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RATIONAL DESIGN METHODOLOGY  
FOR FIRE EXPOSED LOAD  
BEARING STRUCTURES

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## Rational Design Methodology for Fire Exposed Load Bearing Structures

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### SUMMARY

*The state-of-art of reliability studies in the area of fire-exposed structures or structural members is illustrated, taking examples from published papers concerning load-bearing building structures of steel, reinforced concrete, and wood. In parallel, trends are described in the present development of rational structural fire design methods, principally adapted to modern loading and safety philosophy for the non-fire state. Statistically derived results are presented for fire-exposed, insulated steel structures in office buildings, giving the breakdown of the total variance in maximum steel temperature and load-bearing capacity into component variances as a function of the insulation characteristics. The safety index and probability of failure are compared numerically for different fire design procedures. The data presented are examples of the information which is required as input in a qualified systems analysis of fire exposed load-bearing structures.*

### 1. INTRODUCTION

In a general sense, the fire engineering design problem is non-deterministic. Some level of risk — the probability of an adverse event — is virtually unavoidable, and we have to recognize the impossibility of absolute compliance with a preset goal. Performance has to be described and measured in probabilistic terms.

This is one perspective from which we have to judge or appraise the building fire safety code systems now in force. Historically, they were written without actually stating their objective safety level and, still less, without any analytical measurement of the objectives involved. For this reason, there is an urgent need to evaluate the levels of safety inherent

in present local and national fire protection regulations. Lack of knowledge concerning the structure of the analytical models describing the physical process has, up to now, effectively prevented all efforts to assess risk levels quantitatively. Gradually, with expanding modeling capabilities, the potential for a rational, reliability-based design will proportionately increase.

Essential components of a rational design methodology include — in the ideal case [1]:

- (i) analytical modeling of relevant processes; verification of model validation and accuracy; determination of critical design parameters;
- (ii) formulation of functional requirements, independent of choice of design process, expressed either in deterministic or probabilistic terms;
- (iii) determination of design parameter values;
- (iv) verification by reliability analysis that the choice of safety factors leads to safety levels which are consistent with the expressed functional requirements.

The primary aim of the present study is to illustrate the state-of-art of reliability studies in the area of fire-exposed structural members, taking examples from published papers concerning load-bearing structures of steel, concrete, and wood. The study also highlights some trends in the present development of rational structural fire design methods.

### 2. STRUCTURAL SAFETY AND PROBABILISTIC METHODS IN GENERAL

#### 2.1. Fundamental case

Consider a single structural member with a well-defined single failure mode. Assume, further, that the strength or resistance,  $R$ , and load effect,  $S$ , are represented by a pair of statistically independent random variables. Failure in the predetermined mode occurs

when the random model load effect exceeds the random model resistance. The probability of failure  $P_{\text{fail}}$  in this mode is then — Fig. 1:

$$\begin{aligned} P_{\text{fail}} &= P(R - S \leq 0) = F_{R-S}(0) = \\ &= P\left(\frac{(R - S) - (\overline{R} - \overline{S})}{\sigma_{R-S}} \leq -\frac{\overline{R} - \overline{S}}{\sigma_{R-S}}\right) = \\ &= P\left(U \leq -\frac{1}{V_{R-S}}\right) = F_U(-\beta) \end{aligned} \quad (1)$$

where  $F$  = cumulative distribution function,  $P()$  = probability of,  $\overline{R} - \overline{S}$  = mean value of safety margin  $R - S$ ,  $\sigma_R, \sigma_S$  = standard deviation of  $R$  and  $S$ , respectively,  $\sigma_{R-S}$  = standard deviation of safety margin  $R - S$ ,  $U$  = standardized safety margin with  $\overline{U} = 0$  and  $\sigma_U = 1$ ,  $V_{R-S}$  = coefficient of variation of safety margin  $R - S$ ,  $\beta = (\overline{R} - \overline{S})/\sqrt{\sigma_R^2 + \sigma_S^2}$  = safety index, defining the reliability.

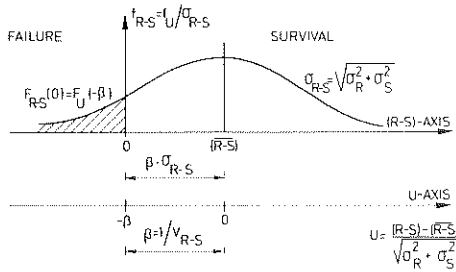


Fig. 1. Definition of safety index,  $\beta$ .

Ideally, the calculated failure probability  $P_{\text{fail}}$  should form the basis for the derivation of design criteria. Now,  $P_{\text{fail}}$  can be evaluated exactly only if the probability density functions of  $R$  and  $S$  are known. In practice, this is seldom the case.

Two main alternatives are available in these circumstances [2, 3]:

- (i) to base a design code format on prescribed distributions for  $R$  and  $S$ ;
- (ii) to acknowledge explicitly the incompleteness of statistical information and disregard the form of the distributions involved.

In the latter case, a design scheme can be based simply on the requirement that some minimum safety margin be maintained. In place of the requirement that a calculated risk of failure must fall below a specified probability, it may be required that the average safety margin,  $\overline{R} - \overline{S}$  or  $\overline{R} - \overline{S}$ , must lie a specified number,  $\beta$ , of standard deviations above zero, *i.e.*,

$$\overline{R} - \overline{S} \geq \beta \sigma_{R-S} \text{ or } \overline{R} > \overline{S} + \beta \sqrt{\sigma_R^2 + \sigma_S^2}. \quad (2)$$

The method is distribution-free and employs only the first and second central moments of relevant stochastic variables, hence the name “second moment code formats”.

The random variables  $R$  and  $S$  are invariable functions of other, more basic variables. The problem is to derive the means and variances of  $R$  and  $S$  from the first and second moments of the basic variables. Exact calculation is only possible when the functional relation between the two sets of variables is a linear transformation. In all other cases, approximate methods must be used. A convenient method is to make a Taylor expansion of  $R$  and  $S$  with the derivatives evaluated at the mean values and truncate the expansion at the linear terms. Assuming that the resistance  $R$  is a function of  $n$  independent stochastic variables  $X_1 \dots X_n$ ,

$$R = R(X_1, X_2 \dots X_j \dots X_n).$$

The first-order approximate values of  $\overline{R}$  and  $\sigma_R$  will then be given by

$$\overline{R} = R(\overline{X}_1, \overline{X}_2 \dots \overline{X}_j \dots \overline{X}_n) \quad (3)$$

$$\sigma_R^2 = \sum_{j=1}^n \left( \frac{\partial R}{\partial X_j} \right)_0^2 \sigma_{X_j}^2. \quad (4)$$

The subscript “0” denotes, in this case, evaluation at mean values.

The formulae must be used with discrimination. A necessary condition for reasonable precision is that the functions  $R, S$ , etc., are, simultaneously, approximately linear in the region close to  $\overline{X}_j$ ,  $j = 1 \dots n$ , as the greater part of the density function mass lies in this area. In more complicated cases, the required central moments must be derived by a Monte Carlo simulation.

## 2.2. Multi-failure mode case

An exact evaluation of the reliability of structural systems having several statistically interdependent failure modes requires lengthy numerical integration. Commonly used approximations of system reliability are based either on the assumption of probabilistic independence of the mode failure events, or on that of their complete statistical dependence.

