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THEORETICAL PREDICTIONS OF FISSION HALF-LIVES OF ELEMENTS WITH Z BETWEEN 92 AND 106 [†]

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Abstract: Ground-state and isomeric fission half-lives are studied for nuclei with Z between 92 and 106. Realistic fission-barrier potentials are established on the basis of a modified liquid-drop model and the modified-oscillator single-particle model, including the effects of reflection asymmetry and axial asymmetry. These barriers, in combination with available experimental half-lives, are used to determine a smooth fission inertial-mass function with only one adjustable parameter. This semi-empirical inertia reproduces the normal fission half-lives in this region to within a factor of 25 on the average. Calculations suggest that the longest-lived doubly even isotope of the element 106 occurs for $N = 152$ with a half-life of around 100 μsec. Furthermore, the hindrance associated with fission of odd-A nuclei is studied for a few selected cases. A particularly large hindrance factor is obtained for $N = 157$ for Fm, No and $Z = 104$ and attributed to the $[615\frac{9}{2}^+]$ neutron orbital. The abrupt drop in half-lives from ^{256}Fm to ^{258}Fm is also discussed and interpreted as the decline of the second-barrier peak below the ground-state level.

1. Introduction

The ability to calculate fission half-lives is essential for theoretical predictions concerning the stability and synthesis of heavy nuclei. However, until recently, calculations [1,2]) have not been very successful in reproducing the known half-lives, and in particular this has been the case when theoretical inertial masses have been used. Calculations in ref. [1]), for example, based on cranking-model inertial masses and theoretical barriers without P_3 and γ corrections, overestimated the known half-lives by 10^{15} to 10^{20}. A more recent and extensive study of fission half-lives in the actinide region by Pauli *et al.* [3]) (with the limitation that it does not take the effect of the γ-degree of freedom into account) gives somewhat better agreement. However, in this treatment the barriers are somewhat arbitrarily lowered below what are currently considered to be the experimental values, by a change in the liquid-drop parameters. One can also argue that although the overall reproduction of ground-state

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and isomeric half-lives is good, the trend of the agreement with increasing Z and N appears less promising for an extension into adjacent regions of unknown heavy elements.

In the last few years rather refined calculations of the fission barriers have been carried out taking into account both reflection asymmetric (e.g. P_3 and P_5) degrees of freedom [4-7]) at the second barrier peak and axially asymmetric degrees of freedom [8, 9]) at the first barrier peak. It therefore seems appropriate at the present time to utilize the wealth of experimental information [10]) on fission half-lives to obtain some semi-empirical information on the fission inertial masses. It is our hope by this approach to develop an alternative method for calculating the fission half-lives of heavy and superheavy elements that are not yet observed.

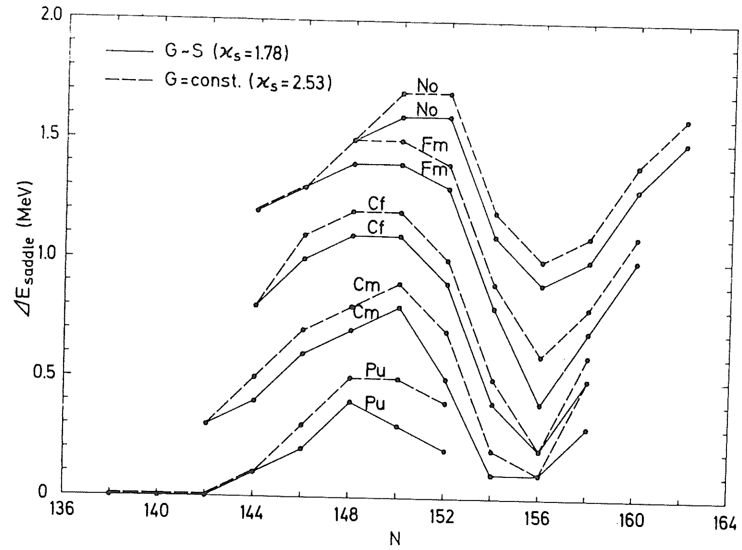

Fig. 1. Effect of γ-distortion on the energy of the first-barrier peak. The figure shows the decrease in energy due to the γ-type of axial asymmetry. The deformation coordinate ε_4 was assumed unchanged.

2. Fission barrier calculations

The theoretical fission barriers used are taken from ref. [7]) when available, or otherwise calculated as described there and in ref. [11]) (the latter reference describing calculations of barriers for odd-even nuclei). Subsequently they have been modified to take into account the effects of the γ-degree of freedom as given in ref. [8]) as well as a readjusted surface-energy term in the liquid-drop energy part of the potential energy as described below.

The fission barrier extrema in ref. [7]) are determined from potential-energy surfaces calculated according to the macroscopic-microscopic method, also denoted the shell-correction method, as developed by Strutinsky [12]). In the calculations P_2 and P_4 distortions and P_3 and P_5 distortions, the latter representing reflection asymmetry, t with increasing Z and N egions of unknown heavy

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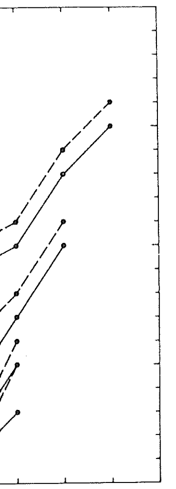

