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# A Method for Evaluation of QRS Shape Features Using a Mathematical Model for the ECG

LEIF SÖRNMO, PER OLA BÖRJESSON, MATS-ERIK NYGÅRDS, AND OLLE PAHLM

Abstract—Automated classification of ECG patterns is facilitated by careful selection of waveform features. This paper presents a method for evaluating the properties of features that describe the shape of a QRS complex. By examining the distances in the feature space for a class of nearly similar complexes, shape transitions which are poorly described by the feature under investigation can be readily identified. To obtain a continuous range of waveforms, which is required by the method, a mathematical model is used to simulate the QRS complexes.

#### I. INTRODUCTION

In automated ECG monitoring, classification of abnormal waveforms is of fundamental interest. In many systems this classification is based on a number of features extracted from the ECG [1]-[3]. Unpleasant results are likely if the features give a bad description of the ECG [4]. Small changes in QRS morphology may cause disproportionately great changes in various features. Thus, it is of great value to obtain a method for evaluating the properties of certain features, indicating their ability or inability to describe the ECG. Such a method will also enable us to compare the properties of simple features

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Fig. 1. Complexes with equal shapes.

against the properties of more complicated ones. This is of great interest since "the ultimate objective is to obtain a pattern space consistent with low dimensionality, retention of sufficient information and enhancement of distance in pattern space . . ."
[5]

A possible way of evaluation is to employ a comprehensive ECG database. From a medical point of view, such an evaluation should always be performed to verify the clinical usefulness of its features. However, to determine the performance of a feature in more detail, it is difficult to organize and manage the large amount of data that is needed. A complement to this approach is to use a mathematical model for simulation of ECG data. If the limitations of this approach are kept in mind, the mathematical model can be a valuable tool in developing well-behaved features. An important advantage when applying an ECG model, as compared to a database, is the ability of varying the QRS morphology in a controllable and continuous way. Representations of the ECG are discussed within various contexts in [3] and [6]-[12].

This paper describes a method which employs simulated ECG data to evaluate features describing the shape of a QRS complex. We define the complexes  $q_0(t)$  and  $q_1(t)$  to have the same shape if and only if

$$q_0(t) = \alpha q_1(\beta t + \tau) \tag{1}$$

for some  $\alpha > 0$ ,  $\beta > 0$ , and  $\tau$ . Thus, according to our definition, the shape of a complex is invariant for a change in amplitude  $(\alpha)$ , width  $(\beta)$ , and reference time  $(\tau)$ . This property is illustrated in Fig. 1 where three different complexes with equal shape are shown. Furthermore, we define a shape feature as any functional  $\Gamma(q(t))$  which is invariant for a change in amplitude, width, and reference time, i.e.,

$$\Gamma(q(t)) = \Gamma(\alpha q(\beta t + \tau)) \quad \alpha, \beta > 0.$$
 (2)

In order to investigate the properties of a shape feature, distances in the feature space between a reference complex and a class of complexes with nearly similar shapes are examined. To determine this class, a measure of dissimilarity is defined which describes the dissimilarity in shape between two QRS complexes as a modified mean-square difference. We expect that for complexes which deviate by the same amount from the reference complex, according to this measure, a negligible variation of the corresponding distances in the feature space should be found. The examination of distances is repeated for reference complexes with different shapes. Finally, the potential of the method is illustrated by the evaluation of a simple shape feature. From the obtained results, the feature is modified to eliminate inherent instabilities.

#### II. MODELING OF QRS COMPLEXES

Earlier investigations [6], [7] indicate that QRS complexes, recorded with various lead configurations from a normal individual, can be well-represented by three orthonormal basis functions, which are essentially mono-, bi-, and triphasic.

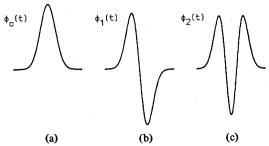


Fig. 2. Basis functions for QRS modeling: (a)  $\phi_0(t)$ , (b)  $\phi_1(t)$ , and (c)  $\phi_2(t)$ .

