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Abstract

A key step to the strut-and-tie method is the selection of an appropriate truss model, due to the static indeterminacy of reinforced concrete there are often several suitable models possible. A method for automatically generating a suitable truss model by using graphic statics is presented. Optimal layouts are found by minimizing the total load path. A formulation of constraints suitable for generating an initial strut-and-tie model confined to an arbitrary polygon with holes is also presented. The performance by using derivative based and derivative free solvers is compared. The method is applied to several examples and the results are compared to existing methods from literature as well as to the principal stress patterns based on finite element analysis. All of the presented examples yield good results and the optimal layouts found can be used as a starting point for further design with the strut-and-tie method.

Keywords: graphic statics, strut-and-tie, reinforcement layout, optimization

1. Introduction

The design and detailing of cracked reinforced concrete members can be performed by reducing the concrete member into a truss-like structure, where some members act in compression (struts) and some in tension (ties). The strut-and-tie model (STM) is a general approach for the design of reinforcement layout for parts a concrete structure where the beam hypothesis is invalid, i.e. at static or geometric discontinuities. Since the regular truss model holds for the beam-like parts of the structure, the STM generalizes the design approach for structural concrete for all parts [1]. A key step to the strut-and-tie method is the selection of an appropriate truss model, due to the static indeterminacy of reinforced concrete there are often several suitable models possible. In this paper, we present a method for automatically generating a suitable truss model by using graphic statics.

Research of graphic statics has increased recently and several research directions are being explored, such as constraint based graphic statics, 3D graphic statics, generalizations that include bending, etc. Of particular interest for this research is the algebraic formulation of graphic statics [2, 3] and structural optimization with graphic statics [4].

There have been several research papers on generating strut-and-tie models, both using topology optimization and the ground structure approach, see [5, 6] and the references therein respectively.

The following motivate research on the use of graphic statics for STM:

- In complex strain fields it may be difficult for a novice designer to select an appropriate truss model, automatic STM generation been researched in [5, 6]. Graphic statics provides an alternative to this.

- STM design tends to lead to solutions containing mechanisms, when searching for alternative solutions this is simple to handle with graphic statics [4].

- The STM network is a pin-jointed structure; graphic statics is a natural approach for modeling such structures.

- The STM is based on the lower bound theorem of plasticity and algebraic graphic statics provides simple and direct means of exploring all possible equilibria. A unique force diagram (equilibrium state) is computed by selection [2], and not indirectly by for instance selecting a set of stiffness parameters.

2. A method for automatic generation of initial truss layouts

The first step in the STM is to identify Bernoulli regions (B-regions) and discontinuity regions (D-regions). B-regions are designed according to beam truss models while D-regions display more complex strain fields. The STM extends the truss model analogy to the D-regions and provides a consistent design approach to the whole structure. In order to apply the STM to a D-region the first step is to identify the section stresses on the region, for instance by a linear elastic finite element analysis (FEA) using a homogeneous isotropic material or by appropriate hand computations, see Fig. 1a. In the case that the whole region is a D-region one only needs to determine the reaction forces at the supports before the method can be applied.

The stresses on the boundary of the D-region are translated to discrete forces which along with external forces serve as the initial points for generating a form diagram, see Fig. 1b, which is where the method deviates from the common approaches used in STM (load-paths or stress trajectories). The initial form diagram generated is a structured/uniform mesh and does not take into account internal stresses, only the position of the external and section forces and the solid structure geometry. Once an initial form diagram has been generated an optimization procedure is run in order to find a form diagram suitable as an initial strut-and-tie model, Fig. 1c. The form diagram in Fig. 1c still has the same topology as in Fig. 1b, however, visualization techniques are used to make only the members with significant force magnitudes visible, thereby illustrating a potential strut-and-tie model. The present work investigates the optimization step of from Fig. 1b to Fig. 1c, using a graphic statics based optimization.