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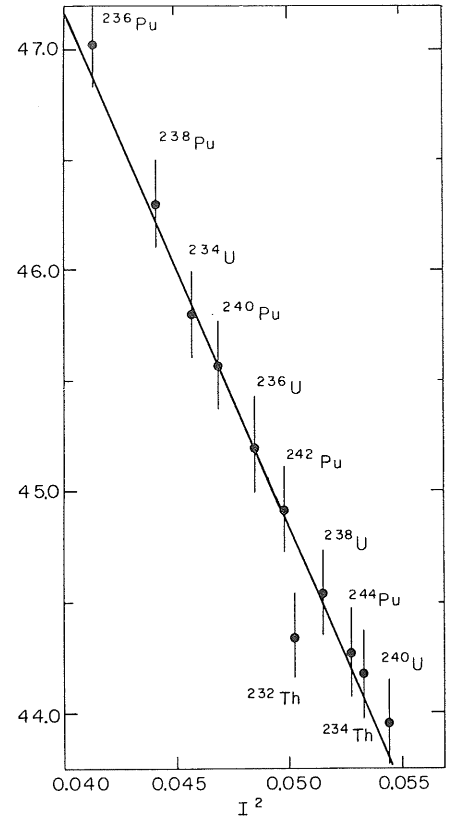

Fig. 2. Value of the parameter ζ of eq. (1), determined for G = constant from eleven experimental values of the second-barrier heights. The straight line represents a least-square fit of the expression $(2a_2/c_3)(1-\kappa_S I^2)$ to these data points. The error bars correspond to an uncertainty of 0.5 MeV in either theoretical or experimental values for the second-barrier peak.

TABLE 1

Semi-empirically determined values of the liquid-drop parameter ζ

Isotope	ζ	E_B^{LD}
^{232}Th	44.35	4.57
^{234}Th	44.17	4.86
^{234}U	45.80	4.55
^{236}U	45.20	4.26
^{238}U	44.54	3.88
^{240}U	43.96	3.62
^{236}Pu	47.02	4.22
^{238}Pu	46.30	3.81
^{240}Pu	45.57	3.40
^{242}Pu	44.92	3.09
^{244}Pu	44.27	2.81

Column one lists the nuclides for which second-barrier data are available. Column two gives the value of ζ that fits the empirically determined barrier height once the theoretical shell correction has been subtracted out. Column three subsequently lists the liquid-drop barrier heights that correspond to the determined ζ-values.

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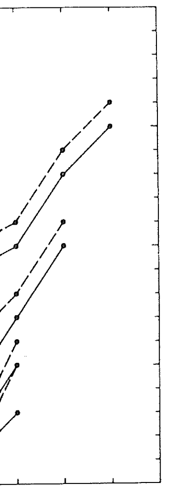

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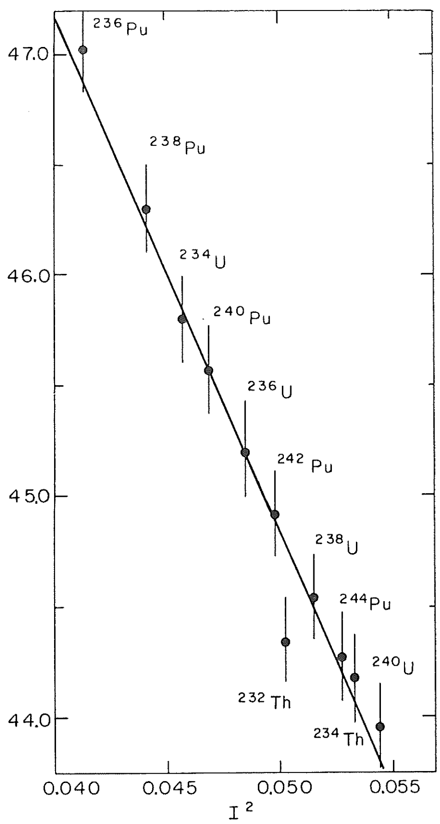

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were considered. In ref. [8]) the liquid-drop model according to Myers and Swiatecki [13]) was used to describe the macroscopic part. The shell correction (microscopic part) was calculated with a modified oscillator single-particle potential. From these calculations it was found that while experimental values of the barrier height were fairly constant as a function of N for fixed Z, the theoretical values increased systematically as a function of N. It has been shown by Larsson *et al.* [8]) and by Götz *et al.* [9]) that it is possible to greatly improve the agreement between theoretical and experimental values at the first barrier peak by the inclusion of the γ-degree of freedom. In the calculations below we have used γ-corrections calculated as in ref. [8]). They are exhibited in fig. 1.

Furthermore Pauli and Ledergerber [6]) have suggested a method to redetermine the liquid-drop parameters from a fit to empirical second-barrier heights while taking into account calculated shell corrections at the second peak. They found that such a redetermination brought their theoretical second-barrier peaks into very good agreement with experiment. Following them we write

$$\begin{aligned}\Delta E_{\mathrm{LD}} &= a_2(1-\kappa_{\mathrm{S}} I^2)A^{\frac{2}{3}}(B_{\mathrm{S}}(\mathrm{def})-1)+\frac{3}{5}\frac{Z^2e^2}{R_0}(B_{\mathrm{C}}(\mathrm{def})-1)\\ &= c_3\frac{Z^2}{A^{\frac{1}{3}}}\left\{\zeta\frac{A}{2Z^2}(B_{\mathrm{S}}(\mathrm{def})-1)+B_{\mathrm{C}}(\mathrm{def})-1\right\},\end{aligned} \tag{1}$$

where $c_3 = \frac{3}{5}e^2/r_0$ and $\zeta = (2a_2/c_3)\ (1-\kappa_{\mathrm{S}} I^2)$. As in ref. [6]) we choose *a priori* $c_3 = 0.720$ MeV. We now take the theoretical values for the second-barrier peak tabulated in ref. [7]), subtract the contributions of the Myers-Swiatecki liquid-drop term and replace it with the expression (1) above. In eq. (1) the surface and Coulomb shape factors B_{S} and B_{C} are determined by the nuclear shape alone and the only unknown quantity is ζ (with c_3 fixed). By requiring experimental and theoretical values for the second barriers to coincide one determines a value of ζ for each one of a number of nuclei. The calculated ζ-values are displayed in fig. 2 as a function of I^2. They are based on a zero-point energy of 0.5 MeV and a pairing strength G that is independent of distortion. These ζ-values and the corresponding liquid-drop barriers are listed in table 1.

