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MUON EXCITATION PROBABILITIES IN MUON-INDUCED FISSION OF URANIUM NUCLEI

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A fissioning muonic atom is represented by two point charges. The time-dependent Schrödinger equation for the muon in the field generated by the two separating point charges is studied. Results of the study indicate that the muon excitation probability is of the order of a few percent. Some of the excited states would correspond to orbits around the lighter fragment.

Negative muons are becoming an increasingly useful tool in the study of nuclear fission [1–3]. After a muon is captured by an atom, it will "cascade down" through its "atomic" orbitals. During this process gamma rays are emitted or the nucleus is excited by inverse conversion. The energy released from the transitions to the lowest muon orbital may lead to fission in heavy nuclei. This process is usually denoted prompt fission because of its short time scale compared to delayed fission following the absorption of the muon by the nucleus.

While in muon absorption the muon is transformed in an inverse beta process, in prompt fission the muon will survive the fission process.

A few general remarks are required as to the prompt fission process. Because the muon binding energy decreases with increasing deformation, the muon atom system will have a higher fission barrier than the conventional atomic system (by about 1 MeV [4]). This may, for example, lead to new shape isomers. A study of the fission barrier in muonic systems may also yield important information about the properties of the fissioning nucleus, such as the sequences of shapes assumed by the nucleus on its way to fission, in particular the deformation of the nucleus at the saddle point [4].

After prompt fission the muon in most cases appears to remain orbiting around one of the two fission fragments. If the motion of the nuclear system were infinitely slow, that is adiabatic, the muon would always occupy the orbital of lowest energy and, consequently, follow the heavier fission fragment. In fact experimental results [2] give indications that the muon remains with the heavy fragment with the largest probability. However the motion of the orbiting muon is probably not adiabatic in the barrier penetration process. Nor is this the case in the later stages of the process of fragment repulsion. One therefore expects that there is a nonnegligible probability of exciting the muon to higher orbitals. Some of the latter orbitals would finally lead over to orbit around the lighter fragment. One may separate the problem of calculating the excitation probability of the muon to higher orbitals during the fission process into two separate parts. First, one should calculate the excitation probability during the penetration of the fission barrier. Second, one should calculate the contribution to the excitation probability as the nucleus deforms from a shape close to the saddle point into two separated fragments. After the penetration of the barrier the excitation probability is governed by the time-dependent interaction dH/dt. The latter quantity can be rewritten $(\nabla_{\bar{R}} H) \cdot d\bar{R}/dt$ in terms of the vector $\bar{R}$ between the two fragments. In this quantity the interaction as a function of separation and the velocity history of the system both enter critically.

Let us first consider the motion beyond the scission point. Here the main feature of the evolution of the system may be understood by considering in some model the mutual Coulomb repulsion between the two fragments. Although the velocity of the system is larger after scission than before, the overlap of the muonic wave functions is there very small and the $\langle b|\partial H/\partial t|a\rangle$ transitional term consequently small. Therefore one expects the effect on the excitation probability of the muon from the part of the trajectory preceeding scission to be very important. Because this contribution depends on the velocity of the fissioning system, it should be sensitive to the effect on the motion from vicosity and also to the sequences of shapes assumed by the fissioning nucleus. A very interesting prospect is associated with a possible measurement of the number of muons ultimately following the heavy and light fragments and of their distribution in higher orbitals. By comparing such measurements to the results of very realistic calculations one might be able to learn about the descent from saddle to scission of the fissioning system.

To investigate in an illustrative fashion the fissioning muonic system we employ here only a very simple model. We thus consider the time-dependent Schrödinger equation for a muon in the field of two separating point charges in a basis of two wave-vectors. Following the time-honoured methods used in molecular physics these two basis functions are generated by the linear combination of atomic orbitals (LCAO) method [5]. We have thus two point charges Z_1 and Z_2, respectively, at a distance R with masses A_1 and A_2 (expressed in units of the nucleon mass). The ground state muon wave functions referring to each separate one of the two point charges are denoted ϕ_1 and ϕ_2. To find the eigenstates of the entire system we consider the new linear combinations

$$u_k(r,R)=\sum_{n=1}^{2} C_{kn}(R)\phi_n \qquad (k=1,2)\,,$$

where $\boldsymbol{r}$ refers to the muon position. The coefficients C_{kn} are determined from the minimization of

$$E_k^u=\langle u_k|H|u_k\rangle/\langle u_k|u_k\rangle\,.$$

This yields two solutions u_1 and u_2 which are approximate solutions to the stationary Schrödinger equation

$$H(r,R)\psi_k(r,R)=E_k\psi_k(r,R)\,.$$

For the muonic motion we use in our calculation the model hamiltonian

$$H=\frac{p^2}{2m}-\frac{Z_1e^2}{4\pi\epsilon_0 r_1}-\frac{Z_2e^2}{4\pi\epsilon_0 r_2}=\frac{p^2}{2m}+V(r,R(t))\,.$$

The distances r_1 and r_2 are obviously the distances to the muon from the two point charges Z_1 and Z_2 and p is the muonic momentum. We calculate $|R(t)|$, the distance between the two point charges, by considering the Coulomb repulsion of the two point charges. Boundary conditions for the separating motion are discussed below. The influence of the muon on $|R(t)|$ is neglected. Thus $|R(t)|$ is predetermined, based on a purely classical calculation.

