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Model-Based Estimation of Cylinder Pressure Sensor Offset using Least-Squares Methods

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Abstract

Two methods for estimating the sensor offset of a cylinder pressure transducer are developed. Both methods fit the pressure data during pre-combustion compression to a polytropic curve. The first method assumes a known polytropic exponent, and the other estimates the polytropic exponent. The first method results in a linear least-squares problem, and the second method results in a nonlinear least-squares problem. The nonlinear least-squares problem is solved by separating out the nonlinear dependence and solving the single-variable minimization problem. For this, a finite difference Newton method is applied. Using this method, the cost of solving the nonlinear least-squares problem is only slightly higher than solving the linear least-squares problem. Both methods show good statistical behavior. Estimation error variances are inversely proportional to the number of pressure samples used for the estimation. The method is computationally inexpensive, and well suited for real-time control applications.

1 Introduction

Crank angle resolved cylinder pressure measurement on internal combustion engines can be made using various kinds of transducer types, of which the piezoelectric, and the optical transducer types are the most prevalent. Their bandwidths are adequate to capture the relevant information in the cylinder pressure trace. All suitable transducer types share one unattractive characteristic, though, in that their DC offset varies in an unpredictable way with time.

This paper develops a new method to estimate and remove the offset from the cylinder pressure measurements. The method amounts to solving the nonlinear least-squares problem of fitting the measured pressure data to a polytropic compression curve. By solving the linear least-squares problem which results from assuming that the parameter causing the nonlinearity is known, the problem is reduced to minimizing a function of one variable. For this purpose, a finite difference Newton method is proposed. The method proposed here is computationally quite inexpensive, and is well suited for real-time applications e.g. where cylinder pressure measurements are used for feedback control. An important advantage compared to previous methods [1] based on two pressure samples is that the estimation variance can be reduced by increasing the number of samples used in the computation.

2 Cylinder Pressure Transducers

A piezoelectric sensing element has to be connected to a charge amplifier, which converts the electrical charge to a voltage or a current. For the piezoelectric transducer type, the DC offset variation is produced by the charge amplifier. The charge amplifier necessarily has some leakage current, which causes the amplifier output to drift over time. This drift is compensated for by a high-pass filter. If the time constant of the high-pass filter is high enough, the filter dominates the drift, and the DC gain of the amplifier is zero. Thermal stress on the sensing element can also distort the measurements.

An optical pressure transducer uses an LED, one or several optical fibers, and a photodetector to measure the light intensity reflected from a metal diaphragm, as it deflects under pressure [4, 5]. The optical pressure transducer has the advantage of being inexpensive compared to the piezoelectric type, since LEDs, optical fibers, and photodetectors are all inexpensive components. Furthermore it does not require a charge amplifier to interface with a data acquisition system. For this transducer, the variation in DC offset is a mechanical phenomenon caused by e.g. thermal stress, aging, and bending of optical fibers.

3 Identification of Compression Parameters when κ is Known

The compression stroke of an internal combustion engine can be modeled as a polytropic process [3], where

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pressure and volume are related according to

$$\frac{p}{p_0} = \left(\frac{V}{V_0}\right)^{-\kappa} \tag{1}$$

p_0 and V_0 are constants representing initial values of pressure and volume respectively.

3.1 Problem Formulation

Assume that cylinder pressure measurements have an unknown but constant offset Δp,

$$p_m = p + \Delta p \tag{2}$$

Combining (1) and (2) yields

$$p_m = \Delta p + CV^{-\kappa} \tag{3}$$

where

$$C = p_0 V_0^{\kappa} \tag{4}$$

Further assume that κ is known.