After the optimization, it is necessary to check that the network follows the principal stress field, due to the limited ductility of the concrete [1]. The computed network might require further rationalizations and all of the other requirements and detailing need to be checked, Fig. 1d.

a) Identify D-region and get section stress   b) Discretize section stress and add initial network   Present work   c) Perform optimization   d) rationalize and proceed with design
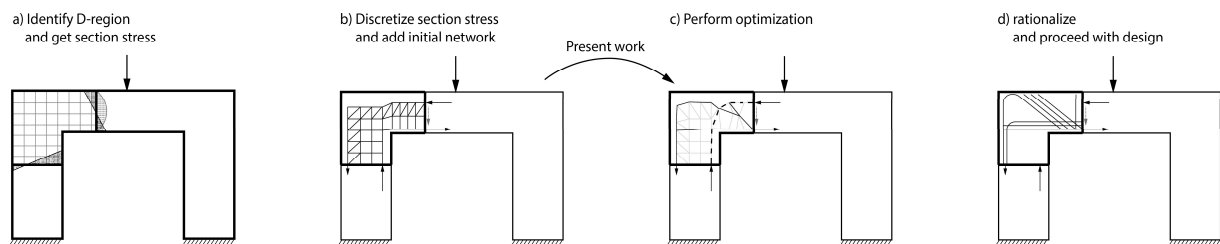
Figure 1: Schematic of the present work.

3. The optimization problem

In [4] graphic statics is used to minimize the total volume of a structure, which is shown to be equivalent to minimizing the total load path in trusses where all members are equally stressed. In this work, we use the total volume as the objective function and formulate the optimization problem as

$$\min_{x} f(x) = \frac{1}{\sigma} \sum L_i L_i^*$$

$$\text{s.t.} \quad \begin{cases} 1 - g(x, p) \leq 0 \\ g(x, q) \leq 0 \\ x \in [x_{\min}, x_{\max}] \end{cases} \tag{1}$$

where x are the design variables, which have upper and lower bounds. σ is the normal stress magnitude, assumed to be equal in all edges. Under these conditions minimization of the total volume is equivalent

to minimization of the total load path, $\sum L_i L_i^*$ [4]. L_i and L_i^* are the length and force magnitude of the i:th edge. $g(\boldsymbol{x}, \boldsymbol{p})$ is a binary function returning one if all of the edges of the form diagram are inside of the polygon defined by the matrix $\boldsymbol{p}$ and zero otherwise. Hence the first constraint requires all of the edges to be inside $\boldsymbol{p}$ while the second constraint requires all the edges to be outside a hole $\boldsymbol{q}$. Additional holes are defined similarly. The implementation of the polygonal constraints is such that the constraints are checked on y user-selected points per edge. Edges that correspond to external forces, section forces, or supports, are not included in the constraints.

The design variables $\boldsymbol{x}$ are the form diagram coordinates along with a set of force densities used to define the magnitude of external loads and section forces (and can be used to determine unique force diagrams in the case of statically indeterminate form diagrams). Using the form diagram coordinates as the design variables makes it simple to formulate the geometric constraints. The bounds for the design variables that are related to the form diagram coordinates are set to the bounding box of $\boldsymbol{p}$ for the coordinates, and for the force densities are set to [-1,1]. The form diagram edges which correspond to external forces or boundary stresses are kept fixed during the optimization, and so are their force magnitudes (keeping the external equilibrium constant) by fixing the respective force densities. In order to compute the force diagram, and to run the optimization, an algebraic formulation of graphic statics as described in [2, 3] is used.

The two constraints are discrete and do not have any derivatives. To solve the optimization problem derivative free solvers from the global optimization toolbox for MATLAB are used.

In order to use gradient based solvers, the two constraints need to be re-formulated. The distance for each point on an edge that is outside the polygon to the nearest point on the polygon is computed and these are summed for each edge. This results in $g_i \leq 0$ new constraints, where g_i is the summed distance for the points of edge i, see Fig. 2. A significant speedup is obtained by subdividing the polygon into a set of convex polygons since any line whose endpoints lie in the same convex polygon will then, by definition, be inside the original polygon.
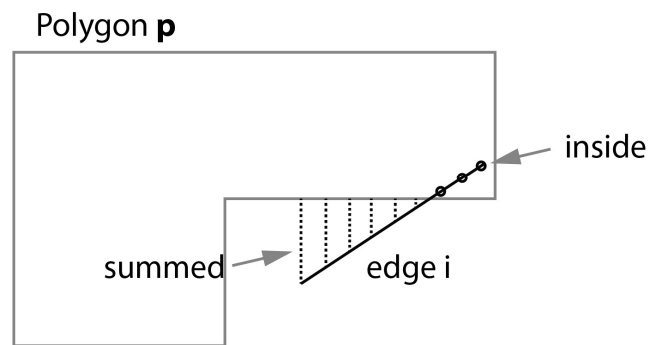

Figure 2: Illustration of constraint.