No study at this level of complexity has been performed for fire-exposed structures. A practical example of normal temperature conditions of multi-mode failure is given in

ref. 3, where the uncertainty of a reinforced concrete beam in flexure is studied. The beam is supposed to have been designed to fail in reinforcement tension. According to ACI design specifications, this is ensured by requiring that the deterministic reinforcement ratio  $\rho \leq 0.75 \rho_b$ , where  $\rho_b$  = the balanced reinforcement ratio. However, since  $\rho$  and  $\rho_b$  are random variables, there is a probability of a flexural compression failure in concrete even when the beam is designed to fail in reinforcement flexural tension. In this case,  $P_{\text{fail}}$  may be evaluated from

$$P_{\text{fail}} = P(\text{failure}|\rho \leq \rho_b) \times P(\rho \leq \rho_b) + P(\text{failure}|\rho > \rho_b) \times P(\rho > \rho_b). \quad (5)$$

For a fire-exposed reinforced concrete structure,  $\rho_b$  will be a function of the temperature level and distribution in particular cross sections. Accordingly, future reliability studies of concrete structures will have to be based on the multi-failure mode concept. This is further emphasized by the possibility of a change in failure mode — for instance, from a flexural failure to a failure with respect to shear, bond or anchorage — during a fire exposure.

For further discussion of general aspects of safety analysis of fire-exposed reinforced concrete members, see ref. 4.

### 2.3. Evaluation of safety factors and load factors

The safety index code format defined earlier results, primarily, in the following design equation for a predetermined value of  $\beta$ ,

$$\bar{R} \geq \bar{S} + \beta \sqrt{\sigma_R^2 + \sigma_S^2}. \quad (6)$$

It is professionally desirable to relate this equation to the traditional code specifications using stress reducing coefficients,  $\phi$ , and load factors (partial factors),  $\gamma_D$  and  $\gamma_L$ , exemplified by the following relation

$$\phi R_n \geq \gamma_D D_n + \gamma_L L_n \quad (7)$$

where all factors or coefficients depend only on the variance of the corresponding random variable and on the value of  $\beta$ .  $D_n$  and  $L_n$  are nominal (characteristic) values of load effect, and  $R_n$ , nominal (characteristic) values of resistance or load-bearing capacity. This division of the uncertainty into smaller,

identifiable parts will be illustrated here only for the load factor format.

Using the separation function [5] twice,

$$(X_1^2 + X_2^2 + \dots)^{1/2} \cong \alpha_{X_1, X_2, \dots} (X_1 + X_2 + \dots) \quad (8)$$

the inequality, eqn. (6), can be rewritten

$$\bar{R} \geq \bar{S} + \beta \alpha_{RS} (\sigma_R + \sigma_S); \bar{S} = \bar{L} + \bar{D} \quad (9)$$

$$\bar{R} (1 - \beta \alpha_{RS} V_R) \geq \bar{L} + \bar{D} + \beta \alpha_{RS} \alpha_{DL} (\sigma_D + \sigma_L) \quad (10)$$

$$\bar{R} (1 - \beta \alpha_{RS} V_R) \geq \bar{L} (1 + \alpha_{RS} \alpha_{DL} \beta V_L) + \bar{D} (1 + \alpha_{RS} \alpha_{DL} \beta V_D) \quad (11)$$

where  $V_X$  denotes the coefficient of variation =  $\sigma_X/\bar{X}$  of the variable  $X$  and  $\alpha$  = linearization factors. Identifying with the inequality, eqn. (6), expressions for the partial factors  $\phi$ ,  $\gamma_D$  and  $\gamma_L$  are given by

$$\phi = (1 - \alpha_{RS} \beta V_R) \frac{\bar{R}}{R_n} \quad (12)$$

$$\gamma_D = (1 + \alpha_{RS} \alpha_{DL} \beta V_D) \frac{\bar{D}}{D_n} \quad (13)$$

$$\gamma_L = (1 + \alpha_{RS} \alpha_{DL} \beta V_L) \frac{\bar{L}}{L_n}. \quad (14)$$

The factors  $\phi$ ,  $\gamma_D$ ,  $\gamma_L$  are based on mean values, but they could just as well have been evaluated on the basis of nominal (characteristic) values.

### 2.4. Definition of component and total system uncertainties

The fire safety engineer faces at least three distinct types of uncertainty. The first is the intrinsic or fundamental uncertainty inherent in physical phenomena and human behaviour; examples could be weather conditions, location and behaviour of individuals at the outbreak of the fire. The second type of uncertainty can be called statistical. It is associated with failure to estimate parameters of statistical distributions representing, for example, the variance of material properties and load characteristics. This uncertainty can be reduced by increasing the sample size. The third kind of uncertainty is caused by the incompleteness of the mathematical model describing the physical reality. The prediction error has to be measured by comparison between theoretical model and experiments.

It must be recognized that lack of statistical data to provide perfectly accurate estimates

of parameters (means, coefficients of variation, etc.) describing stochastic components is *not* an argument against quantification of uncertainty. The incompleteness is only another error factor which must be accounted for, and is subject to quantification in terms of classical or Bayesian statistics.

For the last two categories of uncertainty, a general, systematized scheme for the identification and evaluation of the various sources and types of uncertainty in a differentiated structural fire engineering design has been undertaken in ref. 6 by a practical application to steel structures. The pattern of the identification of uncertainty sources is illustrated by Fig. 2, where boxes with broken lines indicate the origin of component uncertainty. Quantitative estimates will be presented in Section 3. The Figure outlines the approach, according to ref. 6, to the uncertainty assessment: the total uncertainty is divided into components, which must be specified in such a way that a statistically correct comparison between the

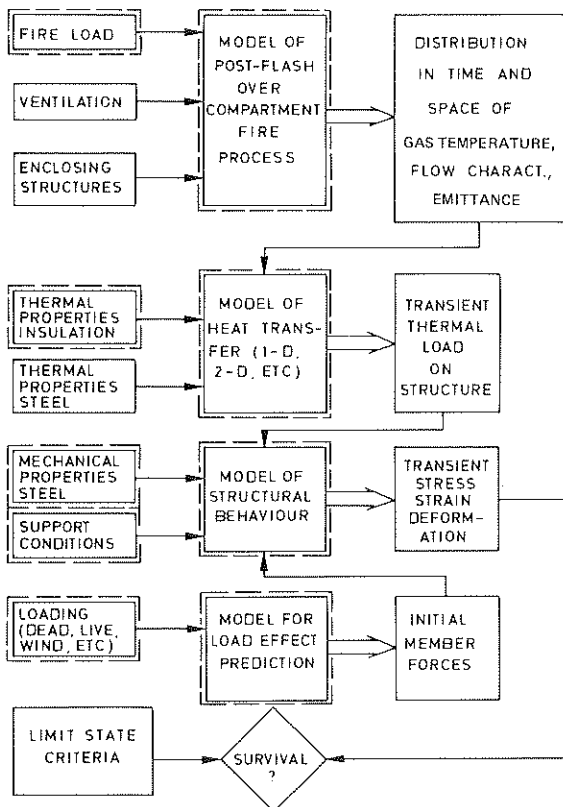


Fig. 2. Total system modeling of a differentiated theoretical design of fire exposed structures. Boxes with broken lines indicate parameters or submodels which are treated as stochastic in Section 3.

employed design theory and experiment is possible. The design theory is the “skeleton”, in relation to which all information must be evaluated. The words “statistically correct” imply that care must be taken to minimize the stochastic interdependence of the different component uncertainties. This, in turn, implies a design theory where the specific elements emulate the physical reality as closely as possible.

