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Published in:
Knowledge and Inquiry: Essays on the Pragmatism of Isaac Levi

2006

Citation for published version (APA):
Olsson, E. J. (2006). Potential Answers - to What Question? In E. J. Olsson (Ed.), *Knowledge and Inquiry: Essays on the Pragmatism of Isaac Levi* (pp. 157-166). Cambridge University Press.

Total number of authors:
1

Potential Answers -- to what Question?

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Key words: induction, decision theory, potential answer, inquiry, abduction, practical reasoning, theoretical reasoning

Throughout his career, Isaac Levi has advocated a unity of reason thesis according to which practical deliberation and theoretical inquiry, while differing as regards values and goals are nonetheless similar from a structural point of view. In support of this thesis he has argued, first and foremost, that problems of induction can be seen as decision problems analogous to practical ones; both kinds of reasoning can be represented within the framework of Bayesian decision theory. In the practical case, the decision-maker's options are practical actions. In the theoretical setting, they are, Levi thinks, acts of accepting potential answers to the inquirer's question. In the paper I argue that his latter view exposes him to criticism unnecessarily.

1. Levi's cognitive decision theory

Suppose that the agent is in an initial state of full belief *K*. There is a question which the agent wants to answer, and so he has identified a set of relevant propositions exclusive and exhaustive relative to *K* in the manner proposed by Levi in his first book from 1967 and defended in later works. This set is his ultimate partition *U*. Given *U*, a set of potential expansions relevant to the inquirer's demands for information can be identified. These potential expansions or, as Levi also calls them, *cognitive options* are formed by adding to *K* an element of *U* or a disjunction of such elements. The result is then closed under logical consequence. Levi assumes that adding some belief contravening proposition is also a potential expansion.

   Suppose, for example, that we would like to determine the color of a given liquid after chemical reaction with another substance. If the experiment is difficult or costly to carry out, the agent might be interesting in trying to settle the matter by predicting the outcome beforehand. Let us assume that from the agent's epistemic point of view only three colors are seriously possible: red, white or blue. The ultimate partition consists of "The color is red," "The color is white" and "The color is blue." The potential expansions are obtained by expanding *K* by one of the elements of the ultimate partition or a disjunction of

such elements, i.e., by "The color is red," "The color is white," "The color is blue," "The color is red," "The color is red or white," "The color is red or blue," "The color is white or blue," "The color is red or white or Blue." Adding the last proposition would be refusing to expand beyond K, as the agent is already committed to accepting it. Finally, expansion by a belief contravening proposition, such as "The color is both red and white" is also considered to be a cognitive option, in which case the inquirer will fall into inconsistency.

Now how are we to decide what potential expansion to implement? Levi's answer is that we should maximize the expected epistemic utility. The expected epistemic utility of adding a proposition A to the belief corpus is representable as $E(A) = \alpha Q(A)+(1-\alpha)Cont(A)$, or some positive affine transformation thereof, where $Q(A)$ is the subjective ("credal") probability of A and $Cont(A)$ the informational value of A. Thus, the expected epistemic utility is a weighted average of the utility functions representing two different desiderata: probability and informational value. Moreover, $Cont(A)$ can be identified with $1-M(A)$ where M is the information determining probability of A. Information determining probability is similar to Carnap's logical probability. Divide $E(A)$ by α and subtract from the result $q = (1-\alpha)/\alpha$. The result will be $E'(A) = Q(A)-qM(A)$.[1] This is a positive affine transformation of the weighted average, and so maximizing this index is equivalent to maximizing the weighted average. The

parameter *q* can be interpreted as the inquirer's degree of boldness. If *q* equals 0, the inquirer is an inductive skeptic; he will always refrain from expanding beyond the current corpus. A reasonable requirement is that falling into contradiction by adding a belief contravening proposition should never be preferred to refusing to expand. Levi shows that this requirement is satisfied so long as $q \leq 1$.

