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CALCULATED GROUND-STATE PROPERTIES OF HEAVY NUCLEI

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Abstract: Ground-state distortions and single-particle corrections are calculated for nuclei with $Z \geq 68$ and $N \geq 106$ by use of the macroscopic-microscopic method as developed by Strutinsky. The microscopic part is calculated primarily by use of the folded Yukawa single-particle potential. Its parameters are redetermined to fit actinide data. The modified oscillator potential is also used in some of the studies. Two methods for calculating the macroscopic energy are investigated. One is the droplet model of Myers and Swiatecki, and the other is a modified liquid-drop model in which the surface-energy term is modified to take into account the finite range of the nuclear force. Single-particle level diagrams for the folded Yukawa potential are also presented. They are plotted as functions of the distortion parameters ϵ , ϵ_4 and ϵ_6 . Theoretical and experimental single-particle levels at the ground state for actinide nuclei are also compared.

1. Introduction

Today many calculations of nuclear properties are carried out by use of the macroscopic-microscopic method, which was developed in its present form by Strutinsky¹⁾. In this method the total potential energy of the nucleus is expressed as the sum of two terms, a macroscopic term and a microscopic term. A recent review of the calculation of fission barriers²⁾ contains some results obtained by use of the folded Yukawa single-particle potential to calculate the microscopic corrections and the droplet model of Myers and Swiatecki³⁻⁵⁾ to calculate the macroscopic part of the energy. A more detailed study of the results obtained for fission barriers is presented in ref.⁶⁾. However, compared to the large distortions involved in fission, much more experimental information is available at ground-state distortions. In this paper we use primarily the folded Yukawa single-particle potential to study single-particle levels, nuclear shapes, and single-particle corrections at ground-state distortions. These quantities depend not only on the single-particle potential but also, to a lesser extent, on the method that is used to calculate the macroscopic energy. We present results obtained by use of the droplet model of Myers and Swiatecki³⁻⁵⁾ and alternatively by use of a modified liquid-drop model in which the surface-energy term is modified to take into account the finite range of the nuclear force⁷⁾. Details on various aspects of the calculations can be found in refs.^{2, 7-9)} and in references quoted

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therein. Here we concentrate on recent results obtained by use of these models, with a brief survey of some of the details of the modified-surface-energy model.

2. Results of the calculations

2.1. SINGLE-PARTICLE LEVEL DIAGRAMS FOR THE FOLDED YUKAWA POTENTIAL

To describe the nuclear shape we use the coordinates ε , ε_4 and ε_6 in the perturbed-spheroid parametrization employed in the modified oscillator model. For most nuclei in the actinide region the two parameters ε and ε_4 describe shapes of lower energy

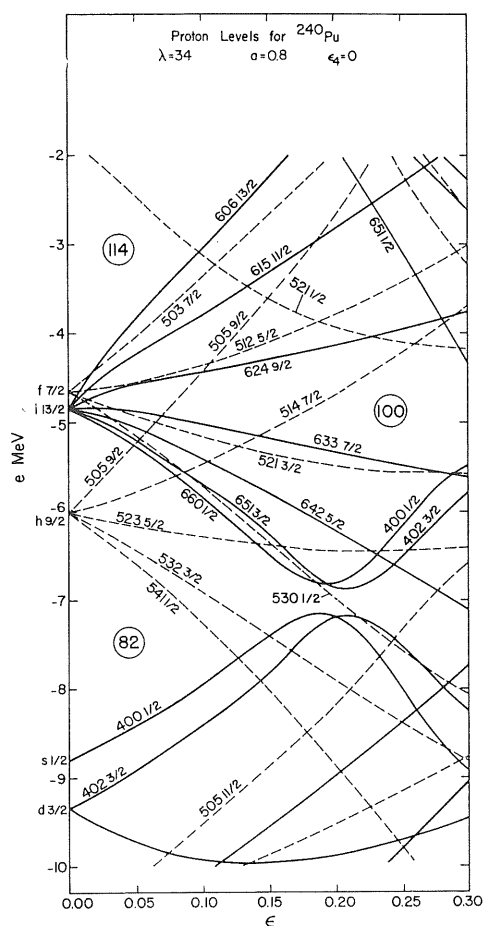


Fig. 1a. Single-proton levels near the Fermi surface of ^{240}Pu as functions of the coordinate ε . The levels are calculated with the folded Yukawa single-particle potential and are labeled by their asymptotic quantum numbers $[N\pi_z, \Lambda\Omega]$. The quantum numbers for the spherical levels are shown at the left.

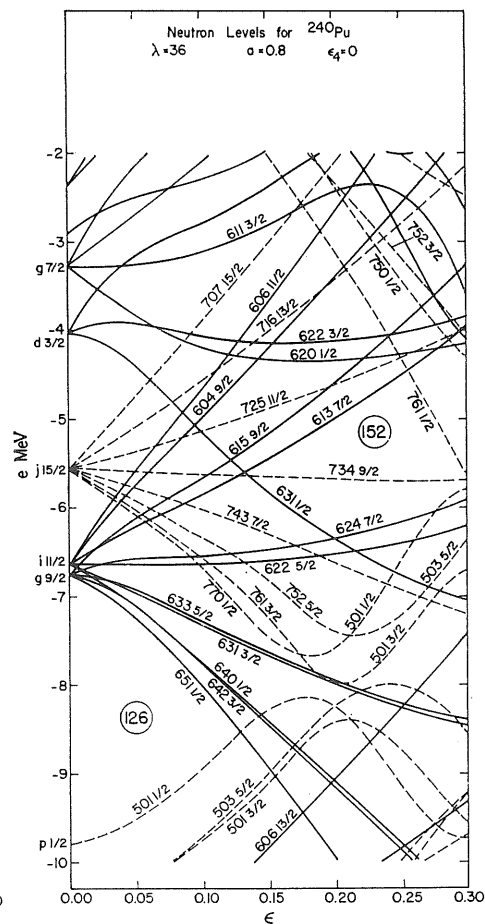


Fig. 1b. Single-neutron levels, analogous to fig. 1a.

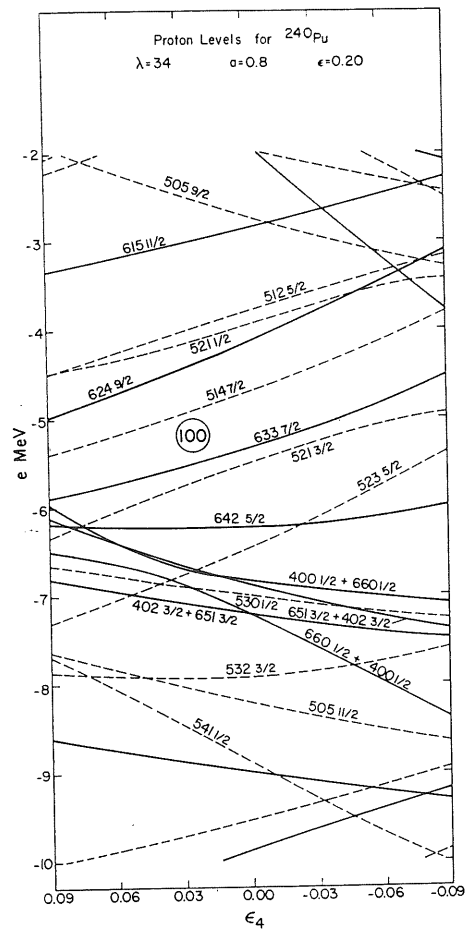


Fig. 2a. Single-proton levels as functions of the coordinate ϵ_4 . The coordinate ϵ has a value that corresponds to the average ground-state shape of the lighter actinides. The larger negative values of ϵ_4 in this figure generate shapes whose ends are highly protruding. For such shapes polar orbitals of the type $[660\frac{1}{2}]$, etc., are most likely to profit. In fact, the orbital seen at the top righthand part of the figure is the orbital usually denoted by $[651\frac{1}{2}]$. However, an analysis of the actual components of the state in question should show that at this small deformation it is dominantly $[660\frac{1}{2}]$ in character. An analysis of the components of other levels of the same type will yield similar results at this small distortion.

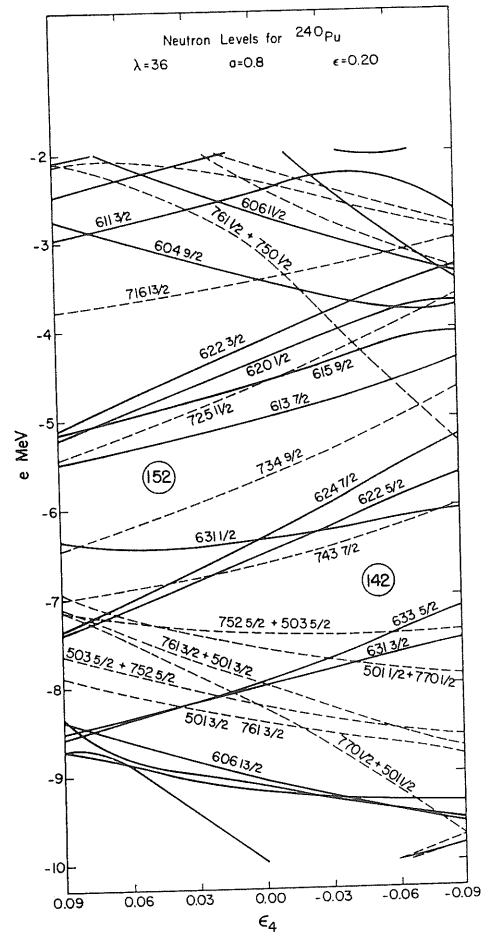


Fig. 2b. Single-neutron levels, analogous to fig. 2a. In analogy to fig. 2a the level denoted by $[761\frac{1}{2}]$ is dominantly of $[770\frac{1}{2}]$ character at this distortion.

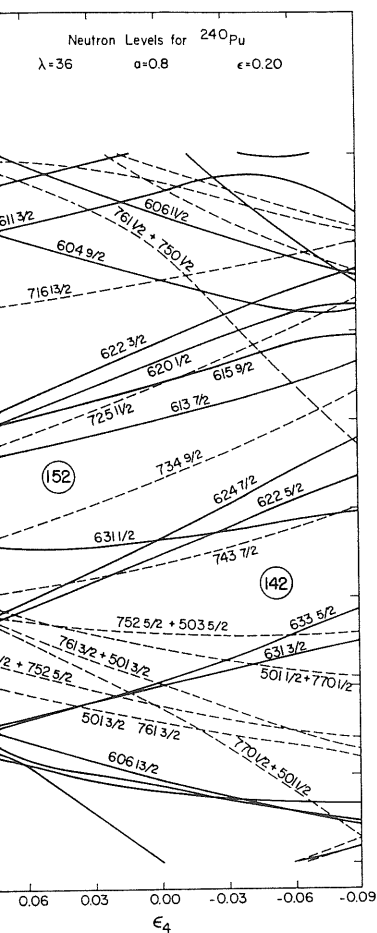


Fig. 2b. Single-neutron levels, analogous to fig. 2a. The level denoted (152) is dominantly of $[770\frac{1}{2}]$ character at this distortion.