In the time-dependent Schrödinger equation

$$H[R(t);r]\,\psi(r,t)=\mathrm{i}\hbar\,\partial\psi(r,t)/\partial t\,,$$

$R(t)$ is then considered a C-number.

From energy conservation it is easy to find $\mathrm{d}R/\mathrm{d}t$ as a function of $\boldsymbol{R}$ while the explicit relation between $\boldsymbol{R}$ and t is analytically complicated and therefore seldom exhibited in the literature. As a point of curiosity one may cite the solution applicable to two point charges pushed apart by the Coulombic repulsion

$$ta=(R^2-Rd)^{1/2}+\tfrac{1}{2}d\ln\left[\frac{2(R^2-Rd)^{1/2}+2R-d}{d}\right]$$

where

$$a=2Z_1Z_2e^2(A_1+A_2)/4\pi\epsilon_0A_1A_2$$

and d is the value of $|R|$ at $t=0$. The quantities A_1 and A_2 are the masses of the two fragments (expressed in units of the nucleon mass).

Subsequently one makes the Ansatz

$$\psi(r,t)=\sum a_k(t)\psi_k(r,R(t))\times\exp\left[-\frac{\mathrm{i}}{\hbar}\int_0^t E_k(t')\,\mathrm{d}t'\right].$$

The time-dependent Schrödinger equation is then equivalent to a corresponding equation for the coefficients $a_k(t)$ [6]:

$$\dot{a}_k=\sum_{n\neq k}\frac{a_n}{E_k-E_n}\exp\left(\mathrm{i}\int_0^t\frac{E_k-E_n}{\hbar}\,\mathrm{d}t'\right)\times\langle\psi_k|(\nabla_R H)\cdot\mathrm{d}R/\mathrm{d}t|\psi_n\rangle\,. \qquad (1)$$

For $(\nabla_R H)$ we find the following expression

$$(\nabla_R H) = -\frac{e^2 Z_1}{4\pi\epsilon_0}\,\frac{A_2}{(A_1+A_2)}\,\frac{r_1}{r_1^3} - \frac{e^2 Z_2}{4\pi\epsilon_0}\,\frac{A_1}{A_1+A_2}\,\frac{r_2}{r_2^3}\,.$$

We have solved eq. (1) for a_k to desired accuracy for our two-level system by standard numerical methods for a system of linear differential equations. Thus, the accuracy of the calculation is, as discussed previously determined by the Ansatz for the hamiltonian and by the limited space of two levels only (both relatively severe approximations). On the other hand, the resulting time-dependent equations are, in principle, treated exactly. The matrix element $\langle u_k | \partial H/\partial t | u_l \rangle$ is calculated by numerical integration. Some care has to be taken in this integration. For large R-values there are important contributions to the integral from region far apart in space, namely the regions around each point charge. For this reason the numerical integration procedure has to include more meshpoints in these regions than elsewhere.

We have studied our illustrative model for a particular set of parameters that corresponds to fission of ^{240}U. We thus consider the two-point system with $Z_1 = 38, A_1 = 100, Z_2 = 54$ and $A_2 = 140$. We study our time-dependent equations for two sets of initial conditions. The value $R = 11$ fm, at which point we start integrating eq. (1), corresponds to the distance between the centers of mass of two touching spherical nuclei with $Z_1 = 38, A_1 = 100, Z_2 = 54$ and $A_2 = 140$.

First we solve the time-dependent equation with $|a_1| = 1.0$ and $|a_2| = 0.0$. This means we assume that the muon was not excited during the penetration of the barrier, i.e. the tunneling process is assumed to be adiabatic. The resulting solution is shown in fig. 1. The bottom two curves in the figure show the change in energy of the two basis states u_1 and u_2 as functions of the distance R between the two asymmetric point charges. The lower curve in the upper part of the diagram shows the change in $|a_2|^2$, also as a function of R. We see that $|a_2|^2$ fluctuates as a function of R, or time, with an amplitude of about 0.005.