The same type of constraint is used for holes, except with the summed distance of the points that are inside of the polygon $\boldsymbol{q}$ rather than outside.

Another approach for constraining edges to be inside the polygon $\boldsymbol{p}$ is to consider the closed faces of the form diagram as polygons and perform a Boolean subtraction of the of the form diagram polygons by the polygon $\boldsymbol{p}$ and subsequently require that the area of what is left should be zero. This approach, however, cannot include holes.

To summarize, the geometric constraints have been implemented in three different ways, first as binary constraints, second as shown in Fig. 2, and finally by using the Boolean subtraction approach.

3.1 Multiple load cases

The minimization of a weighted sum of the individual load case objective functions [5] is performed in order to account for multiple load cases. The objective function becomes

$$f(\boldsymbol{x}) = \psi_1 f_1(\boldsymbol{x}) + \psi_2 f_2(\boldsymbol{x}) + \psi_3 f_3(\boldsymbol{x}) + \cdots \tag{2}$$

where ψ_i is a weight factor and all $f_i(\boldsymbol{x})$ have the same initial form diagram but different magnitudes on external and section forces. This is controlled trough the force densities of the external and section forces. The total load path for the i:th load case is $f_i(\boldsymbol{x})$.

4. Examples and results

In this section, the results of applying the method to some examples are presented. First, some simple models with the purpose to test the method are shown. A convergence study using the presented models is performed, where different optimization algorithms and versions of the constraints are compared. Results for some models with multiple discontinuities and deep beams are shown. Finally, an example with multiple load cases is presented. The section and reaction forces in the examples were computed either by hand or by FEA when required. The initial form diagrams were manually drawn, however, there are meshing techniques available which could be used for this step, but it is out of the scope of the present study.
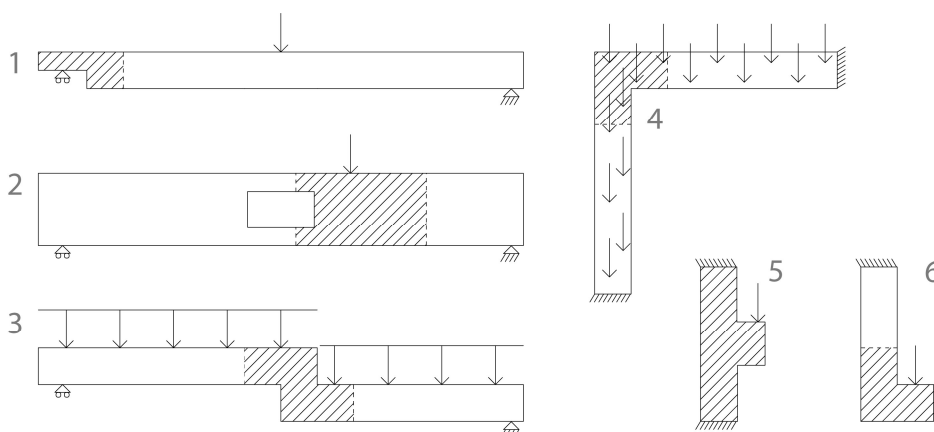
4.1 Models

Figure 3: The structures used for the convergence study.

Here six models used for the convergence study and their solutions are discussed. The models are based on the structures shown in Fig. 3, which also show the D-regions as hatched areas.

4.1.1. Results

The left most column with sub-figures in Fig. 4 are the initial form diagrams. The last column shows the principal stress components from an FEA, the second and third columns show are the form and force diagrams of the solved GD$_1$ solution (which has shown best performance, see section 4.2). Note that only the D-regions for each model are shown. Red is tension, blue is compression. After the optimization the form diagram still has the same topology as the initial form diagram. Highly stressed edges are emphasized by modifying the line weights according to the force magnitudes. Edges with a force magnitude less than 1% of the largest force magnitude are dashed, further emphasizing the load-carrying network. The result of model 6 in Fig. 4 is similar to Fig. 40b in [1], and the result of model 5 is similar to Fig. 11 in [5].