Thus, a linear combination of three orthonormal basis functions  $\phi_t(t)$ ,

$$q(t) = \sum_{i=0}^{2} a_i \phi_i(t),$$
 (3)

seems appropriate for modeling most QRS complexes seen in normal individuals.

To simplify the model, the most significant basis functions as described in [6], [7] are approximated by the set of orthonormal functions defined by

$$\phi_0(t) = \frac{1}{\sqrt{b\sqrt{\pi}}} e^{-t^2/2b^2} \tag{4a}$$

$$\phi_1(t) = -\frac{\sqrt{2}}{\sqrt{b\sqrt{\pi}}} \frac{t}{b} e^{-t^2/2b^2}$$
 (4b)

$$\phi_2(t) = \frac{1}{\sqrt{2b\sqrt{\pi}}} \left( 2 \cdot \frac{t^2}{b^2} - 1 \right) e^{-t^2/2b^2}.$$
 (4c)

The parameter b determines the width of the QRS complex. These functions, shown in Fig. 2, are known as the three first Hermite functions.

To investigate the validity of the model not only for normal ECG's, but also for abnormal patterns, we have tested the model on a material of 200 QRS waveforms, recorded from 126 nonselected patients in a coronary care unit. In this material, the above functions accounted for an average of 98.6 percent of the total energy of the QRS complexes. The results indicate that with the proposed model, it is possible to represent the essential shape of most normal and abnormal patterns seen in ECG monitoring.

Since we are not interested in amplitude and width changes, the four-dimensional space  $(a_0, a_1, a_2, b)$  can be reduced to a two-dimensional one. We assume that the width b and the energy of the complex  $E_q$ 

$$E_q = \int_{-\infty}^{\infty} q^2(t) dt = \sum_{i=0}^{2} a_i^2$$
 (5)

are constant. For convenience, we make both these constants equal to 1. All possible complexes define a sphere with radius 1 in the  $(a_0, a_1, a_2)$  space. We now introduce spherical coordinates  $(r, \varphi, \psi)$  defined by Fig. 3, i.e.,

$$a_0 = r \cos \varphi \cos \psi$$

$$a_1 = r \sin \varphi \cos \psi$$

$$a_2 = r \sin \psi$$
(6)

where  $0^{\circ} \le \varphi \le 360^{\circ}$  and  $-90^{\circ} \le \psi \le 90^{\circ}$ . Since the energy, and thus also the radius r, is constant for all complexes, only

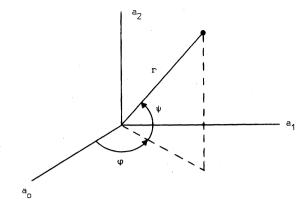


Fig. 3. Definition of spherical coordinates  $(r, \varphi, \psi)$ .

 $\varphi$  and  $\psi$  have to be varied, i.e., we have a two-dimensional parameter space. It should be emphasized that some areas of the sphere are of little interest since they correspond to QRS shapes which are very unlikely to occur in reality.

### III. A MEASURE OF DISSIMILARITY $-\Delta E$

Suppose we have two complexes  $q_0(t)$  and  $q_1(t)$  which are both given by (3), corresponding to some  $(\varphi, \psi)$  and  $(\varphi + \Delta \varphi, \psi + \Delta \psi)$ , respectively. To measure the dissimilarity in shape between  $q_0(t)$  and  $q_1(t)$ , we use the normalized mean-square difference, or in other words, the energy of the difference between the complexes normalized with the energy of  $q_0(t)$ . Since the energy of  $q_0(t)$  was made 1, the measure is defined by

$$\Delta E = \int_{-\infty}^{\infty} |q_1(t) - q_0(t)|^2 dt.$$
 (7)

