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Application of grey relational analysis for evaluating road traffic safety measures: advanced driver assistance systems against infrastructure redesign

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ABSTRACT: Two of the main approaches to improve traffic safety are extensive redesign of the physical road infrastructure and large-scale implementation of advanced driver assistance systems. These approaches are to a large extent substitutes, but also partly complementary. Evaluation of alternative strategic investments in either of the two, and combinations, is complicated by limitations in availability, reliability and accuracy of data. Some of the evaluation methods most commonly used in transportation research are reviewed, and a method that is rather unknown in the western world and that is especially capable of dealing with part of these limitations is introduced. Grey relational analysis (GRA) – a normalisation-based method – provides a simple and transparent evaluation procedure from which a clear-cut ranking order of strategies derives. The application of GRA to the stated evaluation problem is illustrated with a case study in The Netherlands.

1 INTRODUCTION

In The Netherlands in the early 1990s a new concept named inherent safety originated to improve road traffic safety [1]. Infrastructure related measures are the most prominent part of this inherent safety philosophy and are known as the concept duurzaam veilige infrastructuur (DVI, inherently safe infrastructure). It was inspired by the fact that most traffic accidents are caused by human error. To counteract this, the traffic system should be adapted to avoid unintended use of the road infrastructure, encounters at high differences in speed and direction and uncertainty of the road users. The DVI concept was further developed during the mid 1990s and became an integral part of Dutch national traffic policy in 1998. DVI is an extensive and decentralised program, covering several decades and substantial investments to adapt the road network based on the principles of functionality, homogeneity and predictability, and intended to make the road more user-friendly. Main objective is to meet the ambitious Dutch policy targets for 2010: reductions of 30% fatalities and 25% of hospitalisation with respect to the 1998 figures. The DVI principles have been translated to a set of more operational requirements, from which concrete measures can be derived for adapting and upgrading the road network [2]. A first modest implementation of DVI measures has taken place in the years 1998–2002. In the mean time, doubts have arisen if the required investments will indeed bring the expected benefits.

Implementation of advanced driver assistance systems (ADAS) provides another way to improve road traffic safety. These systems support or take over vehicle driving tasks by sensing, communication and actuating devices and are meant to improve the safety, efficiency and comfort of driving. In previous parts of our research, five candidate ADAS functions are selected, which might meet the DVI requirements [3], and their technical feasibility is analysed [4]. Some state-of-the-art technologies are mature like navigation and speed assistance. Other technologies based on radar, lidar, video imaging, communication and/or satellite positioning are promising, but need still considerable improvement in robustness, reliability and cost.

Although complementary effects of ADAS with respect to infrastructure measures may exist, these applications may be largely considered potential substitutes for infrastructure redesign. As such, ADAS applications might offer an attractive and promising alternative to the high cost and long time scale of DVI measures. Through large-scale introduction of selected ADAS applications, safety effects may be achieved sooner and more cost-effectively, and with less negative effects as compared to DVI measures currently implemented [3]. However, possible strategies for this approach are characterised by many uncertainties.

To incorporate more explicit consideration on safety into the decision-making process, a comprehensive (strategic) evaluation framework needs to be built, and an evaluation method needs to be selected, which is able to compare items (i.e. DVI and ADAS) of quite different nature [5].

2 EVALUATION METHODS

2.1 Introduction of evaluation methods

Evaluation methods provide a recipe for analysis and ranking of different available alternatives for achieving a certain goal or objective. Generally, first, a list of relevant attributes of the alternatives is established, creating a (two-dimensional) matrix of alternatives (i) and attributes (k). For each relevant cell of this matrix, a value is established (the value of one attribute for one alternative). Then some operation is applied to rank the alternatives. Each set of attribute values for one alternative constitutes an alternative vector. The essence then is to transform each alternative vector in a coherent way to an appropriate numerical value. After this, the best or optimal alternative can be determined from the ranking, as the one with the highest, lowest or optimal value, depending on the type of problem. A general recipe for the application of evaluation methods may be formulated as follows (not each step is applied by each method): (1) create a decision matrix of alternatives against criteria; (2) establish values for the cells in the matrix; (3) apply a normalisation to the data; (4) establish weights for the various criteria; (5) apply the attribute weights to the data in the matrix; (6) apply some calculations to create an overall numeric value for each alternative vector and (7) rank the alternatives based on the calculated numeric values. The evaluation process is presented in Fig. 1.

Attributes may be expressed in cardinal or ordinal values, and derived from objective measurement or subjective appraisal score. If all attributes are expressed in the same unit (e.g. a monetary unit), then weights for the attribute categories can be omitted (the weights are implicit in the monetisation). Attributes may be aggregated to criteria (attributes may also be called sub-criteria, or just criteria if no classification is applied). Generally, three different types of attributes are distinguished: (1) benefit type or maximisation attributes, for which utility is a monotonically increasing function of value, also called attributes of monotonically increasing utility, or the-higher-the-better attributes; (2) cost type or minimisation attributes, for which utility is a monotonically decreasing function of value, also called attributes of monotonically decreasing utility, or the-lower-the-better attributes and (3) maximum value attributes, for which utility as a function of value has a maximum somewhere in the relevant attribute range, also called non-monotonic attributes.

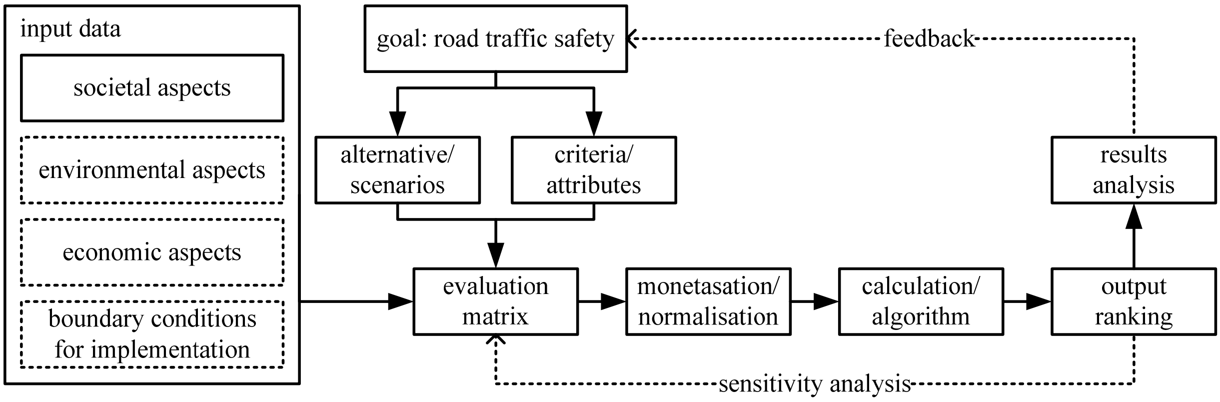

Fig. 1 *General evaluation process*

Different types of normalisation may be used, and their application differs for the type of attribute. Linear normalisation for benefit type attributes divides the attribute value by the maximum value of the attribute range, whereas vector normalisation divides the attribute value by the square root of the sum of the squares of all attribute values in the range (for benefit type attributes) [6]. Cost type attributes first need to be inversed before applying these procedures.

