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Panel Cointegration and the Neutrality of Money*

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Abstract

Most econometric methods for testing the proposition of long-run monetary neutrality rely on the assumption that money and real output do not cointegrate, a result that is usually supported by the data. This paper argues that these results can be attributed in part to the low power of univariate tests, and that a violation of the noncointegration assumption is likely to result in a nonrejection of the neutrality proposition. To alleviate this problem, two new and more powerful panel cointegration tests are proposed that can be used under very general conditions. The tests are then applied to a panel covering 10 countries between 1870 and 1986. The results suggest money and real output are cointegrated, and that the neutrality proposition therefore must be rejected.

JEL Classification: E30; E50; C12; C22; C23.

Keywords: Monetary Neutrality; Panel Cointegration Testing.

1 Introduction

There are few propositions in classical economics that are less controversial than the long-run neutrality of money, which states that permanent changes in the stock of money have no long-run effects on the level of real output. Yet, for an idea so widely accepted among economists, the empirical evidence on the neutrality of money has been very mixed and far from convincing.

Like in many other areas, most early empirical studies on the neutrality proposition focused on United States, and many used conventional reduced-form

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regression analysis. However, it has now become clear that this sort of reduced-form estimates cannot be used as a basis for testing the neutrality proposition, as they do not consider permanent changes in the stock of money. In fact, as shown by Fisher and Seater (1993), meaningful neutrality tests can only be constructed if money and real output are nonstationary, and thus subject to permanent shocks. The authors also show that the proper implementation of such tests generally calls for structural econometric methods.¹

King and Watson (1997) recognize these deficiencies, and develop a new econometric framework for testing the neutrality proposition when the variables are nonstationary but noncointegrated.² Their point of origin is a final-form vector autoregression in the first differences of money and real output, which, in contrast to traditional reduced-form methods, can be used to identify the dynamic response of the variables to the underlying structural disturbances.

The King and Watson (1997) approach to identify these disturbances is eclectic. Rather than imposing a single identifying restriction, such as money exogeneity, they suggest systematically investigating a wide range of restrictions, which leads to the estimation of multiple observationally equivalent structural models. In particular, in this framework, the estimated long-run elasticity of output with respect to money depend critically on what is assumed about one of three other elasticities, namely the short-run elasticity of output with respect to money, the short-run elasticity of money with respect to output or the long-run elasticity of money with respect to output. The idea is to iterate each of these parameters over a reasonable range of values, each time obtaining an estimate of the long-run elasticity of output with respect to money. If money is truly neutral, then these estimates should be close to zero, as monetary shocks should have no long-run effect on real output.

King and Watson (1997) apply their approach to United States, using data that cover the years 1949 to 1990. Based on a wide range of restrictions, they find little evidence against the neutrality of money. Although robust to a wide spectrum of identifying assumptions, this finding is, however, tampered by at least three important caveats.

Firstly, the results hinge on money and output being nonstationary in their levels, and with only 40 years of data, inference on such long-run features is necessarily uncertain. This is especially important as conventional tests for unit roots are known to suffer from low power, as a trend stationary process can be easily mistaken for a unit root in small samples. Many authors have therefore adopted the strategy of using many tests, in hope that this will make their results more robust in this regard.

Another, related, caveat is that, even if it were known that money and output

¹Using structural model, Evans (1996) shows that money should be not neutral in the long-run if it is not neutral in the short-run and the money growth rate is endogenous.

²The King and Watson (1997) approach has not only been very popular for testing money neutrality but also in many other related areas, see Bullard (1999) for a recent survey of this literature.

are indeed nonstationary, the analysis still relies critically on the variables being noncointegrated. This is important not only because a violation makes the vector autoregression of King and Watson (1997) misspecified, thus making their structural estimates suspect, but also because the presence of cointegration is by itself sufficient for rejecting the neutrality proposition, see Fisher and Seater (1993).

Finally, although King and Watson (1997) do verify the noncointegration assumption, their test is implemented as a unit root test on the difference between money and output. As such, this test is also subject to the first critique, and is therefore expected to suffer from low power. Another problem with this testing approach is that a rejection of the no cointegration null does not just imply cointegration but cointegration with a unit slope on money. Hence, a nonrejection could very well be due to cointegration but with a slope that is different from one.

As mentioned, most empirical evidence on the neutrality of money is for United States, and there have been only a few studies based on international data, see Bullard (1999) and the references therein. One of the most notable contributions within this latter field is that by Serletis and Koustas (1998), in which the authors apply the King and Watson (1997) approach to a panel of 10 industrialized countries covering approximately the years 1870 to 1986.

Serletis and Koustas (1998) make a more ambitious attempt than King and Watson (1997) to test the validity of the noncointegration assumption. They do this by subjecting each country in their sample to a battery of cointegration tests, and find that the null hypothesis of no cointegration cannot be rejected, which suggests that the King and Watson (1997) approach is applicable. The ensuing results entail that it is difficult to reject the neutrality proposition under plausible identifying restrictions. The authors also produce evidence to suggest that parts of their results are robust with respect to structural change.

Of course, being based on the same econometric method, the Serletis and Koustas (1998) study is subject to the same line of critique as the King and Watson (1997) study. Moreover, although the cointegration testing approach of Serletis and Koustas (1998) represents a significant improvement upon the King and Watson (1997) approach, it is still expected to suffer from low power, for much of the same reasons that tests for unit roots are expected to have low power.

In his review of the literature on monetary neutrality, Bullard (1999) stresses the importance of accurate unit root and cointegration tests, and suggests that a panel approach might be more appropriate in this respect. This paper is an attempt in this direction. Using the same data as Serletis and Koustas (1998), we begin by showing that the King and Watson (1997) approach is very unlikely to detect deviations from the neutrality proposition when money and output are cointegrated but slowly error correcting, which seems like a very plausible alternative scenario for most applications of this kind.

This finding suggests that the cointegration testing should be considered as an integral part of the neutrality test, and not only as a diagnostic preliminary as is usually the case. Therefore, since conventional tests do not seem to be powerful enough, we develop two new panel cointegration tests that account for the variation in the cross-section, and are expected to produce more accurate results.

However, the new tests are not only appealing from a power point of view. In fact, there are at least three other advantageous features that are worth mentioning here. Firstly, because they derive from a simple error correction generalization of the vector autoregression considered by King and Watson (1997), they seem like a natural extension of what has previously been done. Secondly, the tests are general enough to allow for important features such as cross-sectional dependence, structural breaks as well as unbalanced panels, which are not only highly relevant in this particular study, but in almost every macroeconomic and financial application. Thirdly, since the tests are asymptotically normal, there is no need for a special table of critical values, which make them easy to implement. Another operational advantage is that the distribution of the tests are independent of the regressors.

When we apply the new tests to the Serletis and Koustas (1998) data, we find that the null of no cointegration can be rejected at all conventional significance levels. Thus, in contrast to these authors, we find that permanent changes in the level of money stock do in fact affect real output, which implies that the long-run neutrality of money must be rejected.

The rest of this paper proceeds as follows. Section 2 provides a brief account of the King and Watson (1997) approach, and motivates our study. Section 3 then introduces the new tests and Section 4 is concerned with their small-sample properties. Section 5 presents our empirical results and Section 6 concludes. Proofs of important results are given in the appendix.

