New rate 1/2, 1/3, and 1/4 binary convolutional encoders with an optimum distance profile

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Finally, some comments about the decoding complexity. Suppose that the decoder of woven convolutional codes with outer warp consists of one Viterbi decoder for the inner code and l_o Viterbi decoders for the outer codes. The decoding complexity is proportional to

$$\Gamma = 2^{m_i} + l_o 2^{m_o}. \tag{30}$$

Let us choose $m_i = m_o = \sqrt{m}$. Then, we have

Theorem 2: Suppose that a decoder for a woven convolutional code with outer warp consists of l_o Viterbi decoder for the outer convolutional codes and one Viterbi decoder for the inner convolutional code. If both the outer and inner convolutional codes have memory $\sqrt{m}$, then the complexity of the decoder is proportional to

$$\Gamma = (1 + l_o) 2^{\sqrt{m}}. \tag{31}$$

□

From Theorems 1 and 2 it follows, somewhat surprisingly, that for woven convolutional codes with $m_o = m_i = \sqrt{m}$ the decoding error probability decreases exponentially with m while the decoding complexity increases exponentially only with $\sqrt{m}$.

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New Rate $1/2$, $1/3$, and $1/4$ Binary Convolutional Encoders with an Optimum Distance Profile

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***Abstract*—Tabulations of binary systematic and nonsystematic polynomial convolutional encoders with an optimum distance profile for rate $1/2$, $1/3$, and $1/4$ are given. The reported encoders are found by computer searches that optimize over the weight spectra. The free distances for rate $1/3$ and $1/4$ are compared with Heller's and Griesmer's upper bounds.**

***Index Terms*—Convolutional encoders, free distance, optimum distance profile.**

The *distance profile* [1] $\boldsymbol{d} = [d_0, d_1, \cdots, d_m]$, where d_j is the jth-order *column distance* [2] and m is the memory of the convolutional encoder, is an important distance parameter for convolutional encoders. It is an encoder property but if we limit our interest to consider only encoding matrices $G(D)$ with $G(0)$ having full rank we can regard the distance profile as a code property [3]. When comparing codes with the same rate and memory, we say that a distance profile $\boldsymbol{d}$ is superior to a distance profile $\boldsymbol{d}'$ if $d_i > d_i'$ for the smallest $i, 0 \le i \le m$, where $d_i \ne d_i'$. The code with the superior $\boldsymbol{d}$ will generally require less computation with sequential decoding than the other code [1], [4].

In [5], extensive tables of rate $1/2$ convolutional encoders were given. In Tables I and II we give rate $1/2$ polynomial systematic and nonsystematic convolutional encoders, respectively, with an *optimum distance profile* (ODP encoders), i.e., with a distance profile equal to or superior to that of any other encoder. The generators are written in an octal form according to the convention introduced in [1]. For each value of the memory, we give the encoder with the largest free distance $d_{\rm free}$ among ODP encoders. (The free distance is the minimum Hamming distance between any two differing codewords.) Ties were resolved by comparing their weight spectra, i.e., by successively using the number of low-weight paths $n_{d_{\rm free}+i}$ for $i = 0, 1, \cdots, 9$ as a further optimality criterion. The generators marked with "*" have better spectra than those given in [5].

In an earlier paper [6], systematic convolutional encoders of rate $1/3$ and $1/4$ were published together with a few short nonsystematic encoders of rate $1/3$. Only one spectral component, viz., the number of paths of weight $d_{\rm free}$, was given. Here we give ten spectral components as well as extensive lists of nonsystematic encoders. We list rate $1/3$ and $1/4$ systematic as well as nonsystematic polynomial convolutional ODP encoders. The free distances are compared with Heller's and Griesmer's upper bounds on the free distances for nonlinear trellis and linear convolutional codes, respectively.