Despite the fact that the complexes  $q_0(t)$  and  $q_1(t)$  are generated from (3) with the same width parameter b and the same energy, a lower  $\Delta E$  may result from varying the amplitude, width, and reference time of one of the complexes. Hence, we redefine (7) such that the measure of dissimilarity is given by

$$\Delta E = \min_{\alpha, \beta, \tau} \int_{-\infty}^{\infty} |\alpha q_1(\beta t + \tau) - q_0(t)|^2 dt, \tag{8}$$

i.e., we determine those parameter values  $\alpha_M$ ,  $\beta_M$ , and  $\tau_M$  which minimize  $\Delta E$ . To do this, we take the partial derivatives of  $\Delta E$  with respect to each parameter and set these derivatives equal to zero. Since the integrands of  $\Delta E$  and its derivatives are continuous functions, the differentiation can be performed without problems. Another way to reduce the influence of the amplitude, width, and reference time is to normalize both  $q_0(t)$  and  $q_1(t)$  with respect to  $\alpha$ ,  $\beta$ , and  $\tau$  [13].

## IV. EUCLIDEAN DISTANCES FOR A CLASS OF NEARLY SIMILAR COMPLEXES

In order to describe the properties of a shape feature  $\Gamma$ , we examine the distances in the feature space for a class of complexes with nearly similar shape. It is assumed that  $\Gamma$  is either a scalar or an n-dimensional vector where each component of  $\Gamma = (\Gamma_1, \cdots, \Gamma_i, \cdots, \Gamma_n)$  is defined over an interval of certain length  $c_i \leq \Gamma_i \leq c_i^+$  where  $|c_i^-| \leq \infty$  and  $|c_i^+| \leq \infty$ . Let  $q_0$  be a reference complex located on the sphere, say  $q_0 = 1$ 

Let  $q_0$  be a reference complex located on the sphere, say  $q_0 = q(\varphi_0, \psi_0)$ . Now we want to determine a class of complexes  $\{q_j\}_{j=1}^K$  in which all  $q_j$  have the same degree of dissimilarity compared to  $q_0$ :

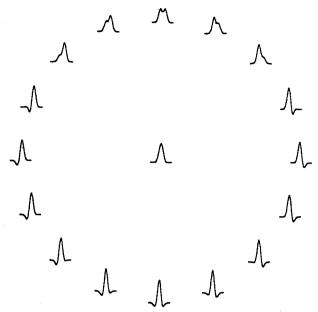


Fig. 4. A class of complexes nearly similar in shape to the reference complex plotted in the center,  $(\varphi, \psi) = (0^{\circ}, 0^{\circ})$ . The constant C, which defines the dissimilarity, is given the value  $4 \cdot 10^{-2}$ .

$$\Delta E(q_0, q_i) = C \qquad j = 1, 2, \cdots, K. \tag{9}$$

This is possible if the constant C is chosen sufficiently small and positive. Thus, in each direction  $\theta_j$  on the sphere from  $q_0$ , there exists a  $q_j$  at a distance  $\delta(\theta_j)$  (measured on the surface of the sphere). We let  $\theta_j = 2\pi/K \cdot j$  and for each  $\theta_j$  determine the smallest distance  $\delta(\theta_j) > 0$ . [Thus,  $q_j$  corresponds to some complex located on the sphere  $(\varphi_0 + \Delta \varphi_j, \psi_0 + \Delta \psi_j)$  and  $(\Delta \varphi_j, \Delta \psi_j) \ddagger (0,0)$ .]

We now proceed with observations in the feature space for the class  $\{q_j\}$  which corresponds to the reference complex  $q(\varphi_0, \psi_0)$ . The Euclidean distance  $d_j(\varphi_0, \psi_0, \theta_j)$  can be computed between the two points in the feature space

$$d_i(\varphi_0, \psi_0, \theta_i) = ||\Gamma(q_i) - \Gamma(q_0)|| \qquad j = 1, 2, \cdots, K \quad (10)$$

where the notation  $\Gamma(q_j)$  denotes that the shape feature is computed for the complex  $q_j$ . In order to obtain a description of the properties of  $\Gamma$  for complexes spread over the entire sphere, the class  $\{q_j\}$  may be determined for reference complexes  $q(\varphi_0, \psi_0)$  where  $0^\circ \leqslant \varphi_0 \leqslant 360^\circ$  and  $-90^\circ \leqslant \psi_0 \leqslant 90^\circ$ . However, due to the above choice of the QRS model and  $\Delta E$ , it is sufficient to consider the area  $0^\circ \leqslant \varphi_0 \leqslant 90^\circ$  and  $-90^\circ \leqslant \psi_0 \leqslant 90^\circ$ . Classes outside this area are obtained by one of the three reflections: change of polarity, and/or reversion of the time scale.