2 Cointegration and the neutrality of money

Let y_{it} and m_{it} denote the logarithm of real output and nominal money supply at time $t = 1, \dots, T$ for country $i = 1, \dots, N$, respectively. Our starting point is a conventional bivariate error correction model for Δy_{it} and Δm_{it} , the first difference of y_{it} and m_{it} , respectively, which can be written as follows

$$\alpha_{yi}(L)\Delta y_{it} = \phi_{yi}(y_{it-1} - \beta_i m_{it-1}) + \gamma_{yi}(L)\Delta m_{it} + e_{yit}, \quad (1)$$

$$\alpha_{mi}(L)\Delta m_{it} = \phi_{mi}(y_{it-1} - \beta_i m_{it-1}) + \gamma_{mi}(L)\Delta y_{it} + e_{mit}, \quad (2)$$

where $\alpha_{yi}(L) = 1 - \sum_{j=1}^{p_i} \alpha_{yij} L^j$ and $\gamma_{yi}(L) = \gamma_{yi} + \sum_{j=1}^{p_i} \gamma_{yij} L^j$ are p_i ordered polynomials in the lag operator L , which governs the short-run dynamics of the first equation. The corresponding polynomials $\alpha_{mi}(L)$ and $\gamma_{mi}(L)$ of the second equation are defined in exactly the same way. The disturbances e_{yit} and e_{mit}

are assumed to be serially uncorrelated, but not necessarily uncorrelated with each other.

The above model is very similar to the King and Watson (1997) model used by Serletis and Koustas (1998). The only difference lies in the first term on the right-hand side, which captures the error correction properties of the data. If ϕ_{yi} and ϕ_{mi} are zero, then there is no error correction, whereas, if at least one of ϕ_{yi} and ϕ_{mi} are less than zero, then there is error correction, and y_{it} and m_{it} are cointegrated with cointegrating slope β_i . Thus, β_i relates to the long-run part of the model, while the remaining parameters relate to the short-run part.

Serletis and Koustas (1998) assume that ϕ_{yi} and ϕ_{mi} are zero, in which case the long-run elasticity of output with respect to a permanent shock in money is given by $\gamma_{yi}(1)/\alpha_{yi}(1)$. Let us denote this parameter by β_{yi} , and let β_{mi} denote the corresponding long-run elasticity of money with respect to a permanent shock in output. Hence, in this notation, the problem of testing the hypothesis of long-run money neutrality is equivalent to testing the restriction that β_{yi} is zero. Note also that, since both the short- and long-run parts of (1) are normalized with respect to output, β_{yi} is identically $\phi_{yi}\beta_i$, which implies that $\phi_{yi} < 0$ is sufficient to reject money neutrality.

Since money is endogenous, however, the above system is not identified so the neutrality restriction is not really testable. In fact, as is well-known, only three of the unidentified parameters $\text{var}(e_{yit})$, $\text{var}(e_{mit})$, $\text{cov}(e_{yit}, e_{mit})$, γ_{yi} and γ_{mi} can be recovered. Thus, to identify this system, two identifying restrictions are required. Serletis and Koustas (1998) assume that e_{yit} and e_{mit} are uncorrelated, which means that only one additional restriction is needed.

A common way of doing this, which we will make use of later, is to assume that money is exogenous so that $\gamma_{mi}(L)$ and ϕ_{mi} are jointly zero, see Fisher and Seater (1993).³ Alternatively, we may follow Serletis and Koustas (1998), and assume a prespecified value for one of the elasticities γ_{yi} , γ_{mi} and β_{mi} . The idea is that if the restriction is true and money is neutral, then the estimates of the remaining parameters should lie close to their true values. In particular, the estimate of β_{yi} should be close to zero. By using this approach, the authors find that the neutrality proposition cannot be rejected.

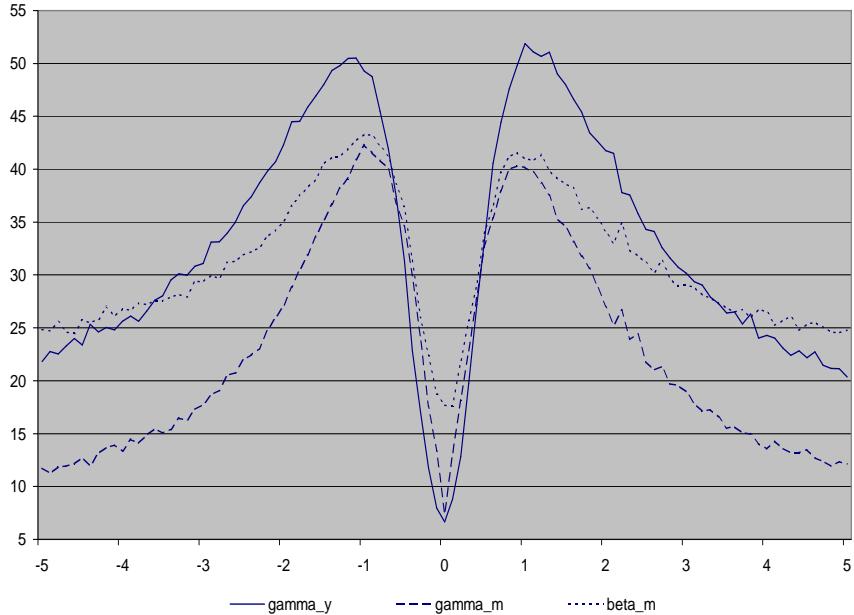
Unfortunately, the properties of the above approach becomes suspect if y_{it} and m_{it} are cointegrated, as the estimated model then becomes misspecified due to an omitted error correction term. To illustrate this effect, we conducted a small simulation exercise using (1) and (2) to generate the data. For simplicity, we set $\alpha_{yi}(L)$ and $\alpha_{mi}(L)$ to unity, and $\gamma_{yi}(L)$ and $\gamma_{mi}(L)$ to zero. We also set ϕ_{mi} to zero so that output is error correcting, while money is a pure unit root process. Thus, in this setup, γ_{yi} , γ_{mi} and β_{mi} are all zero, but y_{it} and m_{it} are cointegrated with slope β_i . The disturbances e_{yit} and e_{mit} are first drawn

³In a recent study, Hatemi and Irandoust (2006) report results suggesting that the exogeneity hypothesis holds for Denmark, Japan and the United States over the period 1961 to 2000.

from the normal distribution and then rescaled to have variance $\text{var}(e_{mit})$ and $\text{var}(e_{yit})$, respectively. We make 1,000 replications of panels of the same size as that used by Serletis and Koustas (1998), and we use their data to calibrate ϕ_{yi} , β_i , $\text{var}(e_{mit})$ and $\text{var}(e_{yit})$.

Figure 1 present the rejection frequencies for testing the hypothesis that β_{yi} is zero, while varying γ_{yi} , γ_{mi} and β_{mi} as in Serletis and Koustas (1998).⁴ The estimates of ϕ_{yi} are all less than zero, with an average of -0.05 , so money is not neutral in this experiment. We therefore expect the test to reject the null with high frequency for all values of γ_{yi} , γ_{mi} and β_{mi} . However, this is not what we observe. In fact, quite oppositely, we see that the power tend to be low, especially in the neighborhood of zero, where the identifying restrictions are true. Another interesting observation is that power can be very low when the identifying restrictions are far from true.

Figure 1: Power as a function of γ_{yi} , γ_{mi} and β_{mi} .



This finding is important because it suggests that the inability of Serletis and Koustas (1998) to reject money neutrality may not reflect the actual data generating process, but rather the low power of their test. A natural way to alleviate this problem would be to check for cointegration before conducting

⁴For comparability with Serletis and Koustas (1998), we estimate (1) and (2) using the instrumental variables estimator of King and Watson (1997). The number of lags is set to five.

the neutrality test. Serletis and Koustas (1998) employ a country-by-country approach whereby each country is subjected to a battery conventional cointegration tests. The results suggest the null of no cointegration cannot be rejected at conventional significance levels.

However, it is well-known that tests of this kind are bound to result in a nonrejection unless the number of time series observations is very large. Thus, the cointegration results of Serletis and Koustas (1998) might also be due to low power. But if these tests are unable to discriminate between the null and alternative hypotheses, then how should we determine which one is true?

The approach taken in this paper is based on using not only the time series dimension of the data, but also the cross-sectional dimension. The idea is that by pooling all the information in the sample, it should be possible to construct more powerful tests.