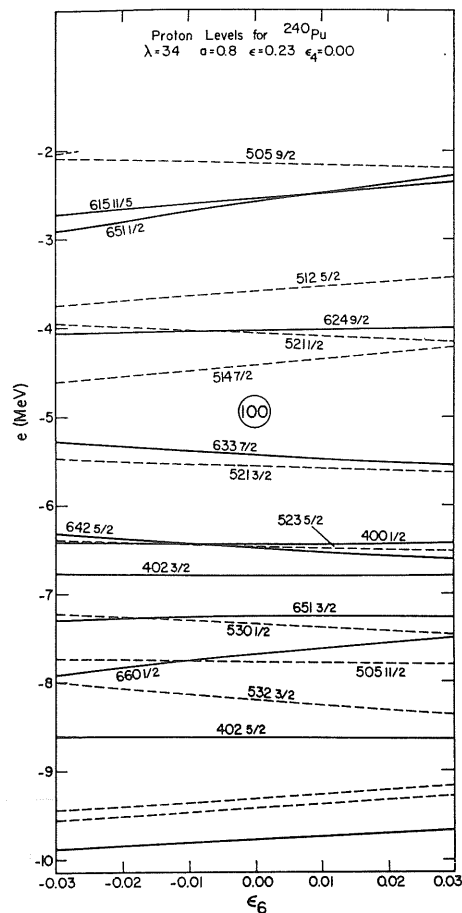


Fig. 3a. Single-proton levels as functions of the coordinate ϵ_6 . The coordinates ϵ and ϵ_4 have values that correspond to the average ground-state shape of the heavier actinides. Note that the orbitals rising most rapidly with ϵ_6 are the ones that contain an appreciable $[660\frac{1}{2}]$ component, namely those labeled by $[660\frac{1}{2}]$ and $[651\frac{1}{2}]$.

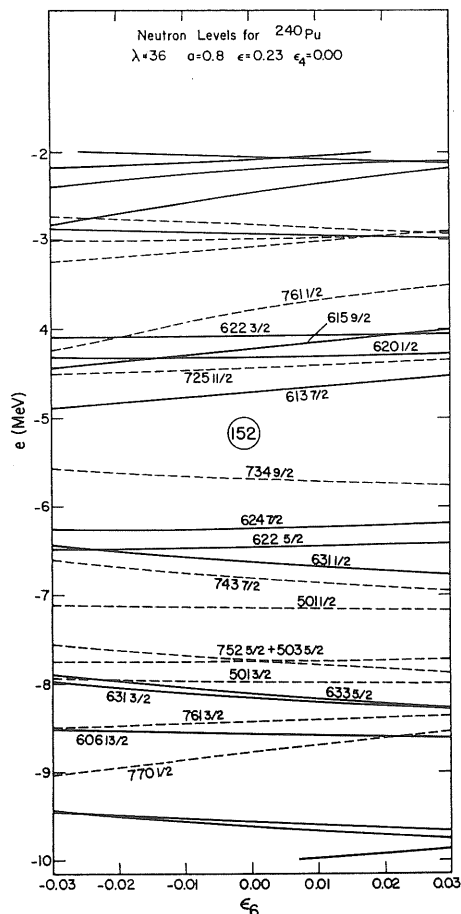


Fig. 3b. Single-neutron levels, analogous to fig. 3a.

than a two-parameter constrained version of the three-quadratic-surface parametrization²⁾). Also, the use of the ϵ -parameters facilitates a comparison with results obtained from the modified oscillator model. In figs. 1a-3b we present single-particle levels as functions of the distortion parameters ϵ , ϵ_4 and ϵ_6 . The parameters of the single-particle potential are the same as those given in ref. 8) except that the range a of the Yukawa folding function (which regulates the surface diffuseness of the potential) and the spin-orbit interaction strengths λ_n and λ_p for neutrons and protons

have been redetermined from adjustments to experimental single-particle levels in actinide nuclei. The resulting values of these constants are $a = 0.8$ fm, $\lambda_n = 36$ and $\lambda_p = 34$. The level diagrams are very similar to those obtained in other models ^{9,10}). We observe for instance gaps at $Z = 100$ and $N = 152$ in figs. 1a and 1b. These gaps are widened by the inclusion of the ε_6 degree of freedom (see figs. 3a and 3b), as is observed also in the modified oscillator model ⁹).

2.2. COMPARISON OF CALCULATED AND EXPERIMENTAL LEVELS

In figs. 4–7 we compare experimental ground-state quasi-particle energies in odd-mass actinide nuclei with values calculated by use of both the modified oscillator

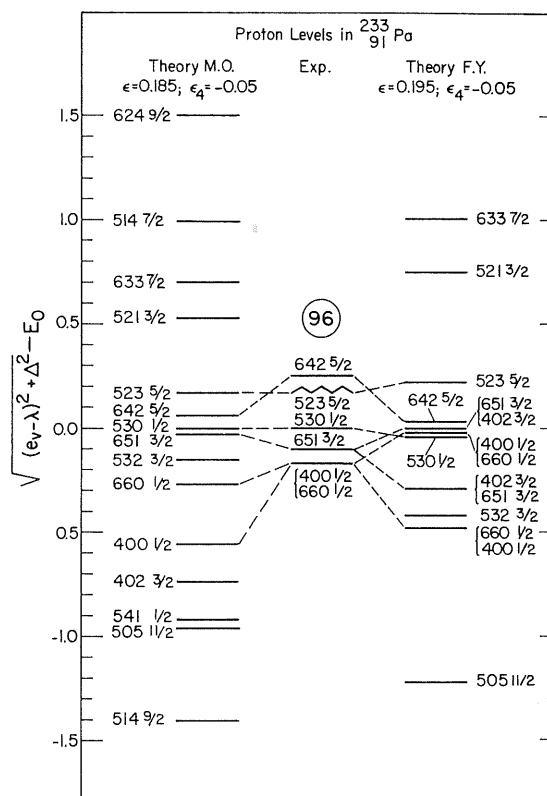
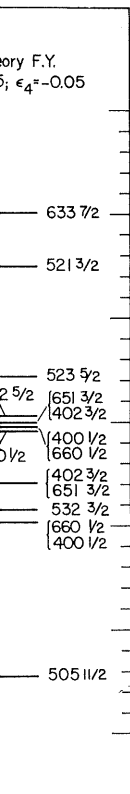


Fig. 4. Proton quasi-particle energies in $^{233}_{91}\text{Pa}$. The theoretical levels are calculated for a distortion that is interpolated between the ground-state distortions of neighbouring even nuclei. The levels are labeled by their asymptotic quantum numbers $[Nn_z l \Omega]$. The folded Yukawa (FY) assignments are determined from figs. 1a–3b. When levels with the same value of Ω are close together and a definite assignment for the dominant component is not possible, the two possible choices are indicated inside a curly bracket. In such cases the more likely assignment is written above the alternative one. The assignments of the modified oscillator (MO) results are determined from diagrams in ref. ⁹) that are similar to figs. 1a–3b. The position of the level marked by a zigzag line has been interpolated from data for neighbouring nuclei.

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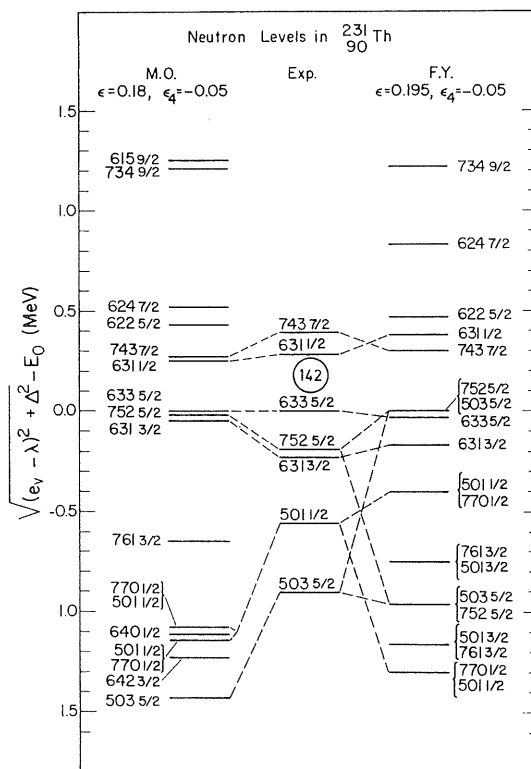


Fig. 5. Neutron quasi-particle energies in ²³¹Th, analogous to fig. 4.

potential and the folded Yukawa potential. When calculating the potential energy in the odd-mass case the interpolated energy for even nuclei is incremented by the quasi-particle energy

$$E_v = \sqrt{(e_v - \lambda)^2 + \Delta^2},$$

where e_v is the single-particle energy of the orbital containing the odd particle, λ is the Fermi energy and Δ is the pairing gap. The quasi-particle energies in figs. 4-7 are taken relative to the ground-state energy (in which the orbital with the lowest quasi-particle energy is filled). Hole states are plotted at negative energies and particle states are plotted at positive energies. In figs. 4 and 5 we investigate proton and neutron levels in the lighter actinides and in figs. 6 and 7 levels in the heavier actinides. The experimental results are taken from a compilation by Hoff ¹¹).

The theoretical levels in figs. 4-7 are calculated for distortions that are interpolated between the ground-state distortions of neighbouring even nuclei. We do not minimize the potential-energy surfaces for each separate value of the spin. Furthermore, the observed experimental band-head energies are considered to be of pure one-quasi-particle character. Thus, no rotational or vibrational corrections are applied. If these and other factors were taken into account in a more elaborate analysis of the theo-

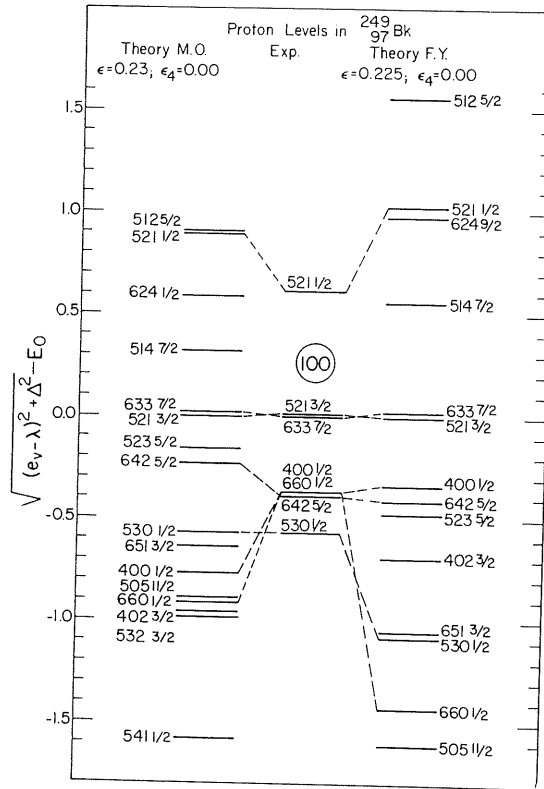
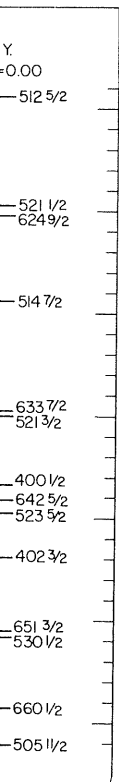


Fig. 6. Proton quasi-particle energies in $^{249}_{97}\text{Bk}$, analogous to fig. 4.

retical and experimental data, the positions of some of the levels could be changed by as much as a few hundred keV. However, within this accuracy, it is still possible to conclude from the comparisons in figs. 4–7 that the agreement between theory and experiment is very good. One may also say that the parameters of the single-particle potentials are “effective” parameters: they are adjusted so that the experimental levels are reproduced although the factors mentioned above are not taken into account explicitly. It is evident from figs. 4–7 that the results calculated from the modified oscillator potential and the folded Yukawa potential are remarkably similar, even though the radial shapes of the two potentials are seemingly different. Also, at larger distortions the level diagrams for the two potentials remain very similar except for the positions of a few levels for which the projection Ω of the single-particle angular momentum on the nuclear symmetry axis is high [cf. level diagrams in refs. ^{6,9}].

The folded Yukawa and modified oscillator single-particle models are compared in fig. 8 with each other and with experimental data for the spectra of odd-mass nuclei adjacent to $^{208}_{82}\text{Pb}$. The notation FY I in fig. 8 refers to the earlier set of parameters determined in ref. ⁸) from adjustments to experimental levels in $^{208}_{82}\text{Pb}$, while FY II refers to the present set of parameters determined from adjustments to experi-



k, analogous to fig. 4.