According to eq. (1) ζ should be a linear function of I^2, which is seen to be well fulfilled in the region studied. This gives strong support to the method used. The parameters κ_{S} and $2a_2/c_3$ are determined from a least-square fit:

$$\kappa_{\mathrm{S}} = 4.1678, \qquad 2a_2/c_3 = 56.6601. \tag{2}$$

For the alternative case of a pairing strength proportional to the nuclear surface area we obtain

$$\kappa_{\mathrm{S}} = 4.0239, \quad 2a_2/c_3 = 57.9913. \tag{3}$$

The error bars in fig. 2 correspond to an uncertainty of 0.5 MeV in either the theoretical or experimental values for the second barrier peak. In the above calculations we have made the approximation that the distortions of the fission barrier extrema are not changed by the refit of the surface-energy term. An estimate shows that this does not affect the results by more than a few hundred keV.

After these modifications the theoretical barriers are in very good agreement with experiment except for the second barrier of ^{232}Th and the first barrier of light Th and U isotopes. It should be pointed out that the modified liquid-drop formula has a limited applicability. Thus the readjusted values of $2a_2/c_3$ and κ_S will obviously not give satisfactory ground-state masses if the other liquid-drop parameters are kept unchanged. To determine a consistent set of liquid-drop parameters one must also simultaneously make a fit to the known nuclear masses. However, even if masses were taken into account in the liquid-drop parameter fit, it seems apparent that one will still obtain a larger value (3 to 4) for the surface symmetry coefficient κ_S than the originally employed value of 1.78 (which was *a priori* assumed equal to the volume symmetry coefficient κ_V). This is consistent with the indication from a later study by Myers and Swiatecki [15]) that κ_S should have a value of the order of 4 to 5. As emphasized by Wilets [14]) one should notice the great importance of the κ_S value for the problem of the possible synthesis of heavy elements along various n-capture paths. Calculations using the recently developed droplet model of Myers and Swiatecki [15]) for the macroscopic part of the potential energy with a set of parameters determined in January 1973 are now in progress. The parameters of that model, which among other refinements treats the surface-symmetry effect in more consistent ways than does the liquid-drop model, correspond to an effective κ_S in the Pu region of about 3. Preliminary results indicate that both ground-state masses and fission barriers for elements in the actinide region are simultaneously reproduced fairly satisfactorily in the droplet model.

3. Semi-empirical fission inertial masses

With the potential-energy surfaces calculated we turn now to the problem of determining the inertial masses associated with the spontaneous-fission process. Several theoretical calculations of the fission inertial masses have been carried out (see for example refs. [2, 3, 14, 16–19])), but since the detailed and unrenormalized applications appear to result in rather erroneous half-lives we shall here employ a semi-empirical approach. Thus we shall attempt to determine some effective fission inertial-mass function from the available experimental half-lives in combination with the theoretical barriers, the latter agreeing remarkably well with empirical data. One may hope, as discussed in the previous section, to obtain an inertial function with a simple distortion dependence from which the main trends of the known half-lives can be reproduced. This would provide us with a basis for what appears as a relatively reliable extension to adjacent regions of nuclei. Since the procedure followed has been described in greater detail elsewhere [20]) we shall here only describe the method briefly.

In the actinide region the fission barrier has usually a first and a second minimum, (I and II, respectively), separated from each other by the first barrier (A) and from the exit region (X) by the second barrier (B), and one may characterize the barrier by the corresponding four extremum points (I, A, II and B) together with a fifth point (X) in the exit region (which latter point we have chosen to lie approximately on the liquid-drop fission path). These five characteristic points are obtained in the $(\varepsilon, \varepsilon_3, \varepsilon_4, \varepsilon_5, \gamma)$-space and then projected onto the ε-axis. The fission-barrier potential curve is subsequently generated from these points by a simple spline method. This approach has the advantage that possible corrections to an extremum-point region, such as P_3-P_5 or γ corrections, can be taken reasonably well into account by just correcting the corresponding characteristic extremum-energy point.

The choice of the actual fission-path coordinate proves to be rather important. The ε-coordinate has a singular behavior for large distortions and thus the corresponding metric does not seem very well suited for an intuitive grasp of the fission problem. Instead, we choose the "equivalent center-of-mass separation", r. The transformation from ε to r is simply given by

$$r = \tfrac{3}{4}R_0 \left(\frac{1+\frac{1}{3}\varepsilon}{1-\frac{2}{3}\varepsilon}\right)^{\frac{2}{3}}, \qquad R_0 = r_0 A^{\frac{1}{3}}. \tag{4}$$

This formula is strictly valid only for purely ellipsoidal shapes and equal-mass fragments, but we shall assume it to hold for more general distortions. The coordinate r has a more appealing asymptotic behaviour. Comparing a barrier plot in ε versus one in r, the transformation gives rise to a stretching of the outer parts of the barrier in terms of r compared to ε. One might argue that ideally the best choice of metric is one in which the inertial mass is independent of the distortion, a description somewhat intermediate between the ε and r representations.