Unfortunately, existing tests suffer from many weaknesses that make them unsuitable for our application. First, most tests are based on the assumption that the cross-sectional units are independent, which is unlikely to hold in the money and output data due to strong inter-economy linkages. Second, most tests become prohibitively difficult to implement when the panel is unbalanced, which is not only relevant for this study but in almost every other macroeconomic application of this kind. Third, as noted by Serletis and Koustas (1998), there is generally no simple way to modify this type of tests for nonstationary data to accommodate structural change, which would seem as a very plausible scenario in this type of data.

This discussion indicate that, before any serious attempt to determine the neutrality of money can be mounted, there is a need to develop more general panel cointegration tests, and the next two sections do exactly that.

3 The panel cointegration tests

The purpose of this section is to device two panel cointegration tests that are consistent with the data generating process of the previous section, and that fit the test requirements given in the above. We begin by describing the model of interest, and then we describe the tests and their asymptotic properties.

3.1 Model and assumptions

The panel model that we consider is given by

$$\alpha_{yi}(L)\Delta y_{it} = \delta'_i d_t + \phi_{yi}(y_{it-1} - \beta_i m_{it-1}) + \gamma_{yi}(L)\Delta m_{it} + e_{yit}, \quad (3)$$

$$\alpha_{mi}(L)\Delta m_{it} = e_{mit}. \quad (4)$$

Note that this is exactly the model used in the previous section. The essential difference is that we have now restricted money to be exogenous by setting

both $\gamma_{mi}(L)$ and ϕ_{mi} equal to zero.⁵ Another difference is the vector d_t , which contains deterministic terms. The conventional elements of d_t include a constant and a linear time trend. However, in this paper, other deterministic components such as polynomial trends and break dummy variables are also possible.

The rest of the assumptions needed for developing our new panel tests are laid out next.

Assumption 1. (Error process.) The disturbances e_{yit} and e_{mit} satisfy the following set of conditions:

- (a) The groups e_{yit} and e_{mit} are mean zero, and independent and identically distributed across both i and t .
- (b) $\text{var}(e_{yit}) > 0$ and $\text{var}(e_{mit}) > 0$.
- (c) $E(e_{ykt}e_{mij}) = 0$ for all k, t, i and j .

Assumption 1 establishes the basic conditions needed for developing the new cointegration tests. Some may appear quite restrictive but are made here in order to make the analysis of the tests more transparent, and will be relaxed later on.

For example, Assumption 1 (a) states that the individuals are independent over the cross-sectional dimension. This condition is not necessary but will be convenient to retain initially as it will allow us to apply standard central limit theory in a very simple manner. A suggestion on how to relax this condition will be given in Section 3.3. Similarly, independence over time is convenient because it facilitates a straightforward asymptotic analysis by application of the conventional methods for integrated processes. In particular, Assumption 1 (a) ensures that an invariance principle holds for each cross-section as T grows. Also, since our approach can be readily generalized to the case when m_{it} is a vector, Assumption 1 (b) ensures that $\text{var}(e_{mit})$ is positive definite suggesting that there cannot be cointegration within m_{it} .

Assumption 1 (c) requires that e_{mit} and e_{yit} are independent. Although this might seem somewhat restrictive at first, our model is actually quite general when it comes to the short-run dynamics. In fact, the only requirement is that the regressors are strongly exogenous with respect to the parameters of interest, which is implicit in our model since the errors are uncorrelated and m_{it} is not error correcting. Weak exogeneity can be readily accommodated by augmenting (3) not only with lags but also with leads of Δm_{it} , so that $\gamma_{yi}(L)$ becomes double-sided.⁶ Moreover, since $\alpha_{yi}(L)$, $\alpha_{mi}(L)$ and $\gamma_{yi}(L)$ are

⁵Fisher and Seater (1993) also assume that ϕ_{mi} is zero, an assumption they refer to as long-run money exogeneity. Our model is more restrictive in the sense that it also sets $\gamma_{mi}(L)$, the short-run response of money to output, at zero. However, as will be argued later in this section, since a nonzero $\gamma_{mi}(L)$ can be accommodated by simply augmenting (3) not only with lags but also with leads of Δm_{it} , this assumption is not particularly restrictive.

⁶Thus, although money need not be strictly exogenous, as in Fisher and Seater (1993), the requirement that ϕ_{mi} is zero cannot be relaxed.

permitted to vary between the individuals of the panel, we are in fact allowing for a completely heterogeneous serial correlation structure.

In the remainder of this section, we present the new panel cointegration tests. We begin by considering the simple case when the cross-sectional units are independent, and then we continue to the case with dependent units.

3.2 Independent units

In constructing the new tests, it is useful to rewrite (3) as

$$\begin{aligned}\Delta y_{it} = & \delta'_i d_t + \phi_{yi} (y_{it-1} - \beta_i m_{it-1}) + \sum_{j=2}^{p_i} \alpha_{yij} \Delta y_{it-j} + \sum_{j=1}^{p_i} \gamma_{yij} \Delta m_{it-j} \\ & + \gamma_{yi} \Delta m_{it} + e_{yit}.\end{aligned}$$

The problem is how to estimate the error correction parameter ϕ_{yi} , which forms the basis for our new tests. One way is to assume that β_i is known and to estimate ϕ_{yi} by least squares. However, as shown by Boswijk (1994) and Zivot (2000), such tests are generally not similar and depend on nuisance parameters even asymptotically.

Alternatively, note that the above regression can be reparameterized as

$$\begin{aligned}\Delta y_{it} = & \delta'_i d_t + \phi_{yi} y_{it-1} + \lambda_i m_{it-1} + \sum_{j=2}^{p_i} \alpha_{yij} \Delta y_{it-j} + \sum_{j=1}^{p_i} \gamma_{yij} \Delta m_{it-j} \\ & + \gamma_{yi} \Delta m_{it} + e_{yit}.\end{aligned}\tag{5}$$

In this regression, the parameter ϕ_{yi} is unaffected by imposing an arbitrary β_i , which suggests that the least squares estimate of ϕ_{yi} can be used to provide a valid test of the hypothesis that $\phi_{yi} = 0$. Indeed, because λ_i is unrestricted, and because the cointegration vector is implicitly estimated under the alternative hypothesis, as seen by writing $\lambda_i = -\phi_{yi}\beta_i$, this means that it is possible to construct a test based on ϕ_{yi} that is asymptotically similar and whose distribution is free of nuisance parameters, see Banerjee *et al.* (1998).

Although this makes inference by least squares possible, in this paper, we propose another approach, which is based on estimating ϕ_{yi} using Δy_{it-1} and Δm_{it-1} as proxies for y_{it-1} and m_{it-1} .⁷ This approach has previously been used by Im and Lee (2005) to test for a unit root.⁸ To our knowledge, however, this is the first time it has been used for testing cointegration. The idea is that by approximating with stationary variables, we can construct statistics with limit distributions that are free of the usual dependence on Brownian motion.

⁷To clarify, the differenced series are not used here because they are good approximations of their levels, but because they eliminate the unit root dependence under the null.

⁸Im and Lee (2005) refer to their test as an instrumental variables approach to unit root testing, which is somewhat misleading because what they do is to approximate the levels of the variable with their first differences, as described above in the text.

The estimated proxy version of (5) is given by

$$\begin{aligned}\Delta y_{it} = & \widehat{\delta}'_i d_t + \widehat{\phi}_{yi} \Delta y_{it-1} + \sum_{j=2}^{p_i} \widehat{\alpha}_{yij} \Delta y_{it-j} + \sum_{j=1}^{p_i} \widehat{\gamma}_{yij} \Delta m_{it-j} \\ & + \widehat{\gamma}_{yi} \Delta m_{it} + \text{error}.\end{aligned}\quad (6)$$

Note that, in this notation, the second polynomial absorbs not only the lags of Δm_{it} in (5) but also our proxy for $\lambda_i m_{it-1}$. Now, let τ_i denote the individual t -statistic for testing the hypothesis that $\phi_{yi} = 0$ in (6). If we let \Rightarrow signify weak convergence, then the asymptotic distribution of this statistic is given in the following theorem.