### 3. RELIABILITY STUDIES OF FIRE-EXPOSED STEEL STRUCTURES

#### 3.1. Design methodology

For more than ten years, a differentiated theoretical procedure has been applied in Sweden, as one alternative for a structural fire engineering design of load-bearing structures and partitions. The procedure constitutes a direct design method, according to Fig. 2, based on gas temperature–time characteristics of the fully developed compartment fire as a function of the fire load density,  $q$ , the ventilation of the fire compartment, and the thermal properties of the structures enclosing the fire compartment. The gas temperature–time curves are illustrated in Fig. 3. The design method is approved for general practical use by the National Board of Physical Planning and Building. To aid practical application, design diagrams and tables are systematically produced giving, directly, on the one hand, the design temperature state of the fire-exposed structure, and on the other, a transfer of this information to the corresponding design load-bearing capacity of the structure; *cf.*, for instance, refs. 7 and 8.

#### 3.2. Structure of reliability study [6]

Using the design data base as a reference frame, a reliability study was undertaken according to the pattern outlined in Fig. 2. The methodology used in this study, published in 1974, is of general character and applicable to a wide class of structures and structural elements.

To obtain usable and efficient final safety measures, the investigation is illustrated numerically for one specified structural element — an insulated, simply supported steel beam of I-cross section as part of a floor or roof assembly. The chosen statistics of dead and

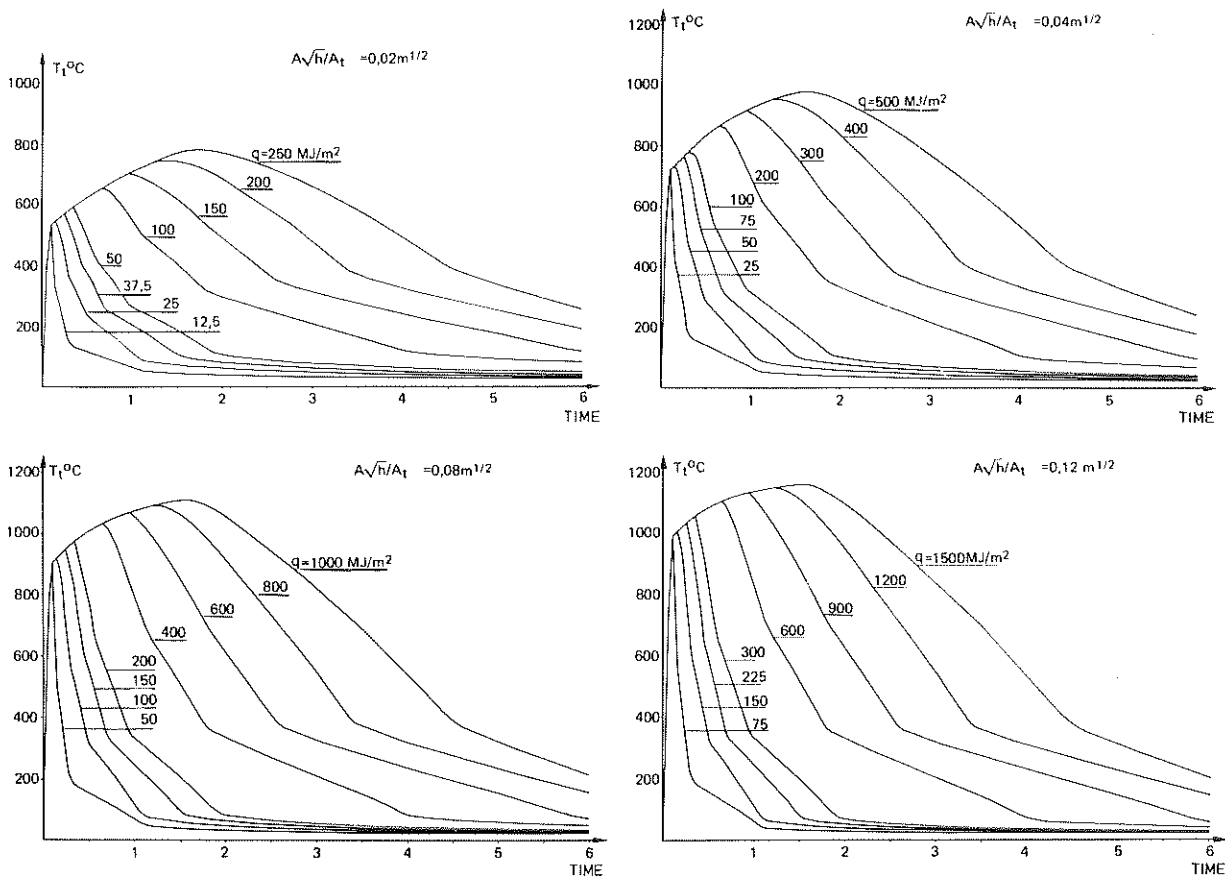


Fig. 3. Design gas temperature-time curves for compartment fires as a function of fire load density  $q$ , and opening factor  $A\sqrt{h}/A_t$ . The curves apply to a fire compartment with certain specified enclosing structure characteristics. From effective values for fire load density and opening factor, other types of enclosing structure can be considered.

live loads and fire load density are representative of office buildings. The assumptions regarding normal temperature design follow approved Swedish procedure. The beam is designed using the concept of allowable stress and with an overall safety factor,  $\gamma_o$  (strength factor) = 1.5. For offices, the present Swedish codes prescribe a value of nominal live load,  $L_n = 2.0 \text{ kN/m}^2$  floor area, irrespective of tributary area. The immovable part of  $L_n = 0.5 \text{ kN/m}^2$  and the moveable part =  $1.5 \text{ kN/m}^2$ . The nominal dead load,  $D_n$ , is made equal to the mean dead load.

By

(a) treating fire load density,  $q$ , as a stochastic parameter determined from statistical investigations,

(b) writing the stochastic variable  $T_{\max}$ , denoting the true maximum steel temperature,

$$T_{\max} = T_n + \Delta T_1 + \Delta T_2 + \Delta T_3 \quad (15)$$

where  $T_n$  = the deterministic value of the maximum steel temperature (design state

temperature), given by design curves for nominal values of fire load density,  $q$ , opening factor of fire compartment,  $A\sqrt{h}/A_t$ , and a thermal insulation parameter,  $\kappa$ ,  $\Delta T_1$  = the uncertainty due to variation in the  $\kappa$ -value,  $\Delta T_2$  = the uncertainty reflecting the prediction error in the theory of compartment fires and heat flow analysis,  $\Delta T_3$  = the correction term reflecting the difference between a natural fire in a laboratory and real life service conditions,

(c) writing the true resistance or load-bearing capacity,  $R$ , of the fire-exposed beam

$$R = (\varphi_n + \Delta\varphi_1 + \Delta\varphi_2)M \quad (16)$$

where  $\varphi_n$  = the design value of the load-bearing capacity, calculated according to creep-deflection theory [7] as a function of  $T_n$ ,  $\Delta\varphi_1$  = the uncertainty, measured by a comparison between the theoretical value,  $\varphi_n$ , and laboratory tests.  $\Delta\varphi_1$  is based on known values of yield strength at room temperature, but includes scatter due to variability of material



properties at elevated temperature, creep parameters, etc.,  $\Delta\varphi_2$  = the uncertainty due to the difference between laboratory tests and *in situ* fire exposure,  $M$  = random factor, expressing uncertainty in material strength, expressed as yield strength at room temperature,

(d) expressing the load effect  $S$  as

$$S = E(L + D) \tag{17}$$

where  $E$  is a random variable expressing the dispersion in load effect prediction.  $L$  and  $D$  describe the basic variability of the live and dead loads, respectively, and, with statistics taken from literature.