Posing the problem as a system identification problem

$$y = \varphi\theta \tag{5}$$

where y is known as the output, φ the regressor vector, and θ the parameter vector. Here,

$$y = p_m, \qquad \varphi = \begin{pmatrix} 1 & V^{-\kappa} \end{pmatrix}, \text{and} \qquad \theta = \begin{pmatrix} \Delta p \\ C \end{pmatrix} \tag{6}$$

Assume crank-angle resolved data is available for cylinder pressure and combustion chamber volume during the compression stroke, and form

$$Y = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} (p_m)_1 \\ \vdots \\ (p_m)_n \end{pmatrix}, \Phi = \begin{pmatrix} \varphi_1 \\ \vdots \\ \varphi_n \end{pmatrix} = \begin{pmatrix} 1 & V_1^{-\kappa} \\ \vdots & \vdots \\ 1 & V_n^{-\kappa} \end{pmatrix} \tag{7}$$

Combining (5) and (7) yields

$$Y = \Phi\theta \tag{8}$$

For equation (8) to hold, it is necessary to select a crank angle interval for which (1) is valid, i.e. combustion chamber closed, and no combustion taking place. This means that all the data has to be collected between the point where the intake valve closes and the point where combustion starts.

3.2 The Least Squares Solution

Assume that the number, n, of cylinder pressure measurements in Y, is larger than the number of unknown parameters in θ, which is two. Then, the least-squares solution to (8) is given by

$$\hat{\theta} = \left(\Phi^T\Phi\right)^{-1}\Phi^T Y = \Phi^+ Y \tag{9}$$

where Φ^+ is the Moore-Penrose pseudo inverse of Φ.

For the problem at hand, the least-squares solution (9) translates to

$$\Delta p = -\frac{S_v S_{pv} - S_{vv} S_p}{nS_{vv} - S_v^2} \tag{10}$$

$$C = \frac{nS_{pv} - S_v S_p}{nS_{vv} - S_v^2} \tag{11}$$

where

$$S_p = \sum_{i=1}^{n} (p_m)_i \tag{12}$$

$$S_v = \sum_{i=1}^{n} V_i^{-\kappa} \tag{13}$$

$$S_{pv} = \sum_{i=1}^{n} (p_m)_i V_i^{-\kappa} \tag{14}$$

$$S_{vv} = \sum_{i=1}^{n} V_i^{-2\kappa} \tag{15}$$

3.3 Statistical Properties of the Least-Squares Estimate

The statistics of the estimates can be analyzed by assuming that Y is a random variable defined by

$$Y = \Phi\theta + V \tag{16}$$

where V represents the measurement noise. If V is assumed to be white Gaussian noise, it can be shown [3] that the estimate is consistent, and that the variances of the estimates are roughly inversely proportional to the the number of measurements as predicted by the central limit theorem.

Figure 1 shows the standard deviation of the estimate for Δp as a function of the number of samples n. The inverse square-root dependence on n, predicted by the central limit theorem is also plotted for comparison. It can be seen that this approximation is quite accurate for large values of n, and can thus be used for a quick approximation of how many samples are required for the desired accuracy.

3.4 Experimental Results

Figure 2 shows the result of applying the presented offset estimation method to the compression stroke of an HCCI engine, with compression ratio 18:1. Estimation is performed based on cylinder- pressure measurements between 135° and 40° before top dead center, and the crank-angle resolution of the pressure sampling is 2.5 samples per degree. The standard deviation of the residual is approximately 500 Pa which is roughly the noise level of the measured pressure signal. Figure 3 shows the cylinder pressure during the intake stroke of the same cycle, compared with the pressure measured

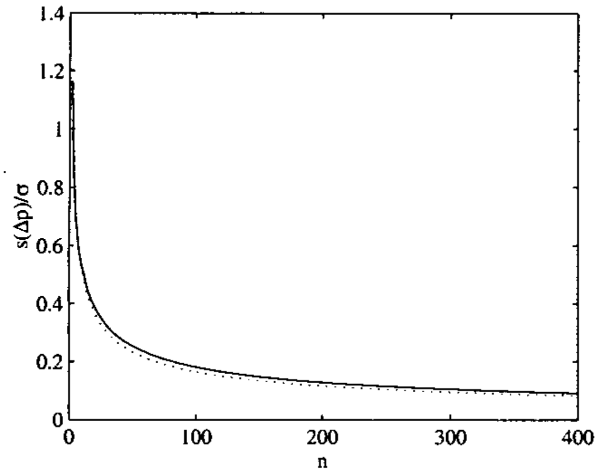

Figure 1: Standard deviation of Δp divided by standard deviation of pressure samples (solid line) plotted versus the number of samples, n. Dotted line plots $a/\sqrt{n}$ with a suitable constant a for comparison. σ represents the standard deviation of the measurement noise