Theorem 1. (Asymptotic distribution under the null.) Under Assumption 1 and the null hypothesis of no cointegration, then

$$\tau_i \Rightarrow N(0, 1) \text{ as } T \rightarrow \infty.$$

The fact that the distribution of τ_i is normal is actually much stronger than one might think. First, since the critical values are readily available using the standard normal table, the new test is much simpler to implement than other cointegration tests where the asymptotic distribution is a nonstandard mixture of Brownian motion. Second, since there is no dependence on the regressors, the same critical values apply regardless of the dimension of m_{it} , which is also different from the conventional test situation. Third, as long as the proxy regression in (6) has been appropriately modified, there is no dependence on the underlying deterministic specification. In particular, in the case of structural break, the asymptotic distribution of the test is free of the usual nuisance parameter indicating the location of the break.

Another advantage with Theorem 1 is that it permits for easy testing in the panel setting. The panel statistic that we consider in this section is given by

$$\tau_N = N^{-1/2} \sum_{i=1}^N \tau_i.$$

This statistic is comparable to most existing panel cointegration tests, and is proposed here as a test of $H_0 : \phi_{yi} = 0$ for all i versus $H_1 : \phi_{yi} < 0$ for at least some i . In view of Theorem 1, since the time series limit of each individual τ_i is standard normal, it is easy to see that the asymptotic distribution of τ_N is also normal. Moreover, because of the normality of τ_i , τ_N is expected to perform better under the null than other tests where the corresponding limit as T grows is usually highly noncentral and skewed. The fact that τ_N is nothing but a sum of individual tests means that unbalanced panels can be very easily accommodated.

It is important that a test is able to fully discriminate between the null and alternative hypotheses. In our case, there are at least two reasons to suspect the power to be low, although still better than for pure time series tests. First, since we are in effect dealing with a stationary regression, $\hat{\phi}_{yi}$ is no longer consistent at the usual rate of T , but rather \sqrt{T} . Second, $\hat{\phi}_{yi}$ is not consistent for ϕ_{yi} under the alternative hypothesis.

To illustrate the second point, suppose that there is no serial correlation, and that $d_t = 0$, in which case (3) can be written as

$$\Delta y_{it} = \phi_{yi}(y_{it-1} - \beta_i m_{it-1}) + \gamma_{yi} \Delta m_{it} + e_{yit}. \quad (7)$$

If we define the signal-to-noise ratio $N_i = (\gamma_{yi} - \beta_i)^2 \text{var}(e_{mit}) / \text{var}(e_{yit})$ and let \rightarrow_p denote convergence in probability, then the limit of the proxy estimator $\hat{\phi}_{yi}$ of ϕ_{yi} under the cointegrated alternative is given in the following theorem.

Theorem 2. (Test consistency.) Under the conditions laid out in the above and the alternative hypothesis of cointegration, then

$$\hat{\phi}_{yi} \rightarrow_p \frac{\phi_{yi}(1 - (1 + \phi_{yi})N_i)}{2 - \phi_{yi}N_i} \quad \text{as } T \rightarrow \infty.$$

It is interesting to evaluate the limit of $\hat{\phi}_{yi}$ depending on whether N_i is zero or not, which in turn depends on whether the common factor restriction $\gamma_{yi} = \beta_i$ is satisfied or not. If it is satisfied, then it is clear that $\hat{\phi}_{yi}$ converges to $\phi_{yi}/2$, which implies that $\sqrt{T}\hat{\phi}_{yi}$ diverges to negative infinity as T grows so the test is consistent. Nevertheless, since $\hat{\phi}_{yi}$ converges to $\phi_{yi}/2$ and not ϕ_{yi} , in small-samples it seems reasonable to expect the power to be lower than for tests based on a consistent estimate of ϕ_{yi} .

On the other hand, if $\gamma_{yi} \neq \beta_i$ and the common factor restriction does not hold, then the direction of the divergence depends on the magnitude of N_i and $-2 < \phi_{yi} < 0$. If $\phi_{yi} \rightarrow -2$, then $\hat{\phi}_{yi}$ goes to minus unity so $\sqrt{T}\hat{\phi}_{yi}$ diverges towards negative infinity as T grows. Note also that if ϕ_{yi} goes to zero, then so does $\sqrt{T}\hat{\phi}_{yi}$, thus corroborating Theorem 1 that τ_i is mean zero under the null. For intermediate values of ϕ_{yi} , the probability limit of $\hat{\phi}_{yi}$ is either positive or negative, depending on the particular combination of N_i and ϕ_{yi} . Thus, in contrast to many existing tests, τ_i and τ_N are double-sided.

One exception from the above analysis is the special case when $N_i > 1$ and ϕ_{yi} is identically $(1 - N_i)/N_i$, in which $\hat{\phi}_{yi}$ converges to zero, suggesting that for this particular value of ϕ_{yi} the test is inconsistent. Thus, in general we need to assume that $N_i < 1$ or that ϕ_{yi} is bounded away from $(1 - N_i)/N_i$, or both. Since the probability that both these conditions are violated is essentially zero, however, this assumption is not very restrictive. In fact, in comparison to the conventional residual-based approach, which requires that the common factor restriction is satisfied, it is actually a great improvement.

3.3 Dependent units

Although the assumption of cross-sectional independence allows for the construction of a very simple test, in applications such as ours, it can be quite restrictive. In this section, we therefore generalize our earlier results to the case with dependence among the cross-sectional units.

In order to do so, however, Assumption 1 (a) must be replaced with something else. Here we assume that the dependence can be described in terms of a common correlation between the individual statistics so that

$$E(\tau_i \tau_j) = \rho \text{ for } i \neq j,$$

where $-1/(N-1) < \rho < 1$ ensures the positive definiteness of the resulting covariance matrix. Note that, if ρ were known, since the variance of $\sum_{i=1}^N \tau_i$ is given by $N + N(N-1)\rho$, we could easily generalize τ_N using

$$\tilde{\tau}_N = (N + N(N-1)\rho)^{-1/2} \sum_{i=1}^N \tau_i.$$

As with τ_N , this statistic is very simple because it is normally distributed. The problem is that the dependence on ρ makes $\tilde{\tau}_N$ infeasible. However, as noted by Hartung (1999), ρ may be estimated consistently as $N \rightarrow \infty$ using

$$\hat{\rho} = \max \left(-\frac{1}{N-1}, \tilde{\rho} \right) \text{ where } \tilde{\rho} = 1 - \frac{1}{N-1} \sum_{i=1}^N \left(\tau_i - \frac{1}{N} \sum_{i=1}^N \tau_i \right)^2.$$

The test statistic recommended by Hartung (1999) may be written as

$$\tilde{\tau}_N = \left(N + N(N-1) \left(\hat{\rho} + \omega \sqrt{\text{var}(\hat{\rho})} \right) \right)^{-1/2} \sum_{i=1}^N \tau_i, \quad (8)$$

where $\omega > 0$ is a weight parameter and $\text{var}(\hat{\rho}) = 2(1 - \hat{\rho})^2/(N+1)$ is the estimated variance of $\hat{\rho}$.⁹ The intuition behind the factor $\omega \sqrt{\text{var}(\hat{\rho})}$ is that because the square root function is concave, the denominator of $\tilde{\tau}_N$ will tend to be underestimated, which can be corrected by adding a small amount of the standard error of $\hat{\rho}$. Again, due to Theorem 1, $\tilde{\tau}_N$ has a limiting normal distribution under the null.

