On the magnetic moment and rotational spectrum of thulium169

Nilsson, Sven Gösta; Mottelson, B R

Published in:
Zeitschrift für Physik

1955

Zeitschrift für Physik, Bd. 141, S. 217—220 (1955).

On the Magnetic Moment and Rotational Spectrum of Thulium^{169}*.

By
B. R. MOTTELSON** and S. G. NILSSON***.

With 2 Figures in the text.

(*Received 20. February 1955.*)

The recent studies of the nuclear properties of $_{69}Tm^{169}$ by means of the optical hyperfine structure investigations performed at Heidelberg [*1*] and by means of the COULOMB excitation reaction [*2*], [*3*] have yielded a considerable amount of data on this nucleus which may be discussed in terms of the unified nuclear model [*4*], [*5*].

For nuclei, such as Tm^{169}, which are well removed from closed shells, the strong deviation of the nuclear shape from spherical symmetry implies that with considerable accuracy one can separate between the intrinsic motion of the nucleus and a collective rotational motion. With each intrinsic state of the system there is then associated a rotational band having states of spin

$$I = K, \quad K+1, \quad K+2, \ldots \tag{1}$$

where K, which is a constant for the band, is the component of total angular momentum along the nuclear symmetry axis. The measured spin [*1*], [*6*] of Tm^{169} of $I = 1/2$ thus shows that the lowest particle configuration in this nucleus has $K = 1/2$. Intrinsic states with $K = 1/2$ are especially interesting since, in this case, there may occur a partial decoupling of the intrinsic spin of the last odd nucleon from the rotational motion. This decoupling, which depends on the intrinsic structure, manifests itself in such properties as the rotational energy spectrum and the magnetic moment, and a determination of these properties may thus provide a rather detailed test of the nuclear model employed.

We shall attempt below to discuss the properties of the lowest intrinsic state of Tm^{169} by assuming that the component of angular momentum, K, along the nuclear symmetry axis is associated with the motion of a single nucleon. We use for this purpose the wave functions obtained by considering particle motion in an axially symmetric deformed potential with the inclusion of an appropriate spin orbit force [*5*].

---

* To Prof. HANS KOPFERMANN on his 60th birthday.

** CERN (European Organization for Nuclear Research), Theoretical Study Division, at the Institute for Theoretical Physics, University of Copenhagen, Denmark.

*** Institute for Theoretical Physics, University of Lund, Sweden.

Since the spectrum of particle states, as well as the properties of a particular state, depend essentially on the deformation, we must have an estimate of this parameter before we are able to consider the more detailed properties of the intrinsic structure. While, in principle, the equilibrium deformation may be calculated by considering the total energy of the system as a function of the deformation and obtaining the minimum value, it appears difficult to obtain reliable estimates in this manner due to our insufficient knowledge of the cohesive forces in the nucleus. Therefore, in the applications of the model considered below, we prefer to use an experimental determination of the deformation, as obtained from the reduced transition probability for electric quadrupole transitions within the corresponding rotational band. From the observed cross sections for Coulomb excitation [2], [3] of the 120 keV level of Tm^{169} (cf. Fig. 1), one thus obtains $|Q_0| = 9 \times 10^{-24}$ cm^2 for the intrinsic quadrupole moment, which corresponds to a deformation parameter $\delta \simeq 0{\cdot}3$. We assume Tm^{169} to have a prolate shape, as is observed for all other nuclei in this region of the periodic table. This value of δ corresponds to a ratio of major to minor axis of 1·3 for the nuclear shape.

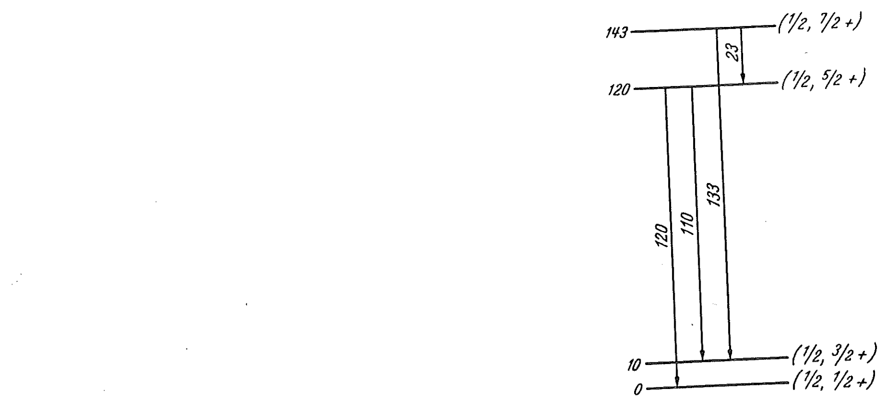


Fig. 1. Lowest rotational band for Tm^{169}. The figure gives the lowest states of Tm^{169} as deduced from the Coulomb excitation studies [2, 3] and from the study [7] of the electron-capture decay of Yb^{169}. The energies are given in keV. The rotational interpretation of these levels suggests the spin assignments given on the right, where the notation is (K, I, parity).

