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Throughput Analysis of Three Multiuser Diversity Schemes

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***Abstract*— The throughput of three slot allocation schemes for the downlink of a wireless TDMA system are investigated under the assumption of block-Rayleigh fading. The schemes use the instantaneous SNR at the users in order to take advantage of the independent fading of the channels. In the first scheme, which has been proposed earlier [1], the base station transmits to the user that experiences the largest SNR in every slot. In the second scheme, which is a variant of an existing scheme [2], the user with highest ratio between instantaneous SNR and mean SNR is transmitted to. Lastly, we propose a scheme that compares the SNR at a user to a threshold in order determine if transmission to that user shall occur. If the SNR is below the threshold, transmission to another user is attempted. The results indicate that considerable throughput gains can be achieved even with limited SNR feedback, although there exists a trade-off between throughput and delay.**

I. Introduction

Radio resources are scarce and expensive. These facts motivate research in techniques that better utilize the radio spectrum than existing systems. The goal is ultimately to increase the spectral efficiency and Quality of Service by only increasing the complexity by a reasonable degree.

One approach that has the potential of accomplishing this goal is Multiuser Diversity, where the fading of channels is exploited instead of combatted. Multiuser Diversity benefits from a large number of users in that many users in a system yields a high probability that at least one of them experiences a large Signal-to-Noise Ratio (SNR) at any given moment. Simply put, transmissions should ideally be made to a user with a high SNR and it is more likely that there exists such a user when the channel is fast fading than when it is slowly fading. The drawback is that in the downlink the SNR at each user must be closely monitored and quickly fed back to the base station. However, this will be possible in soon to be deployed systems [2].

In this paper, the throughput gains of three signal quality based schemes for allocating time slots in the downlink of a TDMA system are derived, evaluated numerically, and compared. They are compared to each other as well as to a static reference scheme, i.e., the Round-Robin scheme, where slot allocations are fixed in time. In the first scheme, a packet is transmitted to the user that experiences the best SNR in each time slot. It was shown in [1] that this was the optimal transmission strategy in the uplink with power control. To mitigate the unfairness that arises in this scheme when users have greatly varying fading statistics, the second scheme dictates that a transmission is performed to the user with greatest instantaneous to mean SNR ratio. In [2], a variant of the this scheme was proposed[1]. Lastly, we propose a scheme that utilizes thresholds in order to determine if transmission to a user should take place or not. If not, transmission to other users may be performed instead. In addition, the delay characteristics, in terms of number of slots between successive transmissions, of the two latter schemes is presented.

In contrast to [1], we consider the downlink without power control and a frequency-flat block-fading AWGN channel is assumed. Furthermore, perfect channel side information is assumed at the receivers. Similar work is also presented in [3], where other schemes are investigated.

The throughput will be expressed in terms of average capacities as well as the throughput of packets using uncoded transmission. The average capacity can be used when no decoding delay constraints are imposed and under this assumption the Shannon capacity in the ordinary sense is obtained [4].

This paper is organized as follows. Section II presents the system model and in Section III the throughputs for the schemes are derived. The delay analysis is performed in Section IV. In Section V the throughput and delay results are presented and, finally, the results are elaborated on in Section VI.

II. System Model

Consider the downlink of a single-cell wireless TDMA system with M mobile users. It is assumed that the channel is frequency-flat, block-Rayleigh fading, and the fading level is constant over blocks of N symbols. Furthermore, the fading levels from block to block are independent and identically distributed for each user and the fading levels are independent between users. It is assumed that a block is equal in duration to a slot of the TDMA scheme. One frame is equal to M slots and in a Round-Robin scheme each user would be assigned the same slot in every frame.

[1]The algorithm described in [2] is called *Proportional Fair Scheduling*. This algorithm takes into account the average throughput of each user during a window of a certain length.

For the ith user the SNR is described by the stochastic variable (SV) Γ_i. The probability density function (PDF) of Γ_i is $f_{\Gamma_i}(\gamma)$ and $F_{\Gamma_i}(\gamma)$ is its cumulative distribution function (CDF). Due to the Rayleigh fading assumption, Γ_i is exponentially distributed with mean $\bar{\gamma}_i$ [5], i.e.,

$$f_{\Gamma_i}(\gamma) = \frac{1}{\bar{\gamma}_i} e^{-\gamma/\bar{\gamma}_i}, \quad \gamma \geq 0. \tag{1}$$

Moreover, it is assumed that the estimations of the SNRs of all users are perfect and that the feedback of the SNRs occurs with no loss in throughput.

