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OCTUPOLE DEFORMATIONS IN THE NUCLEI BEYOND ^{208}Pb

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Calculations of the quadrupole (ϵ_2) - octupole ($\epsilon_3(\epsilon_5)$) potential energy surface for nuclei beyond ^{208}Pb indicates the presence of a very low lying secondary minimum in the *oblate* octupole region of deformation for certain nuclei. The well-known low-lying $K = 0^-$ bands in the Ra-Th region are interpreted as associated with this secondary minimum.

Although reflection symmetric deformations of Y_{20} type are dominant in the rare-earth nuclei, it has been argued by Bohr and Mottelson [1] that with the increase in nucleon quantum numbers in nuclei heavier than those presently observed, the reflection asymmetric octupole deformation would ultimately be energetically favoured over the quadrupole deformation. Thus while for lighter nuclei orbits with $\partial e/\partial l = 2\,\partial e/\partial n$ are approximately degenerate [1], we have for still heavier ones approximate degeneracies of the type $\partial e/\partial l = 3\partial e/\partial n$ becoming frequent, where l and n are the angular and radial quantum numbers respectively. Examples of the former type of degeneracies are 2s with 1d, and 3s, 2d, with 1g, etc., while the latter situation corresponds to the near degeneracy of 2p with 1g, 3p and 2g with 1j, etc.

In the region of nuclei just beyond the double-closed shells in ^{208}Pb, the competition between octupole and quadrupole deformations becomes intense in view of the fact that here $g_{9/2}$ and $j_{15/2}$ come close to each other energywise as do the proton orbitals $f_{7/2}$ and $i_{13/2}$. Whether or not even nuclei ever achieve a stable octupole deformation in this region has not been adequately determined in spite of the fact that reasonably stable nuclei are available for investigation.

Fig. 1a presents two sets of positive and negative parity states which occur regularly in the sequence of nuclei studied. Previously two alternative explanations of these coexistent sets of states have been presented. Both explanations are based on a one-minimum picture of the nuclear potential-energy surface. We shall argue that neither of these alternative appears satisfactory and present an alternative picture with two different potential-energy minima corresponding to two different nuclear shapes.

Corresponding to a pure quadrupole deformation, the wave function for the even-even ground state rotational band is reflection symmetric and contains states of positive parity with only even spin values,

$I^\pi = 0^+, 2^+, 4^+, \ldots$

The odd-A spins are missing as a consequence of the nuclear reflection symmetry. A $K = 0^-$ octupole vibration can then be built on the $K = 0^+$ reflection symmetric ground state giving rise to an

$I^\pi = 1^-, 3^-, 5^-, \ldots$ band.

On the other hand, when an octupole deformation is established, there is obviously no such symmetry. In the special case when the nucleus is completely stable against a transition between one octupole shape and its mirror image, a single band results with

$I^\pi = 0^+, 1^-, 2^+, 3^-, 4^+, \ldots$

In the more general case a tunneling motion is possible between the mirror shapes. Consequently the two bands of opposite parity are displaced in energy with respect to each other, reminiscent of the ammonium molecule, i.e.,

$I^\pi = 0^+, 2^+, 4^+, \ldots$ and $I^\pi = 1^-, 3^-, 5^-, \ldots$

with the even spins energetically favoured.

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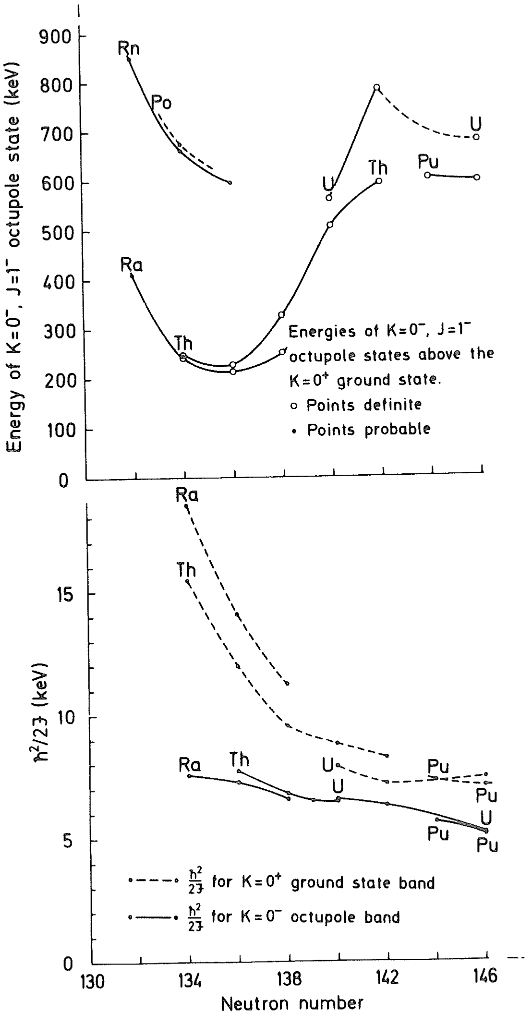

Fig. 1. The upper part (a) shows the energies of the negative parity bands above the 0^+ ground states plotted against neutron number N of a sequence of Ru, Ra, Th, U and Pu nuclei. (b) shows the effective energy moment $\hbar^2/2J$ for the positive and negative parity bands of the same nuclei between Ra and Pu.