The free distance for any binary, rate $R = b/c$ convolutional code encoded by a polynomial, nonsystematic encoding matrix of memory

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TABLE I

$n_{d_{\rm free}+i}, i = 0, \cdots, 9$ for Systematic Rate $R = 1/2$ ODP Encoding Matrices $G = (4\ g_{12})$. For Memories m Marked with "*" These Encoders Have Better Spectra Than Those Given in [5]

			i									
m	g_{12}	$d_{\rm free}$	0	1	2	3	4	5	6	7	8	9
1	6	3	1	1	1	1	1	1	1	1	1	1
2	7	4	2	0	5	0	13	0	34	0	89	0
3	64	4	1	0	6	0	16	0	69	0	232	0
4*	66	5	2	2	1	10	21	29	77	180	332	711
5	73	6	3	0	13	0	55	0	298	0	1401	0
6*	674	6	1	3	4	11	25	53	118	274	654	1430
7*	714	6	2	0	9	0	46	0	248	0	1289	0
8*	671	7	1	5	5	17	35	70	173	452	993	2415
9*	7154	8	4	0	19	0	94	0	542	0	3159	0
10	7152	8	3	0	16	0	79	0	457	0	2618	0
11*	7153	9	3	5	11	26	52	124	317	821	1870	4364
12	67114	9	1	4	10	15	46	104	224	576	1368	3322
13	67116	10	5	0	27	0	124	0	777	0	4529	0
14*	71447	10	4	0	12	0	105	0	517	0	3138	0
15*	671174	10	1	0	16	0	78	0	437	0	2391	0
16	671166	12	13	0	46	0	263	0	1486	0	9019	0
17*	671166	12	13	0	46	0	263	0	1486	0	9019	0
18*	6711454	12	4	0	23	0	154	0	817	0	4896	0
19	7144616	12	3	0	23	0	92	0	556	0	3472	0
20*	7144761	12	1	3	10	25	53	110	263	676	1593	3838
21*	71447614	12	1	0	7	0	66	0	314	0	1842	0
22*	71446166	14	6	0	44	0	189	0	1132	0	6570	0
23*	67115143	14	2	0	38	0	168	0	947	0	5726	0
24*	714461654	15	5	7	23	62	115	256	669	1648	3999	9703
25*	671145536	15	3	7	16	44	112	244	578	1312	3267	8097
26*	714476053	16	8	0	54	0	289	0	1691	0	9609	0
27*	7144760524	16	7	0	73	0	350	0	1971	0	11624	0
28*	7144616566	16	3	0	38	0	134	0	834	0	5052	0
29	7144760535	18	22	0	118	0	695	0	3926	0	22788	0
30*	67114543064	16	1	1	10	15	36	101	225	596	1342	3298
31	67114543066	18	11	0	53	0	307	0	1742	0	10218	0

TABLE II

$n_{d_{\rm free}+i}, i = 0, \cdots, 9$ for Nonsystematic Rate $R = 1/2$ ODP Encoders $G = (g_{11}\ g_{12})$. For Memories m Marked with "*" These Encoders Have Better Spectra Than Those Given in [5]

				i									
m	g_{11}	g_{12}	$d_{\rm free}$	0	1	2	3	4	5	6	7	8	9
2	7	5	5	1	2	4	8	16	32	64	128	256	512
3	74	54	6	1	3	5	11	25	55	121	267	589	1299
4	62	56	7	2	3	4	16	37	68	176	432	925	2156
5*	77	45	8	2	3	8	15	41	90	224	515	1239	2896
6	634	564	10	12	0	53	0	234	0	1517	0	8862	0
7	626	572	10	1	6	13	20	64	123	321	764	1858	4442
8	751	557	12	10	9	30	51	156	340	875	1951	5127	11589
9	7664	5714	12	1	8	8	31	73	150	441	940	2214	5531
10	7512	5562	14	19	0	80	0	450	0	2615	0	15276	0
11	6643	5175	14	1	10	25	46	105	258	616	1531	3611	8675
12	63374	47244	15	2	10	29	55	138	301	692	1720	4199	10245
13	45332	77136	16	5	15	21	56	161	381	879	2095	5085	12207
14	65231	43677	17	3	16	44	62	172	455	1025	2395	5853	14487
15*	727144	424374	18	5	15	21	56	161	381	879	2095	5085	12207
16	717066	522702	19	9	16	48	112	259	596	1457	3460	8257	20562
17*	745705	546153	20	6	31	58	125	314	711	1819	4222	10502	25222
18*	6302164	5634554	21	13	34	72	161	369	914	2167	5318	12937	31241
19	5122642	7315626	22	26	0	160	0	916	0	5154	0	29386	0
20*	7375407	4313045	22	1	17	49	108	234	521	1310	3099	7433	18264
21	67520654	50371444	24	40	0	251	0	1379	0	7812	0	45858	0
22*	64553062	42533736	24	4	27	75	147	331	817	1956	4578	11053	27282
23	55076157	75501351	26	65	0	331	0	2014	0	11359	0	65585	0
24*	744537344	472606614	26	10	45	91	235	465	1186	2882	6790	16618	39794
25	665041116	516260772	27	24	54	125	278	637	1599	3779	9073	21831	52929

