology.

[71] Young, P.C.: Process Parameter Estimation and Self-Adaptive Control, Proc. of the 2nd Symposium on the Theory of Self-Adaptive Control Systems, Teddington, 1965, pp. 118-140.

9. ACKNOWLEDGEMENTS.

The author wants to express his great gratitude to Professor K.J. Åström for his valuable help in the work and for his guidance into the fascinating field of adaptive control.