It was possible to calculate component and total variances, values of safety indices,  $\beta$ , and probability of failure,  $P_{fail}$ , as a function of input parameters. Two different computational methods were used and compared: Monte Carlo simulation and truncated Taylor series expansion.

Here, two examples will be given of the identification of error or variability terms, *viz.*,  $\Delta T_2$  and  $\Delta\varphi_1$ .

$\Delta T_2$ , given by Fig. 4, was obtained by comparing design values of maximum steel temperature and corresponding values from 97 internal, free-standing and insulated columns exposed to natural burn-out tests.

The error term,  $\Delta\varphi_1$  (Fig. 5), was determined by comparing the load-bearing capacity of 41 tested steel beams with the design model capacity.  $\Delta\varphi_1$  is expressed in units  $L_c = \gamma_o(L_n + D_n) =$  the uniformly distributed serviceability limit load of the beam.

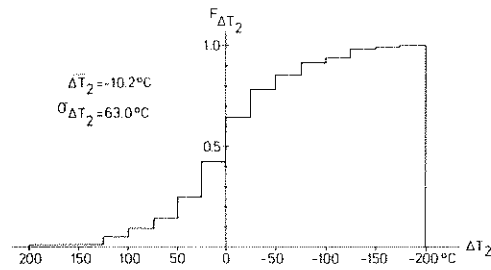


Fig. 4. Cumulative distribution curve of error term  $\Delta T_2$ , describing the prediction of error in compartment fire theory and heat transfer analysis.

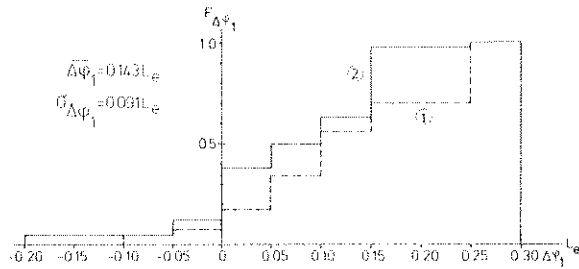


Fig. 5. Cumulative distribution function for the uncertainty term  $\Delta\varphi_1$ . Curves ① and ② refer to different limit state criteria.

The results obtained are further illustrated in Fig. 6, showing the separation of the total variance in maximum steel temperature,  $T_{max}$ , into the component variances as a function of the insulation parameter  $\kappa_n = A_i\lambda_i/(V_s d_i)$ .  $A_i$  is the interior jacket surface area of the insulation per unit length,  $d_i$  is the thickness of the insulation,  $\lambda_i$  is the thermal conductivity of the insulating material, corresponding to an average value for the whole process of fire

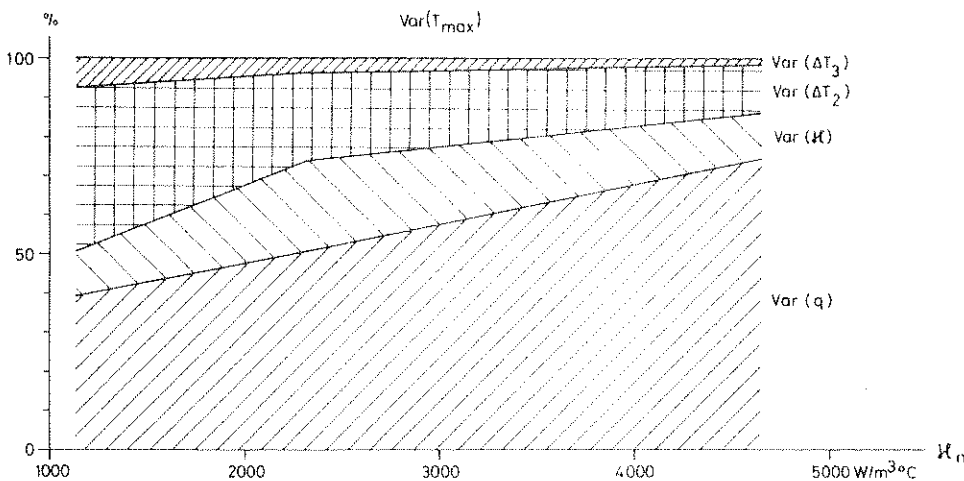


Fig. 6. Separation of total variance in  $T_{max}$  into component variances as a function of insulation parameter  $\kappa_n$  [6].

exposure, and  $V_s$  is the volume of the steel structure per unit length. Increasing  $\kappa_n$  expresses a decreased insulation capacity. The component variances refer to the stochastic character of the fire load density,  $q$ , the uncertainty in the insulation properties,  $\kappa$ , the uncertainty reflecting the prediction error in the theory of compartment fires and heat transfer from the fire process to the structural member,  $\Delta T_2$ , and a correction term reflecting the difference between a natural fire in a laboratory and that in real life service conditions,  $\Delta T_3$ .

Similarly, Fig. 7 illustrates the separation of the total variance in the load-bearing capacity,  $R$ , into component variances as a function of the insulation parameter,  $\kappa_n$ . The component variances refer to the variability in the maximum steel temperature,  $T_{max}$ , the variability in material strength,  $M$ , the uncertainty re-

flecting the prediction error in the strength theory,  $\Delta\varphi_1$ , and the uncertainty due to the difference between laboratory tests and *in situ* fire exposure,  $\Delta\varphi_2$ .

As a final example of computed results, Fig. 8 shows the variation of safety index,  $\beta$ , with insulation parameter,  $\kappa_n$ , for three values of the quantity  $D_n/L_n$  = denoting the ratio of nominal values of dead and live loads, respectively. The opening factor of the fire compartment,  $A\sqrt{h}/A_t = 0.08 \text{ m}^{1/2}$ .

#### 4. RELIABILITY STUDIES OF FIRE-EXPOSED RC-COLUMNS

Studies of reinforced concrete structures are scarce; in fact, the one publication found is a summary work progress report, published in 1979 [9]. Due to the condensed nature of

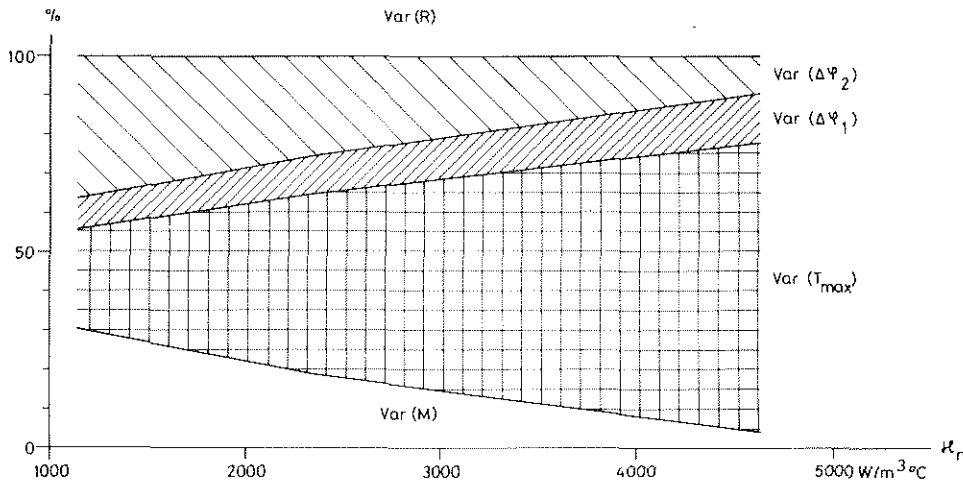


Fig. 7. Break down of total variance in load-carrying capacity  $R$  into component variances as a function of insulation parameter  $\kappa_n$  [6].