TABLE III
$n_{d_{\text{free}}+i}, i = 0, \cdots, 9$ FOR SYSTEMATIC RATE $R = 1/3$ ODP ENCODERS $G = (4\ g_{12}\ g_{13})$

m	g_{12}	g_{13}	d_{free}	i = 0	1	2	3	4	5	6	7	8	9
1	6	6	5	1	1	1	1	1	1	1	1	1	1
2	5	7	6	1	0	2	0	4	0	8	0	16	0
3	64	74	8	2	0	3	0	8	0	19	0	46	0
4	56	72	9	1	2	3	3	1	2	9	24	48	62
5	57	73	10	1	3	1	0	4	10	13	21	35	48
6	564	754	12	4	0	8	0	12	0	39	0	140	0
7	516	676	12	1	2	1	7	8	7	19	20	43	93
8	531	676	13	1	3	3	6	8	9	27	25	46	110
9	5314	6764	15	3	5	2	4	11	11	29	49	80	144
10	5312	6766	16	4	0	8	0	20	0	64	0	168	0
11	5317	6767	16	1	2	4	7	7	15	18	31	66	95
12	65304	71274	17	1	2	6	8	4	12	24	38	65	118
13	65306	71276	18	1	3	4	7	5	9	31	35	71	123
14	65305	71273	19	2	2	3	5	9	23	25	35	65	98
15	653764	712614	20	2	0	8	0	19	0	41	0	138	0
16	531206	676672	20	1	0	5	0	11	0	25	0	106	0
17	653055	712737	22	2	0	7	0	20	0	65	0	176	0
18	5144574	7325154	24	6	0	19	0	40	0	92	0	294	0
19	6530576	7127306	24	2	0	11	0	27	0	71	0	197	0
20	6530547	7127375	26	4	0	21	0	43	0	131	0	307	0
21	65376114	71261054	26	3	0	11	0	23	0	88	0	261	0
22	51445036	73251266	26	1	0	4	0	18	0	49	0	132	0
23	65305477	71273753	28	3	4	3	9	17	25	46	75	142	201
24	514453214	732513134	28	1	0	8	0	29	0	50	0	169	0
25	653761172	712610566	30	2	0	14	0	46	0	100	0	276	0
26	514450363	732512675	31	2	5	9	14	17	25	58	94	136	229
27	6537616604	7126106264	32	10	0	19	0	43	0	140	0	449	0
28	6537616606	7126106264	33	5	13	12	13	23	54	75	145	261	412
29	5312071307	6766735721	33	3	2	5	12	16	31	54	80	119	192
30	51445320354	73251313564	34	1	6	6	5	14	31	43	75	136	200

TABLE IV
$n_{d_{\text{free}}+i}, i = 0, \cdots, 9$ FOR NONSYSTEMATIC RATE $R = 1/3$ ODP ENCODERS $G = (g_{11}\ g_{12}\ g_{13})$