4 Monte Carlo simulations

In this section, we investigate the small-sample properties of the new tests through a small simulation study using the following data generating process

$$\Delta y_t = \phi_y (y_{t-1} - m_{t-1}) + \gamma_y \Delta m_t + e_{yt},$$

⁹Following the recommendation of Hartung (1999), in this paper we use $\omega = 0.2$, which generally led to good test performance in the simulations.

$$\Delta m_t = e_{mt},$$

where all variables are stacked N vectors and $e_{mt} \sim N(0, 1)$. The error e_{yt} is generated by drawing N vectors $e_{yt} = (e_{y1t}, \dots, e_{yNt})'$ from $N(0, \text{cov}(e_{yt}))$, where $\text{cov}(e_{yt})$ is a symmetric matrix with ones along the main diagonal and ρ elsewhere. There are two different values of ρ . If $\rho = 0$, there is no cross-section dependence, whereas if $\rho = 0.8$, then there is strong cross-section dependence. We will refer to these two parameterizations as Cases 1 and 2, respectively. In both cases, $\gamma_y = 1$ so the common factor restriction is satisfied. The effect of a violation of this restriction is studied in Case 3 when $\gamma_y = 3$.

To study the effect of the deterministic component d_t , we consider three models. In Model 1, $d_t = 0$, in Model 2, $d_t = 1$ and in Model 3, $d_t = D_t$, where D_t is a break dummy taking the value one if $t > T/2$ and zero otherwise. All tests are constructed with the lag length chosen according to the rule $4(T/100)^{2/9}$, which seems as a fairly common choice. As suggested earlier, the tests are constructed as double-sided, using the 5% critical value 1.96 to reject the null. The number of replications is 1,000.

The results on the size and size-adjusted power on the 5% level are reported in Table 1. Consider first the results on the size of the tests when $\phi_y = 0$. We see that the two tests perform well with only small distortions in most experiments. One notable exception is Case 2 when τ_N rejects the null too frequently. Of course, since τ_N is constructed under the assumption of cross-section independence, this effect is well expected. The good performance of $\tilde{\tau}_N$ in this case indicates that our suggestion on how to get rid of the dependence works well.

The results on the power of the tests generally coincide with what might be expected based on theory, and can be summarized as follows. First, except for Case 2 when the powers are about equal, we see that τ_N is more powerful than $\tilde{\tau}_N$. Second, the power is increasing in both the sample size and departure from the null, as indicated by ϕ_y . Third, the power of the tests can sometimes be poor if the common factor restriction is not satisfied.

5 Empirical results

In this section, we apply our new tests to check the robustness of the cointegration results provided by Serletis and Koustas (1998). The data that we use for this purpose is taken directly from these authors, and consists annual data on real GDP and money covering approximately the years 1870 to 1986 for 10 countries, Australia, Canada, Denmark, Germany, Italy, Japan, Norway, Sweden, United Kingdom and United States. However, most series have missing observations, which not only makes the panel unbalanced but also reduces the effective number of time series observations, thus making the cross-sectional dimension a very important source of information.

5.1 Unit root tests

Before we carry on with the cointegration testing, we subject the variables to a battery of panel unit root tests. In particular, we use the G_{ols}^{++} , P_m and Z tests of Phillip and Sul (2003), the t_a and t_b tests of Moon and Perron (2004), and the Bai and Ng (2004) P_e^c test, which all permit for cross-sectional dependence by assuming that the variables admit to a common factor representation.

All tests take nonstationarity as the null hypothesis, and all tests except t_a and t_b permit the individual autoregressive roots to differ across countries, which is likely to be important in this kind of heterogenous data. Each statistic is normally distributed under the null hypothesis. Moreover, while G_{ols}^{++} , P_m , t_a and t_b are left-tailed, Z and P_e^c are right-tailed.

For the implementation of the tests, we use the Bartlett kernel, and all bandwidths and lag lengths are chosen according to the rule $4(T/100)^{2/9}$. To determine the number of common factors, we use the Bai and Ng (2004) IC_1 criterion with a maximum of five factors. The results reported in Table 2 indicate that there is an overwhelming support of the unit root null. We therefore conclude that the variables appear to be nonstationary, which corroborates the findings of Serletis and Koustas (1998).

5.2 Cointegration tests

One way to do the cointegration testing is to follow Serletis and Koustas (1998), and to subject each individual pair of time series to a conventional cointegration test. However, as argued in Section 2, this approach is likely to suffer from poor power, in which case a panel test is expected to result in more accurate inference.

Consistent with this story, Serletis and Koustas (1998) are unable to reject the no cointegration null for all countries when using the usual residual-based approach, and for all but two countries when using the Johansen (1988) maximum likelihood approach. Note that this difference may be due to an invalid common factor restriction, which is expected to reduce the power of the residual-based approach. Thus, there are actually two good reasons for believing that the Serletis and Koustas (1998) tests may suffer from low power, the finiteness of the sample and an potentially invalid common factor restriction.

Therefore, to be able to circumvent these problems, we now employ our new panel tests, which are constructed exactly as described in Section 4, with an individual specific intercept in the baseline specification. Table 3 summarizes the results from both the individual and panel cointegration tests. Based on the individual tests, we see that the no cointegration null can be rejected on the 10% level for all but two countries, Germany and Japan. Thus, even on a country-by-country basis, we find only weak evidence of the hypothesis of monetary neutrality. As expected, this finding is reinforced by the panel tests, from which we conclude that the no cointegration null can be safely rejected at all conventional significance levels. Thus, in contrast to Serletis and Koustas

(1998), we find that money is not long-run neutral. We also see that this conclusion is unaffected by the inclusion of a linear time trend.

As a final robustness check of our test results, we consider the possibility of structural change, which, as pointed out by Serletis and Koustas (1998), appears very likely given the wide range of monetary arrangements covered in the sample. Although the timing of the breaks can in principle be determined by visually inspecting the data, in this section, we estimate the breakpoints by minimizing the sum of squared residuals from the proxy regression in (6). As seen from the table, the allowance of a break in the intercept of each regression does not affect the test outcome. Thus, our conclusion remain unaltered even if we permit for structural change.

Table 3 also report the estimated break points obtained by minimizing the sum of squared residuals. The results suggest that all breakpoints have taken place somewhere during the period 1891 to 1950. From an historical point of view, this seems very reasonable. First, there are only two breaks prior to the advent of the First World War, which agrees with the stability of the classical gold standard regime. Second, there is a preponderance of breaks occurring between 1917 and 1950. This accords approximately with the interwar period, and seems consistent with the findings of Serletis and Koustas (1998).

6 Concluding remarks

Most studies on the long-run neutrality of money are based on the assumption that money and real output do not cointegrate, which is typically also supported by the data.

In this paper, we argue that these findings could be partly due to the low power of univariate tests, and that a violation of the noncointegration assumption is likely to result in a nonrejection of the neutrality of money. In order to mitigate this problem, two new and powerful panel cointegration tests based on error correction are proposed. What make these tests advantageous in comparison to the already existing test menu is that they are equipped to handle most of the many challenging features of the money and output data, such as cross-sectional dependence, structural breaks and unbalanced panels.

The tests are applied to a panel of 10 industrialized countries covering the period 1870 to 1986. The results suggest that the null hypothesis of no cointegration between money and real output can be rejected, and thus that the neutrality of money can also be rejected.

The conclusion of this study is therefore that permanent changes in the stock of money have real effects that can persist for appreciable periods of time, which is of course good news for central banks since it implies that they can affect real variables. Although seemingly unrealistic and at odds with fundamental economic theory, there are in fact many rationales for this result.

Firstly, the long-run effect of an increase in money supply on prices could be

dampened by a change in the velocity of money, brought about by for example institutional changes or financial innovations. Indeed, as shown by Bordo and Jonung (1987), the money velocity of many developed countries appear to have followed a nonlinear pattern over time.¹⁰ Between 1870 and 1914, velocity declined before it eventually started to increase again somewhere in the interwar period. The point being that if velocity is unstable, prices may not adjust perfectly to offset monetary changes, in which case real output will ultimately be affected.

