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ADAPTIVE STABILIZATION OF GENERAL, MULTIVARIABLE, CONTINUOUS- OR DISCRETE-TIME LINEAR SYSTEMS.

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DEPARTMENT OF AUTOMATIC CONTROL
LUND INSTITUTE OF TECHNOLOGY
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Adaptive Stabilization of General, Multivariable, Continuous- or Discrete-Time Linear Systems

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Abstract. Let an unknown continuous- or discrete-time, multivariable linear system, possibly non-minimum phase, and of high relative degree, be given. Suppose that we have the a priori information that for a known, nonnegative integer l, there is a (non-adaptive) regulator of order l which stabilizes the system. It is shown that this suffices as a priori information for an adaptive stabilizing controller. An example of such an algorithm is given. The continuous- and the discrete-time versions are given by exactly analogous formulas. This yields a continuous regulator, which does not utilize probing signals. It is based on a dense search through parameter space, and does not utilize high gain properties, as opposed to the "universal regulators" proposed before. In the absence of information of such an l, it is shown how to modify the algorithm to search over the regulator structures, i.e. the controller's dimension.

1. Introduction

During the last year there has been a considerable interest in "universal regulators", see [3] - [4], [6] - [7]. In [7] it was shown for the first time that knowledge about the sign of the "instantaneous gain" was not needed for stabilizing adaptive control of a first order single-input single-output system. In [3] this was generalized to a minimum phase, relative degree one system of arbitrary (finite) order, and in [4] to a square multivariable minimum phase system with CB invertible. The algorithm in [4] is however discontinuous and not given explicitly. Another direction of generalization is [6], which describes a regulator that will stabilize any single-input, single-output, minimum phase system of relative degree not exceeding two.

The main contribution of the papers discussed above is that it has been demonstrated that among the four "classical" assumptions on necessary a priori information for adaptive control of continuous-time, single-input, single-output plants, namely

1) The degree of the plant, n, is known
2) The plant is minimum phase
3) The relative degree $\overline{n}$ is known
4) The sign of the "instantaneous gain" $cA^{\overline{n}-1}b$ is known

points 1) and 4) are not needed. The present work shows that 2) and 3) can be replaced by a weaker condition.

A "Universal Regulator" is presented which only depends on a nonnegative integer l, with the property that there exists a constant, nonadaptive, linear controller of dimension l, which yields internal stability to the controlled system. It is shown in [2] that for continuous time systems, this is necessary a priori information as well (for a regulator of fixed structure). Necessary and sufficient a priori information needed for adaptive stabilization of an unknown multi-input, multi-output linear system has thus been characterized. Obviously, this is not completely independent of 2) and 3): e.g. if 2) and 3) are both true, then in general the least l is $\overline{n}-1$ as is well known.

The regulator presented is based on a dense search through the parameter space. It differs from the previous "universal regulators" in [3] - [4], [6] - [7], in the sense that it is not based on high gain stabilization. Both the continuous- and discrete-time versions are given by exactly analogous formulas.

In Section 2, it is shown that the static feedback problem contains the dynamic feedback problem for a fixed order of the controller dynamics. Section 3 presents the "Universal Regulator", depending only on l. Convergence is proved. In the next section it is shown how the regulator can be modified to search over the structure of the (linear part of the) regulator, still with guaranteed convergence. Some of its properties are discussed in Section 5.

2. A Viewpoint on Dynamic Feedback

In this section we show that, from a certain point of view, dynamic feedback can conceptually be replaced by static feedback. The idea is very simple: we augment the plant by attaching to it a box of integrators, each with its own input and output. Then we apply static feedback from the augmented plant, i.e. the plant together with the integrators. For the continuous time case, the situation is depicted in Figure 1.

