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The instability at the second saddle point of actinide elements towards asymmetric distortions is explained by a decrease in energy of the neutron orbitals [40 $\Lambda\Omega$] (orbitals at the waistline of the nucleus) for asymmetric distortions. These orbitals are situated at the Fermi surface and couple strongly to [51 $\Lambda\Omega$] levels slightly above the Fermi surface.

In a recent publication [1] we exhibited the results of calculations (based on the Strutinsky shell correction method [2]) that in addition to P_2 and P_4 distortions also included the asymmetric P_3 and P_5 degrees of freedom. In the region $N = 130$-150 we encountered for $\epsilon = 0.85$ a tendency to asymmetric distortions. The second barrier peak was found to be reduced by 2 - 2.5 MeV for ^{236}U due to the combined effect of P_3 and P_5 distortion. On the other hand for ^{248}Cf the combined effect in reducing the barrier was less than 0.5 MeV. Finally ^{210}Po was found to be stable at all ϵ-values between 0.0 and 1.0. Recent communications from Pauli et al. [3] indicate very similar results on $P_3 + P_5$ instability reached on the basis of their radially somewhat different potential.

Asymmetry favouring orbitals. Since the early calculations involving only a few representative nuclei, also intermediary nuclei were studied by one of us [4] and the collected results on the possible P_3 and P_5 instability for the point $\epsilon = 0.85$, $\epsilon_4 = 0.12$, which represents the approximate locus of the second barrier peak for actinide nuclei, are found in fig. 1. This figure thus exhibits the transition lines in A and Z for nuclei where the asymmetric degrees of freedom start to affect the height of the second barrier.

Although the tendency to asymmetry was clearly established as a single-particle effect in ref. [1], no detailed "microscopic" explanation was given in terms of specific orbitals. (From the rapid variation in the mass distribution of the fission fragments from ^{210}Po to ^{236}U to ^{252}Fm it is obvious that a shell structure explanation is called for and that this rapid variation hardly can be explained in terms of the smooth variations with Z and A of the liquid-drop model.)

In the calculations of ref. [1] we assumed a potential of the following type [5,6]

$$V = \tfrac{1}{2}\hbar\omega_0(\epsilon, \epsilon_4, \epsilon_1, \epsilon_3, \epsilon_5)\rho^2 \times$$

$$[1 - \tfrac{2}{3}\epsilon P_2 + 2\epsilon_4 P_4 + 2\epsilon_1 P_1 + 2\epsilon_3 P_3 + 2\epsilon_5 P_5] - V_{\mathrm{corr}}$$

$$V_{\mathrm{corr}} = \kappa\hbar\mathring{\omega}_0(2 l_t \cdot s + \mu(l_t^1 - \langle l_t^1\rangle_{\mathrm{shell}}))$$

where the amount of P_1 distortion is determined so as to compensate the center-of mass displacement involved in the addition of P_3 and P_5

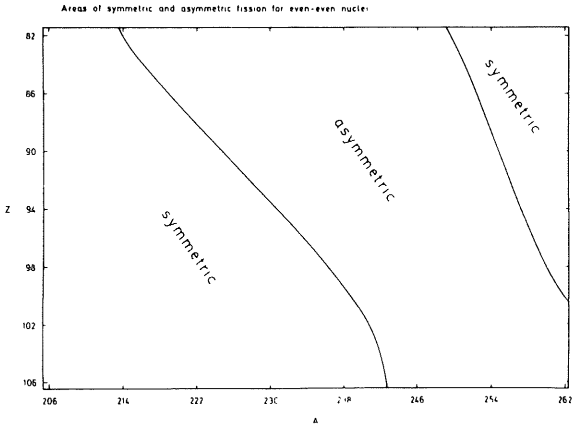

Fig. 1. Map of energy stability at the second barrier peak ($\epsilon = 0.85$, $\epsilon_4 = 0.12$) according to calculations in terms of the asymmetry parameter $\epsilon_3(\epsilon_5)$ for an area of Z- and N-values. It is found that the region of instability of actinide elements involves a limited number of N-values generally up to $N \approx 160$, and starting at $N \approx 132$.