In many cases the solved forms tend to extend to the limits of the constraint-polygon, because of this, the polygon has been reduced in size in models 2-4, i.e. the polygons are not the actual dimensions of the beam, but are offset inward to almost near the positions of the section resultants.
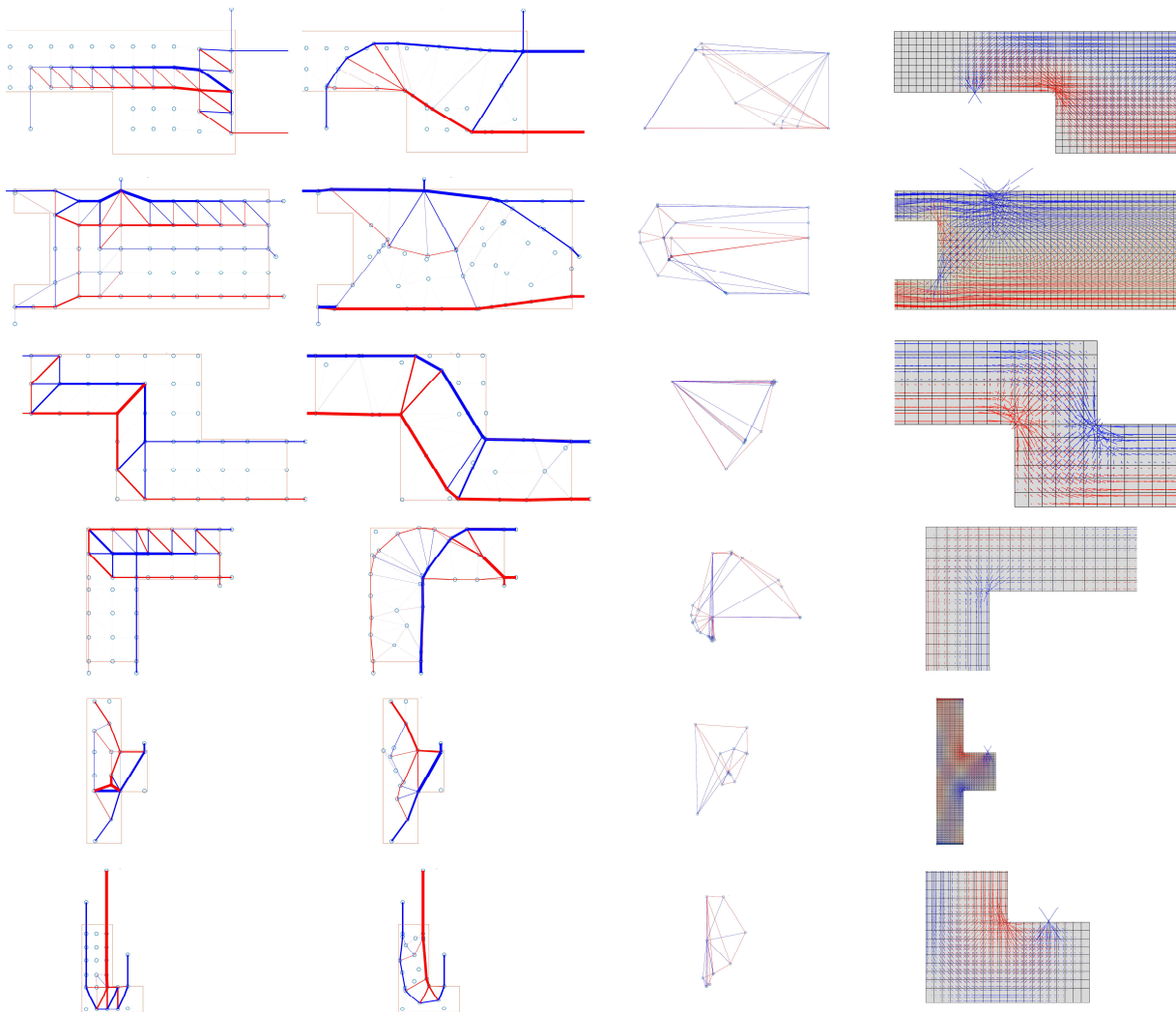
Figure 4: Columns from left to right: Initial form diagram, form diagram after optimization, force diagram after optimization, and finally the principal stresses from FEA. Rows 1-6 are models 1-6 respectively.

