Frequency Domain Properties of Otto Smith Regulators

Åström, Karl Johan

1975

Document Version:
Publisher's PDF, also known as Version of record

Link to publication

Citation for published version (APA):
Åström, K. J. (1975). *Frequency Domain Properties of Otto Smith Regulators*. (Technical Reports TFRT-7082). Department of Automatic Control, Lund Institute of Technology (LTH).

Total number of authors:
1

LUND UNIVERSITY

PO Box 117
221 00 Lund
+46 46-222 00 00

FREQUENCY DOMAIN PROPERTIES OF OTTO SMITH REGULATORS

K. J. ÅSTRÖM

Report 7523(C) September 1975
Department of Automatic Control
Lund Institute of Technology

FREQUENCY DOMAIN PROPERTIES OF OTTO SMITH REGULATORS

K.J. Åström

ABSTRACT

Frequency domain characteristics of Otto Smith regulators are investigated. It is shown that the regulator can be regarded as an ordinary regulator in cascade with a lead network with considerable lead.

TABLE OF CONTENTS

Page

1. Introduction	1
2. Otto Smith's Regulator	2
3. The Regulator Transfer Function	3
4. An Example	7
5. References	15
APPENDIX	16

1. INTRODUCTION

The idea of dead-time compensation introduced by Otto Smith (1957) had little use in analog systems because of the difficulty of implementing the system. Since the regulator can easily be implemented digitally it is now finding increasing use in practical control system. This note presents a simple analysis of the frequency domain properties of the regulator. The note was inspired by a discussion of digital control systems given in a seminar to an industrial audience. Specifically it answers the question: "This is all fine but where does the phase-lead come from?" which was asked by one of the participants.

2. OTTO SMITH'S REGULATOR

Consider the system whose block diagram is shown in Fig. 1.

<u>Fig. 1</u>

Assume that the compensator G_R is chosen in such a way that a suitable performance of the closed loop system is obtained. The closed loop transfer function is then

$$G(s) = \frac{G_R G_P}{1 + G_R G_P}(s) \qquad (1)$$

If the system has an extra time delay this transfer function is changed to

$$G(s) = \frac{G_R G_P e^{-sT}}{1 + G_R G_P e^{-sT}} \qquad (2)$$

If T is sufficiently large the system will always be unstable. To avoid this difficulty Otto Smith (1958) proposed the regulator shown in Fig. 2.

Fig. 2.

If the block diagram of Fig. 2 is redrawn as shown in Fig. 3 it is easily seen that the signals $-y$ and y_c will cancel and the closed loop transfer function becomes

$$G(s) = \frac{G_R G_P e^{-sT}}{1 + G_R G_P} \qquad (3)$$

Fig. 3.

The closed loop system whose transfer function is given by equation (3) is clearly stable for any value of T. Apart from the factor e^{-sT} in the numerator of (3) the transfer function (3) is in fact identical to (1). This means that the regulator shown in the dashed block in Fig. 2 must give a significant phase lead. This will be explored further in the next section.

3. THE REGULATOR TRANSFER FUNCTION

The regulator in the dashed block in Fig. 2 has the transfer function

$$G_R' = \frac{G_R}{1 + G_R G_P (1-e^{-sT})} = \frac{1}{1 + (1-e^{-sT}) G_0} G_R \tag{4}$$

where

$$G_0(s) = G_R(s) G_P(s) \tag{5}$$

The Otto Smith regulator can thus be considered as being a cascade connection of the ordinary regulator (G_R) with a compensator having the transfer function

$$G_c(s) = \frac{1}{1 + (1-e^{-sT}) G_0(s)} \tag{6}$$

The properties of the transfer function G_c will now be explored.