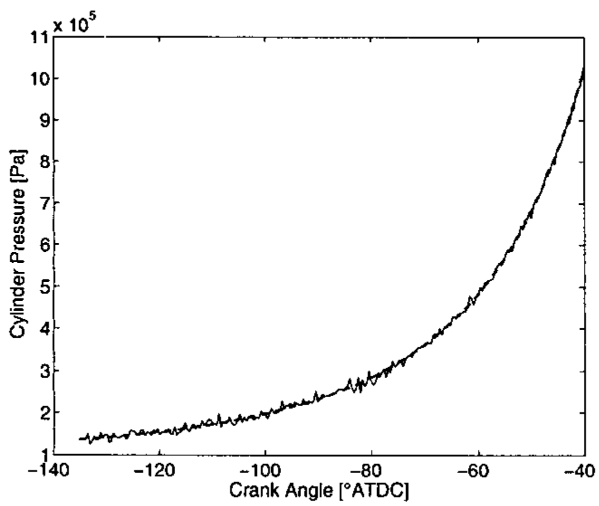

Figure 2: Cylinder pressure (solid line), fitted to adiabatic compression curve (dashed line). Compression ratio 18:1. $\kappa = 1.321$.

in the intake manifold. The correspondence between the pressure measured in the intake manifold and the pressure measured in the cylinder is good. The spike in the cylinder pressure around $-140\,^\circ$ is noise from when the intake valve closes.

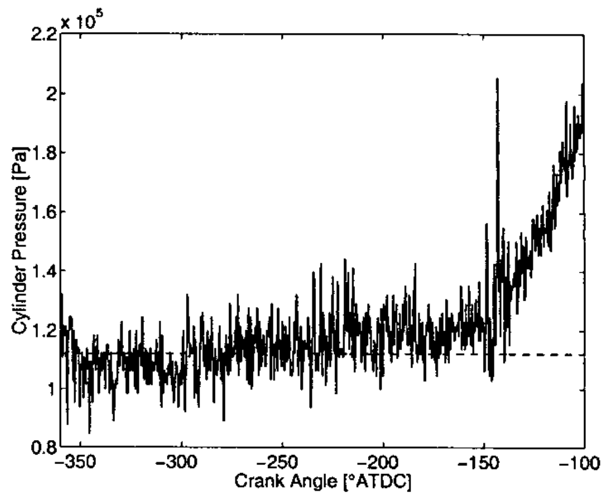

Figure 3: Cylinder pressure (solid line) during the intake stroke of the same cycle as in Figure 2. The dashed line shows the average pressure measured in the intake manifold.

Even though κ can, with good precision, be assumed to be constant during the compression stroke, it is not constant from cycle to cycle. These variations can be caused e.g. by changes in inlet conditions or operating conditions. Since the dependence of κ on operating conditions can be fairly complex, it may be necessary to estimate κ also. This is covered in Section 4.

4 Estimation of the Polytropic Exponent

As mentioned above, it is likely that κ is not known *a priori*. In this case it will be necessary to estimate κ as well. One way of estimating κ is to minimize the RMS error of the pressure trace with respect to κ, i.e.

$$\hat{\kappa} = \arg\min_{\kappa} \left(D^T D\right), \tag{17}$$

where

$$D = Y - \Phi\theta = \left[I - \Phi\left(\Phi^T\Phi\right)^{-1}\Phi^T\right]Y = PY \tag{18}$$

and

$$P = \left[I - \Phi\left(\Phi^T\Phi\right)^{-1}\Phi^T\right] \tag{19}$$

where it is noted that P is a symmetric projection matrix.