A second, related, rationale is that there appears to be an inverse relationship between the stability of velocity and the narrowness of the monetary measure, and that only broad measures of money should be neutral. As an example, using data for the G7 countries, Weber (1994) finds that for broader measures, such as M2 or M3, there is strong evidence in favor of the neutrality of money, while for the narrower M1 measure, the evidence is much weaker. This suggests that our monetary measure, M2, may not be broad enough to ensure that velocity is stable.¹¹

Yet another rationale relates to the fact that monetary neutrality rests on the assumptions of no contracting frictions and, in particular, no unemployment. Lengthy time series data make periods of recession and unemployment more likely, which could well explain why the neutrality of money fails.

¹⁰The long-run behavior of money velocity and its relation with inflation is also studied by Mendizàbal (2006).

¹¹Similar results has been found by Coe and Nason (2004) for Australia, Canada, United Kingdom and United States.

Appendix: Mathematical proofs

This appendix derives the asymptotic properties of the new tests. For ease of exposure, we shall prove the results for the case with no deterministic components.

Proof of Theorem 1

Note that $\hat{\phi}_{yi}$ under the null may be written as

$$\begin{aligned}\hat{\phi}_{yi} &= \left(\sum_{t=2}^T (\Delta y_{it-1}^*)^2 \right)^{-1} \sum_{t=2}^T \Delta y_{it-1}^* \Delta y_{it}^* \\ &= \left(\sum_{t=2}^T (\Delta y_{it-1}^*)^2 \right)^{-1} \sum_{t=2}^T \Delta y_{it-1}^* e_{yit}^*,\end{aligned}$$

where the star notation indicate the projection errors from the vector $w_{it} = (\Delta y_{it-2}, \dots, \Delta y_{it-p_i}, \Delta m_{it-1}, \dots, \Delta m_{it-p_i})'$.

Consider the numerator of $\hat{\phi}_{yi}$. By using the rules for projections, we obtain

$$\begin{aligned}\sum_{t=2}^T (\Delta y_{it-1}^*)^2 &= \sum_{t=2}^T (\Delta y_{it-1})^2 - \sum_{t=2}^T \Delta y_{it-1} w_{it}' \left(\sum_{t=2}^T w_{it} w_{it}' \right)^{-1} \sum_{t=2}^T w_{it} \Delta y_{it-1} \\ &= \sum_{t=2}^T (\Delta y_{it-1})^2 + O_p(\sqrt{T}) O_p(T^{-1}) O_p(\sqrt{T}) \\ &= \sum_{t=2}^T (\Delta y_{it-1})^2 + O_p(1),\end{aligned}\tag{A1}$$

where we have used the fact that Δy_{it-1} and w_{it} are stationary. Now, the null model in Case 1 can be written as

$$\alpha_{yi}(L) \Delta y_{it} = u_{it},\tag{A2}$$

where $u_{it} = \gamma_{yi}(L) \Delta m_{it} + e_{yit}$. From the Beveridge-Nelson (BN) decomposition of $\gamma_{yi}(L)$ as $\gamma_{yi}(L) = \gamma_{yi}(1) + \gamma_{yi}^*(L)(1-L)$, we obtain

$$u_{it} = \gamma_{yi}(L) \Delta m_{it} + e_{yit} = \gamma_{yi}(1) \Delta m_{it} + \gamma_{yi}^*(L) \Delta^2 m_{it} + e_{yit}.\tag{A3}$$

Similarly, the BN decomposition of $\alpha_{mi}(L)$ gives

$$\alpha_{mi}(L) \Delta m_{it} = \alpha_{mi}(1) \Delta m_{it} + \alpha_{mi}^*(L) \Delta^2 m_{it} = e_{mit},$$

or, equivalently

$$\Delta m_{it} = -\frac{\alpha_{mi}^*(L)}{\alpha_{mi}(1)} \Delta^2 m_{it} + \frac{1}{\alpha_{mi}(1)} e_{mit}.$$

This implies that (A3) can be rewritten as

$$u_{it} = \pi_i(1)e_{mit} + e_{yit} + O_p(1),$$

where $\pi_i(1) = \gamma_{yi}(1)/\alpha_{mi}(1)$ so that as $T \rightarrow \infty$

$$\begin{aligned} T^{-1} \sum_{t=2}^T u_{it}^2 &= \pi_i(1)^2 T^{-1} \sum_{t=2}^T e_{mit}^2 + 2T^{-1} \sum_{t=2}^T \pi_i(1)e_{mit}e_{yit} + T^{-1} \sum_{t=2}^T e_{yit}^2 \\ &= \pi_i(1)^2 T^{-1} \sum_{t=2}^T e_{mit}^2 + T^{-1} \sum_{t=2}^T e_{yit}^2 + o_p(1) \\ &\xrightarrow{p} \pi_i(1)^2 \text{var}(e_{mit}) + \text{var}(e_{yit}), \end{aligned} \quad (\text{A4})$$

where we have used Assumption 1 (c) that e_{yit} and e_{mit} are orthogonal.

Another application of the BN decomposition gives

$$\alpha_{yi}(L)\Delta y_{it} = \alpha_{yi}(1)\Delta y_{it} + \alpha_{yi}^*(L)\Delta^2 y_{it} = u_{it},$$

which can be rewritten as

$$\Delta y_{it} = -\frac{\alpha_{yi}^*(L)}{\alpha_{yi}(1)}\Delta^2 y_{it} + \frac{1}{\alpha_{yi}(1)}u_{it} = \frac{1}{\alpha_{yi}(1)}u_{it} + O_p(1).$$

By using (A4) this implies that the limit of (A1) as $T \rightarrow \infty$ can be written as

$$\begin{aligned} T^{-1} \sum_{t=2}^T (\Delta y_{it-1}^*)^2 &= T^{-1} \sum_{t=2}^T (\Delta y_{it-1})^2 + o_p(1) \\ &\xrightarrow{p} \frac{1}{\alpha_{yi}(1)^2} (\pi_i(1)^2 \text{var}(e_{mit}) + \text{var}(e_{yit})). \end{aligned} \quad (\text{A5})$$

Next, consider the denominator of $\hat{\phi}_{yi}$, which can be written as

$$\begin{aligned} \sum_{t=2}^T \Delta y_{it-1}^* e_{it}^* &= \sum_{t=2}^T \Delta y_{it-1} e_{it} - \sum_{t=2}^T \Delta y_{it-1} w_{it}' \left(\sum_{t=2}^T w_{it} w_{it}' \right)^{-1} \sum_{t=2}^T w_{it} e_{it} \\ &= \sum_{t=2}^T \Delta y_{it-1} e_{it} + O_p(\sqrt{T})O_p(T^{-1})O_p(\sqrt{T}) \\ &= \sum_{t=2}^T \Delta y_{it-1} e_{it} + O_p(1). \end{aligned} \quad (\text{A6})$$

If we let $Q_i = (\pi_i(1)^2 \text{var}(e_{mit}) + \text{var}(e_{yit}))/\alpha_{yi}(1)^2$, since $T^{-1} \sum_{t=2}^T (\Delta y_{it-1}^*)^2 \xrightarrow{p} Q_i$ from (A5), we have the following limit as $T \rightarrow \infty$

$$T^{-1/2} \sum_{t=2}^T \Delta y_{it-1}^* e_{it}^* = T^{-1/2} \sum_{t=2}^T \Delta y_{it-1} e_{it} + o_p(1) \Rightarrow N(0, \text{var}(e_{yit})Q_i).$$

Together with (A5) this implies that

$$\begin{aligned}\sqrt{T}\hat{\phi}_{yi} &= \left(T^{-1} \sum_{t=2}^T (\Delta y_{it-1}^*)^2\right)^{-1} T^{-1/2} \sum_{t=2}^T \Delta y_{it-1}^* e_{yit}^* \\ &\Rightarrow N\left(0, \text{var}(e_{yit}) \frac{1}{Q_i}\right).\end{aligned}$$