Hydrodynamical calculations [19, 21]) of the fission inertia [in terms of the coordinate r and under the assumption of "y-family" shapes [22])] yields an inertial function which decreases from the spherical values of $\frac{32}{15}\mu$ to the asymptotic value μ for two separated fragments (cf. fig. 3), μ being the reduced mass of the final two-fragment system. Since these calculations are based on the assumption of irrotational flow of the nuclear matter, they underestimate severely the true inertial mass. More realistic indications of the absolute magnitude of the inertial mass are provided by microscopic calculations [2, 3, 16, 23, 24]). Such calculations yield a fluctuating inertial mass reflecting the specific single-particle structure of the particular nucleus under consideration. For the present first approach, however, we have confined ourselves to consider only a smooth inertial function. It also appears probable that the fissioning nucleus in its motion through deformation space circumvents the higher peaks of the inertia tensor. As can be seen from fig. 3, the microscopic calculations † give a clear indication of the general behaviour of the fission inertial mass: It is always larger than the irrota-

† These results were kindly communicated to us by Krumlinde [23]) and Sobiczewski [24]).

tional mass, but its gross behaviour exhibits the same type of decrease with r. The cranking formula values [23]) with the pairing matrix element G = constant lie far above the semi-empirical values while the quasi-self-consistent expressions [24]) based on a QQ interaction yield a better agreement. In particular this is true for the $G \propto S$ variant of the calculations.

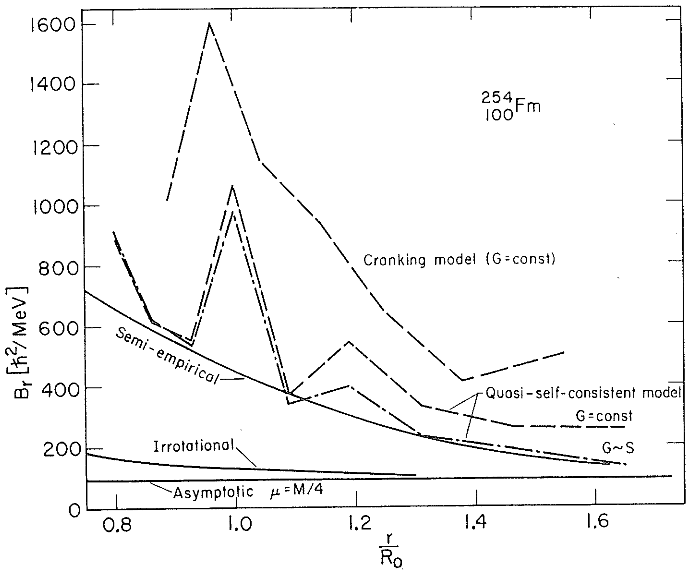

Fig. 3. Comparison of various inertial-mass functions B_r (here shown for ^{254}Fm). The lower curve represents the irrotational-flow calculation [22]), while the kinked upper curves correspond to various microscopic models: Upper dashed: Cranking model, G = constant [ref. [23])]. Lower dashed: Quasi-self-consistent model, G = constant [ref. [29])]. Dot-dashed: Quasi-self-consistent model, $G \propto S$ [ref. [29])]. The smooth curve in between is the determined best one-parameter semi-empirical inertial-mass function (corresponds to $k = 6.5$ in eq. (5)).

These calculations lead us to consider a trial inertial function of the following type

$$B_r = B_r^{\text{rigid}} + k(B_r^{\text{irrot}} - B_r^{\text{rigid}}), \tag{5}$$

where $B_r^{\text{rigid}} = \mu$ is the mass corresponding to a rigid separation of the two fragments, and B_r^{irrot} is the mass corresponding to irrotational flow during the fission process. Thus k is an adjustable parameter describing the contribution to the inertial mass from the internal nuclear motion, k being unity for purely irrotational flow. As mentioned above, we expect from the microscopic calculations k to be considerably larger than that. This inertial function is of the same type as was used by Nix *et al.* [19]). For simplicity we shall here assume equal-mass fragments and furthermore approximate the difference multiplying k by an exponential. The explicit form of B_r thus becomes

$$B_r = \tfrac{1}{4}M[1+k\tfrac{17}{15}e^{-(r-\frac{3}{4}R_0)/d}]. \tag{6}$$

Here M is the mass of the fissioning nucleus and accounts for the general scaling property of the inertial mass. The fall-off parameter d is taken to be that of the irrotational inertia, $d = R_0/2.452$.

4. Fission half-lives

4.1. DOUBLY EVEN NUCLEI

The above trial inertial function, with only one adjustable parameter k, is used in connection with the established fission barriers to fit optimally the spontaneous fission half-lives for all the actinide nuclei (see figs. 4 and 5). From a minimization of the mean logarithmic deviation of the calculated half-lives from the experimental values the parameter k is found to equal 6.5. For this value of k the experimental half-lives are reproduced to within a factor of 25 on the average. Considering the span in half-lives, stretching over 30 decades, we find this parametric fit satisfactory for the present simple approach. We also believe a basis is established for a rather reliable half-life estimate in the close-lying mass regions. In particular, the longest-lived doubly even isotope of element 106 is predicted to occur for $N = 152$ with a half-life around 100 μsec. The prediction for odd-N isotopes of element 106 is discussed in the next section.

The fast fall-off with N of the Fm isotope half-lives (fig. 4) also deserves some comments. Thus between ^{256}Fm and ^{258}Fm there is a shortening in half-lives by a factor of almost 10^8. Theoretically the same fall-off factor occurs instead between ^{258}Fm and ^{260}Fm. The mechanism behind is apparent from fig. 6. Thus for ^{258}Fm the second minimum as well as the second peak remain above the ground-state energy marked by a dashed line (assumed equal to the ground-state potential-energy minimum plus a 0.5 MeV zero-point β-vibrational energy). For ^{260}Fm, on the other hand only the first peak rises above the dashed line, leading to a radical diminishing of the WKB integral and reflected in the rapid fall-off in half-life. Empirically this transition appears to occur between ^{256}Fm and ^{258}Fm.