More formally: Consider the following dynamic feedback problem: Given the plant

$$\dot{x} = Ax + Bu, \qquad x \in \mathbf{R}^n, \qquad u \in \mathbf{R}^m \tag{1C}$$

$$y = Cx, \qquad y \in \mathbf{R}^p$$

and the controller

$$\dot{z} = Fz + Gy, \qquad z \in \mathbf{R}^l \tag{2C}$$

$$u = Hz + Ky$$

It is easy to see that this is equivalent to the static feedback problem

$$\dot{\tilde{x}} = \tilde{A}\tilde{x} + \tilde{B}\tilde{u}$$

$$\tilde{y} = \tilde{C}\tilde{x} \tag{3C}$$

$$\tilde{u} = \tilde{K}\tilde{y}$$

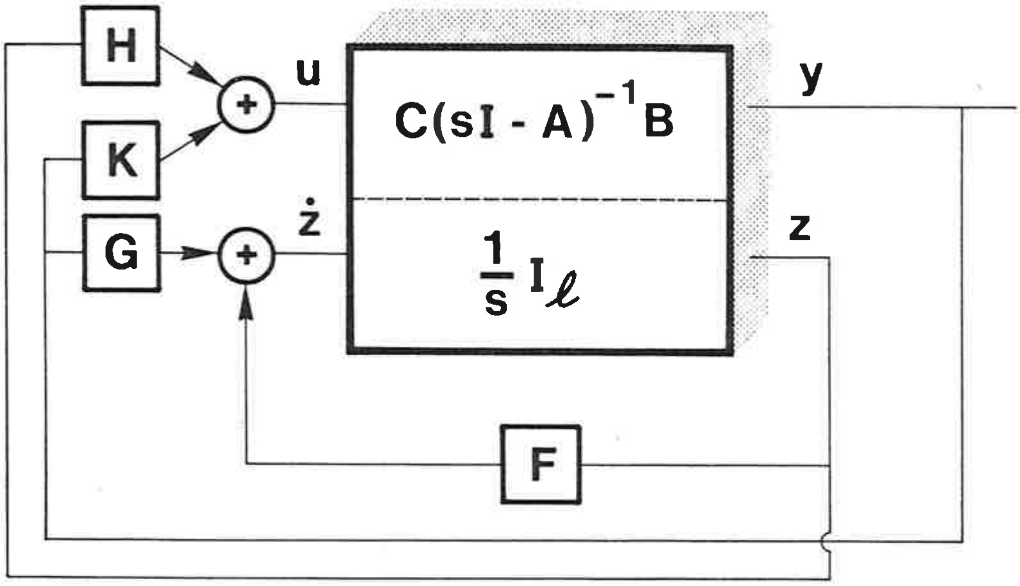

Figure 1. *Dynamic feedback considered as static feedback.*

where

$$\tilde{x} = (x^T z^T)^T$$
$$\tilde{u} = (u^T \dot{z}^T)^T$$
$$\tilde{y} = (y^T z^T)^T$$
$$\tilde{A} = \begin{pmatrix} A & 0 \\ 0 & 0 \end{pmatrix}$$
$$\tilde{B} = \begin{pmatrix} B & 0 \\ 0 & I \end{pmatrix}$$
$$\tilde{C} = \begin{pmatrix} C & 0 \\ 0 & I \end{pmatrix}$$
$$\tilde{K} = \begin{pmatrix} K & H \\ G & F \end{pmatrix}$$

For the discrete time case: Let the plant be

$$x(t+1) = Ax(t) + Bu(t), \qquad x \in \mathbf{R}^n, \qquad u \in \mathbf{R}^m \tag{1D}$$
$$y(t) = Cx(t), \qquad y \in \mathbf{R}^p$$

and the controller

$$z(t+1) = Fz(t) + Gy(t), \qquad z \in \mathbf{R}^l \tag{2D}$$

$$u(t) = Hz(t) + Ky(t)$$

This is equivalent to the static feedback problem

$$\tilde{x}(t+1) = \tilde{A}\tilde{x}(t) + \tilde{B}\tilde{u}(t)$$

$$\tilde{y}(t) = \tilde{C}\tilde{x}(t) \qquad (3D)$$

$$\tilde{u}(t) = \tilde{K}\tilde{y}(t)$$

where $\tilde{x}, \tilde{y}, \tilde{A}, \tilde{B}, \tilde{C}$, and $\tilde{K}$ are as before and $\tilde{u}(t) = \left(u(t)^T z(t+1)^T\right)^T$.