III. THROUGHPUT OF MULTIUSER DIVERSITY SCHEMES

In general, the throughput is dependent on the SNR in each block and in order to obtain the mean throughput, the throughput will be averaged over the PDF of the SNR. Below, the average throughput for each scheme is calculated in terms of capacity. For comparison with the average capacities, the gains for uncoded transmission using binary coherent modulation, e.g., BPSK, is also presented. The probability of successful transmission of a block of N bits is then

$$P_{\text{block}}(\gamma) = \left(1 - Q\left(\sqrt{2\gamma}\right)\right)^N, \tag{2}$$

where $Q(\cdot)$ is the area under the tail of a zero-mean, unit-variance Gaussian PDF. For obtaining the average throughput when using uncoded binary coherent modulation, $\log_2(1+\gamma)$ is replaced with (2) for each scheme.

A. Round-Robin Throughput

If users are statically allocated the same time slot in every frame in an M-user TDMA scheme the average throughput per slot for ith user is [6]

$$\begin{aligned} \bar{C}_i^{\text{RR}} &= \frac{1}{M}\int_0^\infty \log_2(1+\gamma) f_{\Gamma_i}(\gamma) d\gamma \\ &= \frac{e^{\bar{\gamma}_i^{-1}}}{M \ln 2} \mathrm{E}_1\left(\bar{\gamma}_i^{-1}\right), \end{aligned} \tag{3}$$

where $\mathrm{E}_1(x) = \int_x^\infty t^{-1} e^{-t} dt$ [7, p. 287] is the exponential integral function.

B. Max-SNR Throughput

In the Max-SNR scheme, a packet is transmitted to the user that experiences the largest SNR in every slot. This scheme will therefore maximize the aggregate average throughput although in situations where the mean SNRs of the users vary greatly, users with low mean SNRs will experience very low throughputs.

Let $\Gamma_{\max}^{(i)} = \max_{k \neq i}(\Gamma_k)$ be the SV that is the maximum of all SVs Γ_k except Γ_i. The average throughput per slot for the ith user is then

$$\bar{C}_i^{\text{MS}} = \int_0^\infty \log_2(1+\gamma) \Pr\left(\Gamma_{\max}^{(i)} \leq \gamma\right) f_{\Gamma_i}(\gamma) d\gamma. \tag{4}$$

The probability that $\Gamma_{\max}^{(i)}$ is less than γ is

$$\Pr\left(\Gamma_{\max}^{(i)} \leq \gamma\right) = \prod_{\substack{k=1 \\ k \neq i}}^{M} F_{\Gamma_k}(\gamma). \tag{5}$$

C. Weighted-SNR Throughput

In the Weighted-SNR scheme, a packet is transmitted to the user that experiences the largest SNR-to-mean-SNR ratio in every slot. Hence, the user that experiences the highest relative SNR will be transmitted to, and therefore a certain degree of fairness, in terms of the number of packets transmitted to a user, is achieved among the users.

The scheme compares all $\Gamma_k/\bar{\gamma}_k$ and transmits to the user that has the largest such ratio. Let $Y_{\max}^{(i)}$ be the SV that takes the maximum of the SVs $\Gamma_k/\bar{\gamma}_k$, except for $\Gamma_i/\bar{\gamma}_i$, and multiplies this ratio with $\bar{\gamma}_i$, i.e.,

$$Y_{\max}^{(i)} = \bar{\gamma}_i \max_{k \neq i}\left(\frac{\Gamma_k}{\bar{\gamma}_k}\right). \tag{6}$$

The average throughput per slot for the ith user is then

$$\bar{C}_i^{\text{WS}} = \int_0^\infty \log_2(1+\gamma) \Pr\left(Y_{\max}^{(i)} \leq \gamma\right) f_{\Gamma_i}(\gamma) d\gamma. \tag{7}$$

The probability that $\Gamma_{\max}^{(i)}$ is less that γ is

$$\Pr\left(Y_{\max}^{(i)} \leq \gamma\right) = \prod_{\substack{k=1 \\ k \neq i}}^{M} F_{Z_k}\left(\frac{\gamma}{\bar{\gamma}_i}\right), \tag{8}$$

where $Z_k = \Gamma_k/\bar{\gamma}_k$ and $F_{Z_k}(\gamma)$ is the CDF of Z_k. In the case of Rayleigh fading, the Z_k are all exponentially distributed with unit-mean and (7) then becomes

$$\bar{C}_i^{\text{WS}} = \int_0^\infty \log_2(1+\gamma)\left(1 - e^{-\gamma/\bar{\gamma}_i}\right)^{M-1} f_{\Gamma_i}(\gamma) d\gamma. \tag{9}$$

It is noted that $\bar{C}_i^{\text{WS}}$ is independent of the other users' mean SNRs in the Rayleigh fading case.