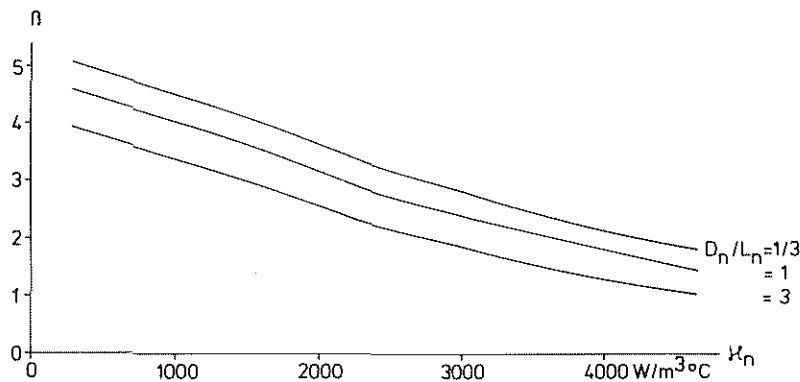


Fig. 8. The variation of safety index  $\beta$  with insulation parameter  $\kappa_n$  and ratio  $D_n/L_n$  for opening factor  $A\sqrt{h}/A_t = 0.08 \text{ m}^{1/2}$ .  $L_n$  is assumed to have a fixed value of  $2.0 \text{ kN/m}^2$  office floor area.

the report, only a few results are shown here. Figure 9 illustrates the break down of total resistance variance into component variances as a function of slenderness ratio,  $\lambda$ , for an eccentrically compressed, reinforced concrete column. The component variances are related to the following stochastic variables:  $f_c$  = strength of concrete at room temperature,  $f_s$  = strength of steel at room temperature,  $b$  = width of cross section,  $h$  = height of cross section,  $x_t$  = position of tensile reinforcement,  $x_c$  = position of compressive reinforcement,  $f_{s,T}$  = yield stress of steel as a function of temperature,  $T$ ,  $K_c$  = thermal conductivity of concrete.

Figure 10, additionally, outlines the dependence of safety index,  $\beta$ , on slenderness ratio,  $\lambda$ , for the column specified in Fig. 9. The  $\beta$  value is applied with reference to a limit state criterion defined as survival of a 60 and 90 min exposure to the standard fire endurance test.

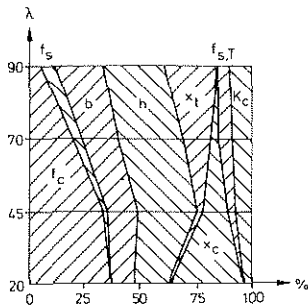


Fig. 9. Separation of total variance in resistance or load bearing capacity,  $R$ , into component variances as a function of slenderness ratio,  $\lambda$ . Eccentrically compressed, reinforced concrete column. Concrete B25, percentage of reinforcement  $\mu = 0.2\%$ ,  $b = h = 30$  cm, eccentricity  $e = 0.2 h$  [9].

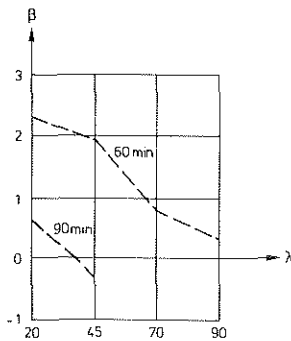


Fig. 10. Variation of safety index  $\beta$  with slenderness ratio  $\lambda$  for column specified in Fig. 9.

## 5. RELIABILITY STUDIES OF FIRE-EXPOSED WOOD STRUCTURAL ELEMENTS

A second moment reliability analysis of fire exposed wood joist assemblies was recently published in ref. 10. By using non-linear least-square regression analysis on 42 full-scale tests, a time-to-failure model was developed, predicting the deterministic value of resistance,  $R$ . The corresponding loading parameter,  $S$ , was defined in this paper as the duration of the ventilation controlled fire predicted by the fire load, window area, and height, assuming constant rate of burning.

Using eqns. (3) and (4), expressions describing total system and component variances were developed, which, when quantified, lead to a determination of the safety index,  $\beta$ .

## 6. SAFETY INDICES AND CONSISTENCY OF SAFETY LEVELS INHERENT IN DIFFERENT DESIGN PROCEDURES FOR FIRE-EXPOSED STEEL STRUCTURES

### 6.1. Standard design procedure, based on fire endurance testing

When calculating the safety index,  $\beta$ , inherent in the standard design procedure, regard must be taken of the variability in the endurance test procedure. This variability was evaluated in ref. 6, and, additionally, a subjective uncertainty measure was put on the influence of the difference between gas temperature-time curves in the furnace and natural fire exposure conditions (rate of heating, absence of decay period). The results for the safety index,  $\beta$ , are illustrated in Fig. 11, similar to Fig. 8.

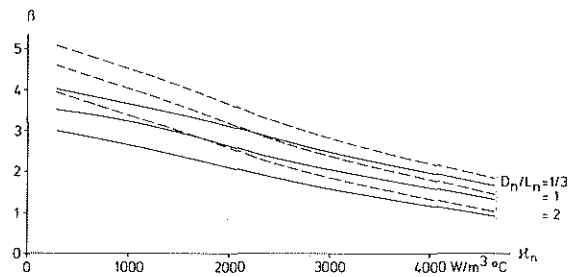


Fig. 11. Variation of safety index  $\beta$  with insulation parameter  $\kappa_n$  and ratios  $D_n/L_n$  for a steel beam designed according to a standard fire endurance test. For comparison, the corresponding curves from Fig. 8 are shown (---).  $A\sqrt{h}/A_t = 0.08 \text{ m}^{1/2}$ .

TABLE 1

Required value of insulation parameter,  $\kappa_{st}$ , and range of variation for safety index,  $\beta$ , implied in different fire endurance ratings according to the standard test

Fire endurance rating (min)	30	60	90	120
Required value of $\kappa_{st}$ ( $\text{W/m}^3 \text{ } ^\circ\text{C}$ )	4080	1415	785	550
Range of variation in $\beta$	0.52 - 2.66	1.77 - 3.69	2.34 - 3.89	2.58 - 3.96

Using the calculated results, the range of variation in safety index  $\beta$  for different fire endurance ratings is given in Table 1, which also shows the required values of the insulation parameter,  $\kappa$ .

The ventilation conditions of the natural fire exposure is assumed to be in the range  $A\sqrt{h}/A_t = 0.04 - 0.12 \text{ m}^{1/2}$ , and the  $D_n/L_n$  ratio in the range 1/3 - 3, creating the  $\beta$  variation in Table 1.