4.2 Convergence study

Table 1 presents the total load paths for the presented models and solution methods used. PS is pattern search with the binary polygon constraints. GD_1 and GD_2 use the MATLAB routine fmincon (using the interior point algorithm), where GD_1 uses the constraint formulated as in Fig. 2, and GD_2 is with the constraint following the Boolean subtraction approach. The genetic algorithm solver was also tested, however, no solutions were found.

Table 1: Total load paths for the different models and solution methods (normalized to initial). Color: Yellow = failed to satisfy constraints. White = Satisfied constraints to within accuracy limits, Red = Exceeds iteration limit

-	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Initial	1.0	1.0	1.0	1.0	1.0	1.0
PS	0.4120	0.5610	0.2647	0.3572	0.5855	0.7203
GD_1	0.2916	0.4268	0.5174	0.3462	0.5811	0.6758
GD_2	N/A	0.4736	0.5071	0.3243	0.3419	0.7146

4.3 Models with multiple discontinuities and deep beams

Here some examples with more complex stress fields are discussed. Fig. 5a shows an example of a frame corner with a corbel exposed to a point load. A FEA solution reveals bending and compression in the wall and bending in the floor, see Fig. 5d. The section stresses in the wall have been discretized into three loads for the truss-network, two that balance the moment, and a third due to the normal force. The method generates a solution where it is seen that reinforcement is required in the top of the floor, and some additional reinforcement on the inside of the wall. Fig. 5c shows a simplified model based on the one from Fig 9b to be used for reinforcement layout computations.
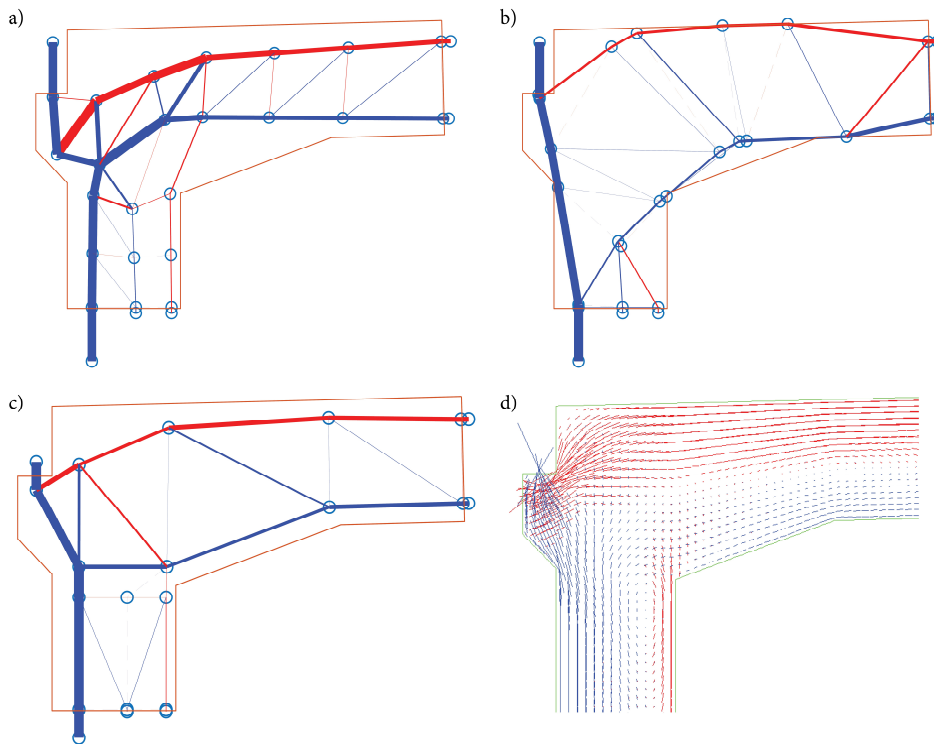

Figure 5: Frame corner with corbel. d) is reproduced from [7].