The first test of the model in the case of Tm^{169} is whether for $Z = 69$ there is available an orbit with $K = 1/2$. From an examination of Fig. 4 of ref. 5, it can be seen that for deformations $\delta \sim 0{\cdot}3$ there exists a unique orbit (no. 43 in the notation of ref. 5) with $K = 1/2$, which should be filled at about $Z = 69$. As we shall see below, this state reproduces in a rather detailed way the observed properties of the lowest intrinsic configuration of Tm^{169}. It may be worth noting that, if we follow this orbit as a function of the deformation, it goes over into an $s_{1/2}$ orbital in the limit of a spherical potential; however, the deformation implies a major modification in the structure of this state and for the deformations of interest the state is mainly composed of $d_{3/2}$, $d_{5/2}$, and $g_{7/2}$ components.

The magnetic moment of Tm^{169} has recently been determined to be [1]

$$\mu = -0{\cdot}20 \pm 0{\cdot}05 \text{ nm}. \tag{2}$$

The value of μ calculated for the above state is shown in Fig. 2a as a function of the deformation. In the calculations of μ, the spin and

G. NILSSON:

as well as the properties of a
e deformation, we must have
are able to consider the more
ure. While, in principle, the
ted by considering the total
ne deformation and obtaining
o obtain reliable estimates in
ledge of the cohesive forces in
herefore, in the applications
onsidered below, we prefer to
nental determination of the
obtained from the reduced
bility for electric quadrupole
n the corresponding rotational
observed cross sections for
ation [2], [3] of the 120 keV
(cf. Fig. 1), one thus obtains
cm² for the intrinsic quadru-
hich corresponds to a deforma-
$\delta \simeq 0{\cdot}3$. We assume Tm^{169} to
shape, as is observed for all
his region of the periodic table.
corresponds to a ratio of major
f 1·3 for the nuclear shape.
st of the model in the case of
er for $Z = 69$ there is available
$K = 1/2$. From an examination
ef. 5, it can be seen that for
$\sim 0{\cdot}3$ there exists a unique
with $K = 1/2$, which should be
e below, this state reproduces
roperties of the lowest intrinsic
h noting that, if we follow this
it goes over into an $s_{1/2}$ orbital
wever, the deformation implies
of this state and for the defor-
composed of $d_{3/2}$, $d_{5/2}$, and $g_{7/2}$

ecently been determined to be [1]

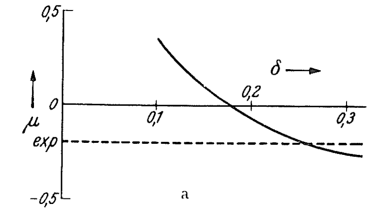
·05 nm. (2)

e state is shown in Fig. 2a as a
calculations of μ, the spin and

orbital g-factors for the intrinsic structure have been taken to be $g_s = 5{\cdot}58$ and $g_l = 1$ for this odd-proton nucleus, and the g-factor for the collective rotation has been assumed to have the value $g_R = Z/A = 0{\cdot}41$. It is seen that, in the region of the observed deformation, the calculated moment agrees well with that observed.

Fig. 1 gives the level scheme for the lowest states of Tm^{169} suggested by the COULOMB exitation studies [2], [3] and by the measurements [7] of γ-rays following the electron capture decay of Yb^{169}. It is seen that the observed energies agree well with the rotational energy spectrum

$$E_I = \frac{\hbar^2}{2\mathfrak{J}}\left\{ I(I+1) + \underline{a}(-)^{I+1/2}(I+1/2) \right\} \tag{3}$$



