Is UK Risky Money Weakly Separable? A Stochastic Approach

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A Stochastic Approach[#]

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Abstract

Using non-parametric weak separability tests that are extended to allow for measurement errors in the data, a broad group of UK monetary assets is found to be weakly separable from consumer goods and leisure over the larger part of the nineties. Financial innovations have made assets with substantial interest rate risk (e.g. unit trusts) more liquid and recent developments in monetary aggregation theory dealt with risk and risk aversion in the calculation of user costs. It is, however, not possible to find any weakly separable group of assets that contains 'risky' assets in the current sample.

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1. Introduction

This paper addresses the question whether or not 'risky' assets should be included in a broad monetary aggregate for United Kingdom over the period 1980Q1-1998Q4. Risky assets are characterised by having a volatile own-rate that is only known with certainty at the end of each period. The returns on assets commonly included in monetary aggregates, such as bank deposits, are less volatile and are often assumed known with certainty at the beginning of each period. With the acceleration of financial innovations, it is legitimate to ask if the frontier between financial assets such as bonds, shares and mutual funds and 'capital certain' assets has become non-existent. Drake, Fleissig and Mullineux (1999) have shown that risky assets are substitutes for capital certain assets by estimating a demand system that includes both capital certain and risky assets using UK data. They also found that the substitutability between risky and capital certain assets decreased when the level of risk aversion increased. Agung, Drake and Mullineux (1998) studied the leading indicator properties of various UK simple sum, Divisia and risky Divisia monetary aggregates over the time period 1979Q1 to 1994Q2. They found that using risky aggregates offers an improvement over both the simple sum and the standard Divisia monetary aggregates. Binner and Elger (2002) have estimated a money demand system that includes the UK personal sector holdings of risky money over the period 1980Q1 to 1999Q4. They found that the risk-adjusted aggregate produced a very stable money demand relationship with coefficient estimates that conformed to economic priors (even though no apparent gain was found over a reference capital certain aggregate). Barnett and Xu (1998, 2000) have investigated the effect of stochastic volatility in interest rates on money velocity. Their findings show that the traditional velocity function becomes unstable if covariances between consumption and interest rates or between money growth and interest rates change over time using simulated data.

In the theory of monetary aggregation, as set out by Barnett (1978, 1980), monetary assets are treated as durable goods directly entering the utility function of the representative consumer. Hence, the question of whether risky assets should be included in a monetary aggregate reduces to study if such a group meets the necessary and sufficient conditions of aggregation. In a utility maximisation framework, it is well known that an aggregate over goods will exist if and only if there exists a *weakly separable* utility function over those goods. Weak separability is crucial, since it ensures that all assets entering in a monetary category sub-utility function are held for their purpose of money and that all substitution effects are internalised. The internalisation of substitution effects ensures the stability of the aggregate over time. In the monetary aggregation literature, weak separability tests have generally been based upon Varian's (1982, 1983) non-parametric (NONPAR)[1] procedure. The NONPAR procedure is developed in a revealed preference framework. A utility function containing goods, leisure and monetary services is tested for compliance with the generalised axiom of revealed preference (GARP) when the monetary assets are disaggregated and when money is treated as an aggregate good. Studies by Swofford and Whitney (1987, 1988), Swofford (1995) and Fisher and Fleissig (1997) on US personal sector data and Patterson (1991), Drake (1994) and Chrystal and Drake (1997) on UK personal sector data are based on this procedure. Spencer (1997) used the procedure to test for consistency with GARP in nine European countries whilst Belongia (2000) used the procedure to identify asset groupings using data from the US, Germany and Japan.

Much criticism has been raised against Varian's non-parametric approach. The results are difficult to interpret since the test is non-stochastic and hence a single rejection suggests rejection of the tested hypothesis as a whole. It may well be the case that the rejection was

[1] NONPAR is the name of a Fortran routine developed by Varian. In the following text, the abbreviation NON-

caused by purely stochastic factors, such as measurement errors. Barnett and Choi (1989) have shown that the test is strongly biased towards rejection using Monte Carlo simulations. In other words, it may be stated that Varian's procedure is likely to produce a type 1 error. Another less well-known problem is related to how the utility- and price indices used in the final stage of the testing procedure are constructed within the NONPAR procedure (de Peretti (2001a,b)). It is not uncommon for the procedure to produce negative values on these indices leading to the misleading conclusion that weak separability should be rejected. de Peretti (2001a,b) has developed a stochastic version of Varian's test that also solves the problem related to calculating the utility- and prices indices. Measurement errors are incorporated in the analysis to discriminate between significant and non-significant violations of GARP. At each step of the procedure, the statistical significance of a violation is tested using a Jarque-Bera normality test. The need to calculate the utility- and price indices within the NONPAR procedure has been eliminated by directly solving the Afriat inequalities utilising results from index number theory.