3. The Universal Regulator

As shown in the preceding section, it suffices to consider adaptive control based on static feedback. A (fixed) regulator is then nothing but a matrix $\in \mathbf{R}^{M \times P}$. Since a (fixed) regulator achieving internal stability to the controlled system places all the eigenvalues in the open left-half plane, (or the open unit disc) and these depend continuously on the parameters of the controller, there is an open set in parameter space yielding a stable system. Equip $\mathbf{R}^{M\times P}$ with the norm

$$||A||^2 = \sum (A)_{ij}^2$$

Thus we identify $\mathbf{R}^{M\times P}$, as a normed space, with $\mathbf{R}^{MP}$, equipped with the Euclidean norm. For the sequel, we let $||.||$ denote the this vector norm, or the corresponding induced matrix norm. Partition $\mathbf{R}^{M\times P} = \mathbf{R}^+ \times S^{MP-1}$ in a natural way. Let the regulator be

$$\tilde{u} = g(h(k))N(h(k))\tilde{y} \qquad (4)$$

$$\dot{k} = ||\tilde{y}||^2 + ||\tilde{u}||^2 \qquad (5C)$$

or

$$k(t+1) = k(t) + ||\tilde{y}||^2 + ||\tilde{u}||^2 \qquad (5D)$$

where

$$N(h) \text{ is “almost periodic” and dense on } S^{MP-1} \qquad (6)$$

and h and g are continuous, scalar functions satisfying

$$h(k) \nearrow \infty, \qquad k \to \infty \qquad (7)$$

$$\text{There exists a } \delta \text{ such that } |dg/dh| < \delta \qquad (8)$$

$$g\left(\{\alpha\nu + (\beta,\gamma)\}_{\nu=n}^{\infty}\right) = \mathbf{R}^+ \quad \text{for } n \in \mathbf{Z}, \alpha \neq 0, \gamma > \beta \qquad (9)$$

$$kg(h(k))\frac{dh}{dk} \to 0, \qquad k \to \infty \qquad (10)$$

We can now formulate our main theorem.

Theorem. *Consider the minimal plant (1). Assume that l is chosen so that there exists a (fixed) stabilizing controller of the form (2), and that the augmentation to the form (3) has beed done. The controller (4) - (5) subject to (6) - (10) will then stabilize the system in the sense that*

$$(x(t), z(t), k(t)) \to (0, 0, k_\infty) \qquad \text{as} \quad t \to \infty$$

where $k_\infty < \infty$.

Remark. One set of functions satisfying (7) - (10) is

$$h(k) = \sqrt{\log k}, \qquad k \geq 1$$

$$g(h) = \sqrt{h}\left(\sin\sqrt{h} + 1\right)$$

A curve $N(h)$ on S^{MP-1}, satisfying (6) can e.g. be realized by the following procedure: First we introduce coordinates on S^{MP-1}, with a variety of lower dimension removed. We use the "spherical coordinates" on S^{MP-1}:

$$x_1 = \sin\theta_{MP-1} \cdots \sin\theta_2 \sin\theta_1$$

$$x_2 = \sin\theta_{MP-1} \cdots \sin\theta_2 \cos\theta_1$$

....

....

$$x_{MP-1} = \sin\theta_{MP-1} \cos\theta_{MP-2}$$

$$x_{MP} = \cos\theta_{MP-1}$$

where

$$(\theta_1, \ldots, \theta_{MP-1}) \in (0, 2\pi) \times (0, \pi)^{MP-2} = D^{MP-1}$$

This is a bijection from D^{MP-1} to a open, dense subset of S^{MP-1}. In order to satisfy (6) put

$$\theta_i = a_i h \qquad i = 1, \ldots, MP-1$$

where $\{a_1, \ldots, a_{MP-1}\}$ are linearly independent over the rational numbers. The curve $N(h)$ is now analogous to a skew line on a torus, hence it is dense and almost periodic [1].

We will prove the theorem only for the discrete time case. The proof for the continuous time case is similar, and can be found in [5].

For the proof we need the following lemma, which is proven in the appendix.

Lemma. *Assume that the linear system (1D) is observable. Then:*
(i) For all x(0), there are constants c_0 and c_1 such that

$$\|x(t)\|^2 \leq c_0 + c_1\left(\sum_{\tau=0}^{t}\|y(\tau)\|^2 + \sum_{\tau=0}^{t}\|u(\tau)\|^2\right) \tag{11}$$

for all $x(0)$, $u(.)$, and $t \geq 0$. Here c_0 does not depend on t or u; and c_1 does not depend on t, $u(.)$ or $x(0)$.