Thus, the lowest lying rotational bands for a nucleus with a stable quadrupole deformation and a low-lying octupole vibration are expected to be very similar to those of a nucleus with a stable octupole deformation (with or without quadrupole deformation in addition) with tunneling between the two mirror shapes. One experimental way to differentiate between the two possibilities is to look for the two-phonon octupole vibration which should exist in the case of the stable pure quadrupole deformation. Two-phonon octupole vibrations with $K^\pi = 0^+$ are expected to have approximately twice the energy of the one-phonon octupole vibration and to decay through the one-phonon state via an E1 cascade to the ground state. A search for the two-phonon octupole vibration [2] and its decay has given negative results. However, in view of the possible large hindrance factors in the alpha decay process which populates the two-phonon octupole vibrational state, this negative result cannot be considered conclusive.

Indeed, what experimental evidence is available [2] cannot be made entirely consistent with either one of these two pictures. On the one hand one has the very low-lying $K = 0^-$ sequences (see fig. 1a), whose energies are not very well explained in the calculations of Vogel [3], assuming they are octupole vibrations. Furthermore, there is a distinct break in the systematics between the Ra nuclei and the Rn nuclei which is quite in contrast to the behaviour of the octupole vibration in the rare-earth region. As one crosses between nominally spherical and deformed nuclei in ^{150}Sm-^{152}Sm and ^{152}Gd-^{154}Gd sequences there is little if any effect in the octupole vibrational energy. Finally, there is the previously mentioned non-observation of the two-phonon octupole vibration. All of these experimental facts favour an interpretation in terms of a stable octupole deformation. On the other hand, the "moments of inertia", $\hbar^2/2\mathcal{J}$ (see fig. 1b) are extremely different for the $K = 0^-$ bands and for the $K = 0^+$ bands and this difference increases in the lightest Ra and Th nuclei. Indeed, these systematics suggest that even in the Rn nuclei, in which there is no $K = 0^+$ ground state rotational band, there still will be a well deformed $K = 0^-$ rotational band. These data argue against an octupole deformation for both positive and negative parity states, and suggest instead the existence of shape isomers in these nuclei. A theoretical foundation for this description is developed later in this paper.

Finally, there is the experimental observation of a fairly large B(E3) of ≈ 70 single particle units in ^{226}Ra with slightly smaller values in Th and U [3]. These values are very large for vibrations, although similar values have been successfully explained by Vogel in terms of the vibrational octupole model. Assuming a transition within a deformed band of stable Y_{30} deformation, they would correspond to $|\beta_3| \approx 0.14$ and $|\epsilon_3| \approx 0.11$.

These seemingly conflicting data can be reconsidered most easily on the basis of a potential-energy picture with two minima, one associated with the even parity and one with the odd parity sequences of states.

To this end we have studied the nuclear potential-energy surfaces in the space of quadrupole and octupole shapes in the region of nuclei in question.

As in refs. [4, 5], we have considered a modified oscillator of the following type:

$$V = \tfrac{1}{2}\hbar\omega_0\rho^2 \times$$
$$\times\,(1 - \tfrac{2}{3}\epsilon_2 P_2 + 2\epsilon_4 P_4 + 2\epsilon_3 P_3 + 2\epsilon_5 P_5 + 2\epsilon_1 P_1)$$

to which $\boldsymbol{l}\cdot\boldsymbol{s}$ and $\boldsymbol{l}^2$ terms are added with strengths [6] appropriate to the region studied. The P_1 term only enters as a center-of-mass correction. For each ϵ_3 value we have selected only one value of ϵ_5 in accordance with the optimizations in the $(\epsilon_3\epsilon_5)$ plane as studied in ref. [4].

We first considered only two deformation coordinates: ϵ_2 on the one hand and $\epsilon_3(\epsilon_5)$ on the other. As P_4, or Y_{40}, favours prolate shapes, while Y_{44}, etc., may be more relevant on the oblate side, we have concluded that the results in terms of ϵ_2, $\epsilon_3(\epsilon_5)$ are in some respect more satisfactory than a calculation in terms of $\epsilon_2(\epsilon_4)$, $\epsilon_3(\epsilon_5)$, which calculation has also been performed. The more complete results will be published later elsewhere.

Potential energy surfaces were calculated for Pb, Po, Rn, Ra, Th, U, and Pu for neutron numbers from 126 to 146. We plot the potential energies as contours in ϵ_2, $\epsilon_3(\epsilon_5)$ coordinates in the ranges shown in the figures. Indeed, the energy surfaces seem to be more A-dependent than either proton or neutron dependent, somewhat in contrast to data in the Rn-Ra region (see fig. 1a).