Since  $\Gamma$  is to be used for classification purposes, it is of interest to study the behavior of  $\Gamma$  when representing complexes with minor variation in shape. By choosing the constant C in (9) such that the variation within the class is of the same order as the beat-to-beat variation for a QRS complex, information may be gained about this behavior. Fig. 4 shows a choice of the class  $\{q_j\}$  where the reference complex is plotted in the center

To obtain a description of the distances  $d_j(\varphi_0, \psi_0, \theta_j)$  for a class, we use the mean [14]

$$m(\varphi_0, \psi_0) = \frac{1}{K} \sum_{j=1}^{K} d_j(\varphi_0, \psi_0, \theta_j)$$
 (11)

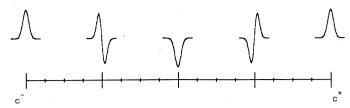


Fig. 5. Transitions between mono- and biphasic QRS complexes. Note that  $c^-$  and  $c^+$  "coincide."

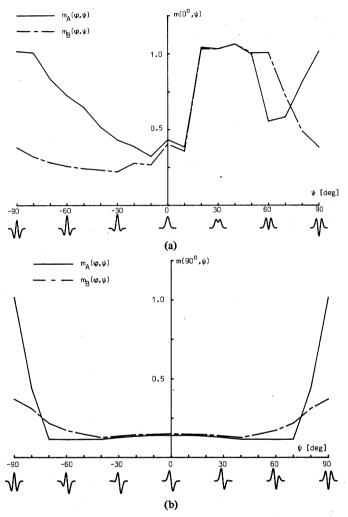


Fig. 6. The mean Euclidean distance  $m(\varphi, \psi)$  calculated for  $\Gamma_A$  and  $\Gamma_B$ . (a)  $\varphi = 0^\circ$ . (b)  $\varphi = 90^\circ$ .

and the within-class scatter

$$S^{2}(\varphi_{0}, \psi_{0}) = \frac{1}{K-1} \sum_{j=1}^{K} (d_{j}(\varphi_{0}, \psi_{0}, \theta_{j}) - m(\varphi_{0}, \psi_{0}))^{2}.$$
(12)

Now, since the shape of each complex in the class in a sense,  $\Delta E$ , deviates by the same amount from  $q_0$ , we desire that for  $\Gamma$  the distance  $d_i(\varphi_0, \psi_0, \theta_i)$  is such that

$$m(\varphi_0, \psi_0) \equiv m \quad \forall \varphi_0, \psi_0$$
 (13a)

and

$$S(\varphi_0, \psi_0) \equiv 0 \quad \forall \varphi_0, \psi_0. \tag{13b}$$

The value of m in (13a) depends on the choice of C, i.e., the dissimilarity in shape within the class. If  $m(\varphi_0, \psi_0)$  varies for

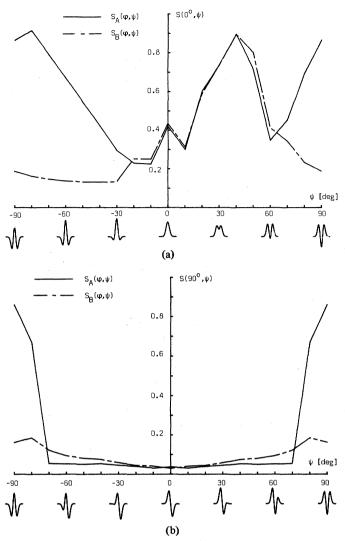
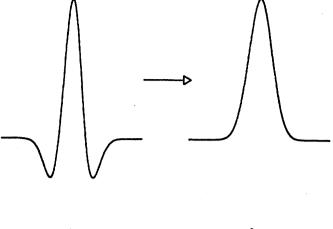