## 6.2. Differentiated Swedish design procedure

### 6.2.1. Nominal value and load factor for dead and live load

The general design inequality can be written

$$R_{n,f} \geq \gamma_{D,f} D_{n,f} + \gamma_{L,f} L_{n,f} \quad (18)$$

where  $R_{n,f}$  = nominal or design value of minimum resistance (load-bearing capacity) during fire exposure,  $D_{n,f}$ ,  $L_{n,f}$  = nominal values of dead and live loads, respectively,  $\gamma_{D,f}$ ,  $\gamma_{L,f}$  = load factors, to be applied to the nominal loads.

For office buildings, the following values for nominal live load  $L_{n,f}$  and load factors  $\gamma_{D,f}$ ,  $\gamma_{L,f}$  are prescribed [7].

Nominal values of live load		Load factors	
Not moveable	Moveable	$\gamma_{D,f}$	$\gamma_{L,f}$
0.35 kN/m <sup>2</sup>	1.00 kN/m <sup>2</sup>	1	1.4

These figures for  $L_{n,f}$  apply to the case where a complete evacuation of personnel during a fire cannot be anticipated. The nominal value of the dead load  $D_{n,f}$  is to be put equal to the mean dead load  $\bar{D}$ .

Measuring  $R_{n,f}$  in  $L_e$ -units, the design inequality can be written

$$R_{n,f} \geq \frac{\gamma_{D,f} D_{n,f} + \gamma_{L,f} L_{n,f}}{\gamma_o(L_n + D_n)} \quad (19)$$

Using the nominal loads, load factors, and overall strength factor given earlier, the design

working stress level during a fire exposure will be, expressed in  $L_e$ -units, 0.658, 0.648 and 0.640 for  $D_{n,f}/L_n = D_n/L_n = 3, 1,$  and  $1/3$ , respectively. This implies that the design maximum steel temperature will be almost independent of the  $D_n/L_n$ -ratio and equal to 520 °C.

### 6.2.2. Nominal value and load factor for fire load density

For those types of building occupancies where a representative fire load survey has been made, the Swedish Building Code stipulates that the nominal value of fire load density,  $q_n$  = a value signifying the 80 per cent level of the corresponding cumulative distribution function.

To this value of  $q_n$  must be added the heat contents,  $q_o$ , of combustible material in the structural elements and of any combustible finishing material such as wall-to-wall carpeting, etc., which are not included in the statistical survey. The load factor applied on the nominal fire load density = 1. For offices, assuming that the heat contents,  $q_o$ , of the structural fire load is negligible,

$$q_n = 138.2 \text{ MJ/m}^2.$$

### 6.2.3. Safety index $\beta$

Table 2 gives the appropriate value of  $\kappa_n$  as well as the resultant range of safety index,  $\beta$ , for various opening factors  $A\sqrt{h}/A_t$ .

The nominal loads,  $D_{n,f}$  and  $L_{n,f}$ , and load factors,  $\gamma_{D,f}$  and  $\gamma_{L,f}$ , were chosen to give a design stress level equal to the level prescribed for the standard fire endurance test, independent of the ratio  $D_n/L_n$ . The differing statistics of the dead and live load effects make the resulting  $\beta$  values dependent on the ratio  $D_n/L_n$ . The next Section will illustrate how statistically consistent load factors can be derived to match a pre-determined safety level (safety index).

TABLE 2

Required value of insulation parameter,  $\kappa_n$ , and range of variation for safety index,  $\beta$ , implied in differentiated Swedish design procedure

	$A\sqrt{h}/A_t \text{ (m}^{1/2}\text{)}$		
	0.04	0.08	0.12
$\kappa_n \text{ (W/m}^3 \text{ }^\circ\text{C)}$	2020	3380	4420
Range of $\beta$	1.66 - 2.56	1.64 - 2.56	1.92 - 2.84

### 6.3. Derivation of statistically consistent partial factors in the differentiated design

The  $\beta$  value derived in Table 2 for  $A\sqrt{h}/A_t = 0.08 \text{ m}^{1/2}$  and  $D_n/L_n = 1$ , is equal to 2.16. Assuming this is a "socially acceptable" level of safety, and utilizing the results exemplified in Section 3.2, the corresponding values for  $\phi_f$ ,  $\gamma_{D,f}$  and  $\gamma_{L,f}$  in eqn. (7) can be calculated by the methodology outlined in Section 2.3. Details of the calculations may be found in ref. 6. The final results are

$$\phi_f = 0.77 \quad (20)$$

$$\gamma_{D,f} = 1.14 \quad (21)$$

$$\gamma_{L,f} = 1.88. \quad (22)$$

Conventionally changing the value of  $\phi_f$  to unity, the values of  $\gamma_{D,f}$  and  $\gamma_{L,f}$  will be  $\gamma_{D,f} = 1.48$  and  $\gamma_{L,f} = 2.44$ , respectively.

Choosing  $\gamma_{D,f} = 1.5$  and  $\gamma_{L,f} = 2.5$ , and applying the load factors on mean values of dead and live load, the degree of utilization of the cross-section will be  $\approx 0.57$ ,  $0.72$ , and  $0.86$  for  $D_n/L_n = 1/3$ ,  $1$ , and  $3$ , respectively, and the corresponding critical steel temperatures  $\sim 540$ ,  $500$  and  $420$   $^\circ\text{C}$ . Design values,  $\kappa_n$ , of the insulation parameter  $\kappa$  will be approximately  $3650$ ,  $3050$  and  $2050 \text{ W/m}^3 \text{ }^\circ\text{C}$ . Table 3 gives a comparison between the range of values of the safety index,  $\beta$ , as compared with the range of values given by present design

TABLE 3

Safety index  $\beta$  and probability of failure  $P_{\text{fail}}$  for different design procedures, applied to an insulated, simply supported steel beam as part of a floor or roof assembly in office buildings

Design procedure	Range of $\beta$	Range of $P_{\text{fail}}$	$(P_{\text{fail}})_{\text{max}}/(P_{\text{fail}})_{\text{min}}$
I. Classification, standard endurance test	1.77 - 3.69	$(1 - 400)10^{-4}$	$\sim 400$
II. Present Swedish design model	1.66 - 2.84	$(23 - 500)10^{-4}$	$\sim 20$
III = II, improved by statistically derived load factors	2.35 - 2.45	$(72 - 95)10^{-4}$	$\sim 1.5$

methods and partial factors. It is emphasized that the load factor evaluation is based on mean values of dead and live load.

The corresponding range of the probability of failure,  $P_{\text{fail}}$ , is also shown in Table 3. Related to this quantity, the difference between the three design procedures is extremely striking, with the respective ratios  $(P_{\text{fail}})_{\text{max}}/(P_{\text{fail}})_{\text{min}} \sim 400$ ,  $20$ , and  $1.5$ . The  $P_{\text{fail}}$  values presented are related to a probability = 1 for a fire outbreak leading to flashover within the fire compartment.

## 7. FUTURE DEVELOPMENT OF A RATIONAL STRUCTURAL FIRE ENGINEERING DESIGN

A further development of the structural fire engineering design procedure described is at present in progress in connection with a three year project aimed at a design manual for fire exposed, load-bearing, reinforced and prestressed concrete structures or structural members. A primary objective of the development is to arrive at a design procedure in regard to fire exposure, which is in principal agreement with modern loading and safety philosophy for the non-fire state. A subsidiary condition is that the level of the functional requirements laid down for the fire engineering design is differentiated with regard to such influences as the occupancy, the height and volume of the building, and the importance of the structure or the structural member for the overall stability of the building. This differentiation must be consistent with the prevalent design method, based on fire classification.