In order test if groups of monetary assets are weakly separable, information on prices and quantities is needed. Aggregate quantities can be readily observed, but rental prices (user costs) must be calculated. Barnett (1978) derived the user cost of money under the assumption that all interest rates on monetary assets are known with certainty. Barnett (1995) derived the Euler equations for monetary assets and a consumer good respectively from an infinite period stochastic constrained utility maximisation problem. Based upon these Euler equations, Barnett and Liu (1995) and Barnett, Liu and Jensen (1997) derived an adjusted user cost for monetary assets under risk and risk aversion. This formula contains no risk (capital certainty) and risk neutrality as special cases, providing a link back to earlier research. Aggregate quan-

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PAR will be used in reference to Varian's non-parametric approach in general.

tities are generally divided by a measure of the population in order to obtain *per capita* holdings. Hence, it is implicitly assumed that a representative consumer exists.

The purpose of this paper is to determine whether the standard components included in the Bank of England Divisia monetary aggregate, national savings and three assets with substantial interest rate risk (bonds, shares and mutual funds) meet the separability requirements needed for aggregation using data from 1980Q1 to 1998Q4.

National savings and all assets included in the Bank of England Divisia index have been assumed capital certain following Drake, Fleissig and Mullineaux (1999). Tests suggested that the risk adjustment of the user costs of the 'risky' assets was so small that it was negible. The representative consumer has therefore been assumed to be risk neutral. Even though this might appear contrary to prior expectations it is a common finding in the literature and it may be explained by the use of data that has been aggregated over consumers. Risk neutrality is convenient for the purpose of performing weak separability tests since the consumers multiperiod constrained utility maximisation problem reduces to a current period constrained utility maximisation problem as shown by Barnett (1995).

The empirical evidence in this study suggests that the entire data set is consistent with utility maximisation only over shorter periods throughout the eighties. Even if shorter samples are used so that overall utility maximisation is accepted, it is difficult to find any meaningful broad set of monetary assets consistent with utility maximisation. The major reason for this is a rapid growth in interest bearing savings deposits that is not reflected in any way by the user cost for this asset. From 1991Q4 to 1998Q4, the data set is consistent with utility maximisation and all “capital certain” assets meet the conditions for aggregation without violations. We are not to find any separable group of monetary assets that contains “risky” assets.

The main policy implication that can be drawn from these results is that the Bank of England should not extend the list of assets included in their Divisia index to include risky assets - at least not until a solution to the equity premium puzzle is available. Then it *may* be the case that at least some of these assets could be included in a monetary aggregate. National savings are found to be weakly separable joint with the other capital certain assets and may therefore be included. Furthermore, monetary aggregates that contain data prior to 1991Q3 should be interpreted with some caution since it is not possible to find any weakly separable group prior to this date that contain assets other than notes and coins and non-interest bearing bank deposits.

The paper is organised as follows. The underlying theory is introduced in section 2. The data is described in section 3. Section 4 contains the empirical results and section 5 concludes the paper.

2. Weak separability and aggregation

2.1 Theoretical overview

The crucial existence condition for the existence of an aggregate over any goods is that it internalises substitution effects. In a utility maximisation framework, this will occur if preferences over those goods are weakly separable. The concept of separable preferences is closely related to the concept of two-stage budgeting. Under two-stage budgeting, the representative consumer budgets in stages. In the first stage he optimally allocates his budget among broad categories (e.g. housing, food, and leisure). In the second stage, the representative consumer allocates his resources within each category. Even though the concepts of two-stage budgeting and separable preferences are similar, they are not identical. As shown by i.e. Gorman (1971), weak separability is only a sufficient condition for the second stage of the budgeting process, i.e. for the existence of a sub-utility function.

Following Strotz (1957) and Gorman (1959), weak separability is defined as follows; Suppose that $x_1,x_2 \in x$, where x is a vector of real consumption goods optimally selected by a consumer. The goods in x_2 are weakly separable from all other goods in x if there exist two twice differentiable functions, $U(x_1,x_2)$ and $U_1(x_2)$. $U(x_1,x_2)$ can then be partitioned as;

$$U = U(x) = W(x_1, U_1(x_2)), \tag{1}$$

where $W(.)$ is a macro-function having the same properties as $U(.)$ and U_1 is a sub-utility function sharing the same properties as $U(.)$ and $W(.)$.

In the theory of monetary aggregation, money is held because it produces an undefinable flow of monetary services. Since a unit of monetary holdings does not depreciate fully over one period, money should be treated as a durable good. Since monetary assets are valued by representative agent for the services they provide, they have a positive value in equilibrium and enter directly in the utility function, see e.g. Poterba and Rotemberg (1987). Hence it is possible to test for weak separability among monetary assets just as for any other good. Extending (1) to incorporate real monetary assets (m) and leisure (l), an aggregate over the assets in m will exist only if $U(x,l,m)$ can be partitioned as;

$$U = U(x,l,m) = W(x,l,U_1(m)), \tag{2}$$

Note that applying the weak separability concept to monetary assets allows one to know, without any *a priori* assumptions, what assets are valued for the purpose of money. In fact, once a monetary budget is allocated (first stage budgeting) the agent optimally allocates over assets yielding varying degrees of the same service. In other words, all assets entering into the monetary budget are money. For example, if a weakly separable utility function rationalising very liquid assets such as notes and coins as well as less liquid assets is found, this collection of assets will be defined as money. As noticed by Swofford (1995, p. 156), "the weak separability criterion is a way to identify money as whatever people view as money".