Define the loss function

$$J = D^T D = Y^T P^T P Y = Y^T P Y \tag{20}$$

The minimum of J can be characterized by

$$\left.\frac{\mathrm{d}J}{\mathrm{d}\kappa}\right|_{\kappa=\hat{\kappa}} = 0 \tag{21}$$

A few steps of algebra yields a simple expression for the derivative,

$$\frac{\mathrm{d}J}{\mathrm{d}\kappa} = -2D^T \frac{\mathrm{d}\Phi}{\mathrm{d}\kappa}\theta \tag{22}$$

Using (7),

$$\frac{\mathrm{d}\Phi}{\mathrm{d}\kappa} = \begin{pmatrix} 0 & -V_1^{-\kappa} \ln V_1 \\ \vdots & \vdots \\ 0 & -V_n^{-\kappa} \ln V_n \end{pmatrix} \tag{23}$$

So finally, using (6)

$$\frac{\mathrm{d}J}{\mathrm{d}\kappa} = 2CD^T \begin{pmatrix} V_1^{-\kappa} \ln V_1 \\ \vdots \\ V_n^{-\kappa} \ln V_n \end{pmatrix} \tag{24}$$

The dependence of J on κ turns out to be nearly quadratic, so a Newton method should converge to the minimum in just a few steps.

4.1 Newton Methods for Optimization

The base Newton optimization method (see e.g. [2]) approximates, at each iteration, the function with its second order Taylor polynomial, for which an analytical solution to the optimization problem exists. This, of course, requires an analytical expression for the second derivative.

In the case that an analytic expression for the second derivative is not available or, as in this case, it is expensive to compute on line, a modified version of the Newton method can be applied. The method used here is called a finite difference Newton method, and estimates the second derivative by a finite difference of first derivatives. Thus, the second derivative, $G(x_k)$, is approximated by

$$G(x_k) \approx H(x_k) = \frac{g(x_k) - g(x_{k-1})}{\delta_{k-1}} \tag{25}$$

where g is the first derivative

Since $g(x_k)$ and $g(x_{k-1})$ are computed anyway, the additional computational effort required for estimating the second derivative is very small.

In [3] it is shown that this method converges superlinearly if the second derivative is Lipschitz in a neighborhood of a local minimizer.

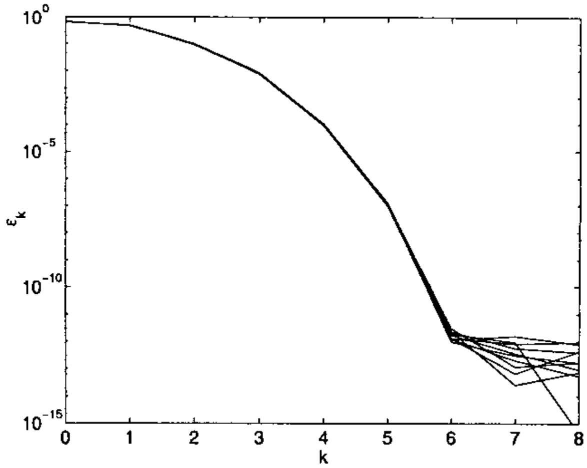

Figure 4: Lin-log plot of ϵ_k for 10 consecutive cycles, clearly indicating the superlinear convergence.

4.2 Evaluation of the Finite Difference Newton Method Applied to the κ-Estimation Problem

Figure 4 shows the error convergence for the estimation of κ on 10 consecutive cycles. The initial guess is intentionally selected far away from the true value in order to better show the superlinear convergence of the method.

Figure 5 shows the estimated pressure sensor offset for 250 consecutive cycles. The initial 50 cycles in Figure 5 indicate that there can indeed be a significant change in the pressure offset over time.