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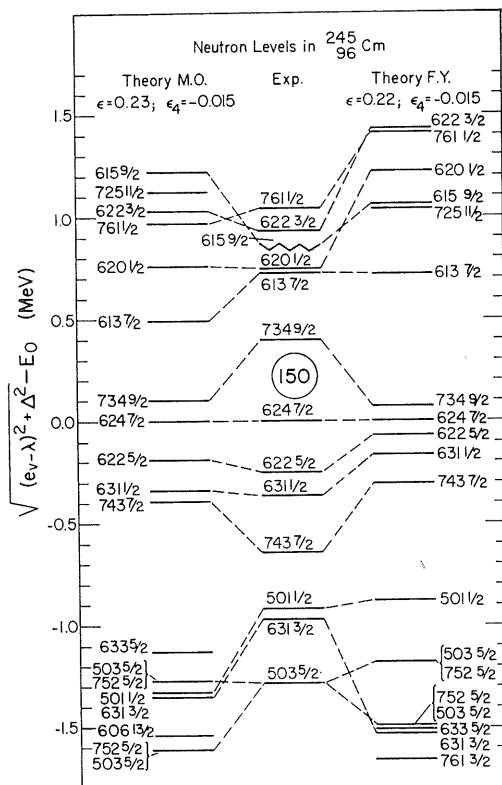


Fig. 7. Neutron quasi-particle energies in ²⁴⁵₉₆Cm, analogous to fig. 4.

mental levels in actinide nuclei. The modified oscillator (MHO) parameters are interpolated between the sets obtained for actinide and rare-earth nuclei. Both of the folded Yukawa versions, but especially FY I, reproduce the experimental levels in ²⁰⁸₈₂Pb somewhat better than the modified oscillator model.

We see that slightly different sets of parameters for the folded Yukawa potential are required to reproduce optimally the experimental levels in the lead and actinide regions, respectively. This suggests that for optimum agreement in any particular region of nuclei a new set of parameters should be determined for that region. For example, for the rare-earth region we find that the values $a = 0.8$ fm, $\lambda_n = 34.5$ and $\lambda_p = 32.0$ are best.

2.3. GROUND-STATE EQUILIBRIUM DISTORTIONS AND SINGLE-PARTICLE CORRECTIONS

Nuclear ground-state shapes and single-particle corrections are calculated for nuclei in the region $68 \leq Z \leq 108$. The ground-state shape is determined by minimizing the potential energy with respect to the distortion parameters ϵ and ϵ_4 . We use the folded Yukawa single-particle potential to calculate the single-particle corrections.

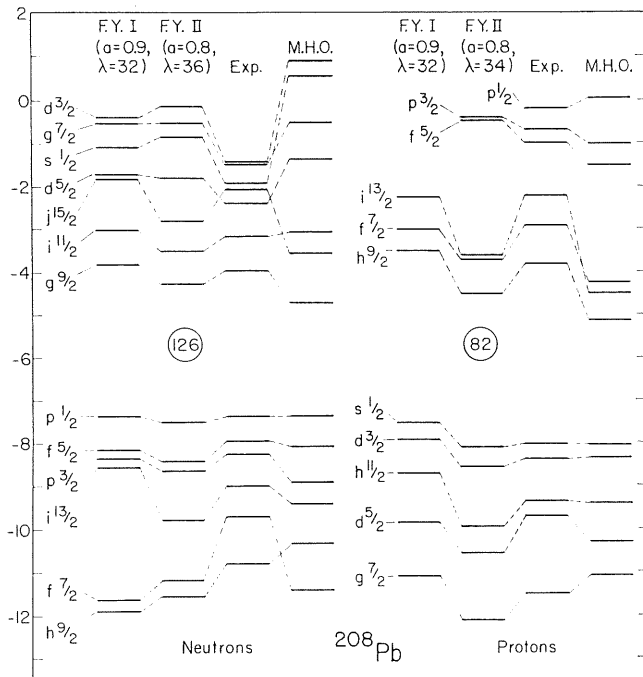
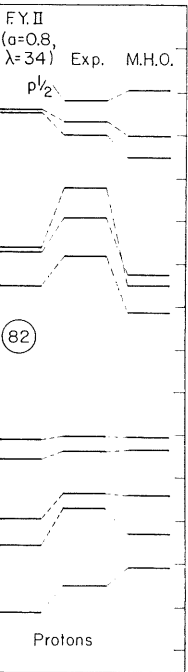


Fig. 8. Single-particle levels in $^{208}_{82}\text{Pb}$. The experimental data are taken from ref. ¹²).

For the macroscopic energy two different methods are used, namely the droplet model ³⁻⁵) and the modified-surface-energy model ⁷). The results are tabulated in tables 1 and 2, respectively, for these two models.

The droplet model includes terms of higher order in $A^{-1/3}$ and in $I^2 = [(N-Z)/A]^2$ than are retained in the liquid-drop model. The constants of these two models are not directly comparable because the shape dependence of the energy is more complicated in the droplet model than in the liquid-drop model. However, some comparisons can be made for limited regions of Z , A and deformation. It is possible to show ¹³), for example, that the constants of the droplet model correspond to an "effective" surface-asymmetry constant $\kappa_s \approx 2.5$ for ^{240}Pu at saddle-point distortions. This value is considerably higher than the frequently used value $\kappa_s = 1.7826$ in the second liquid-drop model of Myers and Swiatecki ¹⁴). Because of this and other differences between the constants, there are differences between the fission-barrier heights and the ground-state masses predicted by the two models. The higher-order terms in the droplet model also account for some of the differences in the results. However, the resulting distortions and single-particle corrections are not appreciably different in the droplet model and the liquid-drop model. Therefore, the comparison made below between the results for the droplet model and the modified-surface-energy model would be similar to a comparison in which the droplet model were replaced by the liquid-drop model.



Experimental data are taken from ref. ¹²).

...s are used, namely the droplet model (7). The results are tabulated in $A^{-1/3}$ and in $I^2 = [(N-Z)/A]^2$. Constants of these two models are of the energy is more complete of the energy is more complete model. However, some considerable deformation. It is possible to droplet model correspond to an frequently used value $\kappa_s = 1.7826$ (latecki ¹⁴). Because of this and differences between the fission- by the two models. The higher- some of the differences in the single-particle corrections are not droplet model and the modified- son in which the droplet model

TABLE 1
Ground-state deformations and single-particle corrections, calculated with the droplet model and folded Yukawa single-particle potential

Z	N	A	ϵ	ϵ_4	β_2	β_4	Single-particle correction (MeV)
68	106	174	0.253	0.053	0.280	-0.037	-2.22
	108	176	0.253	0.063	0.281	-0.049	-2.43
	110	178	0.247	0.067	0.273	-0.054	-1.65
	112	180	0.233	0.067	0.258	-0.057	-0.90
	114	182	0.207	0.063	0.228	-0.058	-0.45
	116	184	0.187	0.057	0.205	-0.053	-0.37
	118	186	0.153	0.050	0.167	-0.050	-0.87
	120	188	-0.120	0.010	-0.125	-0.006	-1.20
	122	190	-0.100	0.013	-0.105	-0.011	-2.52
	124	192	-0.067	0.013	-0.070	-0.014	-3.88
	126	194	0	0	0	0	-5.58
	70	106	176	0.247	0.060	0.273	-0.046
108		178	0.247	0.070	0.274	-0.058	-2.51
110		180	0.240	0.073	0.267	-0.063	-1.84
112		182	0.227	0.077	0.252	-0.070	-1.20
114		184	0.200	0.067	0.221	-0.063	-0.84
116		186	0.180	0.060	0.198	-0.058	-0.86
118		188	0.147	0.050	0.160	-0.051	-1.53
120		190	-0.113	0.010	-0.118	-0.006	-1.92
122		192	-0.100	0.017	-0.105	-0.015	-3.25
124		194	-0.067	0.013	-0.070	-0.014	-4.63
126		196	0	0	0	0	-6.33
72		106	178	0.240	0.060	0.265	-0.047
	108	180	0.233	0.070	0.259	-0.061	-2.07
	110	182	0.227	0.073	0.251	-0.066	-1.57
	112	184	0.200	0.067	0.221	-0.063	-1.21
	114	186	0.193	0.067	0.213	-0.064	-1.14
	116	188	0.153	0.050	0.167	-0.050	-1.52
	118	190	0.147	0.053	0.160	-0.055	-2.40
	120	192	-0.107	0.010	-0.111	-0.007	-2.90
	122	194	-0.100	0.017	-0.105	-0.015	-4.26
	124	196	-0.047	0.007	-0.049	-0.007	-5.70
	126	198	0	0	0	0	-7.44
	74	106	180	0.213	0.053	0.235	-0.045
108		182	0.213	0.063	0.235	-0.057	-1.73
110		184	0.200	0.063	0.220	-0.059	-1.57
112		186	0.193	0.063	0.213	-0.060	-1.44
114		188	0.167	0.057	0.183	-0.056	-1.69
116		190	0.147	0.050	0.160	-0.051	-2.44
118		192	0.140	0.053	0.153	-0.056	-3.40
120		194	-0.100	0.010	-0.105	-0.008	-4.15
122		196	-0.100	0.017	-0.105	-0.015	-5.50
124		198	-0.027	0.003	-0.028	-0.004	-7.09
126		200	0	0	0	0	-8.87
76		106	182	0.200	0.047	0.219	-0.039
	108	184	0.200	0.057	0.220	-0.051	-1.50

TABLE I (continued)

Z	N	A	ε	ε_4	β_2	β_4	Single-particle correction (MeV)
	110	186	0.180	0.053	0.197	-0.050	-1.52
	112	188	0.153	0.047	0.167	-0.046	-1.86
	114	190	0.147	0.047	0.160	-0.047	-2.54
	116	192	0.140	0.050	0.153	-0.052	-3.44
	118	194	0.133	0.053	0.145	-0.056	-4.50
	120	196	-0.100	0.010	-0.105	-0.008	-5.60
	122	198	-0.093	0.017	-0.098	-0.016	-6.97
	124	200	-0.020	0	-0.021	0	-8.72
	126	202	0	0	0	0	-10.51
78	106	184	0.180	0.033	0.196	-0.026	-0.26
	108	186	0.160	0.037	0.174	-0.033	-0.63
	110	188	0.147	0.033	0.159	-0.031	-1.11
	112	190	0.133	0.033	0.144	-0.033	-1.74
	114	192	-0.100	0.013	-0.105	-0.011	-2.75
	116	194	-0.100	0.013	-0.105	-0.011	-4.13
	118	196	-0.100	0.013	-0.105	-0.011	-5.66
	120	198	-0.100	0.013	-0.105	-0.011	-7.22
	122	200	-0.080	0.013	-0.084	-0.013	-8.64
	124	202	-0.020	0	-0.021	0	-10.44
	126	204	0.002	0	0.002	0	-10.88
	128	206	-0.003	0	-0.003	0	-8.42
	130	208	-0.007	0	-0.007	0	-6.14
	132	210	-0.021	0	-0.022	0	-4.07
	134	212	-0.048	0	-0.051	0.001	-2.26
	136	214	0.085	-0.010	0.090	0.015	-1.02
	138	216	0.098	-0.010	0.105	0.016	-0.20
	140	218	0.126	-0.017	0.135	0.027	0.31
	142	220	0.158	-0.020	0.170	0.035	0.36
	144	222	0.176	-0.017	0.190	0.034	0.20
	146	224	0.185	-0.010	0.200	0.027	-0.08
80	106	186	-0.107	0.013	-0.112	-0.011	-0.91
	108	188	-0.100	0.013	-0.105	-0.011	-1.53
	110	190	-0.100	0.017	-0.105	-0.015	-2.34
	112	192	-0.100	0.017	-0.105	-0.015	-3.35
	114	194	-0.100	0.020	-0.105	-0.019	-4.54
	116	196	-0.100	0.020	-0.105	-0.019	-5.92
	118	198	-0.100	0.020	-0.105	-0.019	-7.45
	120	200	-0.100	0.020	-0.105	-0.019	-9.02
	122	202	-0.073	0.017	-0.077	-0.017	-10.52
	124	204	-0.027	0.003	-0.028	-0.004	-12.32
	126	206	-0.003	0	-0.003	0	-12.34
	128	208	-0.007	0	-0.007	0	-9.89
	130	210	-0.012	0	-0.012	0	-7.61
	132	212	-0.021	0	-0.022	0	-5.56
	134	214	-0.044	0.003	-0.046	-0.003	-3.74
	136	216	-0.067	0.003	-0.070	-0.002	-2.24
	138	218	-0.080	0.007	-0.084	-0.005	-1.06
	140	220	-0.094	0.010	-0.099	-0.008	-0.13