2.2 The User Cost of Money

The theory of monetary aggregation was pioneered by Barnett (1978, 1980) under the assumption that there is all rates of return on monetary assets are known with certainty. A unit held of a monetary asset over a period yields an undefinable monetary service flow and by holding that asset an opportunity cost is incurred. Hence it is possible to calculate the user cost of a monetary asset much like equivalent rental prices are calculated for other durable goods. Barnett's (1978, 1980) well-known nominal user cost formula for a monetary asset *i* is defined as;

$$\pi_{it} = \frac{p_t^*(R_t - r_{it})}{(1 + R_t)}, \qquad (3)$$

where p_t^* is a true cost of living index at time *t*, R_t is a benchmark rate at time *t* and r_{it} is the rate of return associated with asset *i* at time *t*.

The benchmark rate is the rate of return on a totally non-monetary asset, that is an asset that does not yield any monetary services whatsoever. Obviously, no such asset exists on the market. It has been suggested that the benchmark rate should be “viewed as being the rate of return on human capital in a world without slavery” (Barnett and Liu (2000, p. 22)). In applied work, it is common to approximate the benchmark rate by the return on a long-run bond or using an envelope method. An envelope benchmark rate often takes the form $R_t=\max(r_{it}:i=1\ldots j)$. Barnett and Liu (2000) promote the inclusion of a large number of assets (j) in determining the benchmark rate and not restricting the construction to the asset components likely to be included in the monetary aggregate.

The theory is complicated by the fact that interest rates are generally not known until the end of each period and consumers are generally assumed to be risk averse. As initially pointed out by Poterba and Rotemberg (198X), this affects the user cost calculation as well as the construction of aggregate monetary quantity aggregates. Barnett and Liu (1995) derived a user cost formula for the case of interest rate risk and risk aversion that can be written in the form $\pi_{it}^{Risk}=\pi_{it}+\Psi_{it}$. Ψ_{it} is a risk adjustment containing unknown functional forms, which makes it complicated to implement practically[2]. If the risk adjustment is zero and the returns are known with certainty, the risk adjusted formula reduces to the user cost formula under capital certainty, providing a link back to earlier research. If returns are risky but consumers are risk neutral, the interest rates in the user cost formula under capital certainty can be replaced by expected interest rates, see Corollary to Theorem 1, Barnett, Liu and Jensen (1997, p. 494). Borrowing standard assumptions from the consumption capital asset pricing model framework, Barnett, Liu and Jensen (1997) have shown how a risk-adjusted user cost formula can be derived that contains no unknown functional forms. This user cost formula is only depend-

[2] For notational convenience, this is the real user cost.

ent on forecasts of the returns on assets with interest rate uncertainty, an Arrow-Pratt measure of relative risk aversion and estimated covariances between the real returns on the monetary asset components and growth in real consumption. Empirical studies often suggest that these estimated covariances are small (close to zero), which make the user cost under uncertainty virtually identical to the user cost under risk aversion. In the finance literature, the finding that the covariances are small is often referred to as an equity premium puzzle following Mehra and Prescott (1985). One explanation as to why the risk adjustment turns out small is that data is used that has been aggregated across consumers. It may well be the case that individual consumers are risk averse, but that the representative consumer is risk neutral (see i.e. Barnett, Liu and Jensen (1997)).

2.3 Testing for weak separability

Testing for the existence of a monetary sub-utility function is often performed as a three-step test of utility maximisation. It is first shown that the entire data set is consistent with utility maximisation, i.e. first test for $U(.)$. Secondly, a sub-utility function must rationalise a group of monetary assets, i.e. test if $U_1(.)$ is consistent with utility maximisation. If both the overall utility and the sub-utility function exist, then test if $U_1(.)$ behaves as an elementary good. In terms of equation (2), a test of whether $W(.)$ is consistent with utility maximisation is required. To test for maximisation behaviour and weak separability, both parametric and non-parametric procedures can be used. Since parametric tests have low power, as shown by Barnett and Choi (1989), this study is based on nonparametric tests as defined by Varian (1982, 1983) and extended by de Peretti (2001a,b). de Peretti's procedure extends the standard non-parametric tests to take into account possible stochastic perturbations in the data and develops a new algorithm to solve a system of inequalities to obtain non-negative aggregate price- and quantity indices used in the final stage of the test procedure. We first review the NONPAR procedure and present the two extensions.