### 7.1. Summary description of design procedure

In summary, the design procedure under development can be described as follows — Fig. 12.

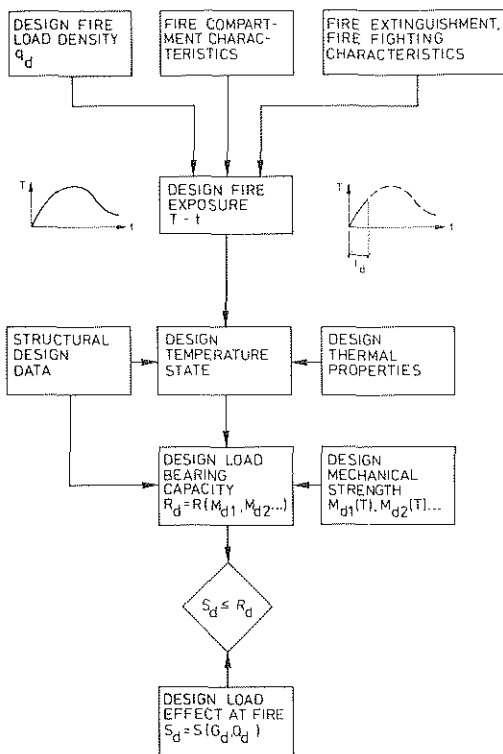


Fig. 12. Reliability based, differentiated, structural fire engineering design procedure under development.

The design fire load density, the fire compartment characteristics, and the fire extinguishment and fire fighting characteristics constitute the basis for the determination of the design fire exposure, given as the gas temperature-time curve,  $T-t$ , of the fully developed compartment fire. Depending on the type of practical application, the load bearing function of the structure or structural member will then be required to comply with either the complete fire process or a limited part of the fire process,  $t_d$ , determined from the time necessary for the fire to be extinguished under the most severe conditions, or from the design evacuation time for the building.

Together with the structural design data, the design thermal properties, and the design mechanical strength of the structural material, the design fire exposure provides the design temperature state and the related design load-bearing capacity,  $R_d$ , for the lowest value of the load-bearing capacity during the relevant fire process.

A direct comparison between the design load-bearing capacity,  $R_d$ , and the design load effect at the fire,  $S_d$ , finally decides whether

or not the structure or structural member can fulfil its required function on exposure to fire.

## 7.2. Functional requirements. Determination of design loads and design load effect

In a design for the ultimate limit state, the functional requirement implies that the design load effect,  $S_d$ , must be smaller than, or equal to, the design load-bearing capacity,  $R_d$ . The load effect then can be, for instance, a moment or a force in a cross section of the structure or an axial force in a structural member. The requirement applies to all relevant types of failure — bending failure, shear failure, instability failure in the form of buckling, lateral buckling, flexural-torsional buckling, etc.

The design consists of an analysis of simultaneous exposure to static loading and fire. The determination of the static loading, and the associated design load effect,  $S_d$ , then follows the procedure according to Fig. 13. The determination begins with characteristic permanent and variable load values,  $G_k$  and  $Q_k$ .  $G_k$  and  $Q_k$  are not identical with  $D_n$  and  $L_n$  used in preceding Sections, thus requiring different notations. The characteristic value of the permanent load,  $G_k$ , will be chosen as the average, and the characteristic value of a variable load,  $Q_k$ , as that corresponding to a probability of excess at least once a year. The characteristic  $Q_k$  values may be differentiated according to whether a complete evacuation of people can be assumed or not in the event of fire.

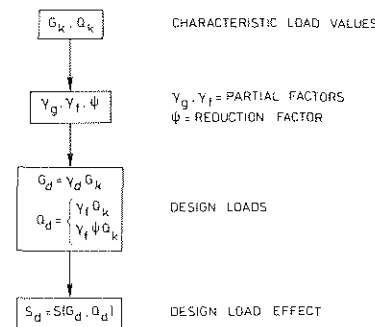


Fig. 13. Procedure for determination of design load effect,  $S_d$ .

A multiplication by partial factors,  $\gamma$ , and reduction factors,  $\psi$ , transfers the characteristic load values to design loads  $G_d$  and  $Q_d$ . By

using the partial factors  $\gamma$ , the following effects are taken into consideration:

- (i) the probability that the load differs unfavourably from the characteristic value,
- (ii) the uncertainty of the model describing the load — for instance, with regard to the distribution of the load over the structure,
- (iii) such uncertainties of the design model which are independent of material.

The partial factors  $\gamma$ , furthermore, depend on the type of loading and the appropriate load combination.

The reduction factors,  $\psi$ , give expression to the relative duration of a variable load. Some examples of  $\psi$  values, specified in a Draft Swedish Building Code, are given in Table 4.

The exposure of a structure or structural member to combined static loading and fire will be considered as an accidental case. The Draft Building Code allows for this when specifying that the design load effect shall be calculated for the most unfavourable combination of the design loads  $G_d$  and  $Q_d$ , with the partial factors,  $\gamma$ , chosen according to Table 5.

The  $\gamma$  values 1.0 and 0.8 for the permanent load are alternative values to be applied in such a way that the most unfavourable load effect is taken into account. The same type of load — for instance, dead load — will always be given the same  $\gamma$  value. The number of

variable loads with  $\psi \leq 0.5$  may be limited to one. No corresponding limitation is allowed for the number of variable loads having  $\psi > 0.5$ .

### 7.3. Categories of structures or structural members. Design fire exposure [11]

As mentioned above, the functional requirements to be laid down for fire engineering design should be differentiated with respect to such effects as the occupancy, the height and volume of the building, and the importance of the structure or structural member to the overall stability of the building. This can be achieved by dividing the structures or structural members into categories, with a related differentiation of the design fire load density,  $q_d$ , and the length of the fire process, to be considered in the design.

In the version of the design procedure under development, three categories, K1, K2, and K3 have been introduced and defined according to Table 6. The Table relates the different categories and the fire endurance in minutes — F30, F60 and F90 — required in the current design, based on classification and results of standard fire endurance tests. For fire safe buildings, the relation applies to the fire endurance requirements specified for the range of the characteristic fire load density  $q_k \leq 200 \text{ MJ m}^{-2}$ . For other types of buildings, the association is generally straightforward.

For the different categories, the design fire exposure will be chosen according to Table 7, specifying the design fire load density,  $q_d$ , in relation to the characteristic fire load density  $q_k$ , and the duration of the fire process. The characteristic fire load density  $q_k$  then, is defined as that value corresponding to a probability in excess of 20%. The related gas temperature-time curves of the fire exposure are specified in accordance with Fig. 3, with due

TABLE 4

Examples of  $\psi$  values, specified in Draft Swedish Building Code, for moveable part of loading  
For unmoveable part of loading,  $\psi = 1$ .