We also solve the time-dependent equation (1) with initial conditions determined in another limit. The two-level occupation amplitudes a_1 and a_2 at the barrier exit point are in this limit calculated in the sudden approximation. For a barrier of 6 MeV the uncertainty relation $\Delta E \cdot \Delta t \geqslant \hbar$ gives a time for the penetration

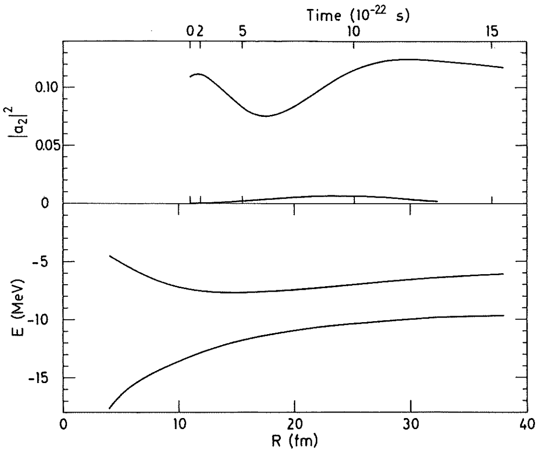

Fig. 1. The lower part of the figure shows the two lowest muon energy levels of a two-center point charge system. The distance between the two point charges is R. For their charge and mass we have $Z_1 = 38, A_1 = 100, Z_2 = 54$ and $A_2 = 140$. The top part of the figure shows the occupation probability of the higher level in our two-level system calculated as a function of R. The integration of the time-dependent Schrödinger equation is started at $R = 11$. The two curves for $|a_2|^2$ refer to the two sets of initial conditions discussed in the text.

of the barrier of about 10^{-22} s which is also approximatively the rotational period of the muon orbital in our point-charge model. In this model we find for the overlap between the muon ground-state wave function and the states at $R = 11$ fm, the point where we start integrating eq. (1), that:

$$(a_1)^2 = |\langle u_1(r, R = 4\ \text{fm}, Z_1 = Z_2 = 46,\ A_1 = A_2 = 120) | u_1(r, R = 11\ \text{fm}, Z_1 = 38,\ Z_2 = 54, A_1 = 100, A_2 = 140)\rangle|^2 = 0.891$$

and

$$(a_2)^2 = |\langle u_1(r, R = 4\ \text{fm}, Z_1 = Z_2 = 46,\ A_1 = A_2 = 120) | u_2(r_1, R = 11\ \text{fm}, Z_1 = 38,\ Z_2 = 54, A_1 = 100, A_2 = 140)\rangle|^2 = 0.058\,.$$

For the muonic ground state orbital we have used the two-center potential described previously, with two equal charges placed 4 fm apart. This choice is made so as to roughly reproduce the ground state quadrupole moment for uranium. From the values of the components of the muon wavefunction at 11 fm it is clear that only half of the probability corresponding to excited states is located in our truncated two-level space. The rest of the probability can be said to lie in higher-lying states. Since we wish to represent all the probability with our two-level system we put somewhat arbitrarily as initial value $a_2^2 = \Sigma_{\upsilon=2}^{\infty} a_\upsilon^2$. Thus as initial values of a_1 and a_2 we have

$$a_1 = 0.94388 + 0 \cdot \mathrm{i}\,, \quad a_2 = 0.33029 + 0 \cdot \mathrm{i}\,.$$

We now proceed to solve the time-dependent Schrödinger equation with this second set of initial conditions. The result is given as the top curve of fig. 1. This curve shows the change in $|a_2|^2$, as a function of the distance R between the two separating point charges. We see that the value of $|a_2|^2$ fluctuates around the value 0.1 with an amplitude of about 0.02. For large values of R the fluctuations disappear and $|a_2|^2$ approaches a final value of about 0.12.

One may ask about the relevance of the present model calculation in a real muonic atom. Since the point charges will pull the muon ground-state wave function closer to the fragment centers compared to a more realistic model, it appears that we have underestimated the overlap between the groundstate wave function at 4 fm with the groundstate wave function at 11 fm and thus *over*estimated our initial value for the excited components of the muon wave function. On the other hand it seems probable that we have *under*estimated the fluctuations in $|a_2|^2$ given by the solution to the time-dependent Schrödinger equation. This is due to the fact that the use of point charges yields too large a value of the denominator $E_k - E_l$ and too small a value of the matrix element $\langle u_k | \partial H/\partial H/\partial t | u_l \rangle$ compared to a more realistic calculation with extended charges. Thus a_k in the time-dependent Schrödinger equation will be underestimated. However, the calculation illustrates some interesting aspects of the fissioning muonic system and should give an order of magnitude estimate of the probability of exciting the muon from the lowest orbital. An excitation probability exceeding a few percent is indicated by the results of the above calculation.

The problem is obviously complicated by other processes not described by the simple two point-charge model. One has, for instance, observed experimentally that after fission the muon has sometimes been captured by an element lighter than any of the two fission fragments [2]. It is therefore valuable in the future to perform a more sophisticated calculation with extended charges and a considerably larger basis. Since the excitation probability is sensitive to the velocity and the path followed in nuclear deformation space in the initial stages of the separation process, such a calculation and a comparison of the results with experimental data may yield information on nuclear viscosity and the shape of the scission configuration.

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