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Indoor Multi-User MIMO: Measured User Orthogonality and Its Impact on the Choice of Coding

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Abstract— **In this paper we study propagation properties of multi-user MIMO systems in narrow indoor corridor environments. From extensive channel measurements, it is observed that the channels to different users often experience very high or very low orthogonality. As a consequence, the gain of optimal dirty-paper-coding, or any other coding scheme, over linear processing techniques becomes insignificant.**

I. Introduction

Multi-user multiple-input multiple-output (MU-MIMO) systems is the wireless industy's current frontier towards satisfying the increasing demand of wireless high-speed services. The optimal signal processing technique in the downlink of a MU-MIMO system is dirty-paper coding (DPC) [1], [2]. Unfortunately, DPC is far too complex for practical implementation, and merely serves as the theoretical benchmark. Most practical signal processing techniques that have been proposed are linear [3]–[5], except for the popular vector-perturbation based schemes [6]. In general, linear processing results in capacity losses (in the Shannon sense) compared to DPC, but is still preferred due to complexity reasons. Previous papers, [7], [8], that compared DPC with linear processing have mainly focused on scaling laws. No attention has been paid to whether realistic propagation properties will change the conclusions.

This paper considers indoor MU-MIMO systems in narrow corridor environments, typically encountered in office buildings, universities, hospitals, etc. We start by performing a theoretical investigation of downlink signal processing techniques in order to establish what channel parameters influence the performance. Then we analyze extensive channel measurements in order to obtain an understanding of the behavior of the predominant parameter(s). Measurements are only available for the 2-user case, which will be the case studied throughout.

The main conclusions of the paper will be the following:

- The measured channels, to different users, often show almost full or no orthogonality.
- Low-complex linear processing performs well in terms of capacity at these orthogonality extremes.
- High-complex processing (e.g. dirty-paper coding) only have a clear advantage in the mid region of orthogonality, but the extra complexity may not motivate the relatively small gains.

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II. Channel Measurements

A large amount of double-directional MIMO channel measurements, in different indoor scenarios, have been carried out in the WILATI+ project. Due to space limitations, we focus on a single case that we consider to be representative. The obtained conclusions from this single case hold true for other scenarios as well.

In their raw form, the measurements contain the combined transfer function of both the propagation channel and the antenna arrays used during the sounding. The sounding antenna arrays used were both cylindrical and spherical. When evaluating MIMO system performance, based on measurements, it is of interest to eliminate as much as possible the influence from different antenna configurations. The channel data for a selected set of measurements have therefore been processed [9] so that they correspond to horizontal 4x4 square antenna arrays being used at both ends. The square arrays consist of vertical $\lambda/2$ dipoles with element spacing $\lambda/2$.

The measurements we discuss in this paper were carried out in a narrow corridor environment, see Figure 1, and the channel sounding equipment is a combination of the sounders from Aalto University/TKK and Lund University, where dynamic multi-link MIMO channel measurements were performed. There are two receivers, Lund University Rx (LURx) and TKK Rx (TKKRx), and two moving transmitters (dashed lineas marked with 1 and 4 in the map). We will assume perfect channel reciprocity and let LURx and TKKRx act as base stations while the moving transmitters will be thought of as users. The scenario consists of two routes along which the transmitters are moved, Route 1 and Route 4. Route 1 starts in the side corridor, rounds the corner, and continues into the main corridor, towards the TKKRx. Route 4 stays entirely in the main corridor and the movement is from the LURx to the TKKRx. Worth noting is that in Route 1, the rounding of the corner takes place after around half the measurement route. This is one of the points of interest in the upcoming analysis,

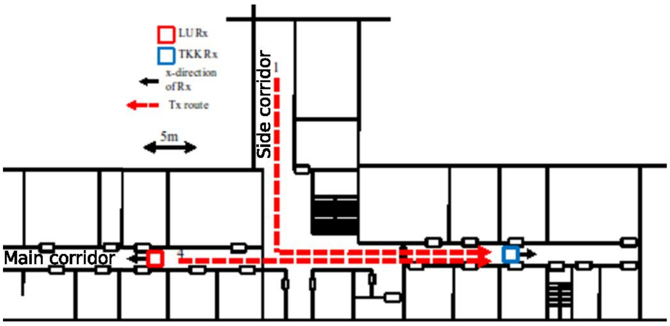

Fig. 1. Map of the measurement scenario. The two users are moving along the dashed lines. The squares show the location of the base stations.

since the propagation changes significantly in this area. The measurements were performed at 5.3 GHz with a bandwidth of 120 MHz. These measurements were then converted, by our project partners at Aalto University, to the 4×4 square array of $\lambda/2$ dipoles and the bandwidth was reduced to 40 MHz.