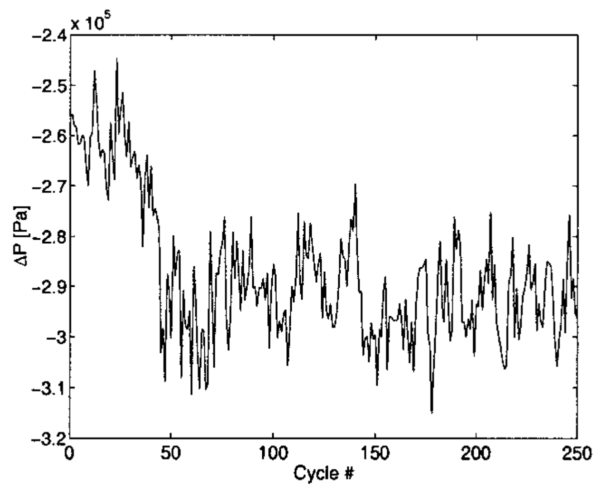

Figure 5: Estimated pressure sensor offset for 250 consecutive cycles.

The statistical properties of the estimates are investigated using Monte Carlo simulation. It is assumed that each cylinder-pressure measurement $p_m(\alpha)$ is a sum of the pressure obtained from polytropic compression, the sensor offset, Δp, and a zero-mean Gaussian Random variable, $V(\alpha)$, with variance σ^2.

$$p_m(\alpha) = C(V(\alpha))^{-\gamma} + \Delta p + V(\alpha) \tag{26}$$

The parameter estimation method is applied to fictitious measurements obtained from this model, and then the sample statistics are investigated. The values used in the simulation are

$$\begin{aligned} \Delta p &= -100\,\mathrm{kPa} \\ p_0 &= 150\,\mathrm{kPa} \\ \kappa &= 1.32 \end{aligned} \tag{27}$$

Figures 6–9 show the results of the simulations in terms of mean values and standard deviations of the estimates of Δp and κ. The standard deviations are somewhat higher than for the linear least-squares estimator. This is to be expected, since one more parameter is estimated from the same data. The standard deviations do, however, drop with the number of measurements in a similar manner as for the linear least-squares estimator.

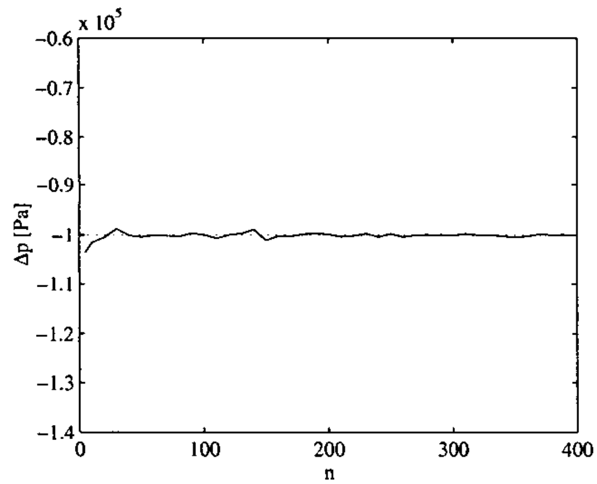

Figure 6: 250-cycle mean value of pressure sensor offset as function of the number of measurements. Dotted line shows nominal value.

5 Conclusions

Both methods presented in this paper allow estimation of the pressure sensor offset.

The linear least-squares estimates of pressure sensor offset , Δp, and initial combustion chamber pressure,

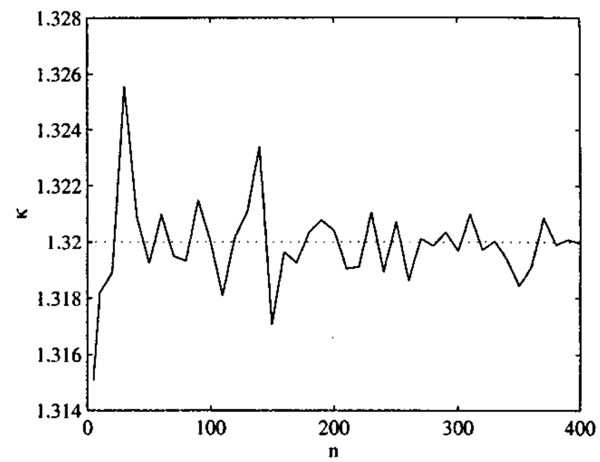

Figure 7: 250-cycle mean value of polytropic exponent estimate as function of the number of measurements. Dotted line shows nominal value.