TABLE I (continued)

β_2	β_4	Single-particle correction (MeV)	Z	N	A	ε	ε_4	β_2	β_4	Single-particle correction (MeV)
0.197	-0.050	-1.52		142	222	0.158	-0.033	0.170	0.052	0.48
0.167	-0.046	-1.86		144	224	0.176	-0.027	0.190	0.046	0.37
0.160	-0.047	-2.54		146	226	0.190	-0.017	0.205	0.036	0.15
0.153	-0.052	-3.44	82	106	188	0	0	0	0	-1.51
0.145	-0.056	-4.50		108	190	0	0	0	0	-2.17
-0.105	-0.008	-5.60		110	192	0	0	0	0	-3.08
-0.098	-0.016	-6.97		112	194	0	0	0	0	-4.20
-0.021	0	-8.72		114	196	0	0	0	0	-5.50
0	0	-10.51		116	198	0	0	0	0	-6.95
0.196	-0.026	-0.26		118	200	0	0	0	0	-8.55
0.174	-0.033	-0.63		120	202	0	0	0	0	-10.30
0.159	-0.031	-1.11		122	204	0	0	0	0	-12.20
0.144	-0.033	-1.74		124	206	0	0	0	0	-14.20
-0.105	-0.011	-2.75		126	208	0.002	0	0.002	0	-13.81
-0.105	-0.011	-4.13		128	210	-0.003	0	-0.003	0	-11.36
-0.105	-0.011	-5.66		130	212	-0.003	0	-0.003	0	-9.07
-0.105	-0.011	-7.22		132	214	-0.003	0	-0.003	0	-6.98
-0.084	-0.013	-8.64		134	216	-0.003	0	-0.003	0	-5.09
-0.021	0	-10.44		136	218	0.002	0	0.002	0	-3.41
0.002	0	-10.88		138	220	0.002	0	0.002	0	-1.98
-0.003	0	-8.42		140	222	0.007	0	0.007	0	-0.79
-0.007	0	-6.14		142	224	0.016	-0.003	0.017	0.004	0.13
-0.022	0	-4.07		144	226	0.176	-0.033	0.190	0.054	0.53
-0.051	0.001	-2.26		146	228	0.190	-0.023	0.205	0.044	0.38
0.090	0.015	-1.02	84	106	190	-0.173	-0.003	-0.179	0.015	1.30
0.105	0.016	-0.20		108	192	0.020	0	0.021	0	0.70
0.135	0.027	0.31		110	194	0	0	0	0	-0.21
0.170	0.035	0.36		112	196	-0.007	0	-0.007	0	-1.33
0.190	0.034	0.20		114	198	-0.007	0	-0.007	0	-2.63
0.200	0.027	-0.08		116	200	-0.013	0	-0.014	0	-4.09
-0.112	-0.011	-0.91		118	202	-0.013	0	-0.014	0	-5.70
-0.105	-0.011	-1.53		120	204	-0.013	0	-0.014	0	-7.44
-0.105	-0.015	-2.34		122	206	-0.007	0	-0.007	0	-9.33
-0.105	-0.015	-3.35		124	208	0	0	0	0	-11.32
-0.105	-0.019	-4.54		126	210	-0.003	0	-0.003	0	-11.45
-0.105	-0.019	-5.92		128	212	-0.003	0	-0.003	0	-9.00
-0.105	-0.019	-7.45		130	214	-0.003	0	-0.003	0	-6.71
-0.105	-0.019	-9.02		132	216	-0.003	0	-0.003	0	-4.62
-0.077	-0.017	-10.52		134	218	-0.007	0	-0.007	0	-2.73
-0.028	-0.004	-12.32		136	220	0.011	-0.003	0.012	0.004	-1.06
-0.003	0	-12.34		138	222	0.098	-0.040	0.105	0.053	0.04
-0.007	0	-9.89		140	224	0.130	-0.047	0.140	0.065	0.54
-0.012	0	-7.61		142	226	0.167	-0.043	0.180	0.065	0.61
-0.022	0	-5.56		144	228	0.181	-0.037	0.195	0.059	0.58
-0.046	-0.003	-3.74		146	230	0.195	-0.027	0.210	0.049	0.48
-0.070	-0.002	-2.24	86	106	192	0.273	0.020	0.300	0.008	1.22
-0.084	-0.005	-1.06		108	194	-0.273	0.013	-0.280	0.014	0.97
-0.099	-0.008	-0.13		110	196	-0.260	0.010	-0.267	0.015	0.58

TABLE 1 (continued)

Z	N	A	ε	ε_4	β_2	β_4	Single-particle correction (MeV)
	112	198	-0.227	0.003	-0.234	0.016	0.16
	114	200	-0.153	-0.003	-0.159	0.013	-0.55
	116	202	-0.100	-0.003	-0.104	0.008	-1.81
	118	204	-0.087	-0.003	-0.091	0.007	-3.33
	120	206	-0.053	0	-0.056	0.001	-4.99
	122	208	-0.013	0	-0.014	0	-6.79
	124	210	-0.007	0	-0.007	0	-8.78
	126	212	-0.003	0	-0.003	0	-9.35
	128	214	-0.003	0	-0.003	0	-6.89
	130	216	-0.007	0	-0.007	0	-4.60
	132	218	-0.012	-0.003	-0.012	0.004	-2.52
	134	220	0.080	-0.033	0.085	0.043	-0.75
	136	222	0.103	-0.043	0.110	0.057	0.16
	138	224	0.135	-0.053	0.145	0.073	0.63
	140	226	0.153	-0.053	0.165	0.076	0.65
	142	228	0.176	-0.050	0.190	0.075	0.50
	144	230	0.190	-0.040	0.205	0.065	0.44
	146	232	0.199	-0.030	0.215	0.054	0.35
88	106	194	0.280	0.017	0.307	0.014	0.84
	108	196	0.300	0.013	0.330	0.023	1.29
	110	198	-0.300	0.020	-0.307	0.012	1.06
	112	200	-0.280	0.017	-0.287	0.011	0.81
	114	202	-0.233	0.007	-0.240	0.013	0.43
	116	204	-0.153	0	-0.159	0.009	-0.38
	118	206	-0.100	-0.003	-0.104	0.008	-1.77
	120	208	-0.100	-0.003	-0.104	0.008	-3.30
	122	210	-0.060	0	-0.063	0.001	-4.67
	124	212	-0.013	0	-0.014	0	-6.56
	126	214	0.002	0	0.002	0	-7.52
	128	216	-0.003	0	-0.003	0	-5.07
	130	218	-0.007	0	-0.007	0	-2.78
	132	220	-0.025	-0.003	-0.027	0.004	-0.72
	134	222	0.103	-0.043	0.110	0.057	0.30
	136	224	0.140	-0.060	0.150	0.082	0.71
	138	226	0.153	-0.060	0.165	0.084	0.64
	140	228	0.172	-0.057	0.185	0.083	0.44
	142	230	0.188	-0.050	0.203	0.077	0.41
	144	232	0.198	-0.040	0.214	0.066	0.29
	146	234	0.200	-0.033	0.216	0.058	0.19
	148	236	0.207	-0.023	0.224	0.047	-0.03
	150	238	0.217	-0.010	0.235	0.033	-0.36
	152	240	0.225	0	0.244	0.022	-0.58
	154	242	0.208	0.003	0.226	0.015	-0.29
	156	244	0.200	0.010	0.217	0.005	-0.10
	158	246	0.190	0.017	0.206	-0.005	-0.04
	160	248	0.183	0.027	0.199	-0.018	-0.15
	162	250	0.183	0.040	0.200	-0.034	-0.36
90	126	216	0.002	0	0.002	0	-5.99

TABLE 1 (continued)

β_2	β_4	Single-particle correction (MeV)	Z	N	A	ε	ε_4	β_2	β_4	Single-particle correction (MeV)
-0.234	0.016	0.16		128	218	-0.007	0	-0.007	0	-3.53
-0.159	0.013	-0.55		130	220	-0.016	-0.003	-0.017	0.004	-1.25
-0.104	0.008	-1.81		132	222	0.098	-0.040	0.105	0.053	0.34
-0.091	0.007	-3.33		134	224	0.140	-0.060	0.150	0.082	0.79
-0.056	0.001	-4.99		136	226	0.153	-0.067	0.166	0.093	0.59
-0.014	0	-6.79		138	228	0.172	-0.063	0.186	0.091	0.30
-0.007	0	-8.78		140	230	0.185	-0.060	0.200	0.090	-0.10
-0.003	0	-9.35		142	232	0.195	-0.050	0.211	0.079	-0.10
-0.003	0	-6.89		144	234	0.200	-0.043	0.216	0.071	-0.21
-0.007	0	-4.60		146	236	0.202	-0.037	0.218	0.063	-0.28
-0.012	0.004	-2.52		148	238	0.210	-0.023	0.227	0.048	-0.46
0.085	0.043	-0.75		150	240	0.220	-0.010	0.239	0.033	-0.75
0.110	0.057	0.16		152	242	0.227	0	0.246	0.022	-0.95
0.145	0.073	0.63		154	244	0.213	0	0.231	0.020	-0.61
0.165	0.076	0.65		156	246	0.200	0.007	0.217	0.009	-0.33
0.190	0.075	0.50		158	248	0.200	0.017	0.217	-0.003	-0.21
0.205	0.065	0.44		160	250	0.195	0.027	0.212	-0.016	-0.23
0.215	0.054	0.35		162	252	0.193	0.040	0.211	-0.032	-0.37
0.307	0.014	0.84	92	126	218	0.002	0	0.002	0	-4.75
0.330	0.023	1.29		128	220	-0.007	0	-0.007	0	-2.30
-0.307	0.012	1.06		130	222	-0.025	-0.003	-0.027	0.004	-0.03
-0.287	0.011	0.81		132	224	0.126	-0.050	0.135	0.068	0.89
-0.240	0.013	0.43		134	226	0.162	-0.063	0.175	0.090	0.66
-0.159	0.009	-0.38		136	228	0.172	-0.070	0.186	0.100	0.18
-0.104	0.008	-1.77		138	230	0.185	-0.067	0.201	0.098	-0.29
-0.104	0.008	-3.30		140	232	0.195	-0.060	0.211	0.091	-0.81
-0.063	0.001	-4.67		142	234	0.200	-0.050	0.216	0.079	-0.72
-0.014	0	-6.56		144	236	0.200	-0.047	0.216	0.075	-0.82
0.002	0	-7.52		146	238	0.203	-0.037	0.220	0.063	-0.86
-0.003	0	-5.07		148	240	0.210	-0.027	0.227	0.052	-1.03
-0.007	0	-2.78		150	242	0.218	-0.013	0.237	0.037	-1.31
-0.027	0.004	-0.72		152	244	0.227	0	0.246	0.022	-1.49
0.110	0.057	0.30		154	246	0.215	0	0.233	0.020	-1.13
0.150	0.082	0.71		156	248	0.207	0.007	0.224	0.010	-0.81
0.165	0.084	0.64		158	250	0.200	0.013	0.217	0.001	-0.64
0.185	0.083	0.44		160	252	0.200	0.027	0.218	-0.015	-0.60
0.203	0.077	0.41		162	254	0.200	0.040	0.219	-0.031	-0.67
0.214	0.066	0.29	94	126	220	0.002	0	0.002	0	-3.83
0.216	0.058	0.19		128	222	-0.007	0	-0.007	0	-1.38
0.224	0.047	-0.03		130	224	0.098	-0.033	0.105	0.045	0.73
0.235	0.033	-0.36		132	226	0.158	-0.050	0.170	0.072	0.97
0.244	0.022	-0.58		134	228	0.181	-0.057	0.195	0.085	0.51
0.226	0.015	-0.29		136	230	0.190	-0.060	0.206	0.090	-0.05
0.217	0.005	-0.10		138	232	0.195	-0.057	0.210	0.087	-0.63
0.206	-0.005	-0.04		140	234	0.199	-0.057	0.216	0.088	-1.24
0.199	-0.018	-0.15		142	236	0.200	-0.050	0.216	0.079	-1.26
0.200	-0.034	-0.36		144	238	0.200	-0.043	0.216	0.071	-1.38
0.002	0	-5.99		146	240	0.207	-0.033	0.224	0.060	-1.48