The measurement routes were sampled and there are 640 snapshots for route 1 and 820 snapshots for route 4. In plots to follow in later sections, a particular user location will be represented by a snapshot index, i.e., snapshot 320 for route 1 means a user location at half the physical measurement route.

III. Performance analysis using channel capacity

A. *Link orthogonality and down-link performance*

We consider the down-link of a MU-MIMO system with 2 users and a base station equipped with M antennas. The $2 \times M$ channel matrix is denoted $\boldsymbol{H}$. The channel model used is

$$\boldsymbol{y} = \sqrt{\rho}\boldsymbol{H}\boldsymbol{s} + \boldsymbol{n},$$

where user k receives the k-th component of the vector $\boldsymbol{y}$, ρ is a measure of the signal-to-noise-ratio (SNR) and absorbs various normalization constants. The channel matrix is assumed to have entries with unit average energy[1], and the vector $\boldsymbol{s}$ is data symbols, each entry with unit average energy. The noise vector $\boldsymbol{n}$ is assumed to comprise zero-mean, unit-variance, circular symmetric complex Gaussian random variables.

It will be useful to define the Gram matrix $\boldsymbol{G}$ associated to matrix $\boldsymbol{H}$, i.e. $\boldsymbol{G} = \boldsymbol{H}\boldsymbol{H}^H$ as

$$\boldsymbol{G} = \begin{bmatrix} 1 & \delta \\ \delta^H & g \end{bmatrix},$$

where the one in the upper left corner is determined by the channel energy being absorbed into the constant ρ. Further, for convenience we assume that $g \leq 1$. It follows that $|\delta| \leq \sqrt{g}$ in order to obtain a positive definite matrix $\boldsymbol{G}$. In what follows we will investigate the loss of simple linear processing compared with the optimal DPC approach with respect to the values ρ, δ, and g. We will focus on sum-rate capacity, and not on the capacity region. We first study the optimal DPC technique.[2]

It has been shown, see e.g. [10], that the DPC capacity is found by solving

$$C_{\text{DPC}} = \max_{\gamma_1,\gamma_2} \log\det\left(\boldsymbol{I} + \rho\boldsymbol{H}^H\boldsymbol{D}_\gamma\boldsymbol{H}\right), \tag{1}$$

subject to

$$\gamma_1 + \gamma_2 = 1.$$

The 2×2 matrix $\boldsymbol{D}_\gamma$ is a diagonal matrix and contains γ_1 and γ_2 on its main diagonal. It can be shown that the optimal value of γ_1 is

$$\gamma_1^{\text{opt}} = \begin{cases} \frac{1}{2} + \frac{1-g}{2\rho(g-|\delta|^2)} & |\delta|^2 \leq \delta_{\text{tr}} \\ 1 & |\delta|^2 > \delta_{\text{tr}}, \end{cases} \tag{2}$$

where

$$\delta_{\text{tr}} = \frac{g(2\rho+1)-1}{2\rho}.$$

Inserting (2) into (1) gives

$$C_{\text{DPC}} = \begin{cases} \log\left(1+\rho\frac{1+g}{2} + \frac{\rho^2(g-|\delta|^2)^2+(1-g)^2}{4(g-|\delta|^2)}\right), & |\delta|^2 \leq \delta_{\text{tr}} \\ \log(1+\rho), & |\delta|^2 > \delta_{\text{tr}}. \end{cases} \tag{3}$$

It may appear as if DPC can have non-zero capacity even for $\rho = 0$ as there is a term that does not depend on ρ inside the logarithm. However, if ρ is small, then $C_{\text{DPC}} = \log(1+\rho)$ since $|\delta|^2 > \delta_{\text{tr}}$ in that case.