D. Random-Select Throughput

In this scheme each user is assigned a fixed slot as well as a threshold. For each slot the SNR at the user assigned to that slot is compared to its threshold. If the SNR is above the threshold a packet is transmitted to the user. However, if the SNR is below the threshold another user is chosen at random and a packet is transmitted to the randomly chosen user, regardless of any threshold.

The benefit of this scheme is reduced complexity in that it, in every slot, only needs the feedback of whether the SNR of one user is above or below a threshold, i.e., one bit of information. The two other schemes require the SNRs of all users in every slot.

In order to simplify the derivation, $\beta_i\left(\gamma_i^{\text{th}}\right)$ is defined as the average throughput for the ith user, given that the SNR exceeds a threshold γ_i^{th}. It can be shown that

$$\beta_i\left(\gamma^{\text{th}}\right) = \frac{1}{M} \frac{\int_{\gamma_i^{\text{th}}}^\infty \log_2(1+\gamma) f_{\Gamma_i}(\gamma) d\gamma}{\Pr\left(\Gamma_i > \gamma_i^{\text{th}}\right)}. \tag{10}$$

The average throughput for the ith user can then be written as the sum of the throughputs in each slot of the frame, i.e., as

$$\underbrace{\Pr\left(\Gamma_i > \gamma_i^{\text{th}}\right)\beta_i\left(\gamma_i^{\text{th}}\right)}_{\text{throughput in } i\text{th slot}} + \sum_{\substack{k=1 \\ k \neq i}}^{M} \underbrace{\Pr\left(\Gamma_k < \gamma_k^{\text{th}}\right)\frac{\beta_i(0)}{M-1}}_{\text{throughput in other slot, } k \neq i}. \tag{11}$$

In (11) the first term arises from the throughput in the slot that the ith user is assigned to, and the second term is the throughput in the other slots. Specifically, the former term is the probability that the SNR exceed the threshold times the throughput given that the SNR exceeds the threshold. The latter term is, for every slot except the ith, the throughput weighted with the probability that the SNR of the kth user is below its threshold times the probability that the ith user is transmitted to instead of the kth.

Inserting (10) in (11) yields the throughput per slot in terms of average capacities

$$\bar{C}_i^{\mathrm{RS}} = \frac{1}{M}\int_{\gamma_i^{\mathrm{th}}}^{\infty} \log_2(1+\gamma) f_{\Gamma_i}(\gamma) d\gamma + \sum_{\substack{k=1\\k\neq i}}^{M} F_{\Gamma_k}\left(\gamma_k^{\mathrm{th}}\right) \frac{\int_0^{\infty} \log_2(1+\gamma) f_{\Gamma_i}(\gamma) d\gamma}{M(M-1)}. \quad (12)$$

The thresholds are chosen to a fraction α_i of the mean of the respective user. This fraction is the same for all users, i.e., $\gamma_i^{\mathrm{th}} = \alpha\bar{\gamma}_i$ for all i. This choice of thresholds will yield a suboptimal scheme but it is nonetheless a reasonable choice. $\bar{C}_i^{\mathrm{RS}}$ now reduces to

$$\bar{C}_i^{\mathrm{RS}} = \left(1-e^{-\alpha}\right)\bar{C}_i^{\mathrm{RR}} + \frac{e^{-\alpha}\ln(\alpha\bar{\gamma}_i+1) + e^{\bar{\gamma}_i^{-1}} \mathrm{E}_1\left(\alpha+\bar{\gamma}_i^{-1}\right)}{M\ln 2}, \quad (13)$$

which is, like the Weighted-SNR scheme, independent of the other users' mean SNRs. It is noted that when α tends toward zero or infinity, $\bar{C}_i^{\mathrm{RS}}$ tends toward $\bar{C}_i^{\mathrm{RR}}$.

IV. Delay Analysis

In this section the delay in terms of the number of slots between successive channel accesses for a user is investigated. This will be done for the Weighted-SNR and Random-Select schemes. It is of little practical interest to examine the delay for the Max-SNR scheme since users with relatively low SNRs will suffer extremely large delays.