terms. In this paper, a simple calculation of the following kind has been performed. Starting from the position $\epsilon = 0.85$, $\epsilon_4 = 0.12$, which, as mentioned, roughly corresponds to the second saddle point in the (ϵ, ϵ_4) plane, we have plotted the single-particle energy levels as functions of $\epsilon_3(\epsilon_5)$. (The notation $\epsilon_3(\epsilon_5)$ implies that for each ϵ_3 we employ the ϵ_5-value that gives minimum

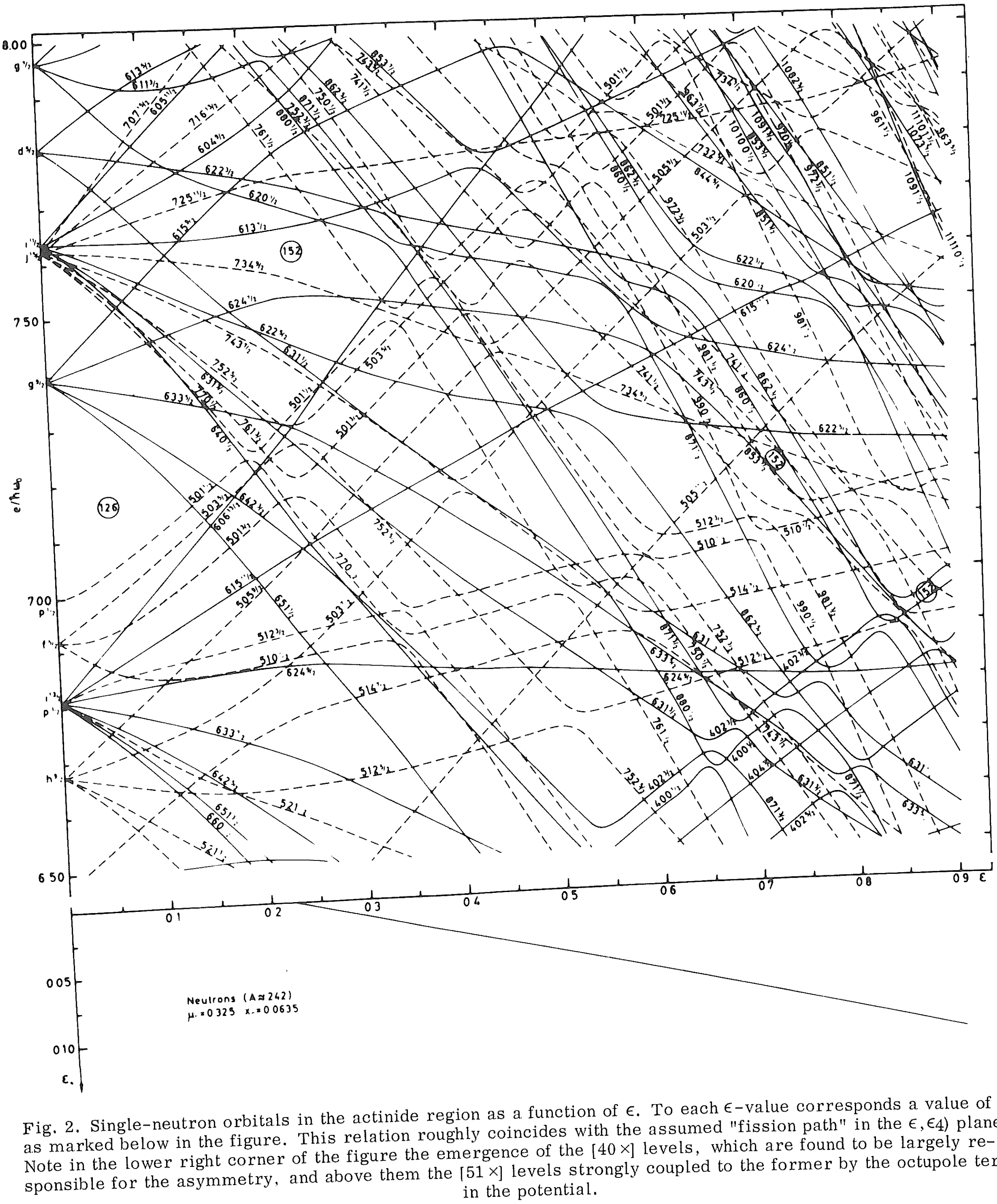

Fig. 2. Single-neutron orbitals in the actinide region as a function of ϵ. To each ϵ-value corresponds a value of ϵ_4 as marked below in the figure. This relation roughly coincides with the assumed "fission path" in the ϵ, ϵ_4) plane. Note in the lower right corner of the figure the emergence of the [40 ×] levels, which are found to be largely responsible for the asymmetry, and above them the [51 ×] levels strongly coupled to the former by the octupole term in the potential.

