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Numerical Solution to $\mathcal{H}^\infty$ Control of Multi-Delayed Systems via Operator Approach

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Abstract

A computational algorithm for the Full Information $\mathcal{H}^\infty$ control problem for multi-delayed LTI systems is derived. The algorithm is based on a new general operator approach in spectral domain developed recently for finite-dimensional LTI plants. A simplicity of spectral operations and explicit formulas for computation make it possible to generalize it to infinite-dimensional plants. In this paper, a complete computational solution for such a plant with several delays in the output, control and disturbance is obtained and illustrated with a simple example.

Keywords: linear systems, $\mathcal{H}^\infty$ control problem, multi-delayed systems, infinite-dimensional systems.

1 Problem statement and basic assumptions

In this paper, a numerical efficient approach is proposed for the standard Full Information $\mathcal{H}^\infty$ control problem for the multi-delayed plant

$$\begin{aligned} a_0(p)y(t) + a_1(p)y(t-\tau) + \cdots + a_m(p)y(t-m\tau) = \\ = \quad b_0(p)u(t) + b_1(p)u(t-\tau) + \cdots + b_m(p)u(t-m\tau) + \\ + c_0(p)v(t) + c_1(p)v(t-\tau) + \cdots + c_m(p)v(t-m\tau) \end{aligned}$$

where $p = d/dt$ is the differentiation operator; $a_k(\cdot)$, $b_k(\cdot)$, $c_k(\cdot)$ are polynomials $0 \le k \le m$; $\tau > 0$ is a constant time delay; $y(t)$ is the system output, $u(t)$ is control and $v(t)$ is the disturbance. Assume the degree $n = \deg a_0$ is the highest among other polynomials in the plant equation.

A stabilizing controller is to be designed to meet the specification

$$\mathcal{F}(y(\cdot), u(\cdot), v(\cdot)) = \int_0^\infty F(y(t), u(t), v(t))\, dt < 0$$

with $F(y,u,v) = |y|^2 + |u|^2 - |v|^2$ for all $v \in L^2(0,\infty)$, $v \not\equiv 0$.

The solution is based on the general approach proposed in [3]. Consider the standard $\mathcal{H}^\infty$ control problem stated for the plant in the "behavioral" form

$$R(p)\, x(t) = 0$$

where $p = d/dt$, x is the N-vector of manifest variables, and $R(d/dt)$ is the Fourier transform of a $n \times N$ generalized function. Assume that $n < N$ and $\operatorname{rank} R(\cdot) = n$.

Given a square nonsingular $N \times N$-matrix Q it is required to find a controller equation

$$C(p)\, x(t) = 0$$

with the Fourier transform $C(p)$ of a $l \times N$ generalized function such that the closed loop system is stable and there exists $\varepsilon > 0$ such that it holds

$$Q(x(\cdot)) = \int_0^\infty x(t)^T Q x(t)\, dt \le -\varepsilon \|x\|_2^2 \qquad (1)$$

for any function $x(\cdot) \in L^2(0,\infty)$ which satisfies the plant and controller equations. Such the controllers will be called contractive.

For the delayed scalar system under consideration it holds

$$\begin{aligned} R(z) &= \begin{pmatrix} a(z) & -b(z) & -c(z) \end{pmatrix} = \\ &= \sum_{k=0}^m \begin{pmatrix} a_k(z) & -b_k(z) & -c_k(z) \end{pmatrix} e^{-k\tau z} \end{aligned}$$

and $Q = \operatorname{diag}(1, 1, -1)$.

In Section 2 the main result is presented for a general control system and the numerical realization described in Section 3. For the general case, the matrix Q is an arbitrary self-adjoint constant nonsingular matrix, it is not assumed to be positive definite in the variables $q = (y,u)$. Both the plant and controller equations define the sets of admissible trajectories of the manifest variables without any reference to the targets of control design. They represent the plant dynamics only and the matrix functions $R(z)$ and $C(z)$ may have arbitrary structure (in particular, polynomial degrees) of their entries. The "behavioral" analysis of the admissible trajectories does not need a transformation to the state space or to other mathematical standard forms [2].

2 Algebraic operator approach

Denote by $\mathcal{R}$ the set of the Fourier transforms of all generalized functions. It contains the standard causal and anti-causal subsets: $\mathcal{R}_-$ (respectively, $\mathcal{R}_+$) is the set of the Fourier transforms of all generalized functions with support sets in $[0,\infty)$ (in $(-\infty,0]$, respectively). The sets $\mathcal{R}_-$ and $\mathcal{R}_+$ are closed with respect to linear operations and to multiplication.