Loading	$\psi$
Live load in dwellings and hotels	0.33
Live load in offices	0.5
Live load in schools	0.8
Live load in assembly rooms	0.8
Live load in libraries	1.0
Snow loading, depending on snow zone	0.6 - 0.8
Wind loading	0.25

TABLE 5

Partial factors,  $\gamma$ , for combined exposure to static loading and fire

Loading	Partial factor, $\gamma$
Permanent load $G_k$	1.0 or 0.8
Variable loads $\psi Q_k$	1.0

TABLE 6

Definition of categories of structures or structural members

Fire endurance in minutes, required in prevalent design, based on classification	Category
—	K0
F30	K1
F60	K2
F90	K3

TABLE 7

Design fire exposure, expressed by the design fire load density  $q_d$

Category of structural member	Design fire load density, $q_d$	Duration of fire exposure
K1	$1.0 q_k$	$\leq 30$ min
K2	$1.0 q_k$	complete fire process
K3	$1.5 q_k$	

consideration being taken of the influence of the thermal properties of the structures enclosing the fire compartment.

By specifying the design fire exposure as described, consideration is taken of:

(i) the probability that the fire load density differs unfavourably from the characteristic value,

(ii) the uncertainty of the analytical model for the determination of the compartment fire and its thermal exposure on the load bearing structure or structural member,

(iii) the uncertainty in specifying the geometry and thermal properties of actual fire compartment materials,

(iv) the safety level required for the respective categories of structure or structural member.

A rough estimation, carried out for some simple types of load bearing structural members, shows that the probability of failure is about one tenth of an order of magnitude at a design for  $q_d = 1.5 q_k$  than for a design where  $q_d = q_k$  [11].

The probability, and the consequences of a fire outbreak are strongly influenced by various types of active fire protection measures such as fire detection systems, sprinkler systems, smoke control systems, roof venting systems, fire alarm systems, and the fire fighting facilities of the fire brigade. The present state of knowledge does not allow for such influences to be included in any sophisticated way in the specification of the design fire exposure. For the described design procedure, discussions are in progress concerning whether the presence of an approved sprinkler system could be taken into account by transferring a structure or structural member to the next lower category.

#### 7.4. Design mechanical strength and design load bearing capacity

The calculation of the ultimate design load bearing capacity,  $R_d$ , of a structure or structural member will be based on the design strength values,  $M_d$ , of the actual structural materials. These strength values are given by the corresponding characteristic strength values,  $M_k$ , divided by a resulting partial factor,  $\gamma_{mn}$ . Normally, the characteristic value is made equal to the lower 5 per cent fractile, as concerns strength.

In a non-fire design for the ultimate limit state, the determination of the design strength follows the procedure according to Fig. 14. The different partial factors  $\gamma_{m1}$ ,  $\gamma_{m2}$ ,  $\gamma_{m3}$  and  $\gamma_n$  express the influences of:

(i) the probability that the value of the material property differs unfavourably from the characteristic value —  $\gamma_{m1}$ ,

(ii) the uncertainty of the model for calculation of the ultimate load bearing capacity, including the influence of such deviations of measurement as are not to be considered separately —  $\gamma_{m2}$ ,

(iii) the uncertainty of the relation between the properties of the material in the structure and the corresponding material properties determined in the test —  $\gamma_{m3}$ ,

(iv) the safety class —  $\gamma_n$ .

The predicted extent of personal and property damage at failure — not serious (class 1), serious (class 2) and very serious (class 3) — decides the choice of the safety class and the connected  $\gamma_n$  value.

By introducing various categories of structure and structural members when specifying the design fire load density and the design fire exposure, the influence of different safety

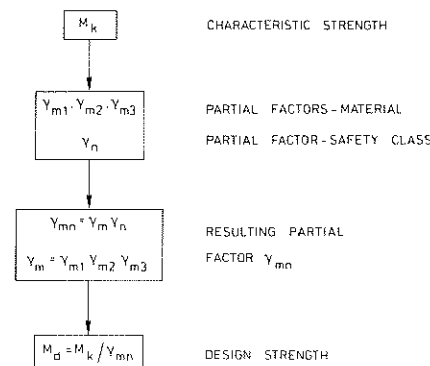


Fig. 14. Procedure for determination of design strength,  $M_d$ , at non-fire ultimate limit state.



classes is already covered. Consequently, the partial factor  $\gamma_n$  is to be made equal to 1 on transferring the described procedure for the determination of the design strength to the fire design situation. The material-related partial factors,  $\gamma_{m1}$ ,  $\gamma_{m2}$ , and  $\gamma_{m3}$ , depend on the type of limit state, type of loading, and type of structural material. This may be exemplified by the values given in ref. 12 where fire exposure is regarded as an accidental loading case, among others. For a fire exposed reinforced concrete structure or structural member, designed according to the ultimate limit state, the resulting material partial factor,  $\gamma_m$ , is prescribed = 1.2 for the compressive strength of concrete and = 1.0 for the tensile strength of the steel reinforcement.

## LIST OF SYMBOLS

$A$	Area of vertical opening in fire compartment	$R$	Resistance or load bearing capacity
$A_i$	Fire exposed surface area of steel member	$S$	Load effect
$A_t$	Total bounding surface area of fire compartment (walls, floor and ceiling)	$T_n$	Nominal (design) value of maximum steel temperature
$A\sqrt{h}$	Air flow factor (ventilation factor)	$T_{max}$	Final value of maximum steel temperature
$A\sqrt{h}/A_t$	Opening factor	$U$	Standardized safety margin
$D$	Dead load intensity	$V_s$	Steel volume of structural member
$D_n$	Nominal value of dead load intensity in non-fire structural design	$\alpha$	Heat transfer coefficient, linearization factor
$D_{n,f}$	Nominal value of dead load intensity in fire structural design	$\beta$	Safety index according to Cornell
$d_i$	Thickness of insulation	$\gamma$	Partial factor
$E$	Load effect prediction error factor	$\gamma_o$	Overall safety factor in allowable stress design
$F$	Probability distribution function (= cumulative distribution function)	$\gamma_D$	Load factor for dead load intensity in non-fire structural design
$G$	Permanent load	$\gamma_{D,f}$	Factor corresponding to $\gamma_D$ in structural fire design
$h$	Height of vertical opening in fire compartment	$\Delta T_1$	Uncertainty term defined by eqn. (15)
$L$	Live load intensity	$\Delta T_2$	Uncertainty term defined by eqn. (15)
$L_e$	Uniformly distributed load level of a simply supported steel beam = $\gamma_o(L_n + D_n)$	$\Delta T_3$	Uncertainty term defined by eqn. (15)
$L_n$	Nominal value of live load intensity in non-fire structural design	$\Delta \varphi_1$	Uncertainty term defined by eqn. (16)
$L_{n,f}$	Nominal value of live load intensity in fire structural design	$\Delta \varphi_2$	Uncertainty term defined by eqn. (16)
$M$	Material uncertainty factor	$\gamma_L, \gamma_{L,f}$	Load factors for live load intensity
$P_{fail}$	Probability of failure	$\kappa$	Insulation parameter ( $A_i/V_s \cdot \lambda_i/d_i$ )
$Q$	Variable load	$\sigma$	Standard deviation
$q$	Fire load density	$\varphi$	Resistance or load-carrying capacity of fire-exposed steel beam
		$\phi$	Strength factor (capacity reduction factor)
		$\psi$	Reduction factor
		$d$	Design
		$f$	Fire
		$n$	Nominal
		$k$	Characteristic
		<i>Superscript</i>	
		$\bar{D}$	Mean value of dead load intensity, etc.

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