2.4 The Standard Non-parametric tests

Since separability tests are simple extensions of utility maximisation tests, we first focus on tests for the latter. Let $x_i=(x^1{}_i,\ldots,x^k{}_i)^T$ be a kx1 vector of consumption goods with associated prices $p_i=(p^1{}_i,\ldots,p^k{}_i)^T$, and let D={$(x_i,p_i)\in(\Re^+)^{2k}$ | $i=1,..,N$} be a data set containing N observations of x_i and p_i. Afriat (1967, 1976) first developed conditions under which a data set D behaves as if generated by the maximisation of a utility function. Afriat's theorem states that it is equivalent to say, for $i,j,r,s,t,q\in\{1,\ldots,N\}$, that[3];

1. There exists a non-satiated utility function that rationalises the data;
2. $\forall$ i,j, there exists numbers $U_i,\lambda_i>0$ that satisfy the Afriat inequalities; $U_i\leq U_j+\lambda_j(p_j\cdot x_i-p_j\cdot x_j)$.
3. There exists a concave, monotonic, continuous, non-satiated utility function that rationalises the data.
4. The data satisfies "cyclical consistency"; that is $p_r\cdot x_r\geq p_r\cdot x_s$,, $p_s\cdot x_s\geq p_s\cdot x_t$, ... , $p_q\cdot x_q\geq p_q\cdot x_r$ implies $p_r\cdot x_r=p_r\cdot x_s$,, $p_s\cdot x_s=p_s\cdot x_t$, ... , $p_q\cdot x_q=p_q\cdot x_r$

Hence, if data are rationalised by a utility function it is possible to assign a utility index (U_i) to each bundle and to rank all bundles. Choices are coherent in time. Since the Afriat inequalities are computationally burdensome for large data sets, Varian (1982) developed another set of conditions. Define first (for $i,j,k,m\in\{1,\ldots,N\}$);

[3] $x\cdot y$ denotes the inner product, i.e. $x\cdot y=x^Ty=\Sigma_i\, x_iy_i$.

- Definition 1 : An observation x_i is strictly directly revealed preferred to x_j written $x_iP^0 x_j$ if $p_i \cdot x_i > p_i \cdot x_j$.
- Definition 2 : An observation x_i is directly revealed preferred to x_j written $x_iR^0 x_j$ if $p_i \cdot x_i \geq p_i \cdot x_j$.
- Definition 3 : An observation x_i is revealed preferred to x_j written $x_iR\, x_j$ if there is a chain of preferences between x_i and x_j such that $p_i \cdot x_i \geq p_i \cdot x_k, , p_k \cdot x_k \geq p_k \cdot x_m, \ldots, p_m \cdot x_m \geq p_m \cdot x_j$.
- Definition 4 : A data set D satisfies the General Axiom Revealed Preferences (hereafter GARP) if $\forall (i,j) \in \{1,\ldots,N\}$ x_iRx_j *implies not* $x_jP^0x_i$.

Varian (1982, p. 948) proved that a data set satisfies cyclical consistency if and only if it satisfies GARP and Afriat's theorem can therefore be rewritten as:

1. There exists a non-satiated utility function that rationalises the data;
2. $\forall$ i,j, there exists numbers $U_i, \lambda_i > 0$ that satisfy the Afriat inequalities $U_i \leq U_j + \lambda_j(p_j \cdot x_i - p_j \cdot x_j)$.
3. There exists a concave, monotonic, continuous, non-satiated utility function that rationalises the data;
4. The data satisfies GARP.

Returning to the separability problem and following Varian (1983), a group of monetary assets will be separable if the three following conditions are satisfied[4]:

1. The entire data set must be consistent with GARP, the existence of $U(.)$.

2. A group of assets must also be consistent with GARP, the existence of $U_1(.)$
3. If the data passes the two previous steps, compute indices from the Afriat inequalities U_i and λ_i for the group of assets and use U_i as quantities and $(\lambda_i)^{-1}$ as prices. $(p^a{}_{i},(\lambda_i)^{-1};x^a{}_{i},U_i)$ is tested for compliance with GARP, where $p^a{}_i$ and $x^a{}_i$ are prices and quantities of all goods, leisure and assets not included in the sub-group of monetary assets that we test for separability.

Nevertheless, this procedure is likely to produce type 1 errors, that is, the probability to reject maximisation (and weak separability) when there is maximisation is high. Two distinct reasons for this can be given. The first one is theoretical and lies in the fact that GARP is non-stochastic, meaning that a single violation of the axiom leads to the rejection of the hypothesis tested. But violations of the axiom may be due to purely stochastic causes as measurement error or optimisation error and are therefore non-significant. Hence, in empirical work, it is crucial to discriminate between significant and non-significant violations. The second reason is related to the test procedure itself. In step three of the procedure, the Afriat inequalities (are solved. Varian (1983) proposed a Fortran routine to solve the Afriat inequalities, but this routine is likely to return negative utility and price indices if variation of prices and quantities are large leading to a rejection of separability.

2.5 Two extensions to Varian's three-step procedure

Is this section, following de Peretti (2001a,b), we first show how to test the significance of violations of GARP, propose a routine to solve the Afriat inequalities and present the final test procedure used in the empirical analysis.

[4] Stage 1is only a necessary conditions for weak separability. Steps 2 and 3 are necessary and sufficient (Theorem 3 in Varian (1983)). Stage 1 is, however, a convenient starting point since it does not involve separating the data into sub-groups.

2.5.1 Testing the significance of violations of GARP

In Varian (1982, 1983), an implicit assumption that the data is measured without errors is made. If this hypothesis is relaxed, we have to take into account that violations of GARP at each step of the separability test procedure may be caused by stochastic factors, so that the data can still be consistent with utility maximisation. Even though the fact that measurement errors or other stochastic considerations may cause violations of the axiom is well known in the related literature (see e.g. Varian (1985)), very few attempts have been made to develop a test to discriminate between significant and non-significant violations.