In the particular case of $g = 1$, the condition on $|\delta|^2$ in (3) becomes $|\delta|^2 \leq 1$, which always holds. Further, for this case C_{DPC} simplifies into

$$C_{\text{DPC}}|_{g=1} = \log\left(1 + \rho + \frac{\rho^2}{4}(1-|\delta|^2)\right). \tag{4}$$

From a complexity point of view, linear processing methods are interesting. The conceptually simplest linear method is to invert the channel matrix by means of the pseudo-inverse of the channel - so called zero-forcing (ZF). With ZF the transmitted signal is

$$\boldsymbol{s} = \frac{1}{\sqrt{\chi}}\boldsymbol{H}^+\boldsymbol{P}\boldsymbol{x},$$

where χ normalizes the energy in $\boldsymbol{s}$ to unity, and the superscript "+" denotes the pseudo-inverse. The matrix $\boldsymbol{P}$ is a power allocation matrix

$$\boldsymbol{P} = \begin{bmatrix} \sqrt{P_1} & 0 \\ 0 & \sqrt{P_2} \end{bmatrix},$$

where P_1 and P_2 are subject to optimization. Without loss of generality, we can set $P_2 = 2 - P_1$, where $1 \leq P_1 \leq 2$ (since $g \leq 1$).

It can be verified that the optimal P_1, subject to the power constraint $\mathbb{E}\,||\boldsymbol{s}||^2 = 2$, is

$$P_1^{\text{opt}} = \min\left(\frac{2-2g+2\rho g-2\rho|\delta|^2}{1-\rho|\delta|^2+\rho g^2+\rho g-2g+g^2-\rho|\delta|^2 g}, 2\right).$$

[1]Subsequently, the channel measurements will be normalized to satisfy this condition when averaged over the routes.

[2]The optimal DPC scheme is far too complex to be implemented in practice, and is merely of theoretic interest. However, there are reduced complexity schemes that can operate close to the DPC limit. Subsequently, when we claim that "we use DPC", we really mean that we use a "close-to-DPC scheme".

A few manipulations yield the capacity with ZF precoding

$$C_{\mathrm{ZF}} = \begin{cases} \log_2\left(\frac{(1+g+\rho g-\rho|\delta|^2)^2}{4g}\right), & g \geq \frac{1+\rho|\delta|^2}{1+\rho} \\ \log_2\left(1+\frac{\rho(g-|\delta|^2)}{g}\right), & g \leq \frac{1+\rho|\delta|^2}{1+\rho}. \end{cases}$$

In the particular case of $g = 1$, C_{ZF} simplifies into

$$C_{\mathrm{ZF}}|_{g=1} = 2\log\left(\frac{2+\rho-\rho|\delta|^2}{2}\right),$$

which coincides with that of DPC for $|\delta|^2 = 0$. Thus, DPC provides no gain over ZF when channel orthogonality is high.

B. Single user capacity

It can be seen in the expression for C_{ZF} that when $|\delta|^2$ grows, C_{ZF} tends to 0. In this case, a solution is to only transmit to the strongest user, in our normalization, user 1. Hence, user 2 gets zero capacity. The sum capacity becomes

$$C_{\mathrm{SU}} = \log(1+\rho),$$

which coincides with that of DPC for $|\delta|^2 = g$ (the subscript "SU" denotes "Single User"). This is natural since DPC is actually performing single user transmission whenever channel orthogonality is low.

By (careful) inspection of the sum-rate expressions, it can be seen that the maximal rate gain of DPC compared with linear precoding is at most 1.5 times. This gain is obtained by setting $g = 1$, $|\delta|^2 = 2/\rho + 1 - \sqrt{(2/\rho+1)^2 - 1}$ and by letting $\rho \to \infty$. Note that we do not refer to the SNR gain, which is unbounded as ρ grows. Further, the rather small gain is due the fact that only two users are present. With more users, the potential gains are higher. The parameters that maximizes the gain of DPC reveals that DPC provides the most advantage whenever channel orthogonality is low. However, the required SNR to get close to the 50% gain is much larger than what is usually encountered in practice. If ρ is set to 10 dB, the maximal gain becomes 1.30, achieved for $\{g, |\delta|^2\} = \{1, .5367\}$. Thus, for realistic SNRs, we expect that the largest gains of DPC compared to linear precoding methods occur whenever channel orthogonality is 'medium'. Further, it can be seen from the equations that the maximal gain reduces when there is a power difference between the two users. Since this is expected in practice, the potential of DPC shrinks even further.