(ii) For $T \geq \nu$, the observability index of the system, c_1 can be taken so

$$||x(t)||^2 \leq c_1 \left(\sum_{\tau=t-T}^{t} ||y(\tau)||^2 + \sum_{\tau=t-T}^{t} ||u(\tau)||^2 \right)$$

for all t, $u(.)$, and $x(t-T)$.

(iii) c_0 and c_1 can be taken so that (11) holds, with the same c_0 and c_1, for all augmentations of the form (1) $\longrightarrow$ (3), i.e. for all l.

Proof of the Theorem. We claim that it is enough to show that k increases to a finite limit k_∞. By (4) - (5D), $\tilde{y}$ and $\tilde{u} \in \ell^2$. Part (ii) of the lemma applied to the plant (3D) yields that $x(t)$ and $z(t) \to 0$, as $t \to \infty$. This proves the claim. For the proof we may thus assume that $k \nearrow \infty$.

First we find an estimation of the norm of x before the system stabilizes: By (5D) and the lemma applied to the system (3D) there exists constants c_0 and c_1 such that

$$||\tilde{x}||^2 \leq c_0 + c_1 k \tag{12}$$

Next we analyse the properties of the regulator matrix curve $g(h)N(h)$: It follows from (6) - (9) that $\{g(h)N(h), h = h(k), k \in \mathbf{R}^+\}$ is a dense subset of the space of $M \times P$ matrices. By (6) and (8), this curve is traversed with a bounded velocity in the parameter h. By assumption, there is a $g_0 N_0$ such that the control law $\tilde{u} = g_0 N_0 \tilde{y}$ stabilizes the system. There is also a $Q = Q^T > 0$ such that $(\tilde{A} + g_0 \tilde{B} N_0 \tilde{C})^T Q (\tilde{A} + g_0 \tilde{B} N_0 \tilde{C}) - Q = -I$. By continuity, the left hand side will be $< -\frac{1}{2} I$ for gN in some neighborhood of $g_0 N_0$.

From this we deduce that there exists infinitely many disjoint open intervals $I_\nu = (\alpha_\nu, \beta_\nu)$, $\nu = 1, 2, \ldots$; a constant $\delta > 0$ such that $\beta_\nu - \alpha_\nu > \delta$ for $\nu = 1, 2, \ldots$; and $(\tilde{A} + g(h)\tilde{B}N(h)\tilde{C})^T Q (\tilde{A} + g(h)\tilde{B}N(h)\tilde{C}) - Q < -\frac{1}{2} I$ for h in any of these intervals.

We now analyze what happens when $h \in I_\nu$ for some ν. Suppose that $h \in I_\nu$ when $t = t_0$. By above, $\tilde{x}^T Q \tilde{x}$ will then be a discrete-time Lyapunov function, and $||\tilde{x}(t)|| \leq c_0 e^{-c_1(t-t_0)} ||\tilde{x}(t_0)||$ for some $c_0, c_1 > 0$ and $t \geq t_0$. This, together with (4), means that there exist constants d_0 and d_1 such that

$$\sum_{t_0}^{\infty} ||\tilde{y}||^2 dt + \sum_{t_0}^{\infty} ||\tilde{u}||^2 dt \leq \left(1 + d_0 \sup_{h \in I_\nu} g\right) \sum_{t_0}^{\infty} ||\tilde{y}||^2 dt \leq d_1 \left(1 + d_0 \sup_{h \in I_\nu} g\right) ||\tilde{x}(t_0)||^2$$

provided that h stays within I_ν, for some ν, for all $t \geq t_0$. In particular, the left hand side exists finite, and the theorem will be proved.

Finally we prove that h will get stuck in some I_ν: The increase of h per unit of time is less than or equal to

$$\sup_{k\in[k(t),k(t+1)]} \left(\frac{dh}{dk}\right) \left(\|\tilde{y}\|^2 + \|\tilde{u}\|^2\right) \leq \sup \left(\frac{dh}{dk}\right) \left(1 + d_0 \sup_{[k(t),k(t+1)]} g\right) \|\tilde{y}\|^2 \leq$$

$$\sup \left(\frac{dh}{dk}\right) d_2 \left(1 + d_0 \sup g\right) \|\tilde{x}\|^2$$

for some d_2. By (10) and (12) this tends to 0, so we will intersect the lower half of I_ν for ν large. While $h \in I_\nu$ h will increase by at most

$$\sup_{h\in I_\nu} \left(\frac{dh}{dk}\right) \sum_{h\in I_\nu} \left(\|\tilde{y}\|^2 + \|\tilde{u}\|^2\right) \leq d_1 \sup_{h\in I_\nu} \left(\frac{dh}{dk}\right) \left(1 + d_0 \sup_{h\in I_\nu} g\right) \|\tilde{x}(t_0)\|^2$$