For typical control loops the open loop gain will be small for high frequencies and high for low frequencies. Assuming that

(i) $\quad |sG_0(s)| \gg 1 \quad$ for $|s| \ll \omega_1$

(ii) $\quad |G_0(s)| \ll 1 \quad$ for $|s| \gg \omega_2$

It then follows from (5) that

$$G_c(s) \approx \frac{1}{sTG_0(s)} \qquad |s| \ll \omega_1$$

$$G_c(s) \approx 1 \qquad |s| \gg \omega_2$$

If

$$G_0(s) \approx \frac{K}{s^n}$$

for small s it follows that

$$G_c(s) \approx \begin{cases} \dfrac{1}{1 + kT} & n = 1 \\[2ex] \dfrac{s^{n-1}}{kT} & n > 1 \end{cases}$$

for small s. If G_0 contains one or more integrators the compensator G_c will thus have a low gain at low frequencies. The gain will decrease with increasing time delay T. At high frequencies the gain of G_c will be equal to one. The amplitude curve thus indicates that the general characteristics of G_c is that of a lead network. For frequencies such that $G_0(s) \approx -1$ it follows from (6) that

$$G_c(s) \approx e^{sT} \qquad (\text{for } G_0 \approx -1) \tag{6}$$

This indicates clearly that the network will give a considerable phase advance.

The approximative analysis thus indicates that the transfer function G_c corresponds to a lead network. The total phase advance will increase with increasing T. If n = 1 the total phase lead will be a multiple of 2π.

4. AN EXAMPLE

A specific example will now be investigated. Let the open loop transfer function be

$$G_0 = \frac{K}{s(s+1)(s+2)}$$

A reasonable value of the gain is $K = 1$. See e.g. Åström (1967 p. 163). The transfer function G_c defined by (6) then has the properties

$$\lim_{s \to 0} G_c(s) = \frac{1}{1 + kT/2}$$

$$\lim_{s \to \infty} G_c(s) = 1$$

The shapes of the Nyquist diagram of the transfer function G_c are indicated in Fig. 4.

Since both $G_c(0)$ and $G_c(i\infty)$ are real the total phase advance is a multiple of 2π. The frequency characteristics of G_c will now be explored further. The frequencies where the Nyquist curve intersects the negative real axis are given by

$$\arg(1 - e^{-i\omega T}) G_0(i\omega) = \pi$$

Hence

$$\frac{\pi}{2} - \frac{R[\omega T]}{2} - \frac{\pi}{2} - \text{arctg } \omega - \text{arctg } \omega/2 = \pi$$

or

$$-\frac{R[\omega T]}{2} = \pi + \gamma$$

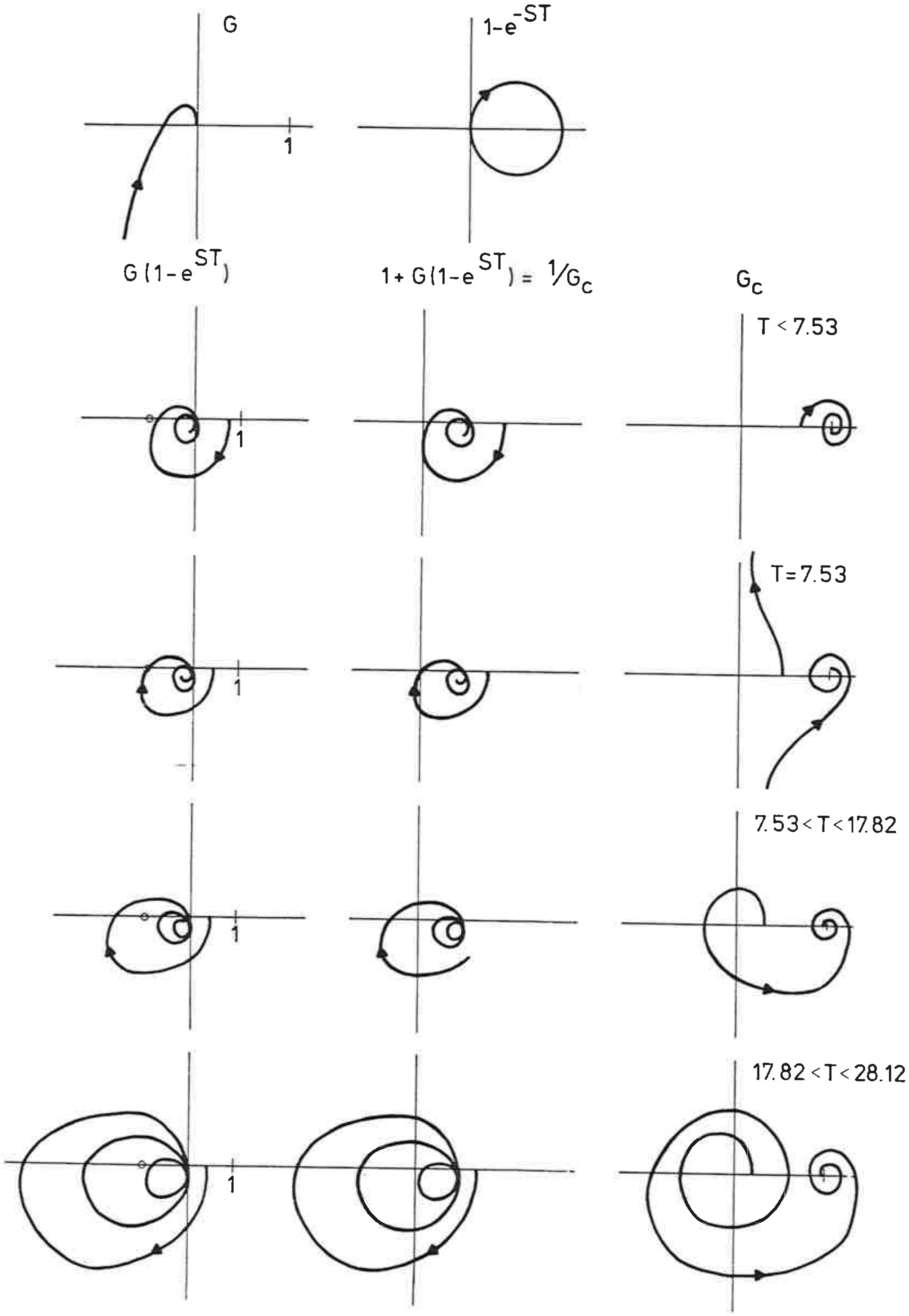
G
$1-e^{-ST}$
1
$G(1-e^{ST})$
$1+G(1-e^{ST}) = 1/G_c$
G_c
$T < 7.53$
$T = 7.53$
$7.53 < T < 17.82$
$17.82 < T < 28.12$
1
1
1
1