A special problem is constituted by the shape-isomeric nuclei. This special group of nuclei was not included in the sample employed when fitting the inertial-mass function. As is seen from fig. 7, the obtained semi-empirical inertial function yields isomeric half-lives being too long by six orders of magnitude on the average. However, the ε_4 degree of freedom is expected to have a relatively large influence on the isomeric fission. In fig. 7 we have displayed the isomeric half-lives when the ε_4 dependence of the coordinate r is taken into account. It is seen that indeed this brings the calculated values into much better agreement with experiment. It should be added that a consistent inclusion of this effect does not appreciably change the good overall fit to the ground-state half-lives.

The large deviations for the U isotopes probably reflect the complicated structure of the second-barrier region as found in the modified-oscillator model [7]. The possible experimental consequences of this structure were first pointed out in ref. [25].

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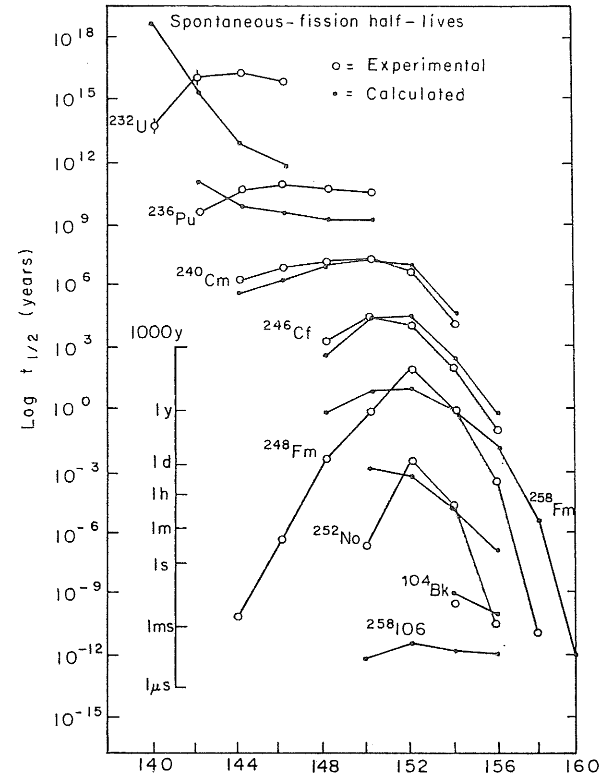

Fig. 4. Spontaneous-fission half-lives. Full circles: experimental values [10]). Open circles: calculated values with the determined semi-empirical inertia shown in fig. 3. The mean logarithmic deviation is 1.4. Also half-lives predicted for the element 106 are shown.

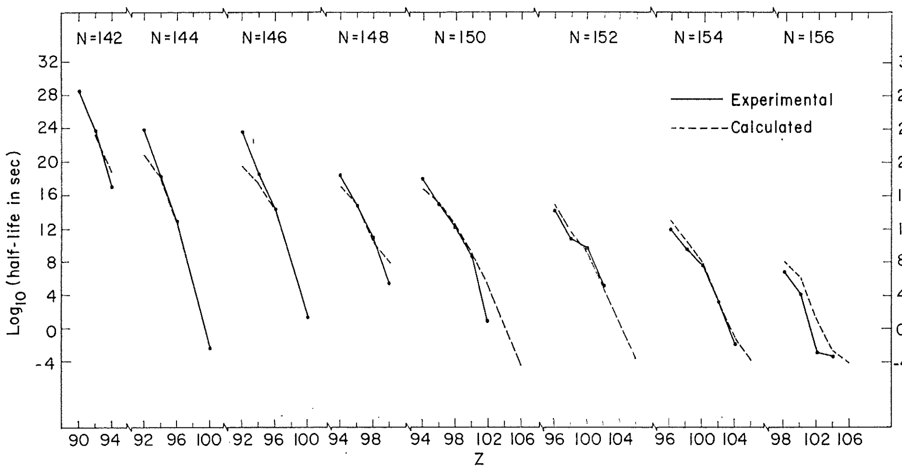

Fig. 5. Experimental and calculated spontaneous fission half-lives as a function of proton number Z for given values of neutron number N.

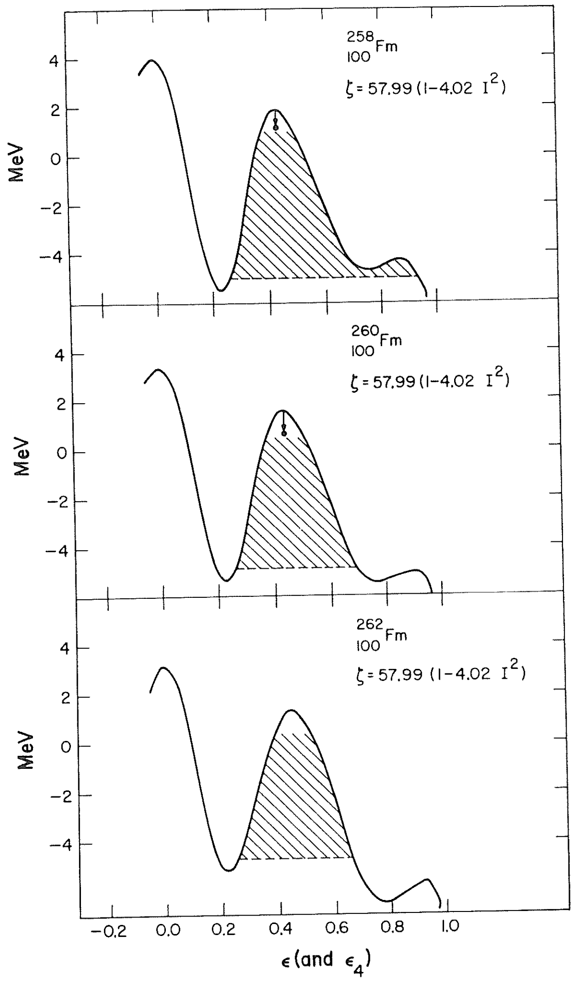

Fig. 6. Fission barrier for heavy Fm isotopes. Beyond ^{258}Fm the second peak and second minimum are below the ground state, leading to a drastic decrease in the fission half-lives.