Assume that under the null hypothesis quantities are generated by optimisation behaviour, given prices and budgets, but are measured with errors. Following Varian (1985), assume that the true data are unobservable and are linked to the observed one by a multiplicative error term (6);

$$x_i^{*j} = x_i^j \left(1 + \varepsilon_i^j\right), \qquad (4)$$

where :

x_i^{*j} is the unobservable true data in period *i* for good *j*,

x_i^j is the observed data in period *i* for good *j*,

ε_i^j is an iid normally distributed error term with zero mean and constant standard error σ.

As in empirical work the magnitude of the error terms (second moment) is unknown, we search for the minimal perturbation in the data to ensure compliance with GARP. If the required adjustment is, given the null, normally distributed, we will conclude that violations are caused by stochastic causes, and then accept maximisation. If, conversely the adjustment is not normally distributed, we will reject the null and conclude to random behaviour or rupture(s) in the utility function. Searching for the minimal perturbation in the data, is achieved solving the quadratic program[5] (7) over x^{*j}_{i} subject to the constraint that the data satisfies GARP (7):

$$obj = \min \sum_{i=1}^{N} \sum_{j=1}^{k} \left(\frac{x_i^{j*}}{x_i^{j}} - 1 \right)^2 \tag{5}$$

subject to

$$U_i \leq U_j + \lambda_j (p_j \cdot x^*_i - p_j \cdot x^*_j) \ \forall \ (i,j) \in \{1,\ldots,N\}.$$

Since, under the null hypothesis, errors are assumed to be normally distributed, the distribution of the error terms can be checked using a standard Jarque-Bera normality test (8), which is distributed as χ^2 with two degrees of freedom:

$$Jarque-Bera = \frac{m}{6}(SK^2 + \frac{1}{4}(K-3)^2), \tag{6}$$

where *SK* is the skewness, *K* is the kurtosis and *m* is the number of bundles replaced times *k*.

Hence the rule of decision is (at a threshold α):

H_0: Jarque-Bera $\leq \chi^2(2)$ critical value. Violations are not significant and maximisation is not rejected.

H_1: Jarque-Bera $> \chi^2(2)$ critical value. Violations are significant and maximisation is rejected.

[5] For computational details see de Peretti (2001a)

The kurtosis measures excess adjustment in few variables. With this procedure, it is possible to discriminate between significant and non-significant violations of GARP.

2.5.2 An alternative procedure to compute utility and price indices.

As discussed in de Peretti (2001b), the procedure used within NONPAR to compute the utility and price indices needed to solve the Afriat inequalities used in step-three of the procedure can produce violations of GARP. The reason is that the procedure can generate negative quantity or price indices. de Peretti (2001b) proposes calculating utility and price indices based upon an index number theoretical approach. The idea is to find utility indices U_i reflecting the evolution of the true and unknown sub-utility function as accurately as possible. Since the chained Törnqvist-Theil quantity (*CQTT*) index is able to track the unknown aggregator function exactly under homotheticity[6] and is able to approximate the variation of the unknown utility function under non-homotheticity (Barnett (1979)), it could be used to compute utility and price indices. Nevertheless, *CQTT* and its dual user cost index *CPTT* will generally not satisfy the Afriat inequalities. Hence *CQTT* and *CPTT* are used as starting values in the quadratic program (7). We search the minimal deviance in *CQTT* and *CPTT* to ensure that *CQTT** and *CPTT** will satisfy the Afriat inequalities. That is, we minimise for *CQTT** and *CPTT** using the quadratic program[7];

$$\min \sum_{i=1}^{N} \left[\left(\frac{CQTT^{*}_{i/b}}{CQTT_{i/b}} - 1 \right)^2 + \left(\frac{\left(CPTT^{*}_{i/b}\right)^{-1}}{\left(CPTT_{i/b}\right)^{-1}} - 1 \right)^2 \right], \qquad (7)$$

subject to

$$CQTT^{*}_{i/b} > 0,$$

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[6] Here, the sub-utility function.

[7] Using this program ensures non-negativity of utility and price indices.

$$CPTT^{*}_{i/b} > 0,$$

and

$$CQTT^{*}_{i/b} \leq CQTT^{*}_{j/b} + \frac{1}{CPTT^{*}_{i/b}} p_j (x_i - x_j),$$

where $CQTT_{i/b}$ and $CPTT_{i/b}$ are chained indices in i with base period b.