TABLE 1 (continued)

Z	N	A	ε	ε_4	β_2	β_4	Single-particle correction (MeV)
	148	242	0.212	-0.023	0.229	0.048	-1.68
	150	244	0.220	-0.010	0.239	0.033	-1.98
	152	246	0.225	0	0.244	0.022	-2.17
	154	248	0.217	0	0.235	0.020	-1.82
	156	250	0.208	0.007	0.226	0.010	-1.47
	158	252	0.202	0.013	0.219	0.001	-1.26
	160	254	0.200	0.023	0.218	-0.011	-1.20
	162	256	0.200	0.037	0.218	-0.027	-1.22
96	126	222	0.002	0	0.002	0	-3.25
	128	224	-0.012	0	-0.012	0	-0.80
	130	226	0.112	-0.033	0.120	0.046	1.03
	132	228	0.222	-0.037	0.241	0.067	0.81
	134	230	0.195	-0.047	0.210	0.074	0.33
	136	232	0.204	-0.047	0.220	0.076	-0.25
	138	234	0.204	-0.047	0.220	0.076	-0.87
	140	236	0.204	-0.047	0.220	0.076	-1.51
	142	238	0.200	-0.043	0.216	0.071	-1.71
	144	240	0.205	-0.037	0.222	0.064	-1.89
	146	242	0.210	-0.027	0.227	0.052	-2.06
	148	244	0.215	-0.020	0.233	0.045	-2.33
	150	246	0.222	-0.007	0.241	0.029	-2.69
	152	248	0.225	0	0.244	0.022	-2.91
	154	250	0.218	0.003	0.237	0.016	-2.55
	156	252	0.210	0.007	0.228	0.011	-2.21
	158	254	0.205	0.013	0.223	0.002	-2.00
	160	256	0.202	0.023	0.220	-0.011	-1.92
	162	258	0.200	0.037	0.218	-0.027	-1.94
98	126	224	0.002	0	0.002	0	-3.03
	128	226	-0.012	0	-0.012	0	-0.58
	130	228	0.250	-0.017	0.272	0.048	0.60
	132	230	0.245	-0.020	0.266	0.051	0.35
	134	232	0.236	-0.023	0.256	0.053	0.01
	136	234	0.218	-0.033	0.236	0.062	-0.47
	138	236	0.213	-0.037	0.230	0.065	-1.07
	140	238	0.213	-0.037	0.230	0.065	-1.71
	142	240	0.207	-0.033	0.224	0.060	-1.87
	144	242	0.210	-0.027	0.227	0.052	-2.15
	146	244	0.215	-0.020	0.233	0.045	-2.43
	148	246	0.220	-0.010	0.239	0.033	-2.83
	150	248	0.225	0	0.244	0.022	-3.31
	152	250	0.227	0.003	0.247	0.018	-3.59
	154	252	0.220	0.007	0.239	0.013	-3.26
	156	254	0.215	0.013	0.234	0.004	-2.94
	158	256	0.210	0.020	0.229	-0.005	-2.77
	160	258	0.207	0.030	0.225	-0.018	-2.74
	162	260	0.203	0.040	0.222	-0.031	-2.79
100	142	242	0.213	-0.023	0.231	0.048	-2.00
	144	244	0.217	-0.017	0.235	0.041	-2.38

TABLE 1 (continued)

β_2	β_4	Single-particle correction (MeV)	Z	N	A	ε	ε_4	β_2	β_4	Single-particle correction (MeV)
0.229	0.048	-1.68								
0.239	0.033	-1.98			146	0.223	-0.007	0.242	0.030	-2.80
0.244	0.022	-2.17			148	0.227	0	0.246	0.022	-3.33
0.235	0.020	-1.82			150	0.228	0.003	0.248	0.018	-3.88
0.226	0.010	-1.47			152	0.230	0.010	0.250	0.011	-4.24
0.219	0.001	-1.26			154	0.225	0.017	0.245	0.001	-3.95
0.218	-0.011	-1.20			156	0.218	0.020	0.238	-0.004	-3.68
0.218	-0.027	-1.22			158	0.213	0.027	0.233	-0.013	-3.57
0.002	0	-3.25			160	0.210	0.037	0.230	-0.026	-3.61
-0.012	0	-0.80			162	0.207	0.047	0.227	-0.038	-3.73
0.120	0.046	1.03	102		142	0.210	-0.017	0.227	0.040	-1.96
0.241	0.067	0.81			144	0.215	-0.010	0.233	0.032	-2.40
0.210	0.074	0.33			146	0.220	0	0.239	0.021	-2.90
0.220	0.076	-0.25			148	0.222	0.003	0.241	0.017	-3.48
0.220	0.076	-0.87			150	0.225	0.010	0.245	0.010	-4.09
0.220	0.076	-1.51			152	0.227	0.017	0.247	0.002	-4.54
0.216	0.071	-1.71			154	0.222	0.023	0.242	-0.007	-4.37
0.222	0.064	-1.89			156	0.217	0.027	0.236	-0.012	-4.22
0.227	0.052	-2.06			158	0.212	0.033	0.231	-0.021	-4.23
0.233	0.045	-2.33			160	0.210	0.043	0.230	-0.033	-4.39
0.241	0.029	-2.69			162	0.205	0.050	0.225	-0.042	-4.63
0.244	0.022	-2.91	104		142	0.212	-0.007	0.229	0.027	-1.83
0.237	0.016	-2.55			144	0.217	0	0.235	0.020	-2.34
0.228	0.011	-2.21			146	0.218	0.003	0.237	0.016	-2.89
0.223	0.002	-2.00			148	0.222	0.010	0.241	0.009	-3.55
0.220	-0.011	-1.92			150	0.223	0.017	0.243	0.001	-4.25
0.218	-0.027	-1.94			152	0.228	0.027	0.249	-0.010	-4.81
0.002	0	-3.03			154	0.222	0.030	0.242	-0.015	-4.75
-0.012	0	-0.58			156	0.218	0.037	0.239	-0.024	-4.72
0.272	0.048	0.60			158	0.213	0.043	0.234	-0.033	-4.83
0.266	0.051	0.35			160	0.210	0.050	0.231	-0.041	-5.12
0.256	0.053	0.01			162	0.207	0.060	0.228	-0.054	-5.48
0.236	0.062	-0.47	106		142	0.215	0.003	0.233	0.016	-1.76
0.230	0.065	-1.07			144	0.217	0.007	0.235	0.012	-2.32
0.230	0.065	-1.71			146	0.222	0.017	0.242	0.001	-2.96
0.224	0.060	-1.87			148	0.225	0.023	0.246	-0.007	-3.70
0.227	0.052	-2.15			150	0.227	0.027	0.248	-0.010	-4.49
0.233	0.045	-2.43			152	0.230	0.037	0.252	-0.022	-5.16
0.239	0.033	-2.83			154	0.225	0.040	0.247	-0.027	-5.21
0.244	0.022	-3.31			156	0.222	0.047	0.243	-0.035	-5.28
0.247	0.018	-3.59			158	0.217	0.050	0.238	-0.040	-5.49
0.239	0.013	-3.26			160	0.212	0.057	0.233	-0.049	-5.88
0.234	0.004	-2.94			162	0.210	0.067	0.232	-0.061	-6.36
0.229	-0.005	-2.77	108		142	0.215	0.013	0.234	0.004	-1.70
0.225	-0.018	-2.74			144	0.217	0.017	0.236	0	-2.31
0.222	-0.031	-2.79			146	0.220	0.023	0.240	-0.008	-3.01
0.231	0.048	-2.00			148	0.223	0.030	0.244	-0.015	-3.82
0.235	0.041	-2.38			150	0.225	0.037	0.246	-0.023	-4.68
					152	0.228	0.043	0.251	-0.030	-5.43

TABLE 1 (continued)

Z	N	A	ε	ε_4	β_2	β_4	Single-particle correction (MeV)
	154	262	0.223	0.047	0.245	-0.035	-5.60
	156	264	0.220	0.053	0.242	-0.044	-5.79
	158	266	0.215	0.060	0.237	-0.052	-6.13
	160	268	0.212	0.063	0.234	-0.057	-6.64
	162	270	0.208	0.070	0.230	-0.065	-7.25

The ground-state single-particle correction is the nuclear ground-state mass relative to the spherical macroscopic energy, which is calculated here by use of the droplet model. In this table the ground-state zero-point energy (which is frequently taken equal to 0.5 MeV) is not included. Single-particle levels for $^{194}_{78}\text{Pt}$, $^{224}_{88}\text{Ra}$ and $^{250}_{98}\text{Cf}$ are used to calculate the potential energy for nearby nuclei. In all three regions the values $a = 0.8$ fm, $\lambda_n = 36$ and $\lambda_p = 34$ are used. Note that a slightly different set of levels were used in the calculations displayed in fig. 9.

The modified-surface-energy model ⁷⁾ was introduced phenomenologically as a generalization of the liquid-drop model to satisfy the following four conditions:

(i) For spherical configurations and large nuclei, the new model should give roughly the same result as the liquid-drop model.

(ii) The new model should be less sensitive to high-multipole ripples on the nuclear surface. The liquid-drop model yields what appears to be an undesirable sensitivity of calculated fission barriers to unphysical fine details of the shape in the neck region.

(iii) Between two separated nuclei there should be an attractive nuclear interaction in addition to the Coulomb repulsion. The range of the effective force from which the surface energy is generated should extend beyond the equivalent sharp surface by an amount that accounts both for the finite range of the nucleon-nucleon interaction and the finite diffuseness of the nuclear matter distribution.

(iv) It should be possible to calculate the new expression for general shapes with reasonable computational effort.

It was shown in ref. ⁷⁾ that these four conditions are satisfied if the shape dependence of the surface energy $c_s A^{2/3} B_s(\xi_v)$ is replaced by the shape dependence of

$$E(\xi_v) = - \frac{V_0}{4\pi a^3} \int_{V(\xi_v)} \frac{e^{-|r-r'|/a}}{|r-r'|/a} d^3r d^3r'. \quad (1)$$

The double integration is over the volume of the shape specified by the deformation coordinates ξ_v ; the magnitude of the volume is kept fixed as the shape is deformed. For a spherical shape, a straightforward evaluation of this integral yields

$$E = V_0 \left[-\frac{4}{3}\pi R_0^3 + 2\pi a R_0^2 - 2\pi a^3 + 2\pi a (R_0 + a)^2 e^{-2R_0/a} \right]. \quad (2)$$

The first term is proportional to the volume and consequently does not depend on the shape of the nucleus. However, because the effective force that is used contains no aspects of nuclear saturation, the volume-energy term is not given correctly and must be replaced by the volume-energy term in the liquid-drop model.