m	g_{11}	g_{12}	g_{13}	d_{free}	i = 0	1	2	3	4	5	6	7	8	9
1	4	6	6	5	1	1	1	1	1	1	1	1	1	1
2	5	7	7	8	2	0	5	0	13	0	34	0	89	0
3	54	64	74	10	3	0	2	0	15	0	24	0	87	0
4	52	66	76	12	5	0	3	0	13	0	62	0	108	0
5	47	53	75	13	1	3	6	4	5	12	14	33	66	106
6	574	664	744	14	1	0	8	0	11	0	35	0	97	0
7	536	656	722	16	1	5	2	6	14	18	34	44	65	125
8	435	526	717	17	1	2	6	7	6	13	30	38	73	140
9	5674	6304	7524	20	7	0	19	0	40	0	99	0	321	0
10	5136	6642	7166	21	4	1	4	14	18	28	39	60	114	225
11	4653	5435	6257	22	3	0	9	0	32	0	70	0	190	0
12	47164	57254	76304	24	2	8	10	15	18	29	74	101	155	267
13	47326	61372	74322	26	7	0	23	0	64	0	166	0	426	0
14	47671	55245	63217	27	6	4	6	21	24	37	69	112	166	328
15	447454	632734	766164	28	1	6	5	17	24	34	67	90	155	266
16	552334	614426	772722	30	3	9	20	21	29	49	83	145	241	409
17	552137	614671	772233	32	7	15	11	21	58	82	94	177	288	523
18	4550704	6246334	7731724	34	28	0	53	0	112	0	357	0	994	0
19	5531236	6151572	7731724	35	8	18	29	32	54	78	130	267	431	693

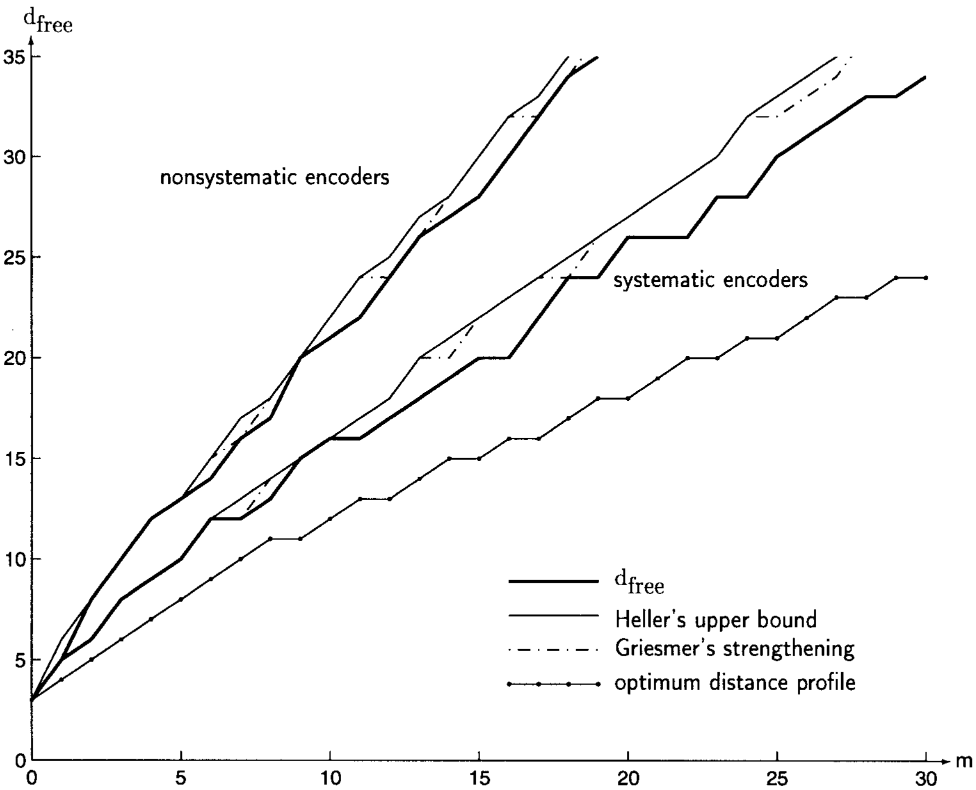

Fig. 1. The free distances for rate $R = 1/3$ systematic and nonsystematic ODP convolutional encoders and comparisons with Heller's and Griesmer's upper bounds and with the optimum distance profile.