In general, evaluation methods for complex decision-making problems make use of multiple criteria (and hence of multiple attributes). Therefore although the name multicriteria analysis (MCA) is generally used for a certain type of methods (sometimes also referred to as multiple attributes decision-making methods), in fact all available evaluation methods for complex decision problems are multicriteria methods. An evaluation method may also try to pursue more than one objective, creating essentially a cubic array of alternatives, attributes and objectives, which makes only sense if certain attributes have different values and/or weights for different objectives.

Apart from the fact that it may often be difficult to establish the right set of attributes, and to find their proper values for each of the alternatives, which requires a thorough understanding of underlying functional relationships of the system in question, the hard part is generally to bring attributes with a very different character to some kind of common denominator, which makes it possible to derive an adequate ranking. It is especially because of the latter aspect that so many different evaluation approaches have been developed. The following two major categories of evaluation methods may be distinguished [see appendix (Section 8.1) for a concise description of some of these methods]:

1. Economics Economics-based methods express attribute values as much as possible in a monetary unit as an objective weight measure. The main representative in this category is cost-benefit analysis (CBA), which uses a monetary unit for all attributes. Cost-effectiveness analysis (CEA) is a more flexible variant, and generally measures costs in monetary units, but the harder to measure benefits in terms of other real units. Planning balance sheet (PBS) and goals-achievement matrix (GAM) are extended monetary methods that express part of the attributes in monetary terms and other attributes in non-

monetary real units or descriptive terms. Other variants (i.e. special cases) of CBA are cost-utility analysis, environmental impact reviews, profitability assessment and fiscal impact analysis.

2. Normalisation-based methods originated to overcome the fact that it is difficult and often impossible to express attributes in monetary or the same units, and because of the lack of adequate techniques to process attributes which are expressed in a range of different units. Instead of putting efforts in valuing benefits and costs or defining better methods to do it, these methods completely abstain from it, and in replacement apply a normalisation to the attribute vectors, by transformation to dimensionless values, which then enables to compare attributes of different character. In addition, in most cases, a subjective set of weights is applied to the attribute categories. These are the methods generally referred to as MCA methods. This paper discusses some of the main methods in this category: analytical hierarchy process (AHP), simple additive weighting (SAW), technique for order preference by similarity to ideal solution (TOPSIS), ÉLimination Et Choix Traduisant la RÉalité (ELECTRE, elimination and choice translating the reality), preference ranking organization method for enrichment evaluations (PROMETHEE) and fuzzy evaluation.

2.2 Requirements for the method to evaluate safety related ADAS and DVI measures

The evaluation case that is addressed in this paper concerns two types of technologies of quite different character, infrastructure and ADAS. For the traffic safety effects of infrastructure measures, data are available. For ADAS safety effects, data are not usually available, but may be estimated in terms of differences with infrastructure effects (see e.g. [5]). Besides traffic safety, the evaluation should take into account a range of other aspects, some of which can be expressed in measurable units with different levels of certainty, whereas others can only be valued by some scoring approach. From these characteristics, three specific requirements derive for the evaluation method to be used. This method should be able to adequately process and aggregate in one evaluation matrix: (1) a wide range of attributes with different value types; (2) both attributes expressed in objective physical measurement units and attributes expressed as subjective scores and (3) both attributes with accurate and reliable values and attributes for which all or part of the available data have a high level of uncertainty with respect to accuracy.

2.3 Assessment of evaluation methods in view of stated requirements

None of the aforementioned methods sufficiently fulfils the requirements for the evaluation. Each method in both categories has its advantages and limitations. No method is able to provide fully satisfying results, and there is often room for arguments. All methods try to provide a ranking of alternatives by calculating a resulting number per alternative. The economics-based methods express attribute values, as much as possible, in a monetary unit. This appears to be very difficult in practice, but the less stringent this condition is applied, the more difficult it becomes to obtain a clear analytical answer. The normalisation-based methods try to remove the issue of incomparable units, but none of them is found on a fundamental theory. Each of these methods is in fact no more than an advanced calculation recipe, and not every method is always able to provide an unambiguous order.

The multitude of attribute value types that need to be addressed in the evaluation of traffic safety measures precludes by nature the use of an economics-based method. The attribute value types comprise both measurement values and scores, which excludes normalisation methods exclusively based on scores. The remaining normalisation-based methods require relatively accurate and reliable inputs which renders them inadequate for the stated evaluation case. In the next section, we present and detail GRA, a normalisation-based evaluation method, and explain that it meets the stated requirements.

3 GREY SYSTEM THEORY AND GREY RELATIONAL ANALYSIS

3.1 Grey system theory

Grey system theory was initiated by Deng [7] in the People's Republic of China. It provides a method for abstract modelling of systems for which the information is limited, incomplete and characterised by random uncertainty. The method of statistics which is generally used to address problems involving random uncertainty, requires relatively large sample sizes for a sufficiently reliable analysis. Grey system theory on the other hand requires only a limited (and from a statistics point of view generally insufficient) sample of discrete sequential (time series) data to reliably model and estimate system behaviour. Grey modelling concentrates on building a differential model from limited data sets [8]. The term grey stands for poor, incomplete and uncertain and is especially used in relation to the concept of information. In system control theory, a system for which the relevant information is completely known is sometimes called a white system, whereas a system for which the relevant information is completely unknown is termed a black system. Any system between these limits then may be called a grey system [9]. Grey system theory typically deals with systems, objects or concepts having well-defined external boundaries but internal uncertainty or vagueness, whereas conversely fuzzy mathematics deals with systems, objects or concepts having a well-defined interior but not well-defined boundaries [10].

GRA is a derived evaluation method, which is based on the concept of grey relational space, one of the elements of grey system theory [11]. The fundamental schema of grey relational space is to integrate two-dimensional Euclidean space (i.e. distance space in grey system theory) and set-point topology (i.e. metric space). Because the latter cannot be directly and intuitively measured in the distance space, GRA specifies an algorithm to determine the distance between a reference series (created from the set) and each of the series of the set (i.e. compared series). Presently, GRA is mainly applied in Chinese speaking areas [12–14], and hardly known in western countries.

3.2 GRA evaluation method

The philosophy of GRA is to find a mathematical way to analyse the correlation between the series that compose a set space. In the case of an evaluation matrix, each alternative (i) can be taken as one series, which consists of a set of criteria. The sets of values of all the alternatives together constitute a grey relational space. By applying a certain algorithm, a clear-cut ranking of the different alternatives (series) is obtained.