total energy.) The entering neutron orbitals are first identified in fig. 2 [7] which shows the single-particle levels as functions of $\epsilon_2(\epsilon_4)$. (The ϵ_4-values in the corresponding calculation are chosen as indicated below in the figure and so as to reproduce the "path" to fission in the (ϵ, ϵ_4)-plane.) Although many quasi-crossings, with orbitals approaching each other closely, occur for

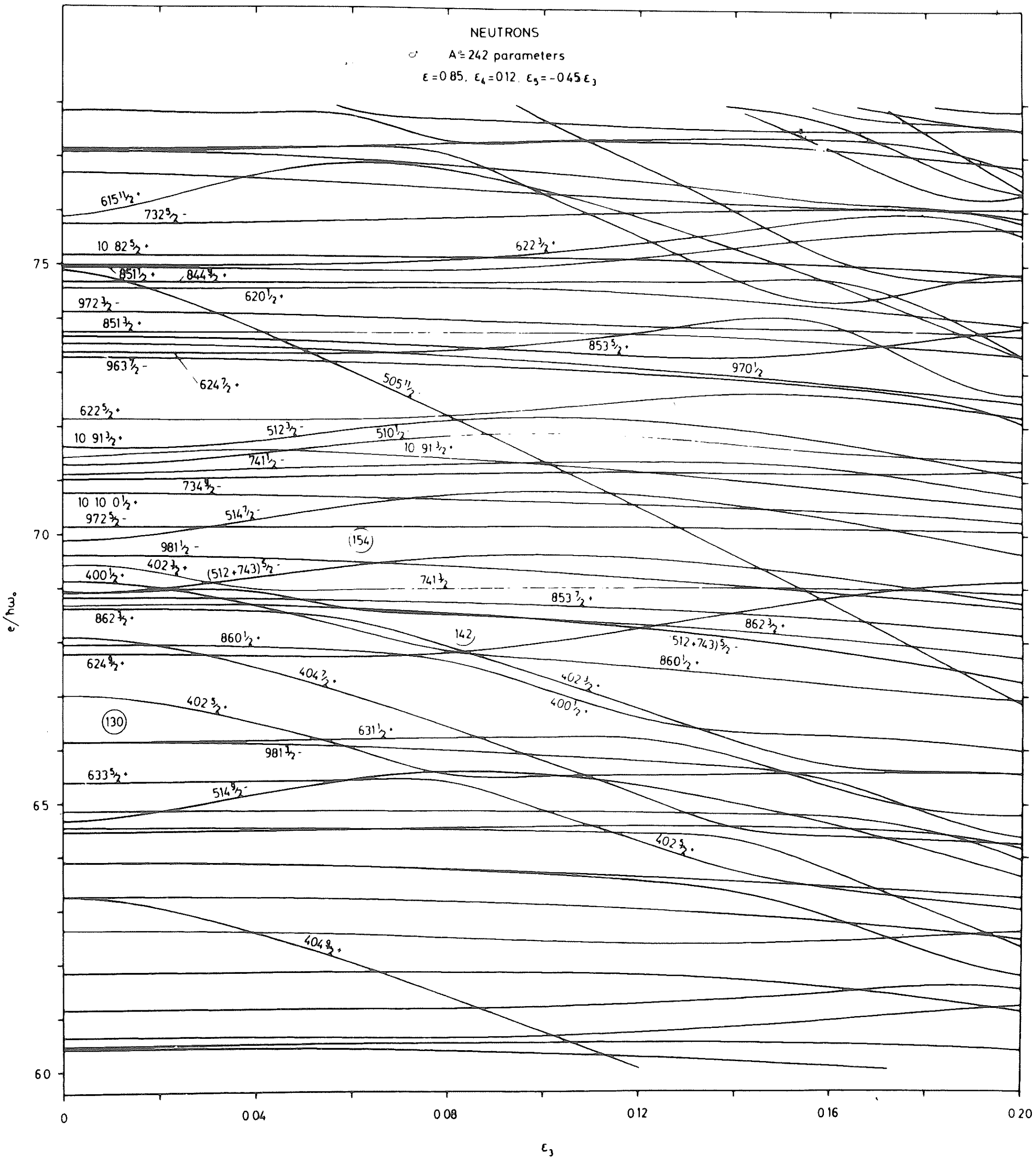

Fig. 3. Single-neutron levels for ϵ = 0.85, ϵ_4 = 0.12 as functions of the asymmetric distortion coordinate $\epsilon_3(\epsilon_5)$. Note the strong downward curvature of the [40 ×] levels as well as of the orbital [505 11/2] and the upward curvature of the [51 ×] levels, which for N=150 are as yet unfilled for the large ϵ-distortions.

these large distortions, one might expect that the so-called asymptotic wave functions are very well realized except very near the quasi-crossings, when the asymptotic characters are shared between the approaching orbitals.

In fig. 3 the effect on these neutron orbitals due to asymmetric $P_3 + P_5$ distortions is exhibited. For ϵ_3 near to zero all levels, as expected, run parallel to the abscissa. However, to second order the levels couple in $\rho^2 P_3$ (and $\rho^2 P_5$) which causes some strongly interacting levels to bend convexly downwards and convexly upwards, respectively.

Clearly exceptional in their behaviour are the [404 7/2], [402 5/2] and [402 3/2] and [400 1/2] levels which bend downwards with increasing asymmetry in fig. 3 as they couple strongly to [514 7/2], [512 5/2], [512 3/2] and [510 1/2], respectively. The latter orbitals are the ones above in fig. 3 that are found to bend upwards. To understand this feature one may replace the operator $\rho^2 P_3$ by the very similar operator $\rho^3 P_3$ the latter of which contains terms proportional to $z(x^2+y^2)$ and to z^3. Of these terms $z(x^2+y^2)$ is associated with the selection rules $\Delta N = 1, 3$, $\Delta n_z = 1$, $\Delta\Lambda = 0$, $\Delta\Omega = 0$, while z^3 has the selection rules $\Delta N = 1, 3$, $\Delta n_z = 1, 3$, $\Delta\Lambda = 0$, $\Delta\Omega = 0$. Both terms are responsible for the coupling between [404 7/2] and [514 7/2]. Analogously [505 11/2] and [404 9/2] bend strongly downwards as they couple to the nearlying orbitals [615 11/2] and [514 9/2]. The first four $N = 4$ levels appear just below the Fermi surface for $N \approx 130$-150 in fig. 2 for $\epsilon \approx 0.7$-0.9 while their "coupling mates" occur just above the Fermi surface for these distortions. For still larger distortions, or alternatively, still smaller N-values the [40×] levels emerge above the Fermi surface, in which case they no more contribute to the potential energy.

"Asymptotic" coupling rules. Let us consider one of the single-particle states involved, as representative of the asymmetry inducing orbitals, namely $[404\ 7/2] \sim (x+iy)^4 \exp(-\rho^2)$. This corresponds to a one-ring density distribution in the $z = 0$ plane (see fig. 4). The "mating" state $[514\ 7/2] \sim z(x+iy)^4 \exp(-\rho^2)$ has a density distribution which may be described as two rings, one above and one below the $z = 0$ plane. Obviously the amplitudes of the upper and lower ring wave functions enter with different signs in this case. The mixing of the two wave functions, [404 7/2] and [514 7/2], obviously leads to a matter distribution asymmetric with respect to the $z = 0$ plane. As the [404 7/2], [402 5/2] etc orbitals are all "waistline states" in the nucleus, their coupling with the $N = 5$ states mentioned leads specifically to a waistline displacement up or down along the nuclear axis. The mechanism found is thus very efficient in generating such asymmetric shapes that to lowest order just amount to a "waistline displacement". The critical role of the orbitals of the type $[N0\Lambda\ \Omega]$ for the occurrence of the instability towards asymmetric distortions, was pointed out by Sven Johansson already in 1961 [8]