To continue the example, suppose that Q("The color is red") = 0.9 and Q("The color is white") = 0.09, so that Q("The color is red or white") = 0.99. Assume, further, that the information determining probability assigned to each element of *U* is 1/3. Hence, M("The color is red") = 1/3 and M("The color is red or white") = 2/3. Setting $q = 0.3$, the expected epistemic utility of expanding by "The color is red" equals Q("The color is red")-qM("The color is red") = 0.9-0.3·1/3 = 0.8. Consider now the weaker proposition "The color is red or white." Its expected epistemic utility will be Q("The color is red or white")-qM("The color is red or white") = 0.99-0.3·2/3 = 0.79. Hence, expanding by "The color is red" has a greater expected epistemic utility than expanding by "The color is red or white." It is easy to verify that expanding by the former in fact maximizes expected epistemic utility. In this case, then, it was possible to settle the matter about the definite color of the liquid from an armchair position, without ever entering the laboratory.

It is not always the case, however, that a definite prediction can be made. A less bold agent, i.e. one with a lower q-value, may not find the risk of predicting the wrong color worth taking. For instance, setting $q = 0.2$, the expected epistemic utility of expanding by "The color is red" equals 0.83, and that of "The color is red or white" equals 0.85. It can be verified that the optimal strategy, the one that maximizes expected epistemic utility, is to expand by "The color is red or white." While the agent can rest assure that the color is either red or white, he cannot say anything more definite about the color, and so he might be tempted to conduct the experiment after all.

As I believe the color example shows, it makes good sense to use Bayesian style decision theory in a purely theoretical context. In recognition of the philosophical implications, Levi proposes a brand of pragmatism according to which "[w]hat is 'pragmatic' … is the recognition of a common structure to practical deliberation and cognitive inquiry in spite of the diversity of aims and values that may be promoted in diverse deliberations and inquiries." (1991, p. 78)

2. Cognitive options based on potential answers

In Levi's theory, the set of cognitive options (potential expansions) relative to a question is the set of all expansions by some element of the ultimate partition or a disjunction of such elements. Semi-formally,

(1) An expansion $K+A$ is a cognitive option relative to a query Q if and only if A is an element of the ultimate partition or a disjunction of such elements.

The disjunctive structure is a feature which earlier decision theoretic accounts of induction lacked. For example, in Hempel's theory, which was devised independently by Levi in one of his early papers, the set of cognitive options was equated with the set of expansions by elements of the ultimate partition plus total suspension of judgment (Hempel 1960, Levi 1962). The possibility of partial suspense was not acknowledged. Thus, in the old Hempel--Levi theory, accepting that the color is either red, white or blue would have been a legitimate option, whereas accepting that it is either red or white would not.

This is not all Levi has to say about the nature of cognitive options; he also believes that they should consists in accepting potential answers to the inquirer's question:

(2) An expansion $K+A$ is a cognitive option relative to a query Q if and only if A is potential answer to Q.

There is plenty of textual support for (2) in Levi's works where the expression "cognitive option," "potential expansion" and "potential answer" are frequently used interchangeably, the result being that no clear distinction is made between (1) and (2). For instance, Levi commits himself to both in one swoop when he writes, recently, that "[t]he set of all potential answers (or the potential expansion strategies) of interest to the inquirer in the context of a given inquiry are linguistically representable as expansions of K by adding some sentence h in L equivalent given K to a disjunction of some subset of d_i's in U_K." (2004, section 2.2)[2]

As a consequence of (1) and (2), we have:

(3) A proposition A is a potential answer to Q if and only if A is an element of the ultimate partition or a disjunction of such elements.

Thus, the elements of the ultimate partition are potential answers to the inquirer's question. So, too, are all disjunctions of such elements.

It is, to be sure, reasonable to consider the elements of the ultimate partition to be potential answers to the inquirer's question. But in what sense is a disjunction of two or more elements of the ultimate partition also a potential answer? Surely, it cannot generally be the case that it is. If we are interested in determining the color of the liquid, we would be quite satisfied with learning that the color is blue, and also with learning that it is white. We would not, however, be entirely satisfied with knowing merely that the color is either red or white. If so, what is the basis for calling that disjunction a potential answer to our question? And what about the uncommitted disjunction expressing merely what we already knew at the outset, namely that the color is either red, blue or white? Surely we would be hard-pressed to call this an answer to our question, and yet, in the presence of (1), this is precisely what Levi's endorsement of (2), and hence also of (3), commits him to.