According to Deng [15], GRA has the following characteristics: (1) only a limited number (at least three values in each series) of data are needed when compared with correlation analysis in statistics; (2) the distribution of the data does not need to be explicitly considered and (3) it provides a simple and transparent calculation procedure. In GRA, the attributes may be of any relevant category, and the original units may be applied, for example physical quantities and scores. Like in other normalisation-based methods, a matrix of i alternatives and k attributes is created, and the attribute vectors need to be expressed in dimensionless (hence comparable) units and similar scales. Different approaches for this normalisation may be used [appendix

(Section 8.2)].

To apply GRA, the input attributes need to satisfy three conditions for comparability of the different series (which are the assumptions of GRA): (1) the difference between the maximum and minimum input values (taken over all attributes) is less than an order of magnitude of two; (2) all attributes are of the same type (benefit, cost or maximum value) and (3) all attributes have the same measurement scale, and if in a quantitative scale, have the same unit or no unit. In the GRA literature, these conditions are (in a not so clear way) referred to as scaling (for the order of magnitude), polarisation (for the attribute type) and non-dimension (for the measurement scale) [11]. If these three conditions are not satisfied, normalisation of the input data prior to GRA processing is required. By applying normalisation, compliance with the three conditions is achieved.

3.3 GRA algorithm

After normalisation, the reference series is identified. This is the base vector of reference values with which all series are compared. Which value for a certain attribute defines the value of the reference series depends on the type of the attribute. In general, for a benefit-type attribute the highest value is taken, for a cost-type attribute the lowest value and for the optimisation or ‘targeted value’ category the predetermined preferred or optimal value.

For each alternative vector (in GRA also called a compared series, because each alternative vector is compared with the reference series), the difference of the reference vector and the alternative vector is calculated

$$\Delta_{0i}(k) = \left|x_0(k) - x_i(k)\right|,\ k = 1, 2, \ldots, n \tag{1}$$

This creates a new ($i \times k$) matrix of difference vectors. From this matrix, for each alternative i and attribute k (i.e. for each i, k cell) the grey relational coefficient for that attribute at point k is calculated. This grey relational coefficient for each element of an alternative vector or compared series is defined as

$$\gamma(x_0(k), x_i(k)) = \frac{\min_{i,k}\left|x_0(k) - x_i(k)\right| + \zeta \max_{i,k}\left|x_0(k) - x_i(k)\right|}{\left|x_0(k) - x_i(k)\right| + \zeta \max_{i,k}\left|x_0(k) - x_i(k)\right|} \tag{2}$$

where $\gamma\,(x_0(k), x_i(k))$ denotes grey relational coefficient of attribute k for alternative i, $x_0(k)$ denotes the element of the referential series for attribute k, $x_i(k)$ denotes the element of the compared series for attribute k, and $\zeta \in (0,1)$ denotes the identification or distinguishing coefficient [11, 16]. When the linear data approach for normalisation [appendix (Section 8.2) (12)–(14)] is applied, the value $\zeta = 1$ is taken. This avoids the discussion concerning the selection of an appropriate value for this coefficient [17]. Formula (2) can be worded as follows: for each i, k value, the sum of the minimum of all values in the k vector, and the maximum of all values in the k vector multiplied by a distinguishing coefficient, is divided by the sum of the value itself and the same maximum multiplied by the same distinguishing coefficient.

The grey relational grade for the compared series x_i in terms of weight w_k is given as

$$\Gamma_{0i} = \sum_{k=1}^{n} w_k \gamma_{0i}(k) \tag{3}$$

where w_k is the k-th weight of $\gamma_{0i} = \gamma(x_0(k), x_i(k))$. The grey relational grades of the different compared series provide a ranking of the alternatives, in which a higher value determines a better alternative.

3.4 Determination of weights

Weighting of the attribute vectors is an issue for each normalisation-based method, and the creation of an acceptable set of weights may generate a lot of discussion and need much effort [18, 19]. However, it is by its nature a subjective exercise, as weights relate to (subjective) preferences or trade-offs, whereas the evaluation method in itself has an objective character. A disadvantage of the use of weights in an early stage of the data processing is that they may change the actual relationships between attributes, the values of which are often unreliable and inaccurate in applications of GRA. We therefore propose to introduce weights only in the last processing step of GRA. The grey relational grade of a compared series, created without applying weights (or taking all weights equal, which is effectively the same) provides an objective measure of how good this series is compared with the reference series, which is based on the best values for each attribute over all alternatives. The set of grey relational grades for the different alternatives provides an objective ranking vector for these alternatives. Weights may be put all equal to $w_k = 1/n$, in a first approximation or if they are considered not relevant, and variation of the weights may be used for sensitivity analysis at a later stage. In this paper, we focus on the (objective) evaluation method itself and its application, and although the usefulness of the identification and application of weights for further processing at an operational level is recognised, a discussion of procedures for establishing weights is considered to be outside the domain of this paper.

3.5 Discussion of GRA

In general, the normalisation-based GRA evaluation method includes some of the positive aspects of both economics and normalisation-based methods, and in addition has its own unique characteristics for evaluation. GRA uses original values of each attribute. In principle, monetised values could still be used if possible, but otherwise any other applicable units or scores can also be applied. It is also allowed to use mixed positive and negative values for the original values of an attribute [20], which may be of use for the referenced application. Other normalisation-based methods that use physical measurement values as inputs for the matrix, generally have a problem to cope with this case.

A key step in normalisation-based methods is to express the attribute vectors in comparable and dimensionless units. The most general normalisation procedures are linear normalisation and vector normalisation, but other methods are applied as well, for example in AHP and PROMETHEE. However, a discussion of why a particular method is used, as well as the influence of the applied normalisation on the data series and their interrelationships, is generally omitted. In fact, the application of different normalisation methods may have an influence on the obtained ranking. In its standard normalisation approach, GRA provides a method of linear normalisation which is slightly more sophisticated than what is normally called linear normalisation. It is prove by Chang [20] that this procedure does not affect the interrelationships between the data series and therefore provides a more robust result.

In fact, all normalisation methods have similarities, but also clear differences. GRA is as efficient as the other methods. Of the other methods discussed in this paper, GRA seems very close to TOPSIS but fundamentally they are not the same. GRA is based on distance space and set-point topology, whereas TOPSIS only applies distance space. GRA takes into account the differences between each alternative (i) for a certain attribute (k) (i.e. Euclidean distance), while at the same time considering the set of attribute values for each alternative and its geometrical structure as a whole (topological metric). The sets of values of all the alternatives together constitute a grey relational space. A clear advantage is that the method requires only relative accuracy of attribute values within each attribute vector, and not absolute accuracy for every attribute value, which provides an essential difference of this method with other presented methods. As far as they use attributes expressed in physical measurement units, the other methods require absolute accuracy for each attribute value on its own.