In Figure 2 we show an example of the sum-rate capacities for ZF, SU, and DPC for $g = .6$ and $\rho = 10$ dB as functions of $|\delta|^2$. As can be seen, there is not much gain of using DPC whenever orthogonality is either very low or very high. In fact, by comparing C_{ZF}, C_{SU} and C_{DPC}, it is easily verified that for a fixed ρ:

$$\lim_{|\delta|^2 \to 0} \frac{SNR_{\mathrm{ZF}}}{SNR_{\mathrm{DPC}}} = 1,$$

and

$$\lim_{|\delta|^2 \to g} \frac{SNR_{\mathrm{SU}}}{SNR_{\mathrm{DPC}}} = 1.$$

The above two equations implies that DPC becomes meaningless as $|\delta|^2 \to 0$ or $|\delta|^2 \to g$.

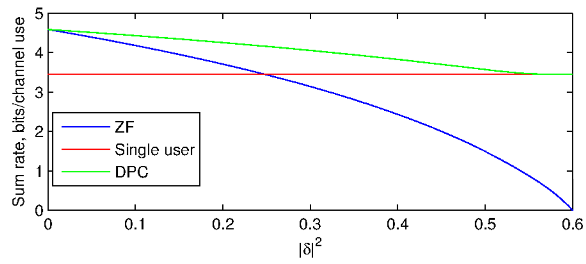

Fig. 2. Sum-rate capacities for ZF, DPC and single user transmission. g=.6 and ρ =10 dB.

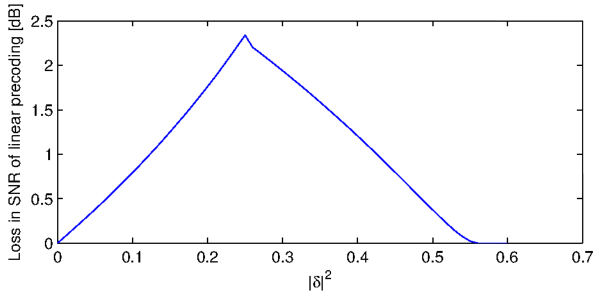

Fig. 3. Loss in SNR (dB scale) of ZF/SU compared with DPC for g=.6 and ρ =10 dB.

In Figure 3 we show the loss in SNR (i.e. the argument inside the logarithm minus 1) of ZF/SU compared with DPC. It can be seen that at 'medium orthogonality', for the selected parameters, the loss is around 2.4 dB. At high and low orthogonality, the loss vanishes.

In this section, we have analytically derived that for 2 single antenna users, the maximal sum-rate gain of DPC as compared to linear precoding is 50 %. This gain is, however, achieved asymptotically as the SNR grows. For practical values of the SNR, the maximal gain is roughly 25-30%. Further, this gain is achieved when channel orthogonality is 'medium', i.e., the propagation channel should neither be very orthogonal nor very close to a singular matrix. Whenever channel orthogonality is either low or high, DPC does not offer any significant gains. We next turn to the channel measurements in order to see how much orthogonality that is present for realistic channels. Remarkably, we will observe that the cases $|\delta|^2 \approx 0$ or $|\delta|^2 \approx g$ are often occuring in realistic indoor channels.

IV. Measurement Results

Within the single measurement scenario investigated in this paper, we provide results for three different setups. In all figures to follow, we plot average values over all sub-carriers.

A. Experiment setup 1

We first consider a single base station setup and let LURx act as base station. We assume two single antenna users that move along their two respective routes. The base station is equipped with 2 transmit antennas, and each user is equipped with a single antenna.

In Figure 4, we plot the orthogonality related coefficient $r = 1 - |\boldsymbol{h}_1\boldsymbol{h}_2^H|^2/(\|\boldsymbol{h}_1\|\,\|\boldsymbol{h}_2\|)$ for all possible user locations.