Despite its counterintuitive consequence (given (1)), Levi has continued to advocate and defend (2). Presumably it plays a significant role in his theory, or it would not be worth retaining. My conjecture is that it has a part to play in his conception of the Peirce--Dewey pragmatist tradition, a tradition to which Levi pledges allegiance. Peirce, according to Levi, "conceived of induction as the task of eliminating potential answers from among those identified via abduction." (1991, p. 77) Levi may be understanding this conception to require

that cognitive options be based on potential answers in the sense of (2). I will return to this issue in the final section.

Given that (2) is worth retaining, the *prima facie* implausibility of (3) raises a problem for Levi. The strategy in the following will be to reconstruct three possible defenses of (3) from Levi's writings.

3. The "modest" defense

Let Q be the question that triggered the investigator's inquiry. Q is his main problematic – the question that motivates his investigation. For example, Q might be the question of whether the color of the liquid will be blue, red or white after the chemical reaction has taken place. Given Q, we can formulate another question, the *modest question relative to Q*, denoted *Mod*(Q): What is the strongest conclusion to be legitimately drawn while trying to answer Q, given the current evidence? For example, in the process of answering the question about the color of the liquid the inquirer may wonder what he can conclude, at most, at the present stage of inquiry. In so doing he is, in my terminology, asking the modest question relative to his main problematic.

The first reconstructed argument refers to *Mod*(Q) in order to support the disjunctive structure of the set of potential answers to Q. Its first premise runs as follows:

(4) A proposition *A* is a potential answer to *Mod*(*Q*) if and only if *A* is an element of the ultimate partition (relative to *Q*) or a disjunction of such elements.

To this the following premise is added:

(5) A proposition *A* is a potential answer to Q if and only if *A* is a potential answer to *Mod*(*Q*).

It is now pointed out that (4) and (5) together entail (3). So much for the bare logical structure of the argument. There can be no question about its validity. But are its premises true?

Let us start with (4). First of all, what are we to mean by a potential answer? The most natural way to think about a potential answer is as a proposition which, once it is accepted as true, terminates inquiry into the matter. Once a potential answer has been accepted, the inquirer no longer has any interest in pursuing the matter any further, and so he will not continue to look for new evidence -- even if the costs of inquiry are negligible. To accept a potential answer is to remove the corresponding question from the research agenda.

Given this understanding of "potential answer" it should be clear that the set of potential answers to $Mod(Q)$ does indeed have a disjunctive structure. The strongest conclusion to be legitimately drawn at a given point in time relative to a question Q may be any disjunction of elements of the latter's ultimate partition. In the worst case, it may be impossible to go beyond the current evidence. In that case, the answer to $Mod(Q)$ is the uncommitted disjunction representing total suspension of judgment.

Suppose, for example, that Q is the question, "What is the color of the liquid?" so that $Mod(Q)$ is the question, "What is the strongest conclusion to be legitimately drawn about the color of the liquid given the current evidence?" Surely "The color is red or white" or any other disjunction is in principle a legitimate answer to the modest question. Such a disjunction, once accepted, gives us exactly what we asked for when $Mod(Q)$ was raised, namely, the strongest conclusion to be legitimately drawn about the color given the present evidence. Every disjunction of elements of Q's ultimate partition is in principle capable of being a complete answer to $Mod(Q)$.

The truth of (4) is, I believe, taken for granted in Levi (1967), where he assumes that the sentences "eligible for acceptance as strongest via induction," which are in effect the potential answers to $Mod(Q)$, are logically equivalent to those sentences that are members of a finite set of sentences M (p. 33). He goes

on to stipulate that the set *M* contain all disjunctions of the members of the ultimate partition (relative to the original question *Q*) (ibid., p. 34). This, of course, is just as it should be, given that he is in fact focusing on the question which I have called *Mod*(*Q*).