GRA can easily be extended to cover multi-objective evaluation problems [21, 22]. The purpose of GRA is not to provide a perfect method for evaluation, but a practical one that has a broader basis than many of the traditional methods. Until now, the focus of GRA is more on the method as such than on its theoretical foundations. Further development of GRA and its foundations is required, and an interesting topic for further research. In general, it may be said that no algorithm can act as a complete substitute for human judgement. GRA is presented in the context of the following statement of Hill [23] 'If it does not sufficiently inform the decision-makers and the public so that they can use the information provided in order to arrive at a more rational decision, evaluation is an academic exercise. For this purpose, evaluation will have to be more context responsive'.

4 APPLICATION OF GRA

We apply the GRA evaluation method to a part of an extraurban route in the Netherlands (Leerdam via Amerongen to Elst). Most parts of the selected route are well built according to the principles of the inherently safe concept, and could be taken as a good example of what the result of infrastructure redesign should be. In this illustration, the method is applied to a specific route. The method can be equally well applied to any other route or to a whole network, and also to just one specific network element (intersection or road section).

4.1 Identification of scenarios for improving road traffic safety

A scenario in this context is synonymous to alternative and is defined as a possible implementation of a single measure (or function) or multiple measures (or functions). As a multitude of scenarios could be designed on the basis of the available measures, certain selection criteria will be chosen to guide this process. It should be taken into account that the process of scenario selection as provided here is illustrative, and that scenarios are selected on the basis of likelihood. For a case study of strategic evaluation, scenarios (S_i, i = 1–6) of infrastructure, ADAS and their combinations are created on the basis of expert knowledge and common logical reasoning, which are summarised in Table 1.

For infrastructure redesign, the selection criteria are costs and implementation period (i.e. short, medium or long). The rationale for this is that these factors have played an important role in establishing the implementation strategy for DVI in The Netherlands. Thus, the two basic DVI scenarios have different focuses: Scenario 1 only takes currently implemented and low cost measures into account and Scenario 2 includes the whole concept of inherently safe, that is the whole range of DVI measures.

As discussed in Lu et al. [4], five ADAS functions, that is navigation system with additional functionality [including traffic message channel (TMC)], speed assistance, lane keeping assistant, collision avoidance and intersection support, have been identified that can match infrastructure measures. These functions are either available on the market today or under development or deployment. The selection criteria for ADAS scenarios are technical and economical feasibility. Therefore two ADAS scenarios can be defined: Scenario 3, that is current ADAS, is based on the most feasible applications from a technology maturity and/or economical feasibility point of view and Scenario 4, that is future ADAS, demonstrates the longer term full potential of the implementation of ADAS applications (restricted to the identified five functions) for traffic safety.

The effects of an ADAS function are dependent on market penetration of the related system, that is in which part of the car population this particular ADAS function is present. The market penetration rate depends on the implementation schema, market-pull or policy-push. Market-pull implies that authorities take no action, but leave the adoption of a system to market forces. Policy-push means that authorities interfere with market forces, for example by fiscal measures or lower car insurance premiums, which may contribute to promote system acceptance if authorities decide for voluntary introduction. Authorities may also choose for mandatory introduction and in addition mandatory use. In this illustration, we assume 100% penetration of the ADAS functions. Different penetration levels could be added as different scenarios. Effects then need to be estimated for these penetration levels, based on estimation of the penetration/effect relationship for a particular ADAS function [8]. The mixed infrastructure and ADAS scenarios are based, for the ADAS part, on the-state-of-the-art technology and assume substitution of those infrastructure functions whose performance can be equally or better met by ADAS functions. The basic arguments for the construction of the combination scenarios are: (1) even ADAS applications need a good infrastructure design based on agreed infrastructure design principles and (2) some infrastructure measures may not be completely matched by ADAS, for example roundabouts, separated bicycle routes and vehicle parking separated from the road. Two combination scenarios are selected on the basis of different criteria, and taking Scenarios 1–4 as benchmark: for Scenario 5, the criterion is minimum costs; and for Scenario 6 the criterion is maximum safety effects.

Table 1: Road traffic safety scenarios

S_1	sober infrastructure redesign – sober 30 km/h zones, speed bumps, intersection channelisation, road category recognisable, parallel roads, absence of parked vehicles, roundabouts, plateaus, separate bicycle lanes, parking places separated from carriageway, semi-paved shoulders, obstacle free zones, cancellation of pedestrian crossings, consistent road markings, reduction of crossings and shoulder protection
S_2	full infrastructure redesign – S_1 together with traffic calming measures, physically separate carriageway roads with duel direction traffic and reconstruction of road sections and junctions
S_3	ADAS functions (state-of-the-art technologies) – navigation (with TMC), (map-based) speed assistance, (magnetic tape based) lane keeping assistance
S_4	ADAS functions (under development or in early stage of deployment) – S_3, together with lane change assistant, anti-collision, intersection support
S_5	combination – roundabouts and S_3 ADAS functions except lane keeping
S_6	combination – roundabouts, parallel roads, particular bicycle lanes and S_3 ADAS functions

4.2 GRA application steps

The process of evaluating the various ADAS and DVI implementation strategies by the application of GRA may be summarised by the following steps:

1. Establish the relevant alternatives and criteria and attributes (sub-criteria). Alternatives are the scenarios of ADAS, DVI and some appropriate combinations; criteria include social, environmental, economic and implementation aspects (Table 2).
2. Give operational definitions for the criteria and attributes (Table 2) to enable the specification of values for each alternative (Table 1).

3. Establish values and create the attributes (*k*) against alternatives (*i*) matrix ($k \times i$) (see Table 3, k = 1–40).
4. Identify the reference series (the ideal alternative), taking into account the (benefit or cost) character of each attribute.
5. Normalise the input data by using (12–14) [appendix (section 8.2)].
6. Calculate the absolute difference between the reference and each compared series by using formula (1) for each alternative *i*.
7. Calculate $\gamma(x_0(k), x_i(k))$ for each difference series, and the grey relational grades Γ_{0i} by using (2) and (3).
8. Rank the alternative scenarios on the basis of the grey relational grades (Table 3, bottom). The ranking provides the evaluation result.