2.5.3 The final extended test procedure

The extended three-step procedure we use to test for weak separability is then the following:

1) Test if the entire data set is rationalised by a utility function running GARP. If violations appear, check their significance using the Jarque-Bera test for normality. If violations are non-significant, go to step two.
2) Select a group of (monetary) assets and run GARP to test for the existence of a sub-utility function. If violations occur in testing compliance with GARP, check their significance. If violations are due to measurement error, accept maximisation and replace bundles by corrected-for-errors bundles. Go to step three.
3) Use quantities corrected-for-errors of measurement for the (monetary) sub-group and compute, using (9), utility and price indices. Test GARP with the quantities and prices of goods, assets and leisure not included in the sub group, plus the utility and price indices computed previously. As this step, there may still be errors of measurement in the goods, assets and leisure not included in the sub-group and violations may appear. If violations are found, run the normality test in a slightly different way. Minimise (7) only over goods, assets and leisure not included in the group, forcing utility indices to be at their "true value", since, in this group, data are corrected for errors of measurement.

If a group of assets passes this three step procedure, they are weakly separable.

3. Data

The data used in the paper is quarterly covering the period 1980Q1 to 1998Q4, which gives a total of 80 observations. The household sector holdings of the standard (henceforth "capital certain") components included in the Bank of England Divisia aggregate and their returns have been downloaded from the Bank of England Monetary and Financial Statistics division website[8]. Recent studies based on UK data have extended the list of capital certain assets to include national savings (NS) and certificates of deposit (see e.g. Chrystal and Drake (1997)). In the current study, only national savings is added to the list of capital certain assets since a consistent personal sector series is available for the entire sample. Certificates of deposits are only available from 1986. Personal sector holdings of national savings have been downloaded from DataStream and the Office for National Savings has contributed the returns on this asset. Data on the household sector holdings of equity (UK quoted shares), bonds (UK government bonds) and unit trusts at market values are available from DataStream. All quantities have been been seasonally adjusted using a X11 procedure and transformed to *per capita* quantities. The Financial Times all-share index is used as a proxy to calculate the return on equity and the Financial Times Actuaries bond index is used as a proxy to calculate the return on bonds. The returns on unit trusts have been calculated as an average quarterly return on all unit trusts. Standard and Poor kindly contributed this data set. The returns on all risky assets, bonds shares and unit trusts, are quarterly returns, whilst the returns on the capital certain as-

[8] Available at http://www.bankofengland.co.uk/mfsd/index.htm (Statistical Abstracts, Part 2). M4 contains notes and coins (N/C), non-interest bearing deposits (NIBD), interest bearing bank sight deposits (IBSD), interest bearing bank time deposits (IBTD) and building society deposits (BSD). See also Fisher, Hudson and Pradhan (1993).

sets are yearly returns reported on a quarterly basis. Therefore, the yearly rates of return on the capital certain assets have been transformed to quarterly returns[9].

Following Drake, Fleissig and Mullineux (1999) and Binner and Elger (2002), the returns on all the assets included in the Bank of England Divisia monetary aggregate as well as national savings are assumed known with certainty. Hence it is possible to calculate their user costs using (3). The calculation of the user costs for the risky assets is dependent on calculating forecasts of the own-rates of the remaining monetary assets. Studies have shown that the returns on "risky" assets are autocorrelated. Hence, it may be possible to forecast these returns, see e.g. Lo and MacKinlay (1988) and Poterba and Summers (1986). Following Agung, Drake and Mullineux (1998) and Drake, Fleissig and Mullineux (1999), the forecasts are based on estimation of autoregressive models (using two lags). One step ahead updated forecasts are then used to construct the actual forecasts used in the index calculation. Binner and Elger (2002) found the effect of the CCAPM-adjustment of the user costs to be small since the estimated covariances used in (5) were 0.00012687, 0.00017035 and 0.00019376 for bonds, shares and unit trusts respectively. Hence, the risk adjustments are of a similar magnitude to likely round-off error in the interest rate data, given any reasonable choice of *Z*. One possible explanation to this discussed in Barnett, Liu and Jensen (1997) is that the data is aggregated over consumers. It may very well be the case that risk-adjustments cancel out across consumers. Based on these considerations, the user costs have not been risk-adjusted (ϕ_{it}=0) and the representative consumer is assumed risk neutral.

The benchmark rate is constructed using an envelope approach over all assets included in this study. Since the expected returns on shares and unit trusts (in particular) are high, it was not

[9] Quarterly returns on the capital certain assets have been calculated using the formula $1+r_q=(1+r_y)^{1/4}$.

possible to find any asset with a return sufficiently high to affect the benchmark rate construction in more than a few periods. In order to avoid zero user costs, 0.5 percentage points have been added to the quarterly envelope benchmark rate. This is approximately equivalent to two percentage points on a yearly rate. The Bank of England currently constructs its benchmark rate by adding two percentage points to the annualised three-month local authority rate.

The data on consumption goods is presented in greater detail in table 1 below. Besides from monetary assets, the goods included in this study belongs to the following broad categories; durable goods, non-durable goods, services and leisure. Leisure is defined as 98 hours minus average hours worked in the week. This number is multiplied by 12 to produce a quarterly figure. The price of leisure is an opportunity cost, approximated by the average hourly wage rate in the industry. For durable goods, stocks and associated rental prices are built using depreciation rates as outlined in Patterson (1991).