The corresponding projection operators from $\mathcal{R}$ to $\mathcal{R}_+$ is denoted by $[\cdot]_+$, and from $\mathcal{R}$ to $\mathcal{R}_-$ by $[\cdot]_-$. Thus, if F, F_+ and F_- are generalized functions with the Fourier transforms f, $[f]_+$ and $[f]_-$, respectively, then for any test function ϕ with the support set in $[0,\infty)$ it holds $\langle F_-,\phi\rangle = \langle F,\phi\rangle$, $\langle F_+,\phi\rangle = 0$; and for any test function ϕ with the support set in $(-\infty,0]$ it holds $\langle F_+,\phi\rangle = \langle F,\phi\rangle$, $\langle F_-,\phi\rangle = 0$.

For any generalized function f denote $f^*(z) = f^T(-z)$ where ^{T} means the transposition. It follows from the definitions that $\mathcal{R}_-^* = \mathcal{R}_+$ and $\mathcal{R}_+^* = \mathcal{R}_-$ (with the appropriate dimensions). The intersection $\mathcal{R}_- \cap \mathcal{R}_+$ is the set of all polynomial functions of the chosen dimension. For any $f \in \mathcal{R}$ the function $[f]_0 = [f]_- + [f]_+ - f$ is polynomial because it belongs to $\mathcal{R}_- \cap \mathcal{R}_+$. Define the projections $[f]_{0+} = [f]_+ - [f]_0 = f - [f]_-$ and $[f]_{0-} = [f]_- - [f]_0 = f - [f]_+$.

Let $0 < m < N$. For any constant $N \times m$-matrix h and a $N \times m$-matrix function $X \in \mathcal{R}$ define the matrix

$$\Phi(z) = h + Q^{-1}[R^*(z)X(z)]_{0-}. \tag{2}$$

Consider the following equation which will be basic to our approach:

$$R(z)\Phi(z) = 0, \tag{3}$$

that is, $\Phi(z)$ is the Fourier transform of a solution to the plant equation.

Lemma 1 *Consider the equation (2) with the additional condition: the function $[R^*X]_{0-}(z)$ tends to zero as $z \to \infty$.*

1. *For any solutions $\Phi_1(z)$ and $\Phi_2(z)$ of the equation (3) defined by the pairs $(h_1, X_1(z))$ and $(h_2, X_2(z))$ it holds*
$$\Phi_1^*(z)Q\Phi_2(z) = h_1^T Q h_2.$$
2. *Assume $m = N - n$ and there exists a solution $\Phi(z)$ of (3) of the dimension $N \times m$ such that the $m \times m$-matrix h^TQh is nonsingular. Then the set of all solutions $x(t)$ to the plant equation can be parameterized by an arbitrary functions $\ell(t)$ in the following way:*

$$\begin{aligned} x(t) &= \Phi(p)(h^TQh)^{-1}\ell(t), && (4)\\ \ell(t) &= \Phi^*(p)Qx(t). && (5) \end{aligned}$$

The variable $\ell(t)$ is called latent in the system behavioral description [2] and the equation (4) is called the image representation of the plant. Given a solution $x(t)$ to the plant equation the corresponding latent variable can be obtained from (5). The latent variable is an essential part of the contractive controller parameterization that will be clear below.

Proof: 1. It follows from the definition (2) of $\Phi(z)$ that the functions

$$\Psi_i(z) = \Phi_i(z) - Q^{-1}R^*(z)X_i(z), \qquad i = 1,2,$$

belong to $\mathcal{R}_+$. The equations $R\Phi_i = 0$ imply

$$\Phi_1^*(z)Q\Phi_2(z) = \Phi_1^*(z)Q\Psi_2(z) = \Psi_1^*(z)Q\Phi_2(z).$$

The second term belongs to $\mathcal{R}_+$ while the third term is in $\mathcal{R}_-$. Hence, this function is polynomial. From the additional assumption $\Phi_i(z) - h_i \to 0$ as $z \to \infty$ it follows that this polynomial is constant and equals to $h_1^TQh_2$.