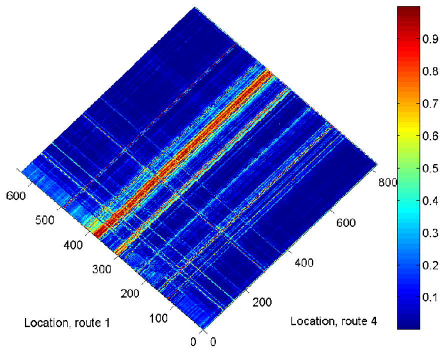

Fig. 4. Orthogonality related coefficient r of measured channels with LURx as base station. The two users are equipped with a single antenna while the base station has two antennas (Experiment setup 1).

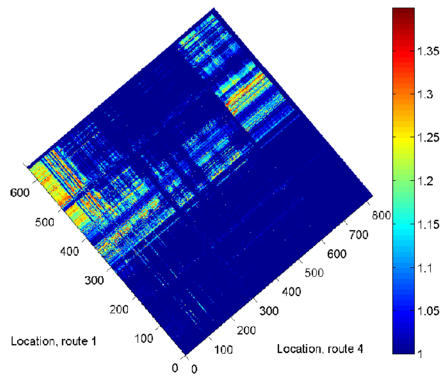

Fig. 5. Gain in sum-rate capacity of DPC over linear precoding in measured dual-link channels (Experiment setup 1).

The vectors $\boldsymbol{h}_1$ and $\boldsymbol{h}_2$ are the channels to user 1 and user 2 respectively. Note that the relationship between r and $|\delta|^2$ is $1 - r = |\delta|^2/g$. It is seen from the figure that $r \approx 0$ for most user locations. This means that $|\delta|^2 \approx g$, so that the two links are heavily correlated for most user locations. As a consequence, the optimal signal processing will be single user transmission to the strongest user. In Figure 5 we show the sum rate gain of DPC compared with linear precoding. As can be seen, the regions where DPC provides gains are few and not significant.

B. Experiment setup 2

We continue to study a single base station, namely LURx, but now let both users move along route 1.[3] The base station is still equipped with 2 antennas. In Figure 6 we plot the orthogonality related coefficient r. We observe that 'medium orthogonality' is seldomly occuring. The situation is similar for $M > 2$, but we omit illustrations due to lack of space.

As can be observed, there are positions along the route where the propagation channel is orthogonal to the channels

[3]Actual channel measurements for two users that simultaneously move along route 1 do not exist. Instead the channel matrices are virtually constructed by combining the different snapshots along route 1 into a MU-MIMO channel matrix. In other words, if we combine measured channels for snapshot x and y, we simulate the case where one user is standing at location x and the other at location y.

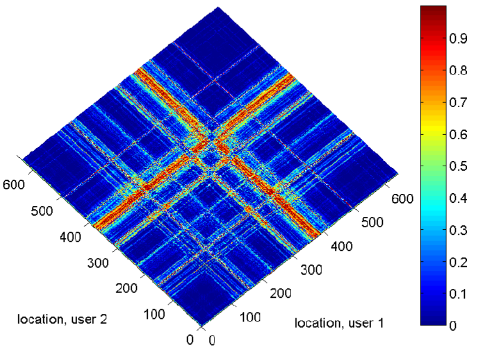

Fig. 6. Orthogonality related coefficient r of 2x2 MU-MIMO (Experiment setup 2).

at all other positions. When not standing at these positions, the channel typically experience very low orthogonality to all other positions. A plausible explanation for this important observation is the following: The propagation can be assumed to be strongly directive in a corridor environment. When moving along the route, the channel vector lies mainly in small subspace. However, at various positions, the channel vectors jumps out of the subspace, so that it becomes orthogonal to that subspace. From visual inspection of the map, it can be seen that he channel vector jumps out of the normal subspace shortly after turning around the corner.

From a signal processing perspective, it is interesting to observe that the orthogonality related coefficient r is either very small or very close to unity. This implies that the gain of DPC is not very significant, as ZF works well for large values of r (red areas) and single user transmission will be used when r is small (dark blue areas). Only in the light blue regions will it be worthwhile to activate DPC. While not illustrated in this paper, a plot of the actual sum-rate gains of DPC over ZF precoding verifies these statements.