Table 1. Data Used in Separability Tests

Type of good:	Codes:
Durables :	
Cars, motorcycles and other vehicles	DURA
Other durables	DURB
Non-durables :	
Food , alcohol and tobacco	NDURA
Clothing and footwear	NDURB
Energy products	NDURC
Other goods	NDURD
Services :	
Rental and water charges	SERA
Catering	SERB
Transport and communication	SERC
Financial services	SERD
Other services	SERE
Leisure	LEIS
Non-risky assets:	
Notes and coins	NC
Non-interest-bearing bank deposits	NIBD
Interest-bearing bank sight deposits	IBSD
Interest-bearing bank time-deposits	IBTD
Building societies deposits	BSD
National Savings	NS
Risky assets :	
Bonds	BONDS
Equities	SHARES
Unit trust	UTRUS

In the Data Appendix, a measure of population is given in Table 1, real quantities in table 2, nominal user costs in table 3.1 and the quantities and user costs are also depicted in Figures 1 and 2.

4 Empirical Results

As stated in the theory section (3.1), weak separability requires that 1) the overall utility is rationalised by a utility function, 2) the monetary aggregate under investigation is rationalised by a sub utility function and 3) that consumption goods, leisure and monetary utility indices satisfying the Afriat inequalities also must be rationalised by a utility function.

When testing if the overall utility function is rationalised by a utility function, 28 violations appear (see table 2 below).

Table 2. Test for an overall utility

Full Period:	1980:Q1 1998:Q4		Step one
Test for an overall utility:			
Number of violations	28		
Value of the objective function	0.2987		
Normality test (Jarque-Bera statistic)	2896.46		
p-value	0.00		
Sub- periods:	1980:Q1 1991:Q3	1991:Q4 1998:Q4	
Test for an overall utility:			
Number of violations	9	0	
Value of the objective function	0.1883		
Normality test (Jarque-Bera statistic)	431.39		
p-value	0.00		

Even though the number of violations of overall utility is not very big, utility maximisation is clearly rejected. Since the full data set is not consistent with utility maximisation, it was divided into two sub-sets. During the period 1980Q1 to 1991Q3, utility maximisation can still not be accepted. Tests revealed that utility maximisation could be accepted if the data set was broken down further into small periods or if "problematic" years were removed. It was still not possible to find any meaningful broad weakly separable groups of monetary assets, the major reason being the rapid growth in IBSD deposits that is not reflected in the user cost for that particular asset. For the second sample, 1991Q4 to 1998Q4, the data is consistent with utility maximisation. There is no violation of GARP.

In the sample that covers the time period 1991Q4 to 1998Q4, all capital certain assets are found to meet the necessary and sufficient conditions for aggregation. It is, however, not possible to find any separable monetary aggregate that contains risky assets. These results are presented in table 3 below;