Table 2: Criteria, attributes and operational value descriptions

Criteria	Attributes	Operational value description
Societal aspects	accident severity	ΔC_f – total fatality reduction rate (1998–2010), as percentage
		ΔC_h – total hospitalisation reduction rate (1998–2010), as percentage
	comfort/convenience	rated from 1 to 10, a higher grade means more comfortable/convenient
	emergency services	rated from 1 to 10, a higher grade indicates better for the services
Environment	reduce emissions	total reduction rate of CO, NOx and HC, as percentage
	reduce noise	rated from 1 to 10, a higher grade means higher noise reduction
Economic aspects	network capacity	rated from 1 to 10, a higher grade means higher contribution for the capacity
	land use	rated from 1 to 10, a lower grade indicates more extra physical space needed
	fuel consumption	reduction of fuel consumption, as percentage
	time spent	total travel time reduction rate, as percentage
	costs	total NPV (net present value) of 2000, in 1million EUR
Implementation	public acceptance	rated from 1 to 10, a higher grade means higher acceptance
	technology difficulty	rated from 1 to 10, a lower grade means fewer technical problems
	policy difficulty	rated from 1 to 10, a lower grade means easier to implement the policy

Since scenarios also have impacts on other factors than mere traffic safety, especially related to societal, environmental and economic aspects and implementation impediments, these factors are taken as main categories of criteria, for each of which one or more attributes are defined. In principle, each input value is as far as possible expressed in physical quantities. If an attribute cannot be easily expressed in physical quantities, a score with scale from 1 to 10 is defined for the attribute. For 'accident severity', currently the research focuses only on fatalities and hospitalisations, due to the lack of data for the other two main categories, slight injury and damage-only.

4.3 Determination of inputs of evaluation matrix

Table 3 presents the 40×6 evaluation matrix (i.e. attributes $k = 40$ and scenarios $i = 6$). The estimated safety effects (attributes 1–28) relate to 14 network elements (intersections and road sections) of the selected route, for each of which two different safety effects are specified, concerning respectively fatality reduction (ΔC_f) and hospitalisation reduction (ΔC_h). The safety effects of infrastructure measures are estimated on the basis of the data of Dutch Institute for Road safety Research (SWOV) [24]. The data cover the period 1998–2010, and the results allow good estimates of the values of attributes for the DVI scenarios (Table 3, k = 1–28 for S_1, S_2, S_5 and S_6). The safety effects of ADAS (see Table 3, k = 1–28 for S_3, S_4, S_5 and S_6) are derived from these values by applying the microscopic comparative analysis approach developed by Lu [5]. Using this method, absolute effects for ADAS functions may be estimated on the basis of estimation of relative effects of ADAS functions compared with DVI functions with similar effects and available data about absolute effects of the concerning

DVI functions. Having then data for the absolute effects of both ADAS and DVI, values for combination scenarios may be obtained as well. The estimated safety effects (attributes 1–28) relate to 14 network elements (intersections and road sections) of the selected route, for each of which two different safety effects are specified, concerning respectively fatality reduction (ΔC_f) and hospitalisation reduction (ΔC_h). Other possibilities to obtain input data for safety effects are, for example by simulation studies, or by real-world implementation pilot studies (before and after studies). The other impacts can be assessed by simulation, be based on literature study, and/or expert judgement. For each of these attributes, in this illustration an aggregated hypothetical value is provided for the whole route (Table 3, k = 29–39). However, in actual evaluations, for all or some of the impacts, specific values may be included for each of the elements (road sections and intersections) of a route or network, whereas partial aggregations may be used as well, as feasible. The values provided in this illustration are based on

Table 3: Outline of input (k = 1–40) and output (grey relational grade) of GRA

k	Attributes $x_i(k)$	Scenarios i (i = 1–6)					
		S_1	S_2	S_3	S_4	S_5	S_6
1	$x_i(1)$, ΔC_h	18.34	25.40	33.00	36.30	33.00	36.30
2	$x_i(2)$, ΔC_f	25.40	35.20	46.10	47.02	46.10	47.02
3	$x_i(3)$, ΔC_h	7.20	8.30	8.50	13.90	8.50	13.90
4	$x_i(4)$, ΔC_f	9.80	11.80	12.10	18.50	12.10	18.50
5	$x_i(5)$, ΔC_h	17.42	24.13	22.94	28.61	22.94	22.94
6	$x_i(6)$, ΔC_f	24.13	33.44	33.38	34.94	33.38	33.38
7	$x_i(7)$, ΔC_h	17.90	17.90	0.70	1.15	0.70	17.90
8	$x_i(8)$, ΔC_f	24.80	24.80	1.40	1.50	1.40	24.80
9	$x_i(9)$, ΔC_h	8.25	11.42	23.96	27.11	23.73	27.11
10	$x_i(10)$, ΔC_f	11.43	15.84	33.93	37.08	33.61	37.08
11	$x_i(11)$, ΔC_h	7.20	7.92	8.50	10.20	8.50	10.20
12	$x_i(12)$, ΔC_f	9.80	10.29	12.10	14.52	12.10	14.52
13	$x_i(13)$, ΔC_h	8.25	11.42	23.96	27.11	23.73	27.11
14	$x_i(14)$, ΔC_f	11.43	15.84	33.93	37.08	33.61	37.08
15	$x_i(15)$, ΔC_h	14.00	14.28	18.30	21.19	18.30	21.19
16	$x_i(16)$, ΔC_f	20.00	20.40	26.10	24.10	26.10	24.10
17	$x_i(17)$, ΔC_h	53.00	53.00	30.00	30.00	53.00	53.00
18	$x_i(18)$, ΔC_f	75.00	75.00	42.00	42.00	75.00	75.00
19	$x_i(19)$, ΔC_h	6.20	6.90	1.10	1.21	1.10	6.90
20	$x_i(20)$, ΔC_f	9.20	10.20	1.10	1.11	1.10	10.20
21	$x_i(21)$, ΔC_h	18.34	25.40	22.60	25.85	22.60	22.60
22	$x_i(22)$, ΔC_f	25.40	35.20	32.10	36.38	32.10	32.10
23	$x_i(23)$, ΔC_h	6.70	8.30	10.00	13.90	8.90	10.00
24	$x_i(24)$, ΔC_f	9.90	11.80	13.30	18.50	11.80	13.30
25	$x_i(25)$, ΔC_h	53.00	53.00	30.00	30.00	30.00	53.00
26	$x_i(26)$, ΔC_f	75.00	75.00	42.00	42.00	42.00	75.00
27	$x_i(27)$, ΔC_h	7.37	9.13	11.20	15.29	9.79	11.20
28	$x_i(28)$, ΔC_f	10.89	12.98	14.63	20.35	12.98	14.63
29	$x_i(29)$ comfort/convenient	2.00	3.00	8.00	9.00	8.00	7.00
30	$x_i(30)$ emergency	2.00	1.00	9.00	10.00	8.00	7.00
31	$x_i(31)$ reduce emission	0.00	0.00	0.90	1.20	1.00	0.90
32	$x_i(32)$ noise reduction	1.00	1.00	4.00	5.00	3.00	4.00
33	$x_i(33)$ throughput	1.00	1.00	4.00	5.00	3.00	4.00
34	$x_i(34)$ land use	2.00	1.00	10.00	10.00	6.00	5.00
35	$x_i(35)$ fuel reduction	0.00	0.00	1.20	1.40	1.60	1.20
36	$x_i(36)$ reduce time spent	0.00	0.00	0.80	1.05	1.00	0.80
37	$x_i(37)$ public acceptance	4.00	4.00	5.00	6.00	6.00	6.00
38	$x_i(38)$ technology	1.00	2.00	3.00	8.00	1.00	3.00
39	$x_i(39)$ policy difficulty	1.00	6.00	5.00	9.00	5.00	6.00
40	$x_i(40)$ costs (MEuro)	0.92	11.42	8.40	28.45	6.28	11.29
	grey relational grade Γ_{0i}	0.596303	0.630823	0.634612	0.890199	0.825519	0.989765