In Fig. 6a, the method is applied to a D-region with a hole. The result in Fig. 6b is similar to one of the two STM models used in [1] on the same geometry. The model is in accordance with the principal stress pattern. The polygon constraints are checked on 10 points per edge, leading to edges that slightly go through the hole in the bottom left of the geometry in Fig. 6b. This can be improved by increasing the number of points per edge used.
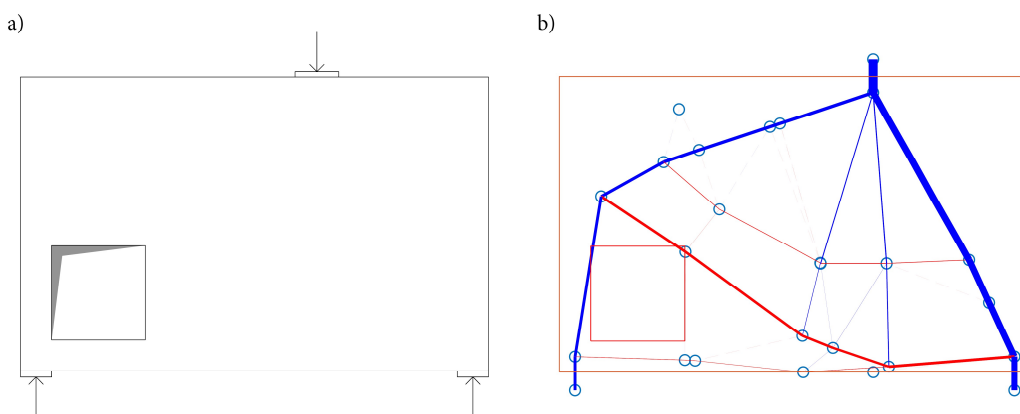

Figure 6: a) method applied to deep beam with a hole. b) the result after optimization.