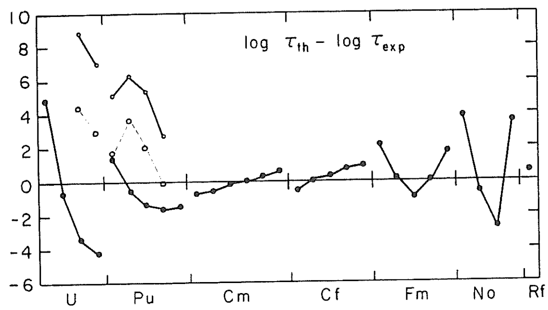

Fig. 7. Deviations of calculated half-lives from experimental values. In addition to the normal half-lives (full circles) also the results for some isomeric states are shown (open circles). The broken lines connect results obtained by including the effect of ε_4 on the coordinate r, while all other points are calculated without this refinement.

In addition to this, the parameterization employed here may be somewhat insufficient for the rather extended barriers for these nuclei.

4.2. ODD-A NUCLEI

The odd-A nuclei are found to have considerably prolonged half-lives (fig. 8) relative to their doubly even neighbours. In fig. 9 we have plotted the logarithm of the relative hindrance factor associated with odd proton and neutron number, respec-

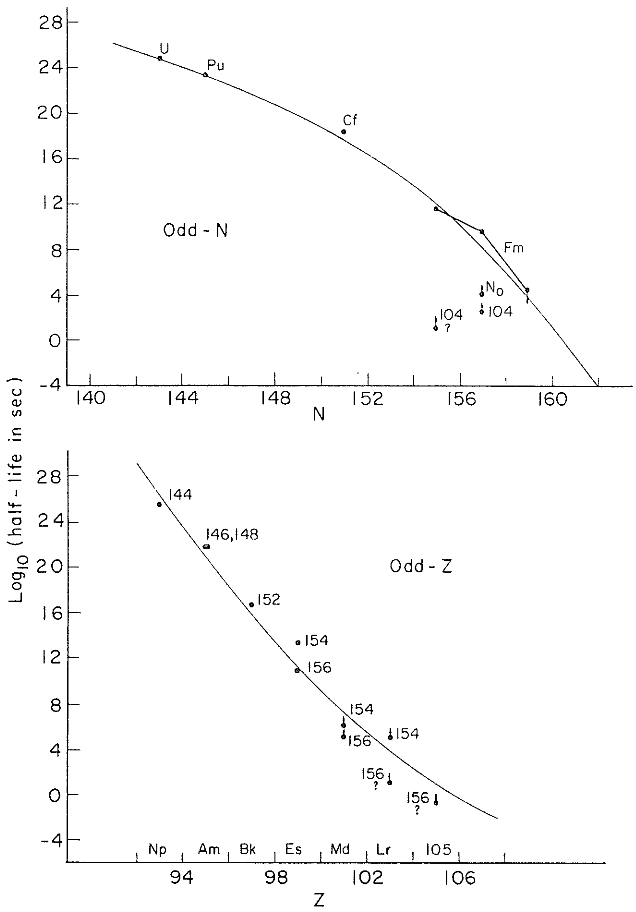

Fig. 8. The spontaneous fission half-lives of odd-N and odd-Z nuclei are plotted as a function of Z and N, respectively. The light line is drawn roughly through the data points to show the general decrease in half-lives with mass number. It is *not* a calculated curve.

tively. In several of the cases the ground-state intrinsic orbital is known and the assignment is then shown in the figure. The hindrance factor is typically of the order of 10^5 but varies in magnitude between 10 and 10^{10}. Particularly large hindrance factors appear to be associated with the $[734\frac{9}{2}]$ and the $N = 157$ orbital, which latter we surmise to be [615$\frac{9}{2}$]. The assignment is unclear since for the calculated ground-state deformation of $\varepsilon = 0.23$ there are several orbitals available close to each other above $N = 152$ (see fig. 10). For the distortion of $\varepsilon = 0.23$ actually $\frac{9}{2}^+$ appears first as the 161st orbital. The reason that we associate [615$\frac{9}{2}$] with the $N = 157$ ground