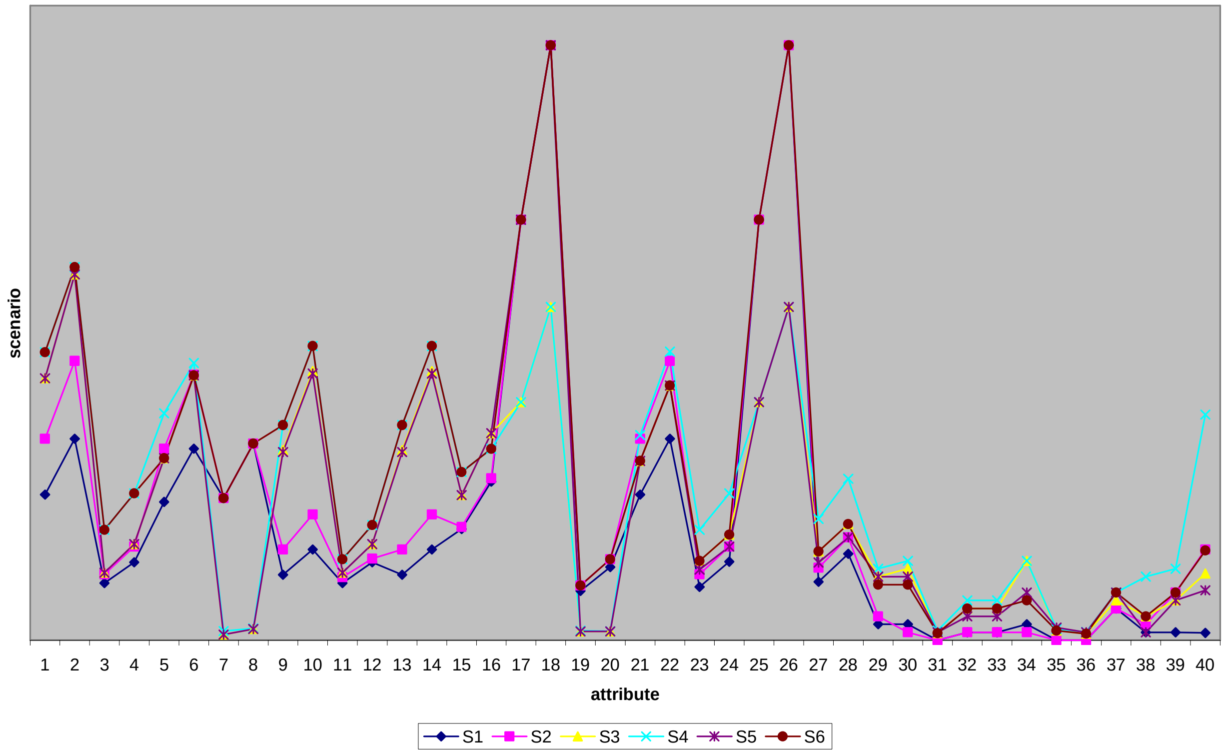

Fig. 2 *Estimation results for all effects (x-axis: attributes, y-axis: effect values)*

expert knowledge resulting from extensive literature study, and result from work performed in the EU funded (FP6) project IN-SAFETY (INfrastructure and SAFETY). The estimates for the attribute costs (Table 3, $k = 40$), are for the infrastructure part based on the estimates provided by the SWOV for the DVI Programme, and for the ADAS part on estimates of the number of vehicles involved, and of feasible system prices. Note again that this is only an illustration of the application of the method and that the current research does not focus on determining all input values (except safety effects) for the evaluation matrix. Fig. 2 shows the results of the estimation for all effects in graphical form.

Then we apply GRA for further processing and comparison of the scenarios to evaluate which are the most favourable options for decision support on alternative investment strategies. The results of this final evaluation of the various ADAS applications, infrastructure redesign and combination strategies are presented at the bottom of Table 3, expressed in terms of the grey relational grade Γ_{0i}.

To investigate the uncertainties and the trade-off between different measures in the macro evaluation, a sensitivity analysis can be performed. Generally, there are two ways of varying the parameters: by applying attribute weights and by variation of the values of certain attributes that have a margin of uncertainty. The weight vector cannot be obtained by modelling itself, but may be obtained from experts views or with other means, for example by applying AHP or other normalisation-based methods. At the operational level, attribute values can be varied by taking costs and safety effects as high, medium and low respectively. We do not further discuss the operation of sensitivity analysis, as this is not a key aspect of the presented GRA method [25].

5 CONCLUSIONS

Evaluation methods are categorised as either economics- or normalisation-based. The comprehensive review of relevant methods for evaluation issues in transport studies indicates that there is no generally acceptable evaluation method. Both economics- and normalisation-based methods in general, and each of the specific methods in particular, have their advantages and limitations, and none of the methods is able to provide fully satisfactory results. The abundance of methods is clearly a measure for the difficulty of the problem of ex-ante evaluation. In fact, each method provides not more than a specific recipe for aggregating the original data (the inputs of the evaluation matrix) and ranking the alternatives.

For selection of an appropriate evaluation method to deal with the evaluation case that is addressed in this paper, three specific requirements are derived. It is then concluded that none of the discussed methods meets these requirements. As an alternative, GRA is presented and discussed. As in other evaluation methods, in GRA an evaluation matrix of *i* alternatives with *k* attributes is created. From the series for each of the alternatives (i.e. the compared series), a reference series is created by data pre-processing. From the original matrix and the reference series, by using the GRA algorithm, a grey relational grade is calculated for each alternative. This number provides an objective measure of the effectiveness of an alternative relative to the other considered alternatives, and the set of grey relational grades provides a ranking of the alternatives, in which a higher grade represents a better alternative. The method does fulfil the three stated requirements. In a general sense, it provides a simple and transparent procedure to compare various alternatives with the theoretical optimal solution within the values provided by the set of all considered alternatives, and to establish a clear-cut ranking order of these alternatives. In a more specific sense, it is especially able to cope with inaccuracy of data, as it requires only relative accuracy of data within an attribute vector, and not absolute accuracy of the specific values. All this does not mean that GRA is a perfect method. It is just a different method with different characteristics and for other purposes. Like the other presented methods, GRA is basically no more that a recipe for evaluation, that is for processing of an evaluation matrix and providing a ranking of the alternatives. Further work on its mathematical foundation may help to strengthen its theoretical basis.