Fig. 4.

where

$$\gamma = \text{arctg}\ \frac{3\omega}{2 - \omega^2}$$

and $R[\alpha]$ denotes the residue of α modulo 2π. The frequencies where the Nyquist curve intersects the negative real axis are thus given by

$$\text{tg}\ \omega T/2 = \frac{3\omega}{\omega^2 - 2}$$

The magnitudes of the transfer function G_c at these frequencies ω_0 are given by

$$\left| (1 - e^{-i\omega_0 T}) G_0(i\omega_0) \right| = 2 \sin \gamma \frac{K}{\omega_0 \sqrt{(1+\omega_0^2)(4+\omega_0^2)}}$$

See Fig. 5.

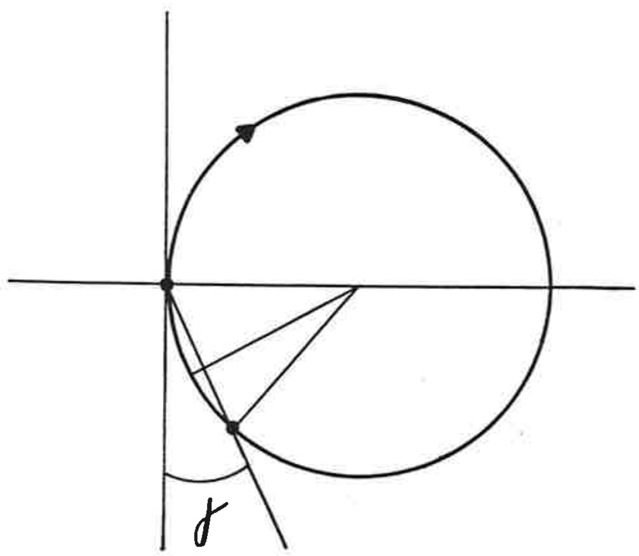

Fig. 5.