What reasons, then, are there for thinking that (5) is true as well? Why should we think that every potential answer to *Mod*(*Q*) is also a potential answer to the inquirer's original question *Q*? I think we have no reason to do so. On the contrary, we have every reason in the world to think that the potential answers to *Mod*(*Q*) that are mere disjunctions of elements of *Q*'s ultimate partition are *not* potential answers to *Q*. Learning that the color is either red or white may answer the question of what we can conclude at most about the color at the present state of inquiry, but it does not answer the original question which demanded the determination of a definite color.

Nonetheless, Levi does commit himself to (5) when he writes, in his 1967 book, that "the set of relevant answers to a given question is determined by the set of sentences in *L* that are eligible for acceptance as strongest via induction from the given evidence." (p. 33) On my reading, this statement entails that the set of potential or, as they were called at the time, relevant answers to the inquirer's original question *Q* is identical with the set of sentences that are eligible for acceptance as strongest via induction from the given evidence. The

latter set clearly coincides with the set of potential answers to what I have called *Mod*(*Q*). But, while Levi clearly endorses (5) in his 1967 book, he does not argue for it there; nor, to my knowledge, does he do so elsewhere.

Although there is clear textual evidence in Levi (1967) for attributing what I have called the modest defense of (3), there is also evidence pointing in the opposite direction. In the second half of the book, he observes that "it seems plausible to suppose that a complete answer would be obtained when an investigator is in a position to accept as true some element of *U*." (p. 143) An inquirer who has not yet accepted a member of the ultimate partition, by means of contrast, "will (provided the costs of inquiry are negligible) continue to look for new evidence, until he can justify a strongest consistent relevant answer to his question." (ibid. p. 145) Levi is here saying, in effect, that the class of complete potential answers to the inquirer's main problematic consists of all and only the elements of the ultimate partition. The inquirer is satisfied if and only if one of the elements of the ultimate partition has been accepted as the answer to his question. From this perspective, (5) must be false.

4. The "abductive" defense

The strategy of justifying the disjunctive closure of potential answers by reference to modest questions seems gradually to have lost its importance in

Levi's thought. In his latest book from 2004, there is no sign of his earlier idea of accepting something "as strongest." The emerging view seems to be that (3) is a fundamental principle which need not be justified in terms of modest questions or anything else. In Levi (1980), it is declared that "the condition that the set of potential answers be generated [in the disjunctive manner explained above] by an ultimate partition can be construed as a principle of abductive logic." (p. 46) In Levi (1984) it is listed as one of two principles of abductive logic (p. 93). The disjunctive structure of the set of potential answers, a property which that set intuitively seems to lack, is here presented as a brute fact. The present strategy is not really a defense of (3) in the sense of an argument in its support. The claim is rather that there is no need for any such argument, a claim that is contradicted by pre--systematic judgment.

In Levi (2004), we are told that "[t]he principles of abduction require, for example, that one be in a position to regard suspension of judgment between rival potential answers to be a potential answer." (section 2.2) The following explanation is furnished:

I contend that we should not recognize as potential answers to the election question [i.e. the question of who will win an election – X, Y or Z] that candidate X will win, that candidate Y will win or that candidate Z will win

without also allowing as potential answers suspension of judgment between any pair or even all three. Philosophers tend to think of suspense or skepticism as an all or nothing affair. Either one suspends judgment between all elements of the ultimate partition or adopts one of them. But the tensions between belief and doubt are more nuanced than the aficionados of the battle between skepticism and opinionation would lead you to think. If someone insists that definite conclusions are to be recommended over suspense, that person should be required to show why suspense is inferior with respect to the goals of the inquiry and, if it is inferior, why some partial skepticism reflecting doubt between elements of some subset of the ultimate partition is not better than opinionation. Ruling out all or merely some forms of suspense as options by stipulation does not meet this demand (section 2.2, notation adapted).

While this is indeed a compelling defense of the disjunctive structure of *cognitive options*, i.e., of (1), it does very little in terms of giving an independent defense of (3). Rather, Levi seems to be presupposing that (2) is true. To be sure, the disjunctive structure of cognitive options supports the disjunctive structure of potential answers *provided that such options are based on potential answers*. But this is a trivial contention. The argument does not succeed in explaining how, contrary to appearance, (3) could be true. The net

effect of the argument is instead to cast doubt on the tenability of (2), once it has been identified as a tacit premise.