Fig. 7. The within-class scatter  $S(\varphi, \psi)$  calculated for  $\Gamma_A$  and  $\Gamma_B$ . (a)  $\varphi = 0^{\circ}$ . (b)  $\varphi = 90^{\circ}$ .

different  $\varphi_0$  and  $\psi_0$ , this variation should be small compared to the longest possible distance  $d_{\max}$  in the *n*-dimensional feature space, where  $d_{\max} = || c^+ - c^- ||$ . Furthermore, if the mean  $m(\varphi_0, \psi_0)$  or even some  $d_j(\varphi_0, \psi_0, \theta_j)$  is equal to zero, there exist at least two complexes with different shapes which cannot be separated by  $\Gamma$ . A large scatter  $S(\varphi_0, \psi_0) \gg 0$  reveals that for small changes in QRS shape, disproportionately great changes can occur in  $\Gamma$ .

# V. EVALUATION OF A ONE-DIMENSIONAL SHAPE FEATURE—AN EXAMPLE

To illustrate the use of the method, we will describe and evaluate a feature with a simple structure, representing the QRS shape with a scalar. In the search for such a one-dimensional representation, certain attributes of the shape must be treated as being of minor importance. Otherwise, it appears almost impossible to gather all relevant information about the shape in a scalar. To overcome these difficulties, we formulate the problem as one of finding a representation which is able to describe the transition between mono- and biphasic complexes (see Fig. 5). Thus, in a sense, the asymmetric component of the complex forms the attribute to be translated into a shape feature. For example, such transitions are simulated by linear combinations of the basis functions in (4a), (4b). The Euclidean distance in this one-dimensional feature space is given by



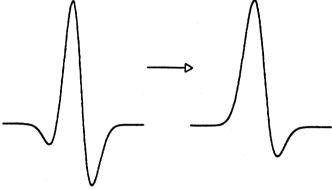


Fig. 8. Examples of transformations which preserve the asymmetric component of the *ORS* complexes.

$$d_j(\varphi_0, \psi_0, \theta_j) = |\Gamma(q_j) - \Gamma(q_0)| \tag{14}$$

where  $c^- \le \Gamma \le c^+$ . Since the points  $c^-$  and  $c^+$  "coincide" (see Fig. 5), the longest possible distance is  $d_{\max} = (c^+ - c^-)/2$ . Then, if (14) yields a distance greater than  $d_{\max}$ , it is modified so that

$$d_i(\varphi_0, \psi_0, \theta_i) = 2d_{\max} - d_i(\varphi_0, \psi_0, \theta_i). \tag{15}$$

A very simple way to obtain information on the shape is to make use of the relation between the peaks of a QRS complex. By means of the QRS model, we can generate complexes which have, at most, three phases, and thus the resulting sequence of peak amplitudes is of a length of three or less. In this sequence, which we denote  $\{M_k\}$ , there exists an index j such that either  $M_j$  or  $M_{j+1}$  is the maximal absolute peak amplitude. Now, let us define the function  $\Gamma_T$  as

$$\Gamma_T = 2 \frac{\min \left[ |M_j|, |M_{j+1}| \right]}{|M_j| + |M_{j+1}|}$$
 (16)

which describes the transition between mono- and biphasic complexes. By normalizing with the peak-to-peak amplitude,  $\Gamma_T$  will be invariant to changes in the amplitude of q(t). Note that in using only the peaks  $M_k$  of the complex,  $\Gamma_T$  is already invariant to changes in width. With the definition in (16),  $\Gamma_T$  equals zero for a monophasic complex, while for a symmetric biphasic complex  $\Gamma_T$  is equal to one. In order to separate the four different transitions in Fig. 5, we first introduce

$$\gamma_M = \max[|M_i|, |M_{i+1}|].$$
 (17)