A. Weighted-SNR Delay

For the Weighted-SNR scheme, the number of slots to the next access is independent of whether the user accessed the channel in the current slot or not. Hence, it suffices to calculate the probability distribution of the number of slots to the next access from an arbitrary slot, and this probability is independent of user. Since relative SNRs are compared, the mean probability of access for any user is $1/M$, independently of the mean SNR. The mean time between accesses is therefore M and the distribution of the number of slots between accesses, K, for the ith user is

$$p_i^{\mathrm{WS}}(k) = \frac{1}{M}\left(1-\frac{1}{M}\right)^{k-1}.$$

B. Random-Select Delay

For the Random-Select scheme, the probability of access for a certain user is dependent on which slot of the frame the system is currently in. To obtain the distribution of the number of slots between accesses for the ith user, $p_i^{\mathrm{RS}}(k)$, we condition on where in the frame the system currently is. Hence,

$$p_i^{\mathrm{RS}}(k) = \sum_{\nu=1}^{M} \frac{1}{M} p_i^{\mathrm{RS}}(k|\nu), \quad (14)$$

since the probability that the system is in the νth slot is $1/M$, for all ν. We calculate the conditioned distribution $p_i^{\mathrm{RS}}(k|\nu)$ only for the first user since by averaging over all slots a distribution valid for all users is obtained.

For convenience, define the probability of access in user one's assigned slot as

$$P_{\mathrm{D}}^{\mathrm{A}} = \Pr(\Gamma_1 > \bar{\gamma}_1) = e^{-\alpha}, \quad (15)$$

the probability of no access in a user one's assigned slot as

$$P_{\mathrm{D}}^{\mathrm{F}} = \Pr(\Gamma_1 \leq \bar{\gamma}_1) = 1 - e^{-\alpha}, \quad (16)$$

the probability of access in an other slot as

$$P_{\mathrm{O}}^{\mathrm{A}} = \frac{1}{M-1}\Pr(\Gamma_1 \leq \bar{\gamma}_1) = \frac{1-e^{-\alpha}}{M-1}, \quad (17)$$

and the probability of no access in an other slot as

$$P_{\mathrm{O}}^{\mathrm{F}} = \Pr(\Gamma_1 > \bar{\gamma}_1) + \left(1-\frac{1}{M-1}\right)\Pr(\Gamma_1 \leq \bar{\gamma}_1) = e^{-\alpha} + \left(1-\frac{1}{M-1}\right)\left(1-e^{-\alpha}\right), \quad (18)$$

where an "other" slot is any slot apart from the one assigned to user one. Recall that it has been assumed that $\alpha_i = \alpha$, for all i. Now $p_1^{\mathrm{RS}}(k|\nu)$ can be written

$$p_1^{\mathrm{RS}}(k|\nu) = \left(P_{\mathrm{O}}^{\mathrm{F}}\right)^{k-1-\lfloor\frac{k-1+\nu}{M}\rfloor}\left(P_{\mathrm{D}}^{\mathrm{F}}\right)^{\lfloor\frac{k-1+\nu}{M}\rfloor} \times \left(P_{\mathrm{D}}^{\mathrm{A}}\right)^{1-h((k+\nu) \bmod M)} \times \left(P_{\mathrm{O}}^{\mathrm{A}}\right)^{h((k+\nu) \bmod M)}, \quad (19)$$

where $h(x)$ is an indicator function

$$h(x) = \begin{cases} 0, & x = 0 \\ 1, & x \neq 0 \end{cases}. \quad (20)$$

The probability distribution of the number of slots between accesses for any user is now given by inserting (19) in (14).

V. Numerical Results

In this section the throughput gains of the three Multiuser Diversity schemes relative to the Round-Robin scheme are presented, as well as the results for the delay analysis. The throughput plots are obtained by normalizing each user's throughput when using one of the Multiuser Diversity schemes with that user's throughput when using the Round-Robin scheme. The gain in aggregate throughput is calculated by normalizing the sum of the users' throughputs with the sum of the throughputs when using the Round-Robin scheme.