C. Experiment setup 3

We next turn to distributed antenna systems. We assume that LURx and TKKRx form a distributed MIMO system and that they are perfectly synchronized. With this we mean that they are fully cooperative and that no restrictions on their ability to exchange information are made. In order to obtain fair comparisons with the co-located antenna system, the two base stations are equipped with a single antenna each. The two users are still equipped with a single antenna. As in Experiment setup 2, we assume that both users are moving along measurement route 1.

In Figure 7, we plot the orthogonality related coefficient r. It is seen that composite channel matrix has much better orthogonality properties with distributed antennas than with co-located antennas (c.f. Figure 6). When both users are placed in the side corridor, channel orthogonality is low. This is intuitive since both base stations share the same propagation properties to that part of the corridor. When both users are placed in the main corridor, the users experience excellent orthogonality properties. This is because the propagation environments to the

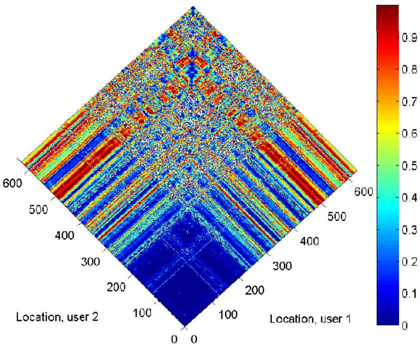

Fig. 7. Orthogonality related coefficient r for distributed MIMO (Experiment setup 3).

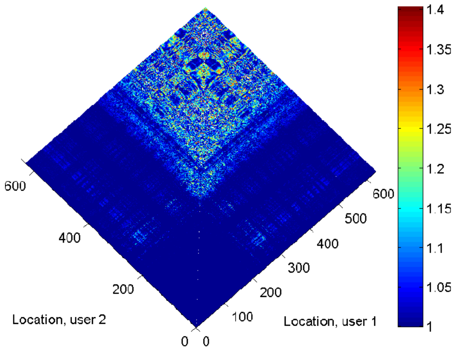

Fig. 8. Gain in sum-rate capacity of DPC over linear precoding for distributed MIMO (Experiment setup 3).

users from the LURx and the TKKRx base stations are very different since the antennas are not co-located. Since one user is closer to the LURx base station, and one is closer to the TKKRx base station, it is natural that orthogonality is high.

In Figure 8 we show the sum-rate gain of DPC over linear processing. The SNR is $\rho = 20$ dB. It is seen that DPC is not worthwhile whenever one or both users are placed in the side corridor, while it provides minor gains when both users are placed in the main corridor. However, these gains occur exactly when the sum-rate is already high for both DPC and linear precoding (sum rates are around 3-4 times as high in the main corridor as compared with the side corridor). When the sum-rate is low for linear precoding, it is low for DPC as well. Hence, DPC is not a technique that can be called in to rescue bad channels.

In this section, we observed through channel measurements that for indoor corridor environments, the channel orthogonality is either very high or very low. This ensures that gains of DPC, or any other scheme, over linear precoding will be small. Further, in indoor corridor environments, there are positions where the channel is nearly orthogonal to all the channels at all other positions. We also observed that with distributed antennas, much better orthogonality properties are obtained.

V. Summary and Conclusions

In this paper we have investigated the performance of downlink MU-MIMO communication in narrow indoor corridor environments. We have seen that the predominant parameter of interest for capacity evaluation is channel orthogonality. Furthermore, the orthogonality between channels to different users plays an important role in the choice of signal processing strategy for the MIMO transmission. If orthogonality is high, low complex linear signal processing (*e.g.* zero-forcing) can achieve the same rates as highly complex optimal methods (*e.g.* dirty-paper coding). The same type of situation occurs in the low orthogonality case where low-complex time sharing between users achieve the same rates as the highly complex optimal methods. Only for 'medium orthogonality' it is beneficial to use the highly complex optimal methods.

Remarkably, the channel measurements show exactly these orthogonality properties: The composite channel from the base station to the two users is either almost singular (low orthogonality) or almost diagonal (high orthogonality). The implication is that in indoor corridor environments, there is not much gain of highly complex algorithms, such as DPC.

Furthermore, the orthogonality plots presented in this paper reveals novel insights into the behavior of wireless indoor channels. In all corridor measurement scenarios, there are locations along the route where the channel to the base station is orthogonal to the channels at all other locations. This novel and interesting fact surely warrants future research efforts.

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