The evaluation result provides the following priority ranking of the six scenarios from high to low, denoted in the special GRA notation: $S_6 \succ S_4 \succ S_5 \succ S_3 \succ S_2 \succ S_1$. The most effective alternative is the combination scenario of ‘current’ ADAS and infrastructure measures (S_6). The scenario of ‘future’ ADAS is the second best (S_4). The next in line is the combination of ‘current’ ADAS and lowest cost infrastructure measures (S_5). The scenarios of infrastructure redesign implementation (S_1 and S_2) are the least effective strategies. One should be careful and not take these results as absolute truths. They are the output of an illustrative application of the method. Better and more sophisticated estimation of some of the input values may lead to a different ranking. The obtained ranking may also be biased by a too optimistic estimation in the SWOV figures of the safety impacts of some infrastructure redesign measures, which are therefore not offset by the high costs and other negative factors (e.g. comfort and convenience, emergency services and land use). Note furthermore that this ranking result only reflects a special case (i.e. a part of an extra-urban road in The Netherlands). The GRA evaluation method is equally applicable to other networks (e.g. a piece of route, a part of a network, a national network or the whole European network) and scenarios of a different composition, for which the ranking results may be different. Especially, the quality of the ranking is essentially dependent on the quality of the input data.

A crucial point to keep in mind is that ADAS applications need a good infrastructure design based on agreed infrastructure design principles. An evaluation method provides a tool for assisting decision making, but no algorithm can act as a complete substitute for human judgement.

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8 APPENDIX

8.1 Overview of prevailing evaluation methods

The following two tables provide summary overviews of the most common and popular economics and normalisation-based evaluation methods, respectively.

Summary overview of some common economics-based methods

CBA	Cost-Benefit Analysis
origin	1844 (France); first serious applications stimulated by US Flood Control Act of 1936
approach	(1) express all benefits and costs in monetary terms; (2) calculate benefit-cost ratio for each alternative; (3) rank efficiency is defined in terms of maximisation of general welfare, and evaluated on the basis of Pareto criterion (welfare is increased when a change makes at least one person better off and no one worse off)
characteristics	fundamental theory based exact approach; cannot deliver this exactness in practical reality
comments	due to various severe limitations; requires high accuracy of the monetised input
CEA	Cost-Effectiveness Analysis
objective	to overcome some of the difficulties of CBA
approach	(1) express benefits in relevant non-monetary unit and (2) calculate effectiveness-cost ratio per alternative
comments	clear-cut ranking order generally not possible when two or more effects are included
PBS	Planning Balance Sheet
founder/year	N. Lichfield, 1956
objectives	to overcome some of the difficulties of CBA; to enable community to choose plan that will maximise the achievement of community welfare goals
approach	(1) separate impacts of each alternative for different groups (avoid double counting); (2) express costs and benefits in monetary unit if possible, or otherwise in any physical unit, score or even in descriptive terms and (3) no calculation of rank order, but use of matrix to support understanding of impacts for different groups
comments	only high level goal addressed, whereas impacts only have meaning in relation to a well-defined objective; resource allocation cannot be resolved in case of conflicting goals or interests
GAM	Goal Achievements Matrix
founder/year	M. Hill, 1968
objectives	to overcome some of the difficulties of CBA; to view costs and benefits always in terms of achievement of operational rather than abstract goals
approach	(1) establish applicable unit of measurement for each identifiable goal, in quantitative or otherwise in qualitative terms; (2) per alternative (course of action), create a matrix of goals against affected groups or sectors; (3) benefits increase and costs decrease the state of achievement of a goal (compensatory character, trade-offs are possible), and may be summed up if expressed in the same unit; (4) per goal, identify the relative weights of the groups and identify the relative weights of each of the goals with respect to the other goals
comments	if all objectives could be expressed in the same unit, the result would resemble a weighted variant of CBA (but not necessarily in monetary terms); comparison of goals expressed in different units remains a problem; it is assumed that weights can be established objectively, which is often not the case in reality; the method does not accommodate for interdependence of objectives; whereas the method is quite capable to express complexity of decision problems, it is also itself complex and thereby costly and does not easily give a clear evaluation result

Summary overview of some common normalisation-based methods

AHP	Analytic Hierarchy Process
founder/year	T.L. Saaty, 1980
characteristics	use of a special approach for normalisation
approach	(1) per attribute, establish the relative importance (weight) of each alternative by pairwise comparison, using a fixed scale of scores ranging from 1 to 9 and their reciprocals; (2) from the scores, calculate a weight vector for each of the attributes, expressing the weight per attribute of the different alternatives (i.e. eigenvector calculation, for which different approaches are in use); (3) use same method (scoring and weight vector calculation) to establish weight vector for the attributes and (4) multiply attribute weight and alternative weight for that particular attribute, and sum the results per alternative, providing an overall priority score for each of the alternative, which can be used for ranking
comments	an advantage is the clear hierarchical structuring of the decision problem, and of the criteria, clarifying their relative importance; a limitation is the use of the artificial 9 points scale; the method faces the rank reversal problem (ranking of alternatives may sometimes be reversed when an extra alternative is added to the existing set
SAW	Simple Additive Weighting
approach	(1) express attributes (any category) in their original units; (2) establish attribute weights by (subjective) expert judgement; (3) normalise each attribute vector (e.g. by linear normalisation); (4) multiply each alternative vector by the attribute weight vector and (5) the sums of the resulting values per alternative vector provide a ranking of the alternatives
comments	the method is indeed simple and straightforward
TOPSIS	Technique for Order Preference by Similarity to Ideal Solutions
founder/year	Hwang and Yoon, 1981
approach	(1) express attributes (any category) in their original units; (2) establish attribute weights by subjective judgement; (3) normalise attribute vectors by vector normalisation and multiply by weight vector; (4) establish vectors of positive ideal values (highest value for benefit, lowest for cost attribute) and negative ideal values (reverse); (5) calculate for each alternative a positive (S^+) and a negative (S^-) ideal separation measure as square root of the sum of the squares of the difference of each attribute value with the value in the positive and negative ideal values vectors, respectively; both the S^+ and the S^- values provide rankings of the alternatives, which may be different; (6) a derived measure called similarity is calculated as $C = S^-/(S^++S^-)$ and provides yet another ranking, which may again be different
comments	the rationale of the method is explained in terms of indifference curves which are expressed by the formula for similarity; the method is easy to understand and to apply
ELECTRE	ÉLimination Et Choix Traduisant la RÉalité
founder/year	B. Roy, 1968
characteristic	outranking based on pairwise comparisons followed by a concordance analysis; an alternative outranks another if it is preferred for at least one attribute and not less preferred for any of the other attributes
approach	(1) express attributes (any category) in their original units; (2) normalise attribute vectors; (3) establish attribute weights by subjective judgement; (4) apply attribute weight vector to alternative vectors and (5) establish kernel of alternatives (each alternative in the kernel is not outranked by another in the kernel, and each alternative outside the kernel is outranked by at least one in the kernel) based on concordance and discordance relationships (not detailed here)
comments	the method has some clear limitations: no ranking of all alternatives is provided and no ranking of alternatives in the kernel; the choice of the average values of concordance and discordance critical indexes as decision thresholds is quite arbitrary; the net concordance and discordance indexes, defined to provide some kind of ranking, are not able to unambiguously solve the ranking issue
PROMETHEE	Preference Ranking Organisation METHod for Enrichment Evaluations
founder/year	J.P. Brans, 1982
characteristic	outranking based on comparison of the pairwise outranking relationships between attributes, after applying a ‘generalised criterion’ to each attribute; in principle many generalised criteria might be defined, in practice six standard generalised criteria are used; these generalised criteria are further specified by parameters, which depend on the situation