TABLE V
$n_{d_{free}+i}, i = 0, \cdots, 9$ FOR SYSTEMATIC RATE $R = 1/4$ ODP ENCODERS $G = (4\ g_{12}\ g_{13}\ g_{14})$

					i									
m	g_{12}	g_{13}	g_{14}	d_{free}	0	1	2	3	4	5	6	7	8	9
1	4	6	6	6	1	0	1	0	1	0	1	0	1	0
2	5	6	7	8	1	0	1	0	2	0	3	0	5	0
3	54	64	74	11	1	0	1	0	5	0	4	0	8	0
4	56	62	72	12	1	0	1	0	5	0	4	0	8	0
5	51	67	73	14	1	0	2	0	5	0	6	0	10	0
6	534	634	754	16	1	3	0	0	3	5	5	9	9	9
7	516	676	732	18	1	3	1	2	3	3	5	10	11	13
8	535	637	755	20	2	4	0	3	2	4	8	9	19	21
9	5350	6370	7554	20	2	0	2	0	9	0	8	0	29	0
10	5156	6272	7404	20	1	0	1	0	4	0	5	0	17	0
11	5351	6371	7557	24	1	0	7	0	7	0	16	0	35	0
12	53514	63714	75574	24	1	0	1	2	1	8	8	3	9	10
13	51056	63116	76472	26	2	0	2	0	5	0	12	0	23	0
14	51055	63117	76473	28	1	0	5	0	4	0	13	0	30	0
15	515630	627350	740424	27	1	0	0	2	3	2	3	5	9	18
16	530036	611516	747332	30	3	0	2	0	6	0	10	0	21	0
17	535154	637141	755775	30	1	0	1	0	5	0	12	0	20	0
18	5105444	6311614	7647074	32	1	0	1	0	3	0	12	0	17	0
19	5105446	6311616	7647072	34	2	0	3	0	4	0	12	0	21	0
20	5105447	6311617	7647073	36	2	2	3	5	1	4	7	9	13	26
21	51054474	63116164	76470730	36	1	0	3	0	4	0	12	0	21	0
22	51563362	62735066	74040356	38	2	0	3	0	5	0	7	0	21	0
23	51054477	63116167	76470731	40	1	1	3	4	1	3	10	10	11	32
24	510544764	631161674	764707304	42	2	0	7	0	5	0	30	0	29	0
25	510544770	631161666	764707302	42	2	0	3	0	10	0	17	0	37	0
26	510544771	631161667	764707303	44	1	0	1	3	2	12	5	10	14	18
27	5105447710	6311616664	7647073024	47	3	3	6	9	13	9	23	25	35	60
28	5105447714	6311616664	7647073032	48	3	4	2	7	8	17	20	33	38	48
29	5105447715	6311616671	7647073025	51	6	3	4	8	15	25	28	39	44	58
30	51054477154	63116166734	76470730324	52	3	2	7	6	9	13	14	35	37	60