4.4 Multiple loads

Fig. 7 shows a double corbel with multiple load cases. Fig. 7a shows the initial form and force diagrams a load on the right, Fig. 7b shows the initial solution for loads on both sides and Fig. 7c shows for a load to the left. The form and force diagrams after applying the optimization method for multiple loads is shown in Figs. 11d-11f respectively. Note that it is the same solution that is shown in Figs. 11d-11f, it is only the external loads that are different, i.e. the form diagrams in Figs. 11d-11f are the same while the force diagrams are different. The result is similar to [5].
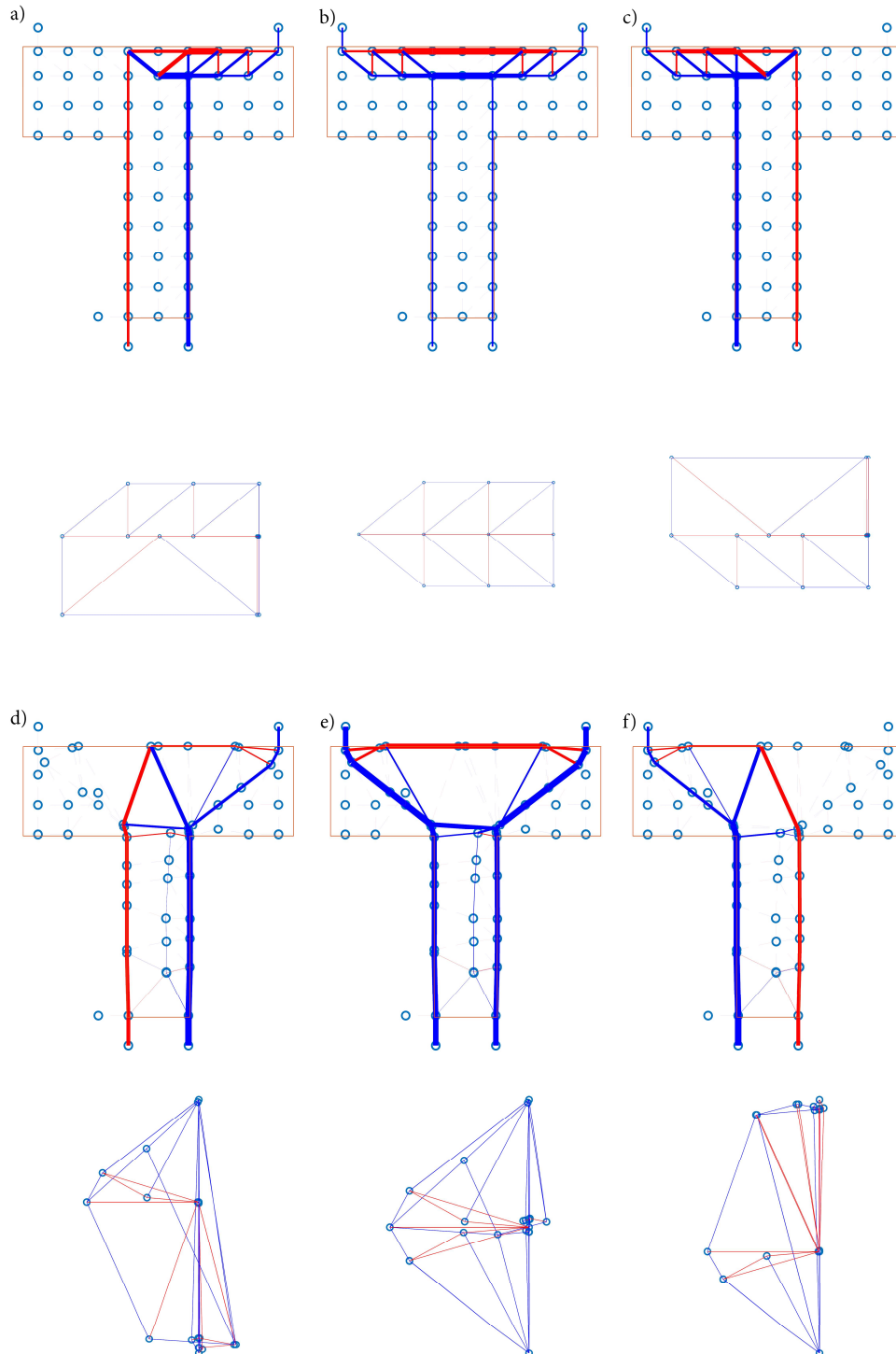

Figure 7: Double corbel with multiple load cases. Sub-figures a) b) and c) are the initial form and force diagrams with right, both, and left external loads respectively. Sub-figures d) e) and f) show the corresponding form and force diagrams after optimization.

5. Discussion

In the strut-and-tie method it is common to spread out a strut with a large force over a larger area, in order to approximate the spread of stress in the concrete better. The objective function used in this paper will minimize the total load path. Maxwell showed that the difference of the compression load path and tension load path is constant, thus, in order to minimize the total load path in the structure in Fig. 8a one only needs to remove the ties, resulting in Fig. 8b. When using the method this should be kept in mind, and large struts should be replaced accordingly, as for instance, the strut in the right part of the example in Fig. 6.
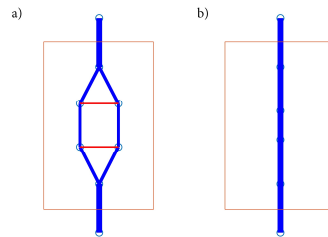

Figure 8: a) compression strut with ties. b) result after optimization using GD_1.

A method for generating initial reinforcement layouts using graphic statics was presented. In many cases, the solutions resemble the principal stress patterns and the method manages to capture appropriate load paths, however, it is still necessary to refine and rationalize the models in subsequent design stages. When there are uncertainties on how to select a good truss geometry for the STM our approach may be used to generate a starting point.

5.1 Further work

There are several aspects requiring further study and automation. The initial form diagram generation needs to be automated. For this, the influence of the initial form diagram needs to be studied further. Most strut-and-tie models are for parts of a structure, and consist of relatively few members, however, in order to support much larger assemblies than the ones presented in the paper the constraints need to be reformulated. Additional work on multiple load cases is required. Finally, the automatic generation of a simplified form diagram after the optimization procedure would be beneficial.

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