approach	(1) express attributes (any category) in their original units; (2) identify for each attribute the applicable generalised and determine the parameter(s) of the preference function; (3) determine weights for each of the attributes; the weights may be set to equal; (4) determine for each pair of alternatives x_1,x_2 the value of the preference function, and sum the values for which x_1 is preferred over x_2, as well as (separately) the values for which x_2 is preferred over x_1; (5) represent these values in the preference index of x_n by x_n; horizontally for each alternative, the row of resulting values being preferred over the other alternatives, vertically the column of values of the other alternatives being preferred over the specific alternative; (6) per alternative, the sum of its rows (the leaving flow) minus the sum of its columns (the entering flow) is calculated, resulting in one number (the net flow) per alternative; (7) the net flows provide the complete pre-order, a complete ranking of the alternatives, of version II of the method; the partial pre-order of version I is said to contain more realistic information, as it may reveal incomparability; this partial pre-order is achieved not by calculating net flows, but by comparing both leaving and entering flows for each alternative pair.
comments	although the method has a quite elegant appearance, the assumption of generalised criteria and use of the preference functions leave the impression that some kind of mysterious magic is applied; because of this, the result is not very transparent
fuzzy	
founder/year	L. Zadeh, 1960s
approach	based on fuzzy set theory and the assumption that in reality crisp attribute values do not exist; therefore attribute values could be expressed, for instance, in linguistic terms, describing sets with fuzzy boundaries, for which crisp values can have memberships from 0 (not an element of the set) to 1 (completely an element of the set), including values in between
comments	fuzzy evaluation is useful as a thinking model, also in combination with other methods; application is complex, costly, not transparent and has a high level of subjectivity

8.2 Overview of GRA normalisation procedures

The GRA community has seen quite extensive discussions on normalisation, that is the data pre-processing, to prove that the original attribute vectors, before normalisation, and the resulting attribute vectors, after normalisation, have a linear relationship, without any distortion [11, 17, 20]. We summarise the main data pre-processing procedures that are proposed and discussed in the GRA literature as follows.

Basic approach: The basic normalisation approach applies division by a specific value selected from the data set, for example the first value, the maximum value, the minimum value or the average value, as follows

- $x_i^*(k) = x_i(k) / x_{i^*}(k)$, $i, i^* = 1,2,\dots,l$ (4)

where $x_i^*(k)$ denotes normalised series, $x_i(k)$ denotes original series, k denotes attribute, and i denotes alternative, and the divisor may take different values, for instance

- the first value of the series $x_{i^*}(k) = x_i(1)$ (5)
- the maximum value of the series $x_{i^*}(k) = \max x_i(k)$ (6)
- the minimum value of the series $x_{i^*}(k) = \min x_i(k)$ (7)
- the average value of the series $x_{i^*}(k) = \overline{x}_i(k)$ (8)

where $\overline{x}_i(k)$ denotes the average value of the series $x_i(k)$.

Effect measurement approach:

- the-larger-the-better: expected effects as large as possible $x_i^*(k) = \frac{x_i(k)}{\max x_i(k)}$ (9)

- the-smaller-the-better: expected effects as small as possible $x_i^*(k) = \frac{\min x_i(k)}{x_i(k)}$ (10)

- optimisation of a specific value between maximum value and minimum value: expected effects as close as possible to a certain specific value (v_s)

$$x_i^*(k) = \frac{\min(x_i(k), v_s)}{\max(x_i(k), v_s)} \tag{11}$$

Linear data approach: The normalisation method of Wu and Chen [17] takes into account the type of the attribute (benefit, cost or optimisation value), and normalises to a scale [0, 1]. For benefit type attributes, the formula is

- the-larger-the-better: $x_i^*(k) = \frac{x_i(k) - \min_k x_i(k)}{\max_k x_i(k) - \min_k x_i(k)}$ (12)

where $\max_k x_i(k)$ is the maximum value of attribute *k* for alternative *i*, and $\min_k x_i(k)$ is the minimum value of attribute *k* for alternative *i*.

- the-smaller-the-better: $x_i^*(k) = \frac{\max_k x_i(k) - x_i(k)}{\max_k x_i(k) - \min_k x_i(k)}$ (13)

- optimisation of a specific value between maximum value and minimum value

$$x_i^*(k) = 1 - \frac{|x_i(k) - x_{ob}(k)|}{\max\{\max_k x_i(k) - x_{ob}(k), x_{ob}(k) - \min_k x_i(k)\}} \tag{14}$$

where $x_{ob}(k)$ denotes the objective value of attribute *k*, which can be determined, e.g. by a certain preference.

Standard approach: For cases in which the presented normalisation methods create a series which still does not satisfy the polarisation condition for direct comparability, Chang [20] provides a new method (and proves its applicability)

- the-larger-the-better $x_i^*(k) = \frac{x_i(k)}{\max x_i(k)}$ (15)

- the-smaller-the-better $x_i^*(k) = -\frac{x_i(k)}{\min x_i(k)} + 2$ (16)

- optimisation of a specific value between maximum and minimum

$$x_i^*(k) = \begin{cases} \frac{x_i(k)}{x_{\exp}} & \text{when } x_i(k) \le x_{\exp} \\ -\frac{x_i(k)}{x_{\exp}} + 2 & \text{when } x_i(k) > x_{\exp} \end{cases} \tag{17}$$

where $x_{\exp}$ denotes the objective value of attribute *k*, which can be determined, for example by maximum, minimum value or a certain value in between.