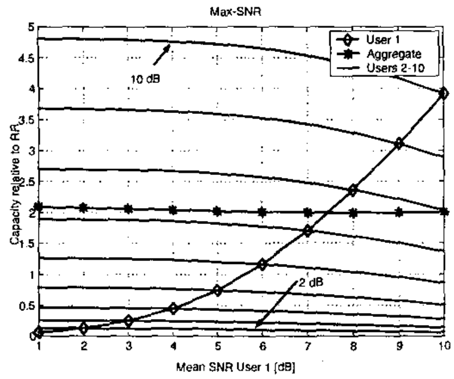

Fig. 1. Gain, in terms of capacities, for Max-SNR scheme for $M = 10$ users. Shown are gains for users with constant mean SNRs (2 dB user at bottom, 10 dB user at top) as well as the gain for user 1 and the gain in aggregate capacity.

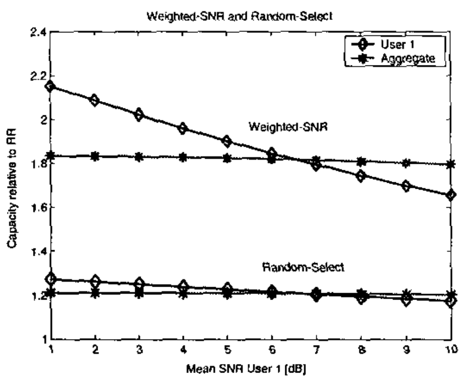

Fig. 2. Gain, in terms of capacities, for Weighted-SNR and Random-Select ($\alpha = 1$) schemes for $M = 10$ users. Shown are the gain for user 1 and the gain in aggregate capacity.

For the throughput plots, the number of users is $M = 10$ and for 9 of the users the mean SNRs are fixed while the mean SNR for a single user is varied between 1 and 10 dB. The other users have constant mean SNRs equal to $2, 3, \ldots, 10$ dB. This way of plotting gives a notion of the interdependence of the users' throughputs and reflects the dynamics in the schemes.

Figure 1 shows the gain of the Max-SNR scheme over the Round-Robin scheme, for the $M = 10$ users, when using the average capacity as a performance measure. In this figure the gain for the users with constant mean SNRs are shown as the mean SNR of user 1 increases, as well as the gain for user 1 and the gain in aggregate capacity. It can be seen that large gains can be made for the considered mean SNR range although only users with high mean SNRs benefit from this scheme. Moreover, increasing the mean SNR of user 1 decreases the gains of the other users. It is also noted that some users suffer a loss in throughput compared to the Round-Robin scheme.

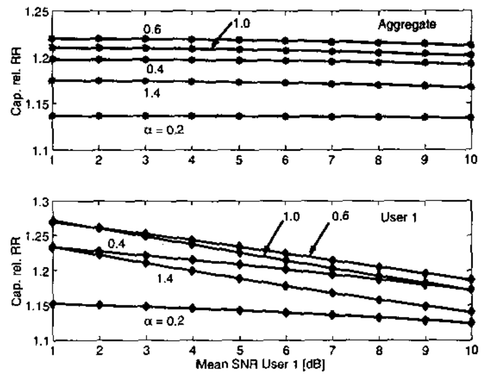

Fig. 3. Impact of α on the gains for the Random-Select scheme for aggregate capacities (upper plot) and for user 1 (lower plot).

As previously mentioned, the throughput for a user when employing the Weighted-SNR and Random-Select schemes are independent of the other users, and therefore also the other users' mean SNRs. Only the number of users, and α for Random-Select, effect the throughput and, in turn, the gains. Hence, the gains for user 1 in Figure 2 and Figure 5 are valid for all users. However, the aggregate throughput curves are dependent on the distribution of the users' mean SNRs.

For the Weighted-SNR scheme in Figure 2, it is seen that the gains in terms of average capacities are more modest than in the case of Max-SNR and a totally different behavior is exhibited. In contrast to the Max-SNR scheme, users with low mean SNRs gain the most in the Weighted-SNR scheme, and the gains are more concentrated than in Figure 1. Most importantly, the gains are greater than one for all mean SNRs, i.e., all users experience gains greater than one. Interestingly, the gain in aggregate average capacity is only slightly less than that of the Max-SNR scheme, which maximizes the aggregate capacity [1].

For the Random-Select scheme, the impact of α on the gains for aggregate capacities and for user 1 as the mean SNR of user 1 is varied is shown in Figure 3. It is observed that the Random-Select scheme is robust to slight changes of α around $\alpha = 1$. Although not presented here, similar behavior is seen for the throughput using uncoded coherent transmission. Therefore, $\alpha = 1$ has been chosen for the Random-Select scheme in Figure 2 and Figure 5. For this scheme, the gains are lower and even more concentrated than in the case of Weighted-SNR and all users experience gains greater than one. Furthermore, users with low SNRs benefit the most in this scheme as well. With a minimal amount of feedback it is seen that the aggregate throughput is increased with 20% compared to that of Round-Robin.