Combining this with the estimate (12), and considering (10) we conclude that for h sufficiently large, the left hand side is less than $\delta/2$. Thus there is a ν such that h will never leave I_ν. This proves the theorem.

4. Searching over the Dimension of the Controller

If no l is known, the algorithm can be modified in the following way: Let the regulator order be $l(h)$, where $l : \mathbf{R}^+ \to \mathbf{Z}^+$ is piecewise constant over lengths $l_i \nearrow \infty$, and $l(\{h > h_0\}) = \mathbf{Z}^+$ for all h_0. Also put $z = 0$ every time the regulator order changes.

By (iii) of the lemma, it is a straightforward verification to check that the proof will still be valid. The details are omitted.

5. Properties of the Universal Regulator

The most obvious property of the regulator described in the previous sections is that it is absolutely useless for every practical purpose, and its value is only on the level of existence proofs, to show that adaptive control with a certain amount of a priori information is possible.

From a certain point of view, the search may involve a vast overkill. For the "dynamic feedback" case we are e.g. also searching through the coordinates of the controller's state space. With more a priori information considerable refinements can be done.

Appendix.

Proof of the lemma. We first prove (i). By adding inequalities over t we see that it is enough to show

$$\|x(t)\|^2 - \|x(t-\nu)\|^2 \leq c_1 \sum_{\tau=t-\nu}^{t} \left(\|y(\tau)\|^2 + \|u(\tau)\|^2\right) dt \tag{13}$$

for some c_1 and all $t \geq \nu$. Further, by using time invariance, it is enough to show (13) for $t = \nu$. In an obvious operator notation

$$x(t) = A^t x(0) + \sum_{\tau=0}^{t} A^{t-\tau} B u(\tau) \equiv L_1^t x(0) + L_2^t u(.)$$

where L_1^t and L_2^t are bounded linear operators. We have

$$||x(\nu)||^2 - ||x(0)||^2 \leq 2||L_1^\nu x(0)||^2 + 2||L_2^\nu u(.)||^2 \leq 2||L_1^\nu x(0)||^2 + 2||L_2^\nu||^2 \sum_{\tau=0}^{\nu} ||u(\tau)||^2 \quad (14)$$

Write $y(t)$ as

$$y(t) = CL_1^t x(0) + CL_2^t u(.) \equiv y_1(t) + y_2(t)$$

Clearly,

$$\sum_0^\nu ||y_1||^2 d\tau \leq 2\sum_0^\nu ||y||^2 + 2\sum_0^\nu ||y_2||^2 \leq 2\sum_0^\nu ||y||^2 + 2\left(||C|| ||L_2^\nu||\right)^2 \sum_0^\nu ||u||^2$$

But observability implies that

$$\sup_{x(0)\neq 0} \frac{||A^\nu x(0)||^2}{\sum_0^\nu ||y_1||^2} = \sup_{x(0)\neq 0} \frac{x(0)^T A^{\nu^T} A^\nu x(0)}{x(0)^T M x(0)} = d_0 < \infty$$

where $M = M^T > 0$. So,

$$||L_1^\nu x(0)||^2 \leq d_0 \sum_0^\nu ||y_1||^2 \leq d_1 \sum_0^\nu ||y||^2 + d_2 \sum_0^\nu ||u||^2$$

for some d_1 and d_2. This, inserted into (14) proves (i).

Because of the estimations in (14), we see that (ii) is already proved for the case $t = T = \nu$. The proof for general t and T is similar.

To show (iii), note that c_1 can be expressed as a function of $||L_2^\nu||$ and d_0, so it is enough to show that these are bounded under all augmentations (1) $\longrightarrow$ (3). But this follows straightforwardly from the form of the augmentation. We leave the details for the reader.

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