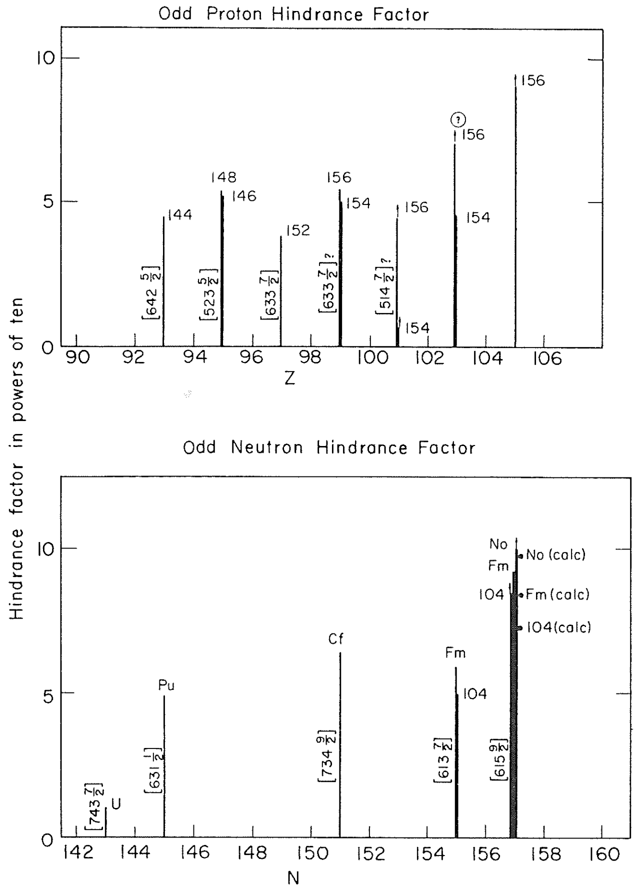

Fig. 9. Spontaneous fission half-life hindrance factors for odd-Z and odd-N nuclei, as obtained by comparing their empirical half-lives with values obtained by interpolation among adjacent doubly even nuclei half-lives. The calculated hindrance factors for $N = 157$ are displayed as dots for comparison.

state with some confidence is the fact that ^{257}Fm is known to decay by an unhindered α-transition to this orbital in ^{253}Cf. In this latter nucleus the orbital assignment is fairly certain. The particular stability at $N = 157$ exhibited for all of the heavy elements, Cf, Fm, No and element 104, was pointed out to us by Seaborg [26]). The relevance of this finding for the production of prospected still heavier elements is obvious and we have been investigating the question of whether $N = 157$ can be expected to yield increased stability also for larger Z-values.

The relative hindrance associated with fission of odd-A elements has been noted for a long time and the effect was first explained by Newton [27] and Wheeler [28] in terms of a "specialization energy". A more quantitative study of this effect in the actinide region was performed by Johansson [29].

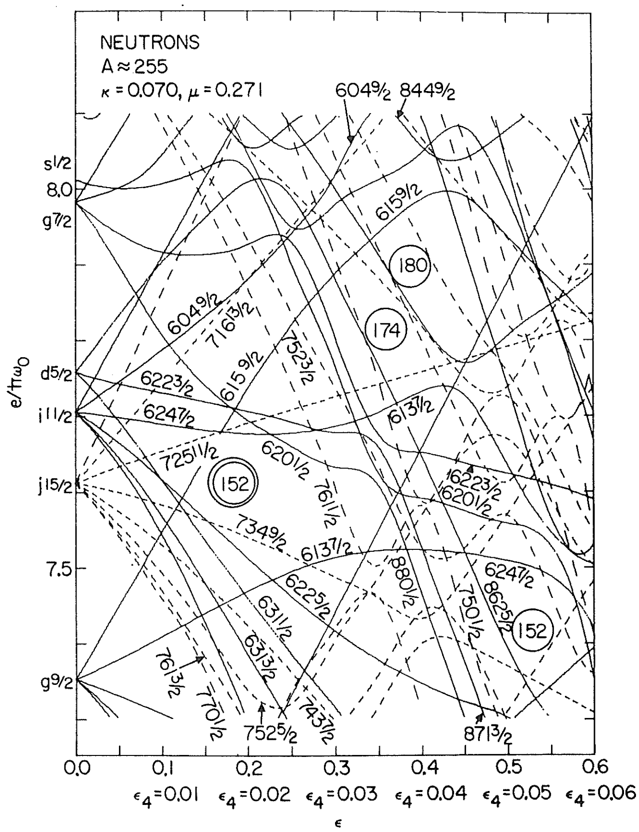

Fig. 10. Single-neutron levels in the region $A \approx 255$ as function of ε. To each value of ε there corresponds a value of ε_4 as indicated below in the figure. The levels are labelled by their asymptotic quantum numbers $[Nn_z\Lambda\Omega]$.

In the odd system the odd particle occupies an orbital of given angular momentum and parity. The quasi-particle energy $\sqrt{(e_\nu-\lambda)^2+\Delta^2}$ in BCS theory represents the difference between the odd system and the interpolated energy based on the doubly even neighbours. For the ground state this quantity is approximately equal to Δ, as the ground-state orbital is the one that occurs in closest vicinity of the Fermi energy λ. For this orbital $(e_\nu-\lambda)^2$ should be negligible compared with Δ^2. For changing deformation the term $(e_\nu-\lambda)^2$ will grow in importance as the e_ν orbital of given Ω and parity may become very distant from the Fermi surface. In the calculations we have accounted for the specialization energy by always choosing the orbital of the given Ω and parity that lies closest to the Fermi surface. This assumption would tend to underestimate the effect. Orbitals that are unique in parity and angular momentum and exhibit a large derivative with respect to the distortion coordinate ε may therefore be good candidates for large specialization energies. This general expectation is brought out quantitatively by the detailed calculations exhibited in

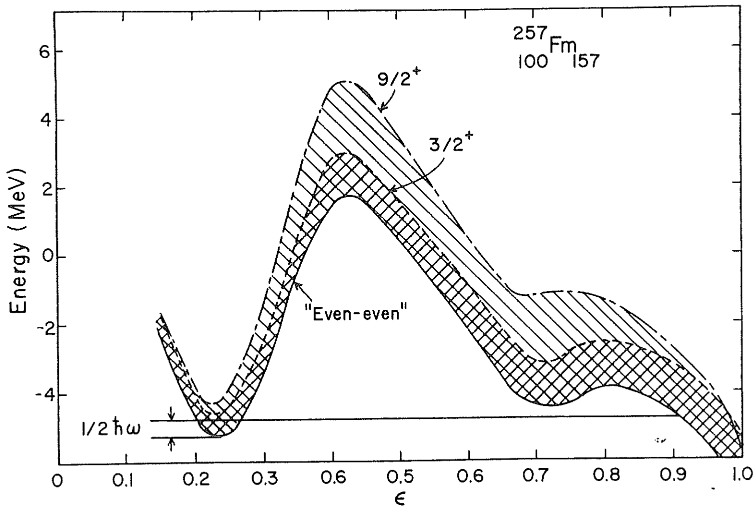

Fig. 11. Fission barriers for ^{257}Fm. The two upper barriers correspond to having the odd particle in the $\frac{9}{2}^{+}$ and $\frac{3}{2}^{+}$ orbitals, respectively, while the lower curve represents the hypothetical even system as obtained by interpolation between ^{256}Fm and ^{258}Fm.