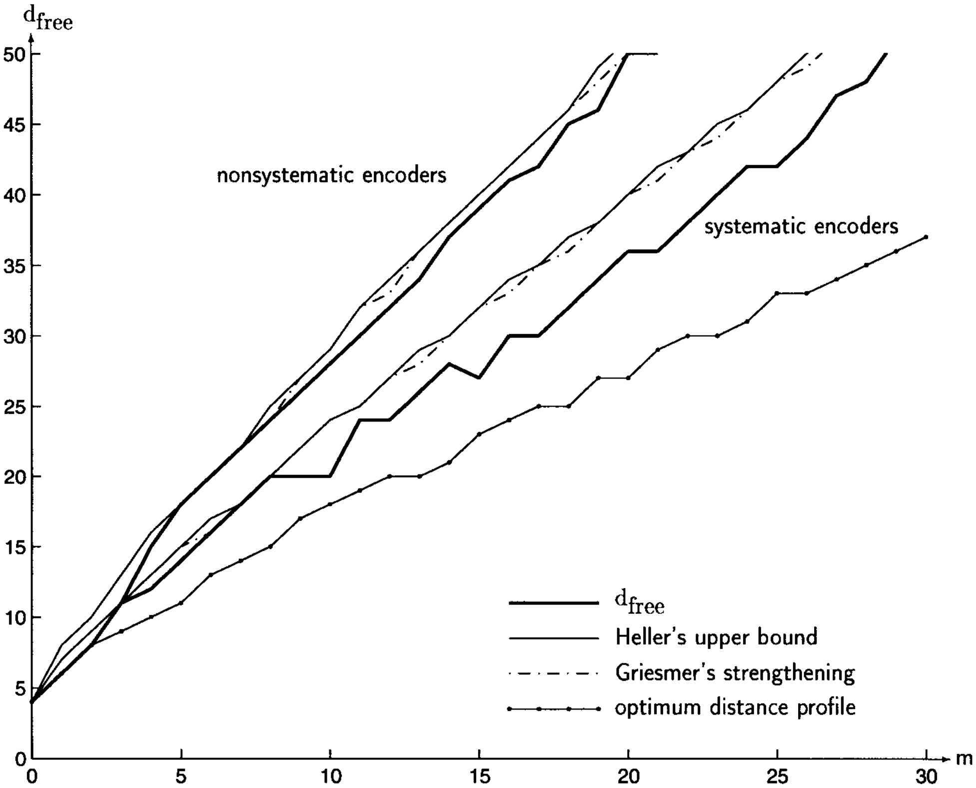

Fig. 2. The free distances for rate $R = 1/4$ systematic and nonsystematic ODP convolutional encoders and comparisons with Heller's and Griesmer's upper bounds and with the optimum distance profile.

TABLE VI
$n_{d_{\rm free}+i}, i = 0, \cdots, 9$ for Nonsystematic Rate $R = 1/4$ ODP Encoders $G = (g_{11}\ g_{12}\ g_{13}\ g_{14})$

						i									
m	g_{11}	g_{12}	g_{13}	g_{14}	$d_{\rm free}$	0	1	2	3	4	5	6	7	8	9
1	4	4	6	6	6	1	0	1	0	1	0	1	0	1	0
2	4	5	6	7	8	1	0	1	0	2	0	3	0	5	0
3	44	54	64	70	11	1	0	0	1	1	2	2	4	4	4
4	46	52	66	76	15	1	2	1	1	1	3	7	4	7	18
5	47	53	67	75	18	3	0	5	0	8	0	10	0	25	0
6	454	574	664	724	20	3	0	4	0	7	0	15	0	27	0
7	476	556	672	712	22	1	5	2	2	4	4	5	15	21	14
8	457	575	663	723	24	1	3	4	7	2	1	6	13	31	29
9	4730	5574	6564	7104	26	3	0	4	0	12	0	20	0	41	0
10	4266	5362	6136	7722	28	4	0	5	0	10	0	17	0	40	0
11	4227	5177	6225	7723	30	4	0	4	0	11	0	23	0	39	0
12	46554	56174	66450	72374	32	1	3	6	9	6	13	10	15	31	37
13	45562	57052	64732	73176	34	1	0	11	0	11	0	33	0	39	0
14	47633	57505	66535	71145	37	3	5	6	10	11	11	25	32	45	56
15	454374	574624	662564	723354	39	5	7	10	4	5	10	15	33	40	62
16	463712	566132	661562	727446	41	3	7	7	10	19	11	21	35	52	75
17	415727	523133	624577	744355	42	1	0	14	0	17	0	24	0	72	0
18	4653444	5426714	6477354	7036504	45	3	5	8	13	16	14	25	42	64	87
19	4654522	5617436	6645066	7237532	46	1	0	13	0	20	0	28	0	89	0
20	4712241	5763615	6765523	7330467	50	13	0	18	0	39	0	91	0	168	0
21	45724414	55057474	65556514	72624710	50	1	7	6	13	15	13	36	36	55	79

m satisfies [7], [8]

$$\text{Heller:}\quad d_{\text{free}} \le \min_{i\ge 1} \left\lfloor \frac{(m+i)c}{2(1-2^{-bi})} \right\rfloor \tag{1}$$

$$\text{Griesmer:}\quad \sum_{j=0}^{bi-1} \left\lceil \frac{d_{\text{free}}}{2^j} \right\rceil \le (m+i)c, \qquad i=1,2,\cdots. \tag{2}$$