Table 3. Results of separability tests for the second sample

<table>
<tr><td>Period</td><td colspan="9">1991:Q4 1998:Q4</td><td></td></tr>
<tr><td></td><td colspan="5">Standard BoE assets</td><td></td><td colspan="3">Risky assets</td><td></td></tr>
<tr><td></td><td>NC</td><td>NIBD</td><td>IBSD</td><td>IBTD</td><td>BSD</td><td>NS</td><td>BOND</td><td>SHARE</td><td>TRUST</td><td></td></tr>
<tr><td><u>Test for a sub-utility function for:</u></td><td>X</td><td>X</td><td>X</td><td>X</td><td>X</td><td></td><td></td><td></td><td></td><td rowspan="5">Step two</td></tr>
<tr><td>Number of violations</td><td colspan="9">0</td></tr>
<tr><td>Value of the objective function</td><td colspan="9"></td></tr>
<tr><td>Normality test (Jarque-Bera statistic)</td><td colspan="9"></td></tr>
<tr><td>P-value</td><td colspan="9"></td></tr>
<tr><td><u>Test for separability</u></td><td colspan="9"></td><td rowspan="6">Step three</td></tr>
<tr><td>Afriat sufficient condition</td><td colspan="9">S</td></tr>
<tr><td>Number of violations</td><td colspan="9">0</td></tr>
<tr><td>Value of the objective function</td><td colspan="9"></td></tr>
<tr><td>Normality test (Jarque-Bera statistic)</td><td colspan="9"></td></tr>
<tr><td>P-value</td><td colspan="9"></td></tr>
<tr><td>Test for a sub-utility function for:</td><td>X</td><td>X</td><td>X</td><td>X</td><td>X</td><td>X</td><td></td><td></td><td></td><td rowspan="5">Step two</td></tr>
<tr><td>Number of violations</td><td colspan="9">0</td></tr>
<tr><td>Value of the objective function</td><td colspan="9"></td></tr>
<tr><td>Normality test (Jarque-Bera statistic)</td><td colspan="9"></td></tr>
<tr><td>P-value</td><td colspan="9"></td></tr>
<tr><td><u>Test for separability</u></td><td colspan="9"></td><td rowspan="6">Step three</td></tr>
<tr><td>Afriat sufficient condition</td><td colspan="9">S</td></tr>
<tr><td>Number of violations</td><td colspan="9">0</td></tr>
<tr><td>Value of the objective function</td><td colspan="9"></td></tr>
<tr><td>Normality test (Jarque-Bera statistic)</td><td colspan="9"></td></tr>
<tr><td>P-value</td><td colspan="9"></td></tr>
<tr><td>Test for a sub-utility function for:</td><td>X</td><td>X</td><td>X</td><td>X</td><td>X</td><td>X</td><td>X</td><td></td><td></td><td rowspan="5">Step two</td></tr>
<tr><td>Number of violations</td><td colspan="9">2</td></tr>
<tr><td>Value of the objective function</td><td colspan="9">0.0028</td></tr>
<tr><td>Normality test (Jarque-Bera statistic)</td><td colspan="9">8.52</td></tr>
<tr><td>P-value</td><td colspan="9">0.01</td></tr>
<tr><td><u>Test for separability</u></td><td colspan="9"></td><td rowspan="6">Step three</td></tr>
<tr><td>Afriat sufficient condition</td><td colspan="9">NS</td></tr>
<tr><td>Number of violations</td><td colspan="9"></td></tr>
<tr><td>Value of the objective function</td><td colspan="9"></td></tr>
<tr><td>Normality test (Jarque-Bera statistic)</td><td colspan="9"></td></tr>
<tr><td>P-value</td><td colspan="9"></td></tr>
<tr><td>Test for a sub-utility function for:</td><td>X</td><td>X</td><td>X</td><td>X</td><td>X</td><td>X</td><td>X</td><td>X</td><td></td><td rowspan="5">Step two</td></tr>
<tr><td>Number of violations</td><td colspan="9">55</td></tr>
<tr><td>Value of the objective function</td><td colspan="9">0.0205</td></tr>
<tr><td>Normality test (Jarque-Bera statistic)</td><td colspan="9">392.16</td></tr>
<tr><td>P-value</td><td colspan="9">0.00</td></tr>
<tr><td><u>Test for separability</u></td><td colspan="9"></td><td rowspan="6">Step three</td></tr>
<tr><td>Afriat sufficient condition</td><td colspan="9">NS</td></tr>
<tr><td>Number of violations</td><td colspan="9"></td></tr>
<tr><td>Value of the objective function</td><td colspan="9"></td></tr>
<tr><td>Normality test (Jarque-Bera statistic)</td><td colspan="9"></td></tr>
<tr><td>P-value</td><td colspan="9"></td></tr>
<tr><td>Test for a sub-utility function for:</td><td>X</td><td>X</td><td>X</td><td>X</td><td>X</td><td>X</td><td>X</td><td>X</td><td>X</td><td rowspan="5">Step two</td></tr>
<tr><td>Number of violations</td><td colspan="9">11</td></tr>
<tr><td>Value of the objective function</td><td colspan="9">0.0137</td></tr>
<tr><td>Normality test (Jarque-Bera statistic)</td><td colspan="9">12.55</td></tr>
<tr><td>P-value</td><td colspan="9">0.00</td></tr>
<tr><td><u>Test for separability</u></td><td colspan="9"></td><td rowspan="6">Step three</td></tr>
<tr><td>Afriat sufficient condition</td><td colspan="9">NS</td></tr>
<tr><td>Number of violations</td><td colspan="9"></td></tr>
<tr><td>Value of the objective function</td><td colspan="9"></td></tr>
<tr><td>Normality test (Jarque-Bera statistic)</td><td colspan="9"></td></tr>
<tr><td>P-value</td><td colspan="9"></td></tr>
</table>

Note: "S" means that the group is weakly separable. "NS" means that the group does not meet the separability condition..

We have also tried to only add all possible combinations of risky assets to the list of capital certain assets (e.g. only unit trusts and unit trusts + bonds). Utility maximisation is strongly rejected (p=0.00) for all combinations. If only unit trusts are added to the list of capital certain assets, there are no GARP violations for the monetary sub-utility function and the Afriat inequalities are satisfied. When these monetary assets are replaces by price and quantity indices, 99 violations appear and utility maximisation is, again, rejected with p=0.00.

5. Conclusions

The results presented in this paper bear strong implications for research on Divisia money using UK data. First of all, it is not possible to find any meaningful broad weakly separable group of monetary assets during the eighties. This clearly implies that studies based upon such data must be interpreted with some caution. Looking at Figures 1 and 2 in the data appendix, it becomes evident that there are large variations in the quantities that are not reflected in the user costs. All capital certain assets are found to be weakly separable over the time period 1991Q4 to 1998Q4. The tests suggest national savings can be added to the list of assets used currently by the Bank of England. Since quite small quantities are held in national savings and the own-rate of national savings is high compared with the other capital certain assets, the impact of including this asset in a monetary services index is small and whether or not to include national savings should be of marginal importance – at least in macro-economic applications. It is furthermore not possible to find any separable group of assets that includes any of the risky assets. While risk neutrality is a convenient finding for the purpose of performing non-parametric weak separability tests, it may be proven wrong when possible future solutions to the equity premium puzzle are incorporated in the model framework.

The main policy implication of this paper is that the Bank of England's Divisia household sector monetary aggregate should be interpreted with some caution before 1991Q4 and that it should not, currently, be extended to include bonds, shares or unit trusts. National savings may, however, be included.

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