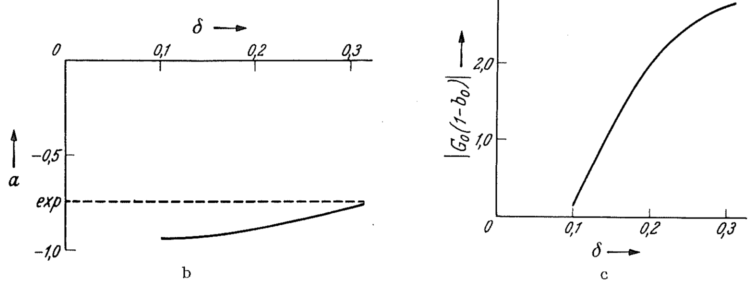


Fig. 2a—c. Properties of the intrinsic state of Tm^{169}. The properties of Tm^{169} are compared with those calculated from the wave functions of ref. 5; the various properties are given as a function of the deformation parameter δ; the value of δ estimated from the observed $E2$ transition probability is $\delta \simeq 0{\cdot}3$.

appropriate to such $K = 1/2$ configurations. While the moment of inertia, $\mathfrak{J}$, depends on the deformation and the nature of the collective flow, the last term in (3) reflects the partial decoupling of the spin of the last odd-nucleon from the rotational motion. The value of $\underline{a}$ implied by the observed spectrum is

$$\underline{a} = -0{\cdot}74, \tag{4}$$

which may be compared with the value calculated from the wave functions of ref. 5 which is shown in Fig. 2b.

The reduced transition probability for $M1$ radiation within a rotational band with $K = 1/2$ depends on the intrinsic structure through two parameters G_0 and b_0, i.e.

$$B(M1) = \frac{3}{64\pi}\left(\frac{e\hbar}{2Mc}\right)^2 \frac{2I+1}{I+1} G_0^2 \left(1 + (-)^{I-1/2} b_0\right)^2 \tag{5}$$

for a transition from a state $I+1$ to I. The observed [2] ratio of 1:10 for the intensity of the 120 keV to the 110 keV γ-ray when combined with the value of the $E2$ transition probability for the 120 keV transition as obtained from the intrinsic quadrupole moment given above yields

$$|G_0(1-b_0)| = 3. \tag{6}$$

This value is compared in Fig. 2c with that calculated from the wave functions of ref. 5. The calculated values of G_0 and b_0 are $-2{\cdot}86$ and $0{\cdot}043$, respectively, for $\delta = 0{\cdot}3$. One may also calculate that the ratio of the intensity of the 23 keV and 133 keV γ-rays is expected to be 1:20. There does not yet seem to be an experimental determination of this relative intensity.

Finally, it should be mentioned that the four nuclear quantities μ, $\underline{a}$, G_0, and b_0 depend on the structure of the intrinsic wave function through only two independent parameters, as soon as we attribute the angular momentum K to a single nucleon. One thus obtains two relations between these four quantities, which are independent of the details of the calculations of ref. 5, but depend only on the coupling scheme underlying these calculations. For an even-parity nucleus with $I_0 = K = 1/2$, one thus obtains

$$G_0 = 3\mu - 1/2\, g_s + g_l\,(1-\underline{a}) - g_R\,(2-\underline{a}) \tag{7a}$$

$$b_0 = -\frac{1}{2G_0}\{3\mu + 1/2\, g_s - g_l\,(1-\underline{a}) - g_R\,(1+\underline{a})\}. \tag{7b}$$

The value of $G_0(1-b_0)$ calculated from (2), (4), and (7), using the g-factors given above, is $-2{\cdot}6$, which is also seen to agree well with the observed value (6).

It is a pleasure for us to acknowledge our many stimulating contacts with Dr. A. BOHR in the course of this work.

References.

[1] LINDENBERGER, K. H., and A. STEUDEL: Submitted to Naturwiss. — [2] HUUS, T., J. H. BJEREGÅRD, B. ELBEK and B. M. MADSEN: To appear in Dan. Mat. Fys. Medd. — [3] HEYDENBURG, N., and G. TEMMER: Private communication. — [4] BOHR, A.: Dan. Mat. Fys. Medd. **26**, Nr. 14 (1952). — BOHR, A., u. B. MOTTELSON: Dan. Mat. Fys. Medd. **27**, Nr. 16 (1953). — [5] NILSSON, S. G.: To appear in Dan. Mat. Fys. Medd. — [6] SCHÜLER, H., u. T. SCHMIDT: Naturwiss. **22**, 838 (1934). — [7] MARTIN, D. S., E. N. JENSEN, F. J. HUGHES and R. T. NICHOLS: Phys. Rev. **82**, 579 (1951).

Druck der Universitätsdruckerei H. Stürtz AG., Würzburg