The shape feature  $\Gamma_A$  is then defined as

$$\Gamma_{A} = \begin{cases} \Gamma_{T} & M_{j} = \gamma_{M} \\ 2 - \Gamma_{T} & M_{j+1} = -\gamma_{M} \\ 2 + \Gamma_{T} & \text{if} & M_{j} = -\gamma_{M} \\ 4 - \Gamma_{T} & M_{j+1} = \gamma_{M} \end{cases}$$
(18)

Since  $0 \le \Gamma_T \le 1$ , we have  $0 \le \Gamma_A \le 4$ . Thus,  $c^- = 0$ ,  $c^+ = 4$ , and  $d_{max} = 2$ .

The mean  $m_A(\varphi, \psi)$  and the within-class scatter  $S_A(\varphi, \psi)$  are shown for  $\Gamma_A$  in Figs. 6 and 7, respectively. Each diagram is plotted for a constant angle  $\varphi$  (0° or 90°). For essentially symmetric triphasic complexes, we observe that  $S_A(\varphi, \psi) \gg 0$ , which indicates that jittering of  $\Gamma_A$  occurs when representing such complexes (see Fig. 7). We may conclude that  $\Gamma_A$  best describes predominantly biphasic complexes (see Fig. 7(b) for  $|\psi| < 70^\circ$ ).

The results demonstrated the inability of  $\Gamma_A$  to represent symmetric triphasic complexes. In order to fit such complexes into the representation in Fig. 5, we must modify the shape feature considerably. This could be done by preserving the asymmetric component of the complex, e.g., to let a symmetric triphasic complex be represented as a monophasic one. This transformation is exemplified in Fig. 8. The peaks  $M_1$  and  $M_3$  are then shrunk until one peak equals zero and the magnitude of the other one equals the difference between  $M_1$  and  $M_3$ . When using this modified sequence, we denote the resulting feature with  $\Gamma_B$ . The results from the evaluation of  $\Gamma_B$  are shown in Figs. 6 and 7 (dashed line). It should be observed that the jittering for symmetric complexes is substantially smaller when using  $\Gamma_B$  instead of  $\Gamma_A$ .

#### VI. CONCLUSIONS

A method for describing properties of shape features has been presented. Essentially, the method embraces three different concepts: a model for generating QRS complexes, a measure for the dissimilarity in shape between different complexes, and the examination of distances in the feature space. Linear combinations of a number of basis functions model the QRS complexes. However, by choosing functions different from the ones in this paper, QRS morphologies of special interest may be studied. The method may possibly be extended to the evaluation of features which, e.g., describe the reference time or the width of a QRS complex, provided that a suitable measure of dissimilarity is defined. The use of the method is illustrated by means of a simple shape feature, in this case revealing an inability of the feature to represent symmetric triphasic complexes.

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# X-Ray Compton Scatter Imaging Using a High Speed Flying Spot X-Ray Tube

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Abstract—A system of Compton scatter medical X-ray fluoroscopy is investigated in this research which uses a specially constructed flying spot X-ray tube. The imaging system uses a narrow pinpoint X-ray beam which scans an object in near real time and creates a penetrating frontal view radiograph of variable penetration into the object. The results of our initial work with X-ray Compton backscatter imaging were presented in a previous paper, and this research seeks to demonstrate an improved X-ray generator which allows the radiographs to be produced much more rapidly. One application of the system may be to provide an X-ray frontal backprojection view of the moving heart epicardial surface which could be useful as a noninvasive diagnostic of cardiac function.

#### INTRODUCTION

X-ray Compton scatter imaging is a method of creating penetrating radiographs by using the radiation that is Compton scattered by an object when illuminated by a beam of high energy X-rays. Our previous work presented a new type of X-ray Compton backscatter radiography as a method of creating a frontal backprojection view of the body interior [4]. The principal advantage of this form of radiography is that it produces an image of different appearance and contrast from conventional radiography and may give the physician new information about the body interior. Since high energy X-rays

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