TABLE 2

Ground-state deformations and single-particle corrections, calculated with the modified-surface-energy model and folded Yukawa single-particle potential

Z	N	A	ϵ	ϵ_4	β_2	β_4	Single-particle correction (MeV)
78	126	204	0.002	0.001	0.002	-0.001	-10.88
	128	206	-0.003	-0.003	-0.003	0.004	-8.42
	130	208	-0.007	-0.003	-0.007	0.004	-6.12
	132	210	-0.021	-0.003	-0.022	0.004	-4.05
	134	212	0.057	-0.008	0.061	0.010	-2.26
	136	214	0.089	-0.016	0.095	0.022	-1.05
	138	216	0.103	-0.020	0.110	0.029	-0.25
	140	218	0.135	-0.032	0.145	0.047	0.20
	142	220	0.167	-0.037	0.180	0.057	0.19
	144	222	0.176	-0.028	0.190	0.048	0.08
	146	224	0.185	-0.016	0.200	0.034	-0.12
80	126	206	-0.003	0.001	-0.003	-0.001	-12.35
	128	208	-0.007	0.001	-0.007	-0.001	-9.89
	130	210	-0.012	0.001	-0.012	-0.001	-7.61
	132	212	-0.025	0.001	-0.027	-0.001	-5.55
	134	214	-0.044	0.005	-0.046	-0.005	-3.73
	136	216	-0.067	0.005	-0.070	-0.004	-2.24
	138	218	-0.085	0.009	-0.089	-0.008	-1.05
	140	220	-0.094	0.013	-0.099	-0.012	-0.13
	142	222	0.162	-0.049	0.175	0.072	0.11
	144	224	0.181	-0.037	0.195	0.059	0.12
	146	226	0.190	-0.028	0.205	0.050	0.03
82	126	208	0.002	0.001	0.002	-0.001	-13.83
	128	210	-0.003	0.001	-0.003	-0.001	-11.37
	130	212	-0.003	0.001	-0.003	-0.001	-9.07
	132	214	-0.003	0.001	-0.003	-0.001	-6.97
	134	216	-0.003	0.001	-0.003	-0.001	-5.08
	136	218	0.002	-0.003	0.002	0.004	-3.39
	138	220	0.002	-0.003	0.002	0.004	-1.96
	140	222	0.011	-0.003	0.012	0.004	-0.77
	142	224	0.162	-0.058	0.175	0.083	0.02
	144	226	0.176	-0.049	0.190	0.074	0.14
	146	228	0.190	-0.037	0.205	0.061	0.15
84	126	210	-0.003	0.001	-0.003	-0.001	-11.47
	128	212	-0.003	0.001	-0.003	-0.001	-9.01
	130	214	-0.003	0.001	-0.003	-0.001	-6.72
	132	216	-0.003	0.001	-0.003	-0.001	-4.62
	134	218	-0.007	-0.003	-0.007	0.004	-2.73
	136	220	0.094	-0.062	0.101	0.079	-1.26
	138	222	0.112	-0.070	0.121	0.092	-0.59
	140	224	0.135	-0.070	0.146	0.095	-0.23
	142	226	0.162	-0.066	0.176	0.093	-0.11
	144	228	0.176	-0.053	0.190	0.080	0.06
	146	230	0.190	-0.041	0.205	0.066	0.13
86	126	212	-0.003	0.001	-0.003	-0.001	-9.37
	128	214	-0.003	0.001	-0.003	-0.001	-6.91

β_2 β_4 Single-particle correction (MeV)

0.245 -0.035 -5.60
 0.242 -0.044 -5.79
 0.237 -0.052 -6.13
 0.234 -0.057 -6.64
 0.230 -0.065 -7.25

ar ground-state mass relative to the of the droplet model. In this table the al to 0.5 MeV) is not included. Single- calculate the potential energy for nearby d $\lambda_p = 34$ are used. Note that a slightly fig. 9.

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pression for general shapes with

are satisfied if the shape dependen- y the shape dependence of

$$\int d^3r d^3r' \frac{1}{a} \quad (1)$$

ape specified by the deformation t fixed as the shape is deformed. of this integral yields

$$[(R_0 + a)^2 e^{-2R_0/a}]. \quad (2)$$

onsequently does not depend on ective force that is used contains r term is not given correctly and liquid-drop model.

TABLE 2 (continued)

Z	N	A	ε	ε_4	β_2	β_4	Single-particle correction (MeV)
	130	216	-0.007	-0.003	-0.007	0.004	-4.62
	132	218	-0.012	-0.003	-0.012	0.004	-2.53
	134	220	0.098	-0.062	0.106	0.080	-1.23
	136	222	0.117	-0.074	0.126	0.098	-0.59
	138	224	0.144	-0.083	0.156	0.112	-0.41
	140	226	0.153	-0.078	0.166	0.108	-0.37
	142	228	0.172	-0.070	0.186	0.100	-0.39
	144	230	0.185	-0.058	0.200	0.086	-0.22
	146	232	0.199	-0.045	0.215	0.073	-0.11
88	126	214	0.002	0.001	0.002	-0.001	-7.55
	128	216	-0.003	0.001	-0.003	-0.001	-5.09
	130	218	-0.012	-0.003	-0.012	0.004	-2.80
	132	220	0.094	-0.058	0.101	0.074	-1.11
	134	222	0.117	-0.074	0.126	0.098	-0.51
	136	224	0.144	-0.087	0.157	0.117	-0.58
	138	226	0.153	-0.087	0.167	0.119	-0.70
	140	228	0.167	-0.083	0.181	0.115	-0.79
	142	230	0.185	-0.067	0.200	0.098	-0.55
	144	232	0.195	-0.057	0.211	0.088	-0.48
	146	234	0.200	-0.047	0.216	0.076	-0.41
	148	236	0.205	-0.034	0.222	0.061	-0.45
	150	238	0.215	-0.018	0.233	0.042	-0.63
	152	240	0.227	-0.002	0.246	0.024	-0.79
	154	242	0.212	0.002	0.230	0.017	-0.49
	156	244	0.200	0.011	0.217	0.003	-0.29
	158	246	0.197	0.024	0.214	-0.013	-0.29
	160	248	0.197	0.044	0.215	-0.037	-0.54
	162	250	0.198	0.064	0.219	-0.060	-1.05
90	126	216	0.002	0.001	0.002	-0.001	-6.02
	128	218	-0.007	0.001	-0.007	-0.001	-3.56
	130	220	-0.016	-0.003	-0.017	0.004	-1.28
	132	222	0.107	-0.070	0.116	0.091	-0.39
	134	224	0.144	-0.087	0.157	0.117	-0.50
	136	226	0.153	-0.095	0.167	0.130	-0.99
	138	228	0.162	-0.091	0.177	0.126	-1.24
	140	230	0.176	-0.087	0.192	0.122	-1.47
	142	232	0.190	-0.070	0.206	0.104	-1.08
	144	234	0.198	-0.060	0.215	0.092	-1.00
	146	236	0.200	-0.051	0.216	0.080	-0.89
	148	238	0.205	-0.041	0.222	0.069	-0.89
	150	240	0.215	-0.021	0.233	0.046	-1.00
	152	242	0.227	-0.005	0.246	0.028	-1.12
	154	244	0.215	-0.002	0.233	0.022	-0.77
	156	246	0.202	0.005	0.219	0.011	-0.49
	158	248	0.200	0.021	0.218	-0.009	-0.41
	160	250	0.200	0.041	0.219	-0.032	-0.56
	162	252	0.200	0.060	0.220	-0.056	-0.98
92	126	218	0.002	0.001	0.002	-0.001	-4.78

TABLE 2 (continued)

β_2	β_4	Single-particle correction (MeV)	Z	N	A	ϵ	ϵ_4	β_2	β_4	Single-particle correction (MeV)
-0.007	0.004	-4.62								
-0.012	0.004	-2.53								
0.106	0.080	-1.23								
0.126	0.098	-0.59								
0.156	0.112	-0.41								
0.166	0.108	-0.37								
0.186	0.100	-0.39								
0.200	0.086	-0.22								
0.215	0.073	-0.11								
0.002	-0.001	-7.55								
-0.003	-0.001	-5.09								
-0.012	0.004	-2.80								
0.101	0.074	-1.11								
0.126	0.098	-0.51								
0.157	0.117	-0.58								
0.167	0.119	-0.70								
0.181	0.115	-0.79								
0.200	0.098	-0.55								
0.211	0.088	-0.48								
0.216	0.076	-0.41								
0.222	0.061	-0.45								
0.233	0.042	-0.63								
0.246	0.024	-0.79								
0.230	0.017	-0.49								
0.217	0.003	-0.29								
0.214	-0.013	-0.29								
0.215	-0.037	-0.54								
0.219	-0.060	-1.05								
0.002	-0.001	-6.02								
-0.007	-0.001	-3.56								
-0.017	0.004	-1.28								
0.116	0.091	-0.39								
0.157	0.117	-0.50								
0.167	0.130	-0.99								
0.177	0.126	-1.24								
0.192	0.122	-1.47								
0.206	0.104	-1.08								
0.215	0.092	-1.00								
0.216	0.080	-0.89								
0.222	0.069	-0.89								
0.233	0.046	-1.00								
0.246	0.028	-1.12								
0.233	0.022	-0.77								
0.219	0.011	-0.49								
0.218	-0.009	-0.41								
0.219	-0.032	-0.56								
0.220	-0.056	-0.98								
0.002	-0.001	-4.78								
			94							
				128	220	-0.007	0.001	-0.007	-0.001	-2.33
				130	222	0.098	-0.058	0.106	0.075	-0.27
				132	224	0.144	-0.078	0.156	0.107	-0.18
				134	226	0.158	-0.091	0.172	0.125	-0.86
				136	228	0.167	-0.095	0.182	0.132	-1.55
				138	230	0.176	-0.091	0.192	0.128	-1.95
				140	232	0.185	-0.087	0.202	0.124	-2.29
				142	234	0.195	-0.070	0.211	0.104	-1.68
				144	236	0.200	-0.060	0.217	0.093	-1.60
				146	238	0.200	-0.054	0.216	0.084	-1.46
				148	240	0.205	-0.041	0.222	0.069	-1.43
				150	242	0.213	-0.024	0.231	0.050	-1.52
				152	244	0.225	-0.005	0.244	0.028	-1.61
				154	246	0.217	-0.002	0.235	0.022	-1.26
				156	248	0.207	0.005	0.224	0.012	-0.93
				158	250	0.200	0.015	0.217	-0.001	-0.78
				160	252	0.200	0.034	0.218	-0.024	-0.86
				162	254	0.200	0.054	0.220	-0.048	-1.17
				126	220	0.002	0.001	0.002	-0.001	-3.86
				128	222	-0.007	0.001	-0.007	-0.001	-1.41
				130	224	0.107	-0.058	0.116	0.075	0.23
				132	226	0.162	-0.074	0.176	0.104	-0.02
				134	228	0.172	-0.083	0.186	0.116	-0.76
				136	230	0.176	-0.087	0.192	0.122	-1.46
				138	232	0.185	-0.083	0.202	0.119	-1.99
				140	234	0.195	-0.078	0.211	0.115	-2.48
				142	236	0.200	-0.064	0.217	0.097	-2.05
				144	238	0.200	-0.057	0.216	0.089	-2.03
				146	240	0.203	-0.047	0.220	0.077	-1.95
				148	242	0.207	-0.038	0.224	0.065	-1.98
				150	244	0.215	-0.021	0.233	0.046	-2.13
				152	246	0.223	-0.005	0.242	0.028	-2.24
				154	248	0.217	-0.002	0.235	0.022	-1.90
				156	250	0.208	0.005	0.226	0.013	-1.55
				158	252	0.203	0.015	0.221	0	-1.37
				160	254	0.202	0.031	0.220	-0.020	-1.39
				162	256	0.200	0.051	0.219	-0.044	-1.63
			96	126	222	0.002	0.001	0.002	-0.001	-3.26
				128	224	-0.012	0.001	-0.012	-0.001	-0.82
				130	226	0.126	-0.053	0.135	0.072	0.55
				132	228	0.176	-0.062	0.190	0.090	0.13
				134	230	0.181	-0.070	0.196	0.102	-0.55
				136	232	0.185	-0.074	0.201	0.108	-1.22
				138	234	0.195	-0.070	0.211	0.104	-1.82
				140	236	0.199	-0.066	0.216	0.100	-2.41
				142	238	0.200	-0.057	0.216	0.089	-2.29
				144	240	0.200	-0.051	0.216	0.080	-2.35
				146	242	0.207	-0.041	0.224	0.069	-2.37

TABLE 2 (continued)