The τ_i statistic is given by

$$\tau_i = \frac{\hat{\phi}_{yi}}{\sqrt{\text{var}(\hat{\phi}_{yi})}}.$$

Thus, in order to obtain the limit of this statistic, we need to evaluate $\text{var}(\hat{\phi}_{yi})$, which may be written as

$$\text{var}(\hat{\phi}_{yi}) = \left(\hat{\sigma}_i^{-2} \sum_{t=2}^T (\Delta y_{it-1}^*)^2\right)^{-1}.$$

Consider $\hat{\sigma}_i^2$. The limit as $T \rightarrow \infty$ of this term is given by

$$\begin{aligned}\hat{\sigma}_i^2 &= T^{-1} \sum_{t=2}^T e_{it}^{*2} \\ &= T^{-1} \sum_{t=2}^T e_{it}^2 - T^{-1} \sum_{t=2}^T e_{it} w_{it}' \left(\sum_{t=2}^T w_{it} w_{it}'\right)^{-1} \sum_{t=2}^T w_{it} e_{it} \\ &= T^{-1} \sum_{t=2}^T e_{it}^2 + T^{-1} O_p(\sqrt{T}) O_p(T^{-1}) O_p(\sqrt{T}) \\ &= T^{-1} \sum_{t=2}^T e_{it}^2 + o_p(1) \xrightarrow{p} \text{var}(e_{yit}).\end{aligned}$$

This, together with (A5), implies

$$T\text{var}(\hat{\phi}_{yi}) = \left(\hat{\sigma}_i^{-2} T^{-1} \sum_{t=2}^T (\Delta y_{it-1}^*)^2\right)^{-1} \xrightarrow{p} \text{var}(e_{yit}) \frac{1}{Q_i},$$

which ensures that

$$\tau_i = \frac{\hat{\phi}_{yi}}{\sqrt{\text{var}(\hat{\phi}_{yi})}} = \frac{\sqrt{T}\hat{\phi}_{yi}}{\sqrt{T\text{var}(\hat{\phi}_{yi})}} \Rightarrow N(0, 1).$$

This completes the proof. ■

Proof of Theorem 2

The proxy estimator of ϕ_y in (6) is given by

$$\hat{\phi}_y = \left(T^{-1} \sum_{t=2}^T (\Delta y_{it-1}^*)^2 \right)^{-1} T^{-1} \sum_{t=2}^T \Delta y_{it-1}^* \Delta y_{it}^*, \quad (\text{A7})$$

where Δy_{it-1}^* and Δy_{it}^* are the errors from projecting Δy_{it-1} and Δy_{it} onto $w_{it} = (\Delta m_{it}, \Delta m_{it-1})'$.

The numerator of (A7) can be expressed as

$$\begin{aligned} T^{-1} \sum_{t=2}^T (\Delta y_{it-1}^*)^2 &= T^{-1} \sum_{t=2}^T (\Delta y_{it-1})^2 - T^{-1} \sum_{t=2}^T \Delta y_{it-1} w_{it}' \\ &\cdot \left(T^{-1} \sum_{t=2}^T w_{it} w_{it}' \right)^{-1} T^{-1} \sum_{t=2}^T w_{it} \Delta y_{it-1}. \end{aligned} \quad (\text{A8})$$

Now, (6) can be written in first differences as

$$\begin{aligned} \Delta y_{it} &= (\phi_{yi} + 1) \Delta y_{it-1} + \lambda_i \Delta m_{it-1} + \gamma_{yi} \Delta^2 m_{it} + \Delta e_{yit} \\ &= \frac{1}{1 - (\phi_{yi} + 1)L} ((\lambda_i - \gamma_{yi}) \Delta m_{it-1} + \gamma_{yi} \Delta m_{it} + e_{yit} - e_{yit-1}). \end{aligned}$$

By expanding Δy_{it}^2 , and some algebra, we get

$$\begin{aligned} (\Delta y_{it})^2 &= \gamma_{yi}^2 (\Delta m_{it})^2 + \frac{1}{1 - (\phi_{yi} + 1)^2 L} \phi_{yi}^2 (\gamma_{yi} - \beta_i)^2 (\Delta m_{it-1})^2 \\ &+ e_{yit}^2 + \frac{1}{1 - (\phi_{yi} + 1)^2 L} \phi_{yi}^2 e_{yit-1}^2 + \dots, \end{aligned}$$

where the remaining terms are cross-products with zero expectation. Thus, we can show that as $T \rightarrow \infty$

$$\begin{aligned} T^{-1} \sum_{t=2}^T (\Delta y_{it-1})^2 &\xrightarrow{p} \left(\gamma_{yi}^2 + \frac{1}{1 - (\phi_{yi} + 1)^2} \phi_{yi}^2 (\gamma_{yi} - \beta_i)^2 \right) \text{var}(e_{mit}) \\ &+ \left(1 + \frac{1}{1 - (\phi_{yi} + 1)^2} \phi_{yi}^2 \right) \text{var}(e_{yit}). \end{aligned}$$

Also, we have that

$$\begin{aligned} T^{-1} \sum_{t=2}^T \Delta y_{it-1} w_{it}' &\xrightarrow{p} (0, \gamma_{yi} \text{var}(e_{mit})), \\ T^{-1} \sum_{t=2}^T w_{it} w_{it}' &\xrightarrow{p} \begin{pmatrix} \text{var}(e_{mit}) & 0 \\ 0 & \text{var}(e_{mit}) \end{pmatrix}. \end{aligned}$$

Putting everything together, we obtain the following limit of (A8) as $T \rightarrow \infty$

$$\begin{aligned}
T^{-1} \sum_{t=2}^T (\Delta y_{it-1}^*)^2 &\xrightarrow{p} \left(\frac{1}{1 - (\phi_{yi} + 1)^2} \phi_{yi}^2 (\gamma_{yi} - \beta_i)^2 \right) \text{var}(e_{mit}) \\
&+ \left(1 + \frac{1}{1 - (\phi_{yi} + 1)^2} \phi_{yi}^2 \right) \text{var}(e_{yit}) \\
&= \frac{1}{1 - (\phi_{yi} + 1)^2} (-2\phi_{yi} \text{var}(e_{yit})) \\
&+ \phi_{yi}^2 (\gamma_{yi} - \beta_i)^2 \text{var}(e_{mit}). \tag{A9}
\end{aligned}$$

Consider next the denominator of (A7). We have

$$\begin{aligned}
\Delta y_{it} \Delta y_{it-1} &= \gamma_{yi} \phi_{yi} (\gamma_{yi} - \beta_i) (\Delta m_{it-1})^2 \\
&+ \frac{1}{1 - (\phi_{yi} + 1)^2 L} \phi_{yi}^2 (\phi_{yi} + 1) (\gamma_{yi} - \beta_i)^2 (\Delta m_{it-2})^2 \\
&+ \phi_{yi} e_{yit-1}^2 + \frac{1}{1 - (\phi_{yi} + 1)^2 L} \phi_{yi}^2 (\phi_{yi} + 1) e_{yit-2}^2 + \dots,
\end{aligned}$$

where, as before, the remainder includes only cross-products that have zero expectation. It follows that

$$\begin{aligned}
T^{-1} \sum_{t=2}^T \Delta y_{it} \Delta y_{it-1} &\xrightarrow{p} \left(\gamma_{yi} \phi_{yi} (\gamma_{yi} - \beta_i) + \frac{1}{1 - (\phi_{yi} + 1)^2} \phi_{yi}^2 (\gamma_{yi} - \beta_i)^2 \right) \\
&\cdot \text{var}(e_{mit}) + \left(\phi_{yi} + \frac{1}{1 - (\phi_{yi} + 1)^2} \phi_{yi}^2 (\phi_{yi} + 1) \right) \\
&\cdot \text{var}(e_{yit}).
\end{aligned}$$