5. The "partial" defense

Hitherto "answer" has been taken to mean "complete answer." Let us by an *incomplete* answer mean a proposition that, while falling short of answering the question completely, still goes some way toward answering it. By a *partial* answer I will mean one which is either complete or incomplete, i.e., one which takes us *at least* one step closer to a complete resolution. The final line of defense takes (3) to be true provided that "potential answer" is taken to mean "partial potential answer."

To be sure, most disjunctions of partial answers are themselves partial answers in this sense. For instance, "The color is red or white" is not only a disjunction of partial answers; it is also a partial answer itself. In concluding that the color is either red or white we are in effect excluding the possibility of the color being blue, and so we are closer to obtaining a definite answer than we used to be.

Nonetheless, it does not hold generally that disjunctions of partial answers are themselves partial answers. The one exception is total suspense. While "The color is red or white" and "The color is white or blue" are both partial answers,

this does not hold for their disjunction "The color is either red, white or blue."
Accepting the latter means remaining in status quo, and so it obviously does not
take us any closer to accepting a definite answer than we were before.

There are traces of the "partial" defense in Levi (1967). There he writes that
"a relevant [potential] answer is, among other things, a sentence in L whose
truth value is not decidable via deduction from the total evidence." (p. 33).
Clearly, not being decidable via deduction from the total evidence is
tantamount to being a partial answer in our sense. The qualification "among
other things" presumably refers to the case of complete suspension which,
although it is regarded as a potential answer by Levi, *is* decidable via deduction
from the total evidence. On my reconstruction, Levi is here hinting at the
interpretation of potential answers as partial answers while downplaying the
fact that complete suspense does not fit into this picture.

6. On the place of potential answers in the pragmatist's conception of induction

Why would Levi want to subscribe to (2) in the first place given that (3) follows
once (1) is assumed true as well? I have already hinted that the answer may be
sought in Peirce's account of induction, or in Levi's conception thereof.
According to Peirce, as Levi understands him, induction is the task of
eliminating potential answers from among those identified by abduction. Levi

might have been led to regard it as essential to the Peircean view that the cognitive option be based on potential answers in the sense of (2). This is the only explanation I can come up with for Levi's continued endorsement of (2) and his assigning it the status of a fundamental abductive principle in his later works.

Yet, the satisfaction of (2) is in fact *not* required by the Peircean view. To see this, we recall that in all cases, except total suspense, employing Levi's inductive method means, in effect, rejecting the elements of the ultimate partition that are incompatible with the accepted proposition given our background knowledge. Hence, as Levi himself has often pointed out, his theory of induction can equivalently be described as a method for how, if possible, to reject elements of the ultimate partition. It follows that in order to achieve compliance with Peirce it is sufficient to require, unproblematically, that the elements of the ultimate partition be potential answers. It need not be stipulated, in addition, that disjunctions of such elements are potential answers.

In summary, in holding that all cognitive options correspond to accepting some potential answer to the inquirer's main problematic Levi exposes himself to severe criticism from the standpoint of common sense. If I am right, he does so unnecessarily: neither the Peircean conception of inquiry nor any other element of Levi's theory that I have been able to identify depends crucially on

the thesis that cognitive options should be based, in all cases, on potential answers to the inquirer's main problematic. Giving up the troublesome contention -- which is the policy I recommend in the end -- would, in Levi's jargon, not incur any significant loss of informational value.

Notes

1. Here is an alternative path to this equation. In deliberate expansion, the decision maker evaluates the epistemic utility of expanding K by adding A without importing error and with importing error. The former is $1-qM(A)$ and the latter is $-qM(A)$. The expected epistemic utility of adding A is determined by multiplying the first term by the credal probability that A and the second term by the credal probability that $\neg A$. Then the sum of these products is taken which equals $Q(A)-qM(A)$.

2. U_K is the ultimate partition relativized to the corpus K. For a random sample from his earlier work, see Levi (1984), pp. 92, where he contends that determining the cognitive options and identifying the list of potential answers are equivalent procedures.

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