Figures 4 and 5 show the gains in terms of throughput of packets ($N = 200$) using uncoded transmission. These figures correspond to Figures 1 and 2, respectively. It can be seen that the results are quite similar to the corresponding figure for average capacities. However, Figure 5 indicates that for

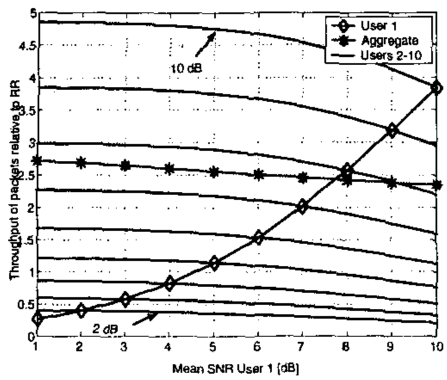

Fig. 4. Gain, in terms of throughput of packets ($N = 200$), for the Max-SNR scheme for $M = 10$ users. Shown are gains for users with constant mean SNRs (2 dB user at bottom, 10 dB user at top) as well as the gain for user 1 and the gain in aggregate throughput of packets.

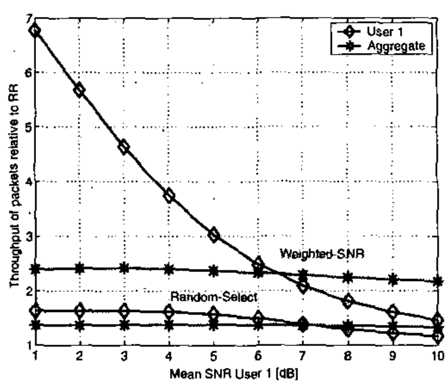

Fig. 5. Gain, in terms of throughput of packets ($N = 200$), for Weighted-SNR and Random-Select ($\alpha = 1$) schemes for $M = 10$ users. Shown are the gain for user 1 and the gain in aggregate throughput of packets.

the Weighted-SNR scheme the gains are larger for the low-SNR users and that the gain for the aggregate throughput of packets decreases as the mean SNR of user 1 increases. This illustrates the more realistic, compared to average capacities, dependence of the throughput on the SNR.

The probability that the number of slots between successive accesses exceeds $2M$ is plotted against M in Figure 6 for the Weighted-SNR and Random-Select schemes. For the latter scheme, α is varied and it is seen that $\alpha = 0.4$ yields good results both in terms of throughput and delay. Hence, this parameter provides an efficient tool for trading delay for throughput.

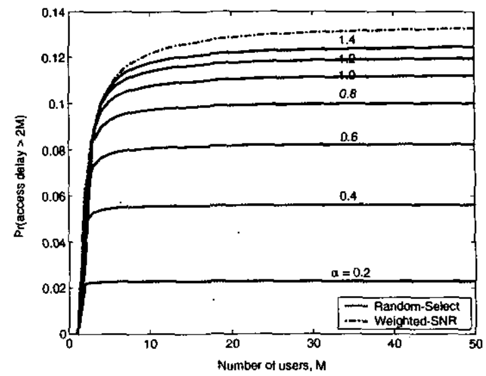

Fig. 6. Probability that the number of slots between accesses exceeds $2M$ for the Weighted-SNR and Random-Select schemes.

VI. Conclusions

In this paper we have derived expressions for the throughputs for three Multiuser Diversity schemes for the downlink of a slotted TDMA system, and it was seen that considerable gains could be achieved by use of these schemes. However, the gains come at a cost and that cost is the feedback of the SNRs of the users. In the Max-SNR and Weighted-SNR schemes it was required that the SNRs of all users were fed back to the transmitter. A new scheme was proposed, called Random-Select, which was simpler and more easily implemented. It only required the feedback of whether the SNR for one user was above or below a threshold for every slot, i.e., one bit of information. Lower but still significant gains of around 20% were therefore observed for this scheme. The Random-Select scheme provided another advantage in that it could be tuned to provide varying degrees of access delay. However, access delay had to be traded for throughput.

An other important issue is the notion of fairness among the users. In contrast to the Max-SNR scheme, the two schemes Weighted-SNR and Random-Select are fair in that they provide gains for all users regardless of mean SNRs.

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