Z	N	A	ε	ε_4	β_2	β_4	Single-particle correction (MeV)
	148	244	0.212	-0.028	0.229	0.054	-2.51
	150	246	0.217	-0.015	0.235	0.038	-2.75
	152	248	0.223	-0.002	0.243	0.024	-2.93
	154	250	0.218	0.002	0.237	0.019	-2.59
	156	252	0.212	0.008	0.230	0.009	-2.25
	158	254	0.207	0.018	0.225	-0.004	-2.07
	160	256	0.205	0.031	0.224	-0.020	-2.09
	162	258	0.203	0.051	0.223	-0.043	-2.30
98	126	224	0.002	0.001	0.002	-0.001	-3.03
	128	226	-0.012	0.001	-0.012	-0.001	-0.59
	130	228	0.236	-0.028	0.256	0.059	0.51
	132	230	0.218	-0.041	0.236	0.071	0.18
	134	232	0.190	-0.058	0.205	0.087	-0.36
	136	234	0.195	-0.062	0.211	0.093	-1.00
	138	236	0.199	-0.058	0.216	0.089	-1.65
	140	238	0.199	-0.058	0.216	0.089	-2.28
	142	240	0.200	-0.047	0.216	0.076	-2.25
	144	242	0.205	-0.041	0.222	0.069	-2.40
	146	244	0.212	-0.028	0.229	0.054	-2.56
	148	246	0.217	-0.018	0.235	0.042	-2.86
	150	248	0.223	-0.005	0.242	0.028	-3.28
	152	250	0.227	0.002	0.246	0.020	-3.56
	154	252	0.222	0.008	0.241	0.011	-3.24
	156	254	0.215	0.015	0.234	0.002	-2.96
	158	256	0.212	0.024	0.231	-0.011	-2.84
	160	258	0.208	0.038	0.228	-0.027	-2.92
	162	260	0.205	0.054	0.225	-0.047	-3.16
100	142	242	0.203	-0.038	0.220	0.064	-2.17
	144	244	0.210	-0.028	0.227	0.053	-2.44
	146	246	0.218	-0.015	0.237	0.039	-2.76
	148	248	0.225	-0.002	0.244	0.024	-3.25
	150	250	0.227	0.002	0.246	0.020	-3.80
	152	252	0.230	0.011	0.251	0.009	-4.17
	154	254	0.225	0.018	0.245	0	-3.91
	156	256	0.220	0.024	0.240	-0.009	-3.70
	158	258	0.215	0.034	0.235	-0.022	-3.66
	160	260	0.212	0.044	0.232	-0.034	-3.82
	162	262	0.208	0.057	0.229	-0.050	-4.13
102	142	244	0.203	-0.028	0.220	0.052	-1.98
	144	246	0.210	-0.018	0.227	0.041	-2.35
	146	248	0.217	-0.005	0.235	0.026	-2.79
	148	250	0.220	0.002	0.239	0.019	-3.37
	150	252	0.223	0.008	0.243	0.012	-3.98
	152	254	0.228	0.021	0.249	-0.003	-4.46
	154	256	0.223	0.028	0.244	-0.012	-4.34
	156	258	0.218	0.034	0.239	-0.021	-4.27
	158	260	0.215	0.044	0.236	-0.033	-4.37
	160	262	0.212	0.054	0.233	-0.046	-4.67

TABLE 2 (continued)

β_2	β_4	Single-particle correction (MeV)	Z	N	A	ϵ	ϵ_4	β_2	β_4	Single-particle correction (MeV)	
0.229	0.054	-2.51	104	162	264	0.208	0.064	0.230	-0.058	-5.11	
0.235	0.038	-2.75		142	246	0.207	-0.015	0.224	0.036	-1.74	
0.243	0.024	-2.93		144	248	0.212	-0.005	0.229	0.025	-2.21	
0.237	0.019	-2.59		146	250	0.215	0.002	0.233	0.018	-2.75	
0.230	0.009	-2.25		148	252	0.220	0.011	0.239	0.007	-3.41	
0.225	-0.004	-2.07		150	254	0.225	0.021	0.245	-0.004	-4.14	
0.224	-0.020	-2.09		152	256	0.228	0.031	0.250	-0.015	-4.76	
0.223	-0.043	-2.30		154	258	0.225	0.038	0.246	-0.024	-4.77	
0.002	-0.001	-3.03		156	260	0.220	0.044	0.241	-0.033	-4.83	
-0.012	-0.001	-0.59		158	262	0.217	0.054	0.238	-0.045	-5.06	
0.256	0.059	0.51		160	264	0.213	0.060	0.235	-0.053	-5.50	
0.236	0.071	0.18		162	266	0.208	0.070	0.230	-0.066	-6.08	
0.205	0.087	-0.36		106	142	248	0.210	-0.002	0.228	0.021	-1.60
0.211	0.093	-1.00			144	250	0.213	0.005	0.232	0.014	-2.15
0.216	0.089	-1.65	146		252	0.222	0.018	0.242	-0.001	-2.80	
0.216	0.089	-2.28	148		254	0.225	0.028	0.246	-0.012	-3.58	
0.216	0.076	-2.25	150		256	0.228	0.034	0.250	-0.019	-4.42	
0.222	0.069	-2.40	152		258	0.230	0.041	0.252	-0.027	-5.17	
0.229	0.054	-2.56	154		260	0.227	0.047	0.249	-0.035	-5.31	
0.235	0.042	-2.86	156		262	0.223	0.054	0.246	-0.044	-5.49	
0.242	0.028	-3.28	158		264	0.218	0.060	0.241	-0.052	-5.83	
0.246	0.020	-3.56	160		266	0.215	0.067	0.238	-0.061	-6.37	
0.241	0.011	-3.24	162		268	0.212	0.077	0.235	-0.073	-7.06	
0.234	0.002	-2.96	108		142	250	0.213	0.015	0.232	0.002	-1.53
0.231	-0.011	-2.84			144	252	0.220	0.024	0.240	-0.009	-2.16
0.228	-0.027	-2.92			146	254	0.222	0.031	0.242	-0.017	-2.91
0.225	-0.047	-3.16		148	256	0.225	0.038	0.246	-0.024	-3.77	
0.220	0.064	-2.17		150	258	0.227	0.044	0.249	-0.031	-4.68	
0.227	0.053	-2.44		152	260	0.230	0.051	0.253	-0.038	-5.51	
0.237	0.039	-2.76		154	262	0.225	0.057	0.248	-0.047	-5.80	
0.244	0.024	-3.25		156	264	0.222	0.064	0.245	-0.056	-6.11	
0.246	0.020	-3.80		158	266	0.218	0.070	0.242	-0.064	-6.58	
0.251	0.009	-4.17		160	268	0.213	0.073	0.236	-0.069	-7.24	
0.245	0	-3.91		162	270	0.212	0.080	0.235	-0.077	-8.05	
0.240	-0.009	-3.70									
0.235	-0.022	-3.66									
0.232	-0.034	-3.82									
0.229	-0.050	-4.13									
0.220	0.052	-1.98									
0.227	0.041	-2.35									
0.235	0.026	-2.79									
0.239	0.019	-3.37									
0.243	0.012	-3.98									
0.249	-0.003	-4.46									
0.244	-0.012	-4.34									
0.239	-0.021	-4.27									
0.236	-0.033	-4.37									
0.233	-0.046	-4.67									

The ground-state single-particle correction is the nuclear ground-state mass relative to the spherical macroscopic energy, which is calculated here by use of the modified-surface-energy model. In this table the ground-state zero-point energy (which is frequently taken equal to 0.5 MeV) is not included. Single-particle levels for $^{224}_{88}\text{Ra}$ and $^{250}_{98}\text{Cf}$ are used to calculate the potential energy for nearby nuclei. In both regions the values $a = 0.8$ fm, $\lambda_n = 36$ and $\lambda_p = 34$ are used.

One then arrives at the following expression for the macroscopic energy in the modified-surface-energy model:

$$E_{\text{macro}} = -a_v(1 - \kappa_v I^2)A + E(\xi_v) + \frac{4}{3}\pi V_0 R_0^3 + \frac{3}{5} \frac{e^2 Z^2}{r_0 A^{\frac{1}{3}}} \left[B_c(\xi_v) - \frac{5}{6}\pi^2 \frac{d^2}{r_0^2 A^{\frac{2}{3}}} - \frac{0.7636}{Z^{\frac{1}{3}}} \right]. \quad (3)$$

The shape-dependent function $B_C(\xi_v)$ is the ratio of the Coulomb energy of the deformed equivalent sharp surface nucleus to that of the spherical nucleus. The quantity d that appears in the surface-diffuseness correction to the Coulomb energy is the parameter in a Fermi function that specifies the charge distribution; following refs. ^{14,15}) its value is taken to be

$$d = (2.4 \text{ fm}) / (2 \ln 9) = 0.5461 \text{ fm},$$

in accordance with the Stanford electron scattering experiments. The last term in eq. (3) is the exchange correction to the Coulomb energy ¹⁶⁾.

The strength parameter V_0 is determined by equating the second term in eq. (2) to the surface energy of a spherical nucleus; this yields

$$2\pi a r_0^2 V_0 = c_s = a_s(1 - \kappa_s I^2),$$

where we use the result that the equivalent sharp surface nuclear radius R_0 is given by

$$R_0 = r_0 A^{\frac{1}{3}}.$$

The preliminary values chosen in ref. ⁷⁾ for the four constants that enter eq. (1) are

$$r_0 = 1.16 \text{ fm}, \quad a = 1.4 \text{ fm}, \quad a_s = 24.7 \text{ MeV}, \quad \kappa_s = 4.0.$$

We determine preliminary values for the remaining two constants a_v and κ_v in eq. (3) by adjusting them to reproduce optimally the experimental ground-state masses of 133 even nuclei in the region $62 \leq Z \leq 102$. In this determination the single-particle corrections are calculated with the modified oscillator potential; for each nucleus a constant average pairing energy of -2.3 MeV is subtracted. The resulting values are $a_v = 16.48509$ MeV and $\kappa_v = 2.32427$. The rms deviation between the calculated and experimental ground-state masses is 1.5 MeV.

From tables 1 and 2 we see that the calculated ground-state values of ε are approximately equal in the droplet model and the modified-surface-energy model. However, for the lighter actinide nuclei such as ${}^{226}_{90}\text{Th}$, the calculated ground-state value of ε_4 is about 40 % larger in absolute value for the modified-surface-energy model than for the droplet model. Also, the calculated ground-state energy (relative to the spherical macroscopic energy) is lower by about 1.5 MeV in the modified-surface-energy model than in the droplet model. This is because for high-multipole distortions the macroscopic energy increases much less rapidly in the modified-surface-energy model than in the droplet model.

For higher multipole distortions there is an even larger difference between the two models. We have investigated this for ε_6 distortions in the modified oscillator model. With ε and ε_4 held fixed at the values $\varepsilon = 0.25$ and $\varepsilon_4 = 0.00$, the ε_6 distortion coordinate is varied; the resulting changes in the energies for ${}^{252}_{100}\text{Fm}$ are given in table 3. We observe that for these fixed values of ε and ε_4 the single-particle correction is unstable against ε_6 distortions. This is consistent with figs. 3a and 3b, where it is seen

TABLE 3
Variation of energies for ${}_{100}^{252}\text{Fm}$ with ε_6 , for $\varepsilon = 0.25$ and $\varepsilon_4 = 0$

ε_6	$\Delta E_{s.p.}$ (MeV)	$\Delta E_{\text{droplet}}$ (MeV)	$\Delta E_{m.s.e.}$ (MeV)	$\Delta E_{s.p.} + \Delta E_{\text{droplet}}$ (MeV)	$\Delta E_{s.p.} + \Delta E_{m.s.e.}$ (MeV)
0	0	0	0	0	0
0.01	-0.49	0.06	0.02	-0.43	-0.47
0.02	-0.95	0.33	0.15	-0.62	-0.80
0.03	-1.40	0.82	0.36	-0.58	-1.04
0.04	-1.81	1.50	0.66	-0.31	-1.15
0.05	-2.18	2.40	1.05	0.22	-1.13

The changes in the single-particle correction (column 2) are added to the changes in the macroscopic energy calculated with the droplet model and the modified-surface-energy model (columns 3 and 4, respectively) to give the changes in the total energy calculated with these two models (columns 5 and 6). The single-particle correction is calculated with the modified oscillator potential.

that the gaps at $Z = 100$ and $N = 152$, respectively, increase for positive ε_6 distortions. The equilibrium value of ε_6 is twice as large with the modified-surface-energy model as with the droplet model. Also, the ground state is lowered twice as much by the ε_6 degree of freedom in the former model as in the latter.