Since $T^{-1} \sum_{t=2}^T \Delta y_{it} w'_{it} \xrightarrow{p} (\gamma_{yi} \text{var}(e_{mit}), \phi_{yi} (\gamma_{yi} - \beta_i) \text{var}(e_{mit}))$ as $T \rightarrow \infty$, we get

$$\begin{aligned}
T^{-1} \sum_{t=2}^T \Delta y_{it}^* \Delta y_{it-1}^* &= T^{-1} \sum_{t=2}^T \Delta y_{it} \Delta y_{it-1} - T^{-1} \sum_{t=2}^T \Delta y_{it} w'_{it} \\
&\cdot \left(T^{-1} \sum_{t=2}^T w_{it} w'_{it} \right)^{-1} T^{-1} \sum_{t=2}^T w_{it} \Delta y_{it-1} \\
&\xrightarrow{p} \left(\frac{1}{1 - (\phi_{yi} + 1)^2} \phi_{yi}^2 (\gamma_{yi} - \beta_i)^2 \right) \text{var}(e_{mit}) \\
&+ \left(\phi_{yi} + \frac{1}{1 - (\phi_{yi} + 1)^2} \phi_{yi}^2 (\phi_{yi} + 1) \right) \text{var}(e_{yit}) \\
&= -\frac{1}{1 - (\phi_{yi} + 1)^2} \phi_{yi}^2 \text{var}(e_{yit}) \\
&+ \frac{1}{1 - (\phi_{yi} + 1)^2} \phi_{yi}^2 (\phi_{yi} + 1) (\gamma_{yi} - \beta_i)^2 \text{var}(e_{mit}).
\end{aligned}$$

This result, together with (A7) and (A9), establishes the proof. \blacksquare

Table 1: Size and size-adjusted power on the 5% level for the cointegration tests.

Case	Model	N	T	$\phi_y = 0$		$\phi_y = -0.1$		$\phi_y = -0.2$		$\phi_y = -0.3$	
				$\tilde{\tau}_N$	τ_N	$\tilde{\tau}_N$	τ_N	$\tilde{\tau}_N$	τ_N	$\tilde{\tau}_N$	τ_N
1	1	5	100	5.9	6.3	18.7	22.1	53.8	73.2	86.8	97.2
		200	5.5	5.2	26.2	37.8	77.9	96.8	99.7	100.0	100.0
		10	100	4.1	7.0	31.2	40.5	75.3	94.4	98.0	100.0
	2	200	3.4	5.4	49.3	71.0	97.8	100.0	100.0	100.0	100.0
		5	100	8.1	8.8	13.8	21.3	44.3	67.5	75.3	97.6
		200	6.4	6.2	26.1	39.9	77.8	95.5	99.4	99.9	100.0
2	3	10	100	6.9	9.2	27.2	36.2	71.2	93.1	95.7	100.0
		200	3.6	5.1	47.3	69.0	95.2	99.8	100.0	100.0	100.0
		5	100	6.8	6.1	17.5	25.4	51.9	74.9	83.0	97.4
	3	200	5.2	6.7	31.9	38.4	78.0	94.2	99.5	100.0	100.0
		10	100	5.2	7.2	30.7	42.8	74.3	95.7	97.6	100.0
		200	3.8	7.2	49.5	68.9	95.3	99.9	100.0	100.0	100.0
3	2	2	1	5	100	8.5	32.0	8.9	11.1	27.3	26.5
		200	7.7	30.6	14.5	14.2	46.3	47.4	84.9	87.5	87.5
		10	100	10.6	48.4	9.0	8.2	25.1	26.4	54.5	59.3
	3	200	7.4	45.8	12.4	15.2	49.6	53.4	88.3	91.6	91.6
		5	100	9.5	31.1	8.2	11.1	19.7	27.5	43.2	54.4
		200	8.5	31.5	15.5	18.2	46.7	54.4	87.2	91.5	91.5
3	10	10	100	8.7	45.1	10.6	9.3	28.0	29.9	55.5	61.0
		200	8.9	46.3	13.1	12.9	45.3	50.8	86.1	89.1	89.1
		5	100	8.2	29.3	9.8	12.3	24.5	31.0	54.7	66.6
	10	200	7.5	31.9	14.1	15.5	45.3	53.9	81.6	87.9	87.9
		100	10.1	48.0	9.2	9.8	25.4	29.0	50.6	56.5	56.5
		200	8.5	46.2	13.1	14.4	46.6	51.8	88.7	91.8	91.8

Continued overleaf

Table 1: Continued.

Case	Model	N	T	$\phi_y = 0$		$\phi_y = -0.1$		$\phi_y = -0.2$		$\phi_y = -0.3$	
				$\tilde{\tau}_N$	τ_N	$\tilde{\tau}_N$	τ_N	$\tilde{\tau}_N$	τ_N	$\tilde{\tau}_N$	τ_N
3	1	5	100	6.5	6.5	8.5	8.8	8.0	8.8	42.8	60.1
		200	5.3	4.7	10.2	14.7	10.6	17.2	67.2	91.5	
		10	100	4.0	5.9	9.7	12.4	9.7	12.0	67.3	88.6
	2	200	3.0	4.0	16.9	24.4	18.5	26.5	92.6	99.6	
		5	100	7.3	8.0	4.1	3.6	8.6	9.2	39.8	57.0
		200	4.5	5.7	9.9	9.0	13.9	17.4	71.5	90.6	
3	3	10	100	6.0	9.1	4.6	5.3	10.3	12.8	60.0	85.2
		200	4.3	7.7	11.6	14.1	17.8	23.7	89.0	99.5	
		5	100	8.7	7.6	3.5	3.2	7.0	7.7	34.3	53.1
	10	200	5.2	6.3	9.1	9.9	9.8	13.8	66.7	91.7	
		100	6.0	7.9	5.5	5.8	11.1	12.7	63.3	86.8	
		200	4.1	5.4	10.8	16.2	17.6	28.7	90.7	99.5	

Notes: The cross-section is independent in Case 1 and dependent in Case 2. In Case 3, the common factor restriction is violated. Model 1 refers to the test with no deterministic component, Model 2 refers to the test with an intercept and Model 3 refers to the test with a break in the intercept. The value ϕ_y refers to the error correction parameter.

Table 2: Panel unit root tests.

Study	Test	Output		Money	
		Value	<i>p</i> -value	Value	<i>p</i> -value
Bai and Ng (2004)	P_e^c	−0.211	0.583	0.495	0.310
Phillips and Sul (2003)	G_{ols}^{++}	2.150	0.984	6.918	1.000
	Z	0.129	0.551	4.093	1.000
	P_m	0.105	0.458	−1.902	0.971
Moon and Perron (2004)	t_a	0.039	0.516	1.052	0.854
	t_b	0.469	0.681	9.484	1.000

Notes: The tests are computed using the Bartlett kernel. All bandwidths and lag lengths are set equal to $4(T/100)^{2/9}$. The maximum number of common factors is set to three.

Table 3: Cointegration tests.

Country	Constant		Trend		Intercept break	
	Value	p-value	Value	p-value	Value	p-value
Australia	-1.824	0.068	-1.929	0.054	-1.648	0.099
Canada	1.660	0.097	1.617	0.106	1.720	0.085
Denmark	2.927	0.003	2.821	0.005	3.407	0.001
Germany	0.909	0.364	0.888	0.375	0.675	0.500
Italy	3.526	0.000	2.804	0.005	2.752	0.006
Japan	0.691	0.490	-0.158	0.874	-1.092	0.275
Norway	2.438	0.015	2.390	0.017	3.600	0.000
Sweden	1.808	0.071	1.777	0.076	2.921	0.003
United Kingdom	4.412	0.000	4.310	0.000	5.066	0.000
United States	3.619	0.000	3.254	0.001	3.892	0.000
τ_N	6.905	0.000	6.087	0.000	7.292	0.000
$\tilde{\tau}_N$	6.377	0.000	5.621	0.000	6.734	0.000

Notes: The tests are computed based on a lag length of $4(T/100)^{2/9}$. The breaks are estimated by minimizing the sum of squared residuals from the estimated proxy regression.

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