For systematic encoding matrices we have the corresponding bounds [3]

$$\text{Heller:}\quad d_{\text{free}} \le \min_{i\ge 1} \left\lfloor \frac{(m(1-R)+i)c}{2(1-2^{-bi})} \right\rfloor \tag{3}$$

$$\text{Griesmer:}\quad \sum_{j=0}^{bi-1} \left\lceil \frac{d_{\text{free}}}{2^j} \right\rceil \le (m(1-R)+i)c, \qquad i=1,2,\cdots. \tag{4}$$

In Table III we list rate $1/3$ systematic polynomial ODP encoders for memories $1 \le m \le 30$. The corresponding nonsystematic encoders for memories $1 \le m \le 19$ are listed in Table IV. In Fig. 1 the free distances are compared with Heller's and Griesmer's upper bounds. For comparison we also show the optimum distance profile. (The distance profile is always the same for systematic and nonsystematic encoders [1], [3].)

Rate $1/4$ systematic polynomial ODP convolutional encoders for memories $1 \le m \le 30$ are listed in Table V and rate $1/4$ nonsystematic polynomial ODP convolutional encoders for memories $1 \le m \le 21$ are listed in Table VI. Finally, in Fig. 2 the free distances are related to Heller's and Griesmer's bounds.

The new convolutional codes combine a large free distance with an optimum distance profile and, thus might be attractive for use in various communication systems.

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The Weighted Coordinates Bound and Trellis Complexity of Block Codes and Periodic Packings

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***Abstract*—Weighted entropy profiles and a new bound, the weighted coordinates bound, on the state complexity profile of block codes are presented. These profiles and bound generalize the notion of dimension/length profile (DLP) and entropy/length profile (ELP) to block codes whose symbols are not drawn from a common alphabet set, and in particular, group codes. Likewise, the new bound may improve upon the DLP and ELP bounds for linear and nonlinear block codes over fields. However, it seems that the major contribution of the proposed bound is to the study of trellis complexity of block codes whose different coordinates are drawn from different alphabet sets. The label code of lattice and nonlattice periodic packings usually has this property. The construction of a trellis diagram for a lattice and some related bounds are generalized to periodic packings by introducing the fundamental module of the packing, and using the new bound on the state complexity profile. This generalization is limited to a given coordinate system. We show that any bounds on the trellis structure of block codes, and in particular, the bound presented in this work, are applicable to periodic packings.**

***Index Terms*—Entropy/dimension profiles, entropy/length profiles, lattices, periodic packings, trellis complexity.**

I. INTRODUCTION

Trellis diagrams suggest an efficient framework for soft-decision decoding algorithms for codes and lattices, such as the maximum-likelihood or the maximum *a posteriori* algorithms. Trellis complexity is a fundamental descriptive characteristic of both codes and lattices since it reflects the decoding complexity of these algorithms. The investigation of trellis diagrams of linear block codes has been an active research area during the last decade. Less attention has been directed to group codes and lattices in recent literature hitherto.

Under a given symbol permutation, any group code has a unique minimal biproper trellis [14]. An algorithm for computing the minimal trellis for a group code over a finite Abelian group has been presented by Vazirani *et al*. [27]. This algorithm extends the work of Kschischang and Sorokine [15] which treats linear codes over fields. The generalization of Vazirani *et al*. introduces the notions of p*-linear combinations* and p*-generator sequences*. The trellis product of the codewords of a p-generator sequence is minimal if and only if this sequence is *two-way proper*. A two-way proper p-generator sequence is a generalization of the trellis-oriented generator matrix [6], [15], for linear block codes over fields.

Measures of trellis complexity of block codes over a fixed alphabet set are bounded by the *entropy/length profile* (ELP) [18] which extends the *dimension/length profile* (DLP) of linear codes [7] to nonlinear codes. Several studies have addressed the problem of finding efficient permutations that meet the DLP bound, and hence minimize measures of trellis complexity (e.g., [3], [12], [13]). There is no measure equivalent to the DLP and ELP for block codes whose symbols are taken from alphabets of different sizes, such as

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