Consider, for example, the Ra isotopes with neutron numbers from 126 to 146. For ^{214}Ra with 126 neutrons, the potential energy surface indicates a rigid, spherical nucleus. With the addition of neutrons the rigidity is decreased. Thus already for ^{218}Ra, an octupole deformation is indicated for the ground state of this nucleus, although there is no quadrupole deformation, $(\epsilon_2 \approx 0)$. The equilibrium octupole deformation, ϵ_3, reaches its largest value at ^{224}Ra, presumably as a result of the completion of the $g_{9/2}$ orbital for $N = 136$. This is actually the neutron number for which all of the nuclei in fig. 1a have the most low-lying 1^- state. With the addition of 4 more neutrons at ^{228}Ra, the potential energy surface has become particularly soft in ϵ_2 as well as ϵ_3(see fig. 2). With ϵ_4 included in the calculations (not shown), a prolate minimum is established.

With the addition of 2 more neutrons giving ^{230}Ra, the very soft potential-energy surface exhibits a reflection symmetric quadrupole ground state and a reflection asymmetric *oblate* secondary minimum. The addition of $\epsilon_4 P_4$ lowers the prolate minimum considerably relative to the oblate-octupole minima. The fact that these secondary minima involve an *oblate octupole* deformation is particularly interesting because such a shape has not been previously suggested to our knowledge. For ^{232}Ra and ^{234}Ra (see fig. 2), the two minima become somewhat more distinct while the secondary minimum increases in energy somewhat with respect to the ground-state minimum.

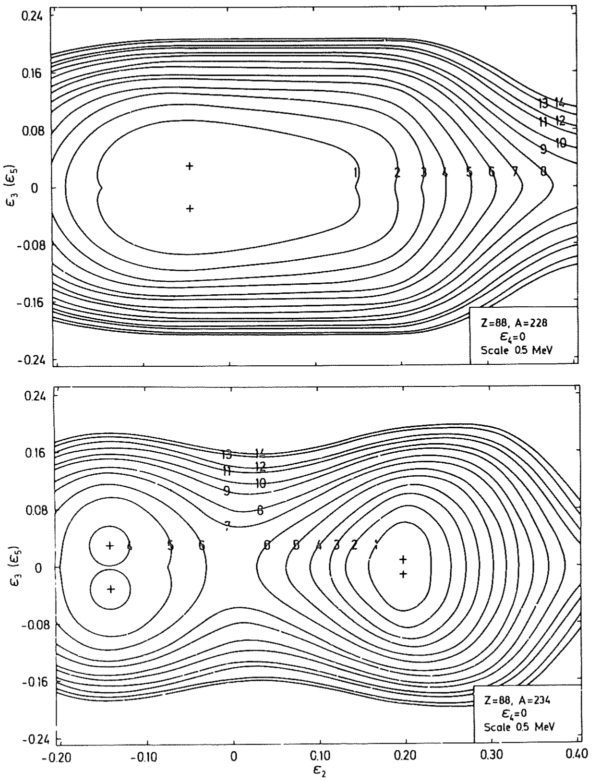

Fig. 2. The potential-energy surface in terms of ϵ_2 (elongation) and $\epsilon_3(\epsilon_5)$ (asymmetry) for the nuclei ^{228}Ra and ^{234}Ra. Note that the separation between contour lines is 0.5 MeV. Crosses show the minima in the potential-energy surface.

We thus like to suggest that the long-known mysterious $K = 0^-$ rotational bands seen in the Rn, Ra region of nuclei are actually associated with *octupole-oblate shape isomeric minima.* The magnitude of ϵ_3 for the minima lies between 0.02 and 0.05 for most of these nuclei which is less than half of the ϵ_3 value of about 0.11 that is obtained from the observed B(E3)-value under the assumption of equal deformation of the 3^- and 0^+ states. In our case even larger ϵ_3-values may be called for.

In the calculations, the barrier between the octupole oblate mirror image minima also comes out less than 100 keV high in all cases, which is hardly enough to establish stable octupole shapes.

The depths of the minima are fairly dependent on the details of the single-particle scheme assumed, in particular on the distance between the $g_{9/2}$ and $j_{15/2}$ sub-shells. Of considerable importance is probably the fact that other multipole distortions associated with Y_{33}, Y_{32} and Y_{31} may be expected to play a large role on the oblate side. An argument in favour of this supposition is the fact that on the oblate side near $\epsilon_2 = -0.15$, the (707 15/2) neutron orbital originating from the $j_{15/2}$ spherical subshell comes in very close proximity to the (604 9/2) orbital originating from $g_{9/2}$. These orbitals have a large matrix element of $r^3 Y_{33}$ between them. The case is similar for the proton orbitals (606 13/2) and (503 7/2). The inclusion of these degrees of freedom in the calculations is presently being considered. The influence of Y_{33}, Y_{32} and Y_{31} is probably to deepen the octupole minima and to push them farther apart which would thus be more consistent with the larger B(E3) values indicated by experiments. Such an influence should also initiate the appearance of oblate octupole shape isomeric minima at earlier neutron numbers as required by the experimental systematics (fig. 1a).

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