But

$$\sin \gamma = \frac{3\omega_0}{\sqrt{\omega_0^4 + 5\omega_0^2 + 4}} = \frac{3\omega_0}{\sqrt{(\omega_0^2+1)(4+\omega_0^2)}}$$

Hence

$$\left| (1 - e^{-i\omega_0 T}) G_0 (i\omega_0) \right| = \frac{6K}{(1+\omega_0^2)(4+\omega_0^2)}$$

The intersection will be to the left of the point -1 if

$$6K > (1+\omega_0^2)(4+\omega_0^2)$$

For K = 1 this gives

$$\omega_0 < \sqrt{(\sqrt{33}-5)/2} = 0.6102$$

Furthermore,it follows from Fig. 5 that

$$\omega_0 T = 2\pi n - 2\gamma$$

Hence

$$\gamma = n\pi - \omega_0 T/2$$

Introducing $\omega_0 = 0.6102$ and using (8) the following numerical values are obtained

$$T = 10.3n - 2.77$$

The time delay corresponding to integer values of n are listed below.

n	T
1	7.53
2	17.82
3	28.12
4	38.42
5	48.72
6	59.01
7	69.31
8	79.61
9	89.90
10	100.2

This table gives the limits of the time delay for the Nyquist curve to make 1, 2, 3, ... revolutions around the origin. Notice that for the values of T given in the table above the function $1 + (1 - e^{-sT})G_0(s)$ will vanish for certain frequencies which means that the transfer function G_c becomes infinite. In Fig. 6, Fig. 7 and Fig. 8 are shown the Bode diagrams for T = 5, 12 and 22 corresponding to n = 0, 1 and 2.

Fig. 6 - Bode diagram for the transfer function

$$G_c(s) = \frac{1}{1 + (1-e^{-sT})G_0(s)}$$

for $G_0(s) = \dfrac{1}{\cancel{s}(s+1)(s+2)}$ and $T = 5$.

given

Fig. 7 - Bode diagram for the transfer function

$$G_c(s) = \frac{1}{1 + (1-e^{-sT})G_0(s)}$$

for $G_0(s) = \dfrac{1}{\cancel{s}(s+1)(s+2)}$ and T = 12.

glimer

Fig. 8 - Bode diagram for the transfer function

$$G_c(s) = \frac{1}{1 + (1-e^{-sT})G_0(s)}$$

for $G_0(s) = \dfrac{1}{S(s+1)(s+2)}$ and $T = 22.$

gemen

5. REFERENCES

Smith, O.J.M. (1957)
Closer Control of loops with Dead Time.
Chem. Eng. Progr. 53 (1957) 217-219.

Smith, O.J.M. (1958)
Feedback Control Systems.
McGraw Hill, New York.

Åström, K.J. (1967)
Reglerteori.
Almqvist & Wiksell, Stockholm.

APPENDIX

The following FORTRAN program was used to evaluate the transfer function.

```
C         FREQUENCY ANALYSIS OF OTTO-SMITH COMPENSATOR
C         KJA 740805 REVISED 750616
C
1         WRITE (9,100)
100       FORMAT (' WRITE GAIN TIME DELAY W0 W1 AND NP')
          IC=1
          AK=RTTFF(IC)
          T=RTTFF(IC)
          W0=RTTFF(IC)
          W1=RTTFF(IC)
          NP=RTTFF(IC)
          WRITE(6,101) T,AK
101       FORMAT ('1TIME DELAY =',F7.3,'SEC','     GAIN =',F6.3)
          WRITE (6,102)
102       FORMAT ('0OMEGA      ABSG       ARGG DEG  LOGABSG
          1ARGG RAD')
          PHI=3.14259265
          DO 10 J=1,NP
          W=W0+W1*FLOAT(J)
C
          R1=W
          A1=PHI/2.
          R2=SQRT(1.+W*W)
          A2=ATAN2(W,1.)
          R3=SQRT(4.+W*W)
          A3=ATAN2(W,2.)
          S1=1. - COS(W*T)
          S2=SIN(W*T)
          R4=SQRT(S1*S1+S2*S2)
          A4=ATAN2(S2,S1)
          R5=AK*R4/(R1*R2*R3)
          A5=A4-A1-A2-A3
          S1=1.+R5*COS(A5)
          S2=R5*SIN(A5)
          R=1./SQRT(S1*S1+S2*S2)
          A=-ATAN2(S2,S1)
          RLOG=ALOG10(R)
          ADEG=A*180./PHI
10        WRITE (6,103) W,R,ADEG,RLOG,A
103       FORMAT (F10.4,F10.5,F10.2,2(1PE13.5))
          GO TO 1
          END
```