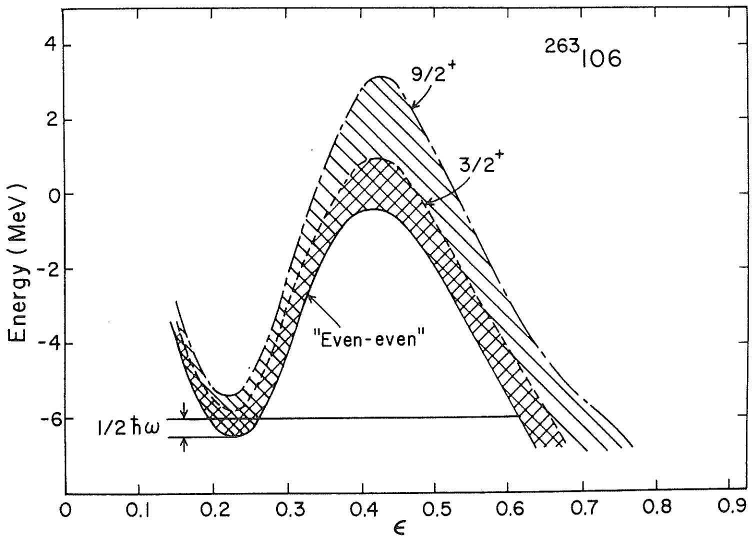

Fig. 12. Same as fig. 11 for ^{263}106. Note that the second barrier is absent even in the case of the odd particle occupying the $\frac{9}{2}^{+}$ orbital.

figs. 11 and 12. There the fission barriers, with only ε and ε_4 taken into account, are calculated for ^{257}Fm and ^{263}106, respectively.

For $N = 157$ the orbital nearest the Fermi surface for $\varepsilon = 0.23$ is $[622\frac{3}{2}]$. The corresponding specialization energy is found to increase the barrier on the average by 0.5–1 MeV. The exclusive $\frac{9}{2}^+$ orbital, on the other hand, gives rise to a specialization energy contribution of up to 2.5 MeV.

Based on these theoretical barriers we have calculated the extra hindrance factor increasing the half-lives of nuclei having $\frac{9}{2}^+$ as the odd-particle orbital under the two simplifying assumptions: Firstly, degrees of freedom in addition to ε and ε_4, namely ε_3 and ε_5 (reflexion asymmetry) and γ (axial asymmetry) can be neglected for the calculation of the effect of the increase in the potential-energy surface. Secondly, the odd-particle influence on the inertial mass involved in the barrier penetration may be neglected. Under these assumptions we obtain hindrance factors for the different $N = 157$ cases as shown in fig. 9 to the right of the experimental bars drawn in the figure for three $N = 157$ nuclei. The agreement appears surprisingly good. Thus for Fm, No and $Z = 104$ the theoretical calculations including only the potential-energy effect appear to reproduce the empirical findings very well. For the element, $Z = 106$, $N = 157$, however, the calculations predict a much smaller hindrance factor ($\approx 10^3$) due to the fact that its barrier has only one peak as seen in fig. 12.

In view of the somewhat unsatisfactory simplifying assumptions made we do not expect more than qualitative agreement for the odd-A effect. Thus Szymanski *et al.* [17]) report in a preliminary calculation on the average a 10–30 % increase in $B_{\varepsilon\varepsilon}$ due to the presence of an odd particle. If this result is substantiated in a more detailed calculation, this effect alone would increase the extra inhibition on the odd case by a factor of 10–10^5 and could be nearly as important an effect as the specialization energy.

The inclusion of axial asymmetry enters the problem in the following way. As shown by Larsson *et al.* [8]), the first barrier for the heavier of the actinides is displaced 10–20° into the γ-plane. At this distortion the quantum number-K, on whose conservation the whole specialization energy concept is based, is only approximately conserved and the single-particle wavefunctions of given K show mixing of components with $K \pm 2$. The hindrance due to "specialization" is therefore weakened.

In addition, due to the inclusion of reflexion asymmetric distortions at the second-barrier peak, we may expect similar impurities from parity mixing. This latter effect is relatively less serious as the mixing occurs first at the second barrier.

The effects last mentioned lead us to believe that in our calculations we have generally somewhat overestimated the specialization energy, although there are some approximations mentioned that work in the opposite direction. Although the agreement with experiments appears good, there is probably room for contributions due to the effect of the odd particle on the mass tensor, which effect we have so far neglected.

5. Conclusions

With the potential-energy surfaces available from calculations it is shown that a large number of empirical ground-state fission half-lives can be reproduced to within one and a half orders of magnitude on the average in terms of a simple smooth inertial mass function with one adjustable parameter. The latter is determined from the half-life data. From this fit it appears that reasonably credible predictions of half-lives of isotopes of $Z = 106$ can be made. The longest-lived doubly even isotopes are predicted to occur for $N = 152$ and are of the order of 100 μsec. The extra hindrance, associated with odd-A elements and encountered empirically for $N = 157$ isotopes of the elements between $Z = 100$ and 104, is found to be of less significance for the $Z = 106$ case, although a hindrance factor of the order of 10^3 is still expected.

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