Theoretical and experimental ground-state single-particle corrections²⁾ are compared in fig. 9 for nuclei with neutron number $N \geq 120$. The theoretical values are calculated by use of the folded Yukawa single-particle potential and the droplet model. From the figure it is apparent that the deviations between theory and experiment are about 1 MeV on the average; the maximum deviation occurs for ${}_{90}^{222}\text{Th}$, where it is 2.6 MeV in magnitude. It is of interest to note that the differences in the ground-state single-particle corrections calculated by use of the droplet model and the modified-surface-energy model are also of this order. We noted earlier that it is possible to reproduce experimental ground-state masses in the region $62 \leq Z \leq 102$ with a rms deviation of 1.5 MeV by use of a modified oscillator single-particle potential and the modified-surface-energy model. However, in that determination only the parameters a_v and κ_v for the volume-energy term were varied. Until all the parameters are varied simultaneously to reproduce nuclear masses, fission-barrier heights and interaction-barrier heights, it is not possible to know how accurately these quantities ultimately can be calculated in the modified-surface-energy model.

There also exists experimental information on the shapes of actinide nuclei in their ground states. Recently, it has become possible to determine higher multipoles in the shape of the nuclear surface from Coulomb-excitation experiments that populate the 0^+ , 2^+ and 4^+ states in the ground-state rotational band¹⁸⁾. The experiments and theoretical results are often analyzed by assuming a homogeneous distribution of nuclear charge inside a sharp surface defined by

$$R = R_0(\beta_v)(1 + \beta_2 Y_{20} + \beta_4 Y_{40} + \dots).$$

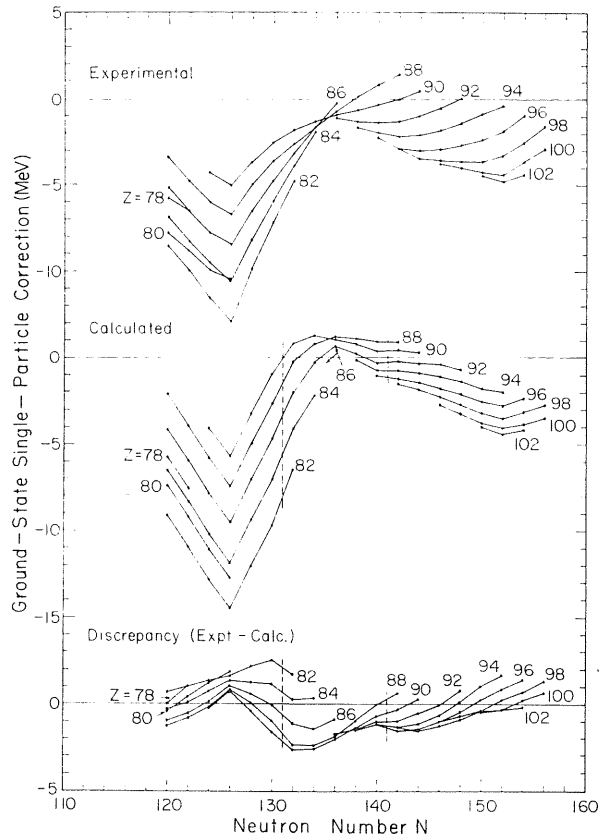
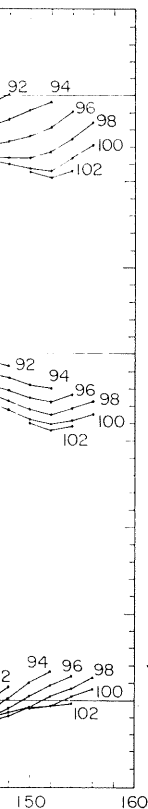


Fig. 9. Comparison of experimental ground-state single-particle corrections for even nuclei with values calculated by use of the folded Yukawa potential and the droplet model²⁾. The ground-state single-particle correction is the nuclear ground-state mass relative to the spherical macroscopic energy, which is calculated here from the droplet model of Myers and Swiatecki. The experimental masses are taken from ref. 17). The calculated masses are obtained by minimizing the potential energy with respect to a separation coordinate and a necking coordinate in two different shape parametrizations, as discussed in ref. 2). Single-particle levels for $^{208}_{82}\text{Pb}$, $^{224}_{88}\text{Ra}$ and $^{250}_{98}\text{Cf}$ are used to calculate the potential energy for each nucleus in the left-hand, middle and right-hand regions, respectively; these regions are indicated by the dashed vertical lines. In all three regions the values $a = 0.8$ fm, $\lambda_n = 36$ and $\lambda_p = 34$ are used. A constant ground-state zero-point energy of 0.5 MeV is included for each nucleus. The lower portion of the figure gives the discrepancy between the experimental and calculated masses.

In figs. 10 and 11 we compare experimental and theoretical results in terms of the parameters β_2 and β_4 , respectively. The calculated values are also tabulated in tables 1 and 2. The calculated values of β_2 in fig. 10 are determined by use of the folded Yukawa single-particle potential and the modified-surface-energy model. When the droplet model is used the resulting values of β_2 are practically the same, as seen from tables 1 and 2. However, for the parameter β_4 displayed in fig. 11 there is an appreciable difference between the results calculated with the modified-surface-energy



...le corrections for even nuclei with
...e droplet model²). The ground-state
...relative to the spherical macroscopic
...ers and Swiatecki. The experimental
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...els for $^{208}_{82}\text{Pb}$, $^{224}_{88}\text{Ra}$ and $^{250}_{98}\text{Cf}$ are
...e left-hand, middle and right-hand
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...t ground-state zero-point energy of
...figure gives the discrepancy between
...masses.

...eoretical results in terms of the
...ues are also tabulated in tables
...etermined by use of the folded
...urface-energy model. When the
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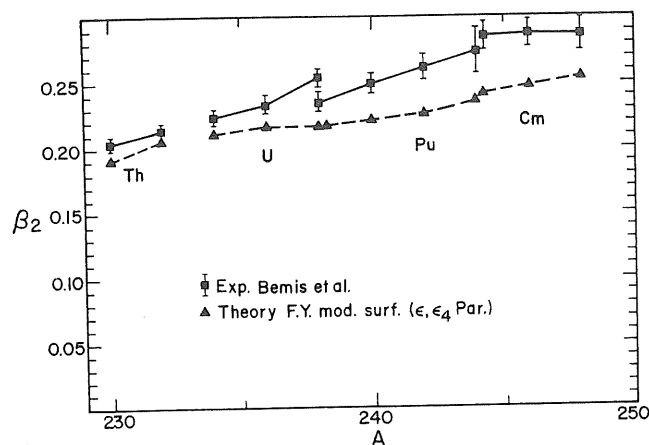


Fig. 10. Calculated and experimental values of β_2 for actinide nuclei. The experimental values are taken from ref. ¹⁸).

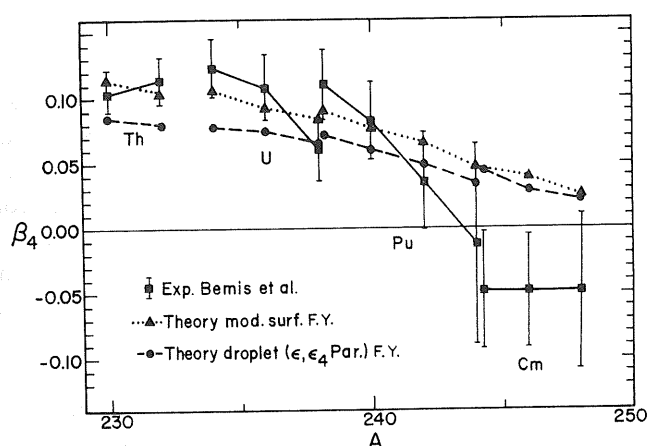


Fig. 11. Calculated and experimental values of β_4 for actinide nuclei. The experimental values are taken from ref. ¹⁸).

...model and those calculated with the droplet model. This is analogous to differences
...noted earlier for the parameter ϵ_4 , which is similar to β_4 . The modified-surface-
...energy model agrees somewhat better with the trend exhibited by the experimental
...data, at least for the lighter actinides. However, the errors in the experimental values
...of β_4 are so large that there is little conflict between the experimental results and the
...theoretical results for either of the two models.

...On the other hand, the theoretical values of β_2 in fig. 10 are consistently lower
...than the experimental values. A more direct way of comparing experiment and theory
...would be to calculate the electric quadrupole and hexadecapole moments from the
...single-particle wave functions at the ground state and compare them with the experi-

mentally determined moments. For the modified oscillator model, B. Nilsson¹⁹⁾ has calculated the quadrupole moment Q_2^{mass} based on the proton wave functions (i.e. the charged matter distribution) appropriate to the ground-state shapes. When compared with the quadrupole moment Q_2^{pot} calculated by assuming a homogeneous charge inside the nuclear equipotential surface the result for all nuclei studied was $Q_2^{\text{mass}} < Q_2^{\text{pot}} < Q_2^{\text{exp}}$. Thus the discrepancy between the experimental and theoretical results becomes even larger when the single-particle wave functions of the modified oscillator model are used.

As pointed out by Brack²⁰⁾ the folded Yukawa model differs from the modified oscillator model in particular in the sense that in the calculations the Coulomb field is explicitly included in the single-particle potential. According to Brack the relation $Q_2^{\text{mass}} < Q_2^{\text{pot}}$ is therefore no more valid in the folded Yukawa model. One should consequently expect some improvement on this point once the single-particle wave functions are employed in the calculation of Q_2^{mass} .

3. Summary and conclusions

We have calculated a variety of ground-state properties for heavy nuclei by use of the macroscopic-microscopic method. The microscopic properties were calculated primarily by use of the folded Yukawa single-particle potential. Some comparisons were made with results obtained by use of the modified oscillator potential. Two models were also investigated for the macroscopic energy, namely the droplet model and the modified-surface-energy model. We compared such calculated and experimental quantities as intrinsic energy levels, nuclear ground-state masses, and ground-state quadrupole and hexadecapole distortions. All these quantities were reproduced by the calculations reasonably well, except that the theoretical values of β_2 are systematically somewhat smaller than the experimental values. There are some differences between the predictions of the droplet model and the modified-surface-energy model, but it is not possible for us to express a preference for one or the other of these alternative macroscopic models solely from the present investigation.

We would like to thank R. W. Hoff for permission to use his unpublished compilation of experimental quasi-particle energies, and C. E. Bemis, Jr. and R. W. Hoff for discussions concerning these data. P. Möller and S. G. Nilsson are grateful to the Los Alamos Scientific Laboratory for the kind hospitality extended to them.

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oscillator model, B. Nilsson¹⁹) has the proton wave functions (i.e. ground-state shapes. When compared by assuming a homogeneous result for all nuclei studied was the experimental and theoretical wave functions of the modified

model differs from the modified oscillations the Coulomb field is according to Brack the relation Yukawa model. One should compare the single-particle wave func-

DISCUSSION

Properties for heavy nuclei by use of microscopic properties were calculated in the potential. Some comparisons were made with the modified oscillator potential. Two models, namely the droplet model and the modified-surface-energy model, were compared. There are some differences between the two models. In one or the other of these alternative investigations.

We would like to use his unpublished communication. E. Bemis, Jr. and R. W. Hoff and S. G. Nilsson are grateful to the National Science Foundation for the hospitality extended to them.

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