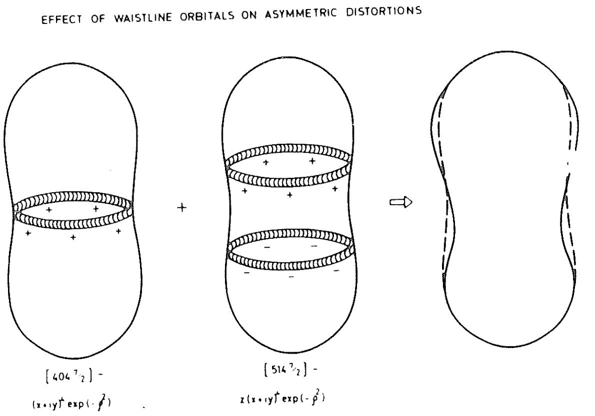

Fig. 4. Sketch of the matter density distributions corresponding to the asymptotic wave functions [404 7/2] $\sim (x+iy)^4 \exp(-\rho^2)$, and $[514\ 7/2] \sim z(x+iy)^4 \exp(-\rho^2)$, and the effect of a mixture of the latter orbital into the former one on the shape of the nuclear surface.

Analogous regions in N and Z. A corresponding situation is expected for nuclei that are one shell lighter in their neutron number in which case the orbitals [303 5/2], [301 3/2], [301 1/2] should be involved and couple strongly by "asymptotic" selection rules to [413 5/2], [411 3/2] and [411 1/2]. These orbitals enter as proton orbitals for the actinide nuclei. In that case they appear to be entirely filled for $\epsilon = 0.8$ and with $Z \approx 80$ - 90 also the last of the coupling mates or the [41×] orbitals are being filled. Hence the protons appear to give a very small contribution to the asymmetry effect in the calculations. However, as mainly only fission fragment spectra of elements along the stability line are available from experiments, one cannot from experimental findings entirely exclude the alternative situation that the [40×] orbitals e.g., occur at a little lower energy in the neutron case and the [30×] orbitals conversely at a little higher energy in the proton case. In this situation in the actinide region the proton number Z and not the neutron number N should become the decisive factor.

The rare-earth region. The shell structure trend towards asymmetric instability maybe expected to develop for large ϵ in the rare-earth region particularly for lower values of N and Z. Here the barrier extends to much larger distortions where the description in terms of solely ϵ and ϵ_4 is far less satisfactory. Then the more general parameterisation suggested by other groups working in this field along similar lines is clearly needed [9].

The superheavy element region. Also in the superheavy region a similar situation occurs for $\epsilon \approx 0.6$, which is there the position of the second barrier peak. In this case the orbitals [505 9/2], [503 7/2], [503 5/2], [501 3/2] and [501 1/2] and their [61×] mates are involved (see fig. 2). Actually the corresponding computer calculations of the total potential-energy surfaces give there an instability towards asymmetric distortions of the second fission barrier first for $N \geqslant 184$ [10]. In this case the second peak (not always very distinctly developed in this region) is situated at a smaller ϵ-value, $\epsilon \approx 0.6$, than in the actinide cases and the reduction due to asymmetry is also smaller and for this reason much less important for the fission half lives than in the actinide region.

Thus in the superheavy case for $N \geqslant 184$ we seem to leave the asymmetry-favouring orbitals unpopulated at the distortion that there corresponds to the secondary peak. On the other hand, a shift downward of some of the particular asymmetry-driving orbitals e.g. due to the replacement of the $\boldsymbol{l}\cdot\boldsymbol{s}$ term with a term proportional to $(\boldsymbol{s}\cdot\boldsymbol{p}\times\nabla V)$ may make the secondary barrier of the superheavy elements somewhat more sensitive to the mentioned asymmetry effect.

Nuclei on the r-process path. From fig. 1 we draw the important conclusion that very neutron rich actinide elements with $N \geqslant 160$ are not expected to show asymmetric instabilities. Thus the neutron rich nuclei on the r-process path are not expected to have their second barrier peaks reduced by asymmetry effects. However, their fission barrier heights are sensitive to the unsufficiently known surface symmetry energy term. Furthermore, the model dependence of the position of the [40×] states may be responsible for some uncertainity of these predictions.

Conclusion. This analysis based on the simple coupling rules of the asymptotic wave functions thus appears to give a simple understanding of and strong support to the conclusions reached in the calculations of ref. [1]. The strong couplings classified as allowed by the asymptotic selection rules involving a few conspicious orbitals are responsible for the entire asymmetry effect. These orbitals are shown to be particularly effective as they correspond to nucleons situated at the very waistline of the nucleus for the large elongations corresponding to the second-barrier peak shapes.

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