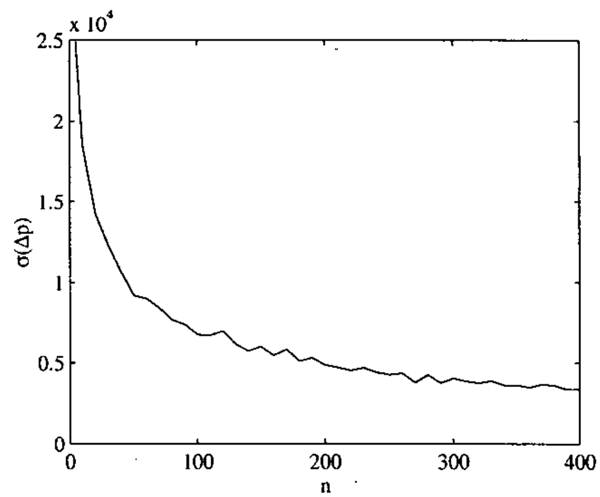

Figure 8: Estimated standard deviation of pressure offset estimate as function of the number of measurements. Standard deviation obtained from Monte Carlo simulations.

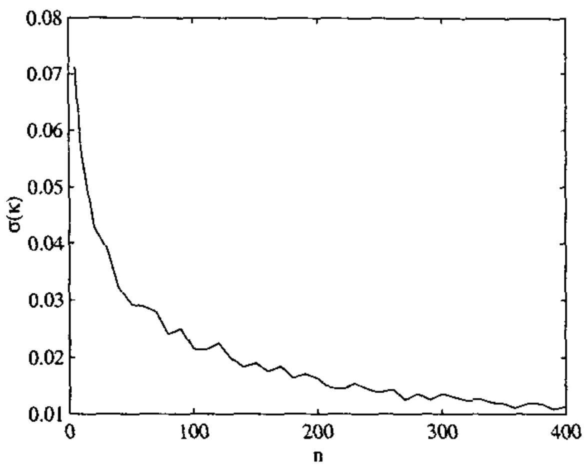

Figure 9: Estimated standard deviation of polytropic exponent estimate as function of the number of measurements. Standard deviation obtained from Monte Carlo simulations.

p_0, described in Section 3, have good statistical properties for the case that the polytropic exponent, κ is known. The standard deviations of the estimates roughly drop as $1/\sqrt{n}$ with the number of samples, n. This means that with as few as 50 samples, the standard deviations of the estimates are reduced to one fifth of the standard deviation obtained with present methods based on only two samples.

With unknown polytropic exponent, the nonlinear least-squares method proposed in Section 4 can be applied. Each iteration of the finite difference Newton method to find the least-squares estimate of κ involves applying the linear least-squares method once, and computing the derivative of the loss function once. The cost of computing the actual Newton step is negligible. The cost for computing the derivative is the same as for applying the linear least-squares method. Both problems involve solving a linear system of equations with the same left-hand side though, so information from one can be used for the other. Thus, the cost for solving both these problems is essentially the same as for just applying the linear least-squares method once. Furthermore, the Newton method converges in a few steps, so the total cost is only a few times the cost of applying the linear least-squares method. This makes the method suitable for real-time applications e.g. where cylinder pressure measurements are used for feedback control.

The standard deviations of the estimates are somewhat higher for the nonlinear least-squares estimates. This is to be expected since one more parameter is estimated. If κ is known to be constant or if it varies little from cycle to cycle, the estimate for κ can be low-pass filtered, and the filtered estimate can be used for linear least-squares estimation of the other two parameters. Sufficient filtering should result in similar standard deviations as for the "known κ" case.

Acknowledgments

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