2. It can be directly verified that

$$\begin{pmatrix} R(z) \\ \Phi^*(z)Q \end{pmatrix}^{-1} = (Q^{-1}R^*(z)P(z)^{-1}, \Phi(z)(h^TQh)^{-1})$$

where $P(z) = R(z)Q^{-1}R^*(z)$ is a nonsingular matrix. Therefore the system

$$\begin{aligned} R(p)x(t) &= 0,\\ \Phi^*(p)Qx(t) &= \ell(t) \end{aligned}$$

is equivalent to the equation

$$x(t) = \Phi(p)(h^TQh)^{-1}\ell(t),$$

that completes the proof of Lemma 1. ∎

The basic function $\Phi(z)$ defined in (2) can be multiplied from the right by any nonsingular matrix S. Choose the matrix in such a way that S^Th^TQhS becomes diagonal and normalized. Assume this has been done, therefore, the new matrix h satisfies

$$h^TQh = \begin{pmatrix} I_{m-k} & 0 \\ 0 & -I_k \end{pmatrix} = J.$$

Split the latent variable according to this partition:

$$\ell(t) = \begin{pmatrix} U(t) \\ V(t) \end{pmatrix}.$$

Denote the Fourier transforms of the corresponding functions by $\tilde{x}(z)$, $\tilde{\ell}(z)$, $\tilde{U}(z)$, $\tilde{V}(z)$ provided they are in $L^2(0,\infty)$. It follows from Lemma 1 that

$$\tilde{x}^*(z)Q\tilde{x}(z) = \tilde{U}^*(z)\tilde{U}(z) - \tilde{V}^*(z)\tilde{V}(z)$$

for any complex z. This obviously implies that

$$U(t) = 0$$

is one of the contractive controllers and it solves the control problem if $k = l$ and the closed loop system is stable. It can be shown that for LTI finite-dimensional plants this is the central controller in the Full Information $\mathcal{H}^\infty$ control problem because spectral equations have state-space analogues through direct algebraic relations [1]. The equation

$$U(t) = D(p)V(t), \qquad \|D\|_{\mathcal{H}^\infty} < 1,$$

gives a parameterization of the class of all contractive controllers. This is the complete solution to the problem stated in the "behavioral" language.

Consider the LTI plant with the fixed output, control and disturbance variables. Assume, respectively, that $x = (y, u, v)$ and n is the dimension of the output y. Then the equation $U(t) = 0$ is implicit because the function U contains the disturbance v which is undesirable in a controller. The explicit equations can be obtained in the following way. Partition the matrix $\Phi(z)$ according to the dimensions of y, u and v:

$$\Phi(z) = \begin{pmatrix} \Phi_{yu}(z) & \Phi_{yv}(z) \\ \Phi_{uu}(z) & \Phi_{uv}(z) \\ \Phi_{vu}(z) & \Phi_{vv}(z) \end{pmatrix}.$$

The subscripts indicate the dimensions of the matrices. The image plant representation (4) with $U(t) = 0$ imply

$$\left\{ \begin{array}{rcl} y(t) & = & \Phi_{yv}(p)V(t), \\ u(t) & = & \Phi_{uv}(p)V(t). \end{array} \right.$$

This is an explicit representation of the central controller. The class of all contractive controllers can be described by the system

$$\left\{ \begin{array}{rcl} y(t) & = & \{\Phi_{yv}(p) + \Phi_{yu}(p)D(p)\}\, V(t), \\ u(t) & = & \{\Phi_{uv}(p) + \Phi_{uu}(p)D(p)\}\, V(t) \end{array} \right. \tag{6}$$

with $\|D\|_\infty < 1$. The number of scalar disturbances acting on the linear system can be reduced so that it does not exceed the number of equations or the number of the controlled outputs which are usually the same. Therefore, the vector $V(t)$ can be determined from the first equation in (6) and then substituted into the second equation. This results in the explicit controller transfer function from y to u.

The standard $\mathcal{H}^\infty$ control problem includes a minimization of the contraction level of the closed loop system. Assume the quadratic form Q depends on the level γ:

$$Q_\gamma(y,u,v) = Q_0(y,u,v) - \gamma^2\|v\|^2.$$

It is required to minimize γ for which the inequality (1) can be achieved by some stabilizing controller. The following assumptions are standard for the plant equation:

A1. The disturbance variable v in the behavior is free, that is, for any $v \in L_2(0,\infty)$ there exists a function $q \in L_2(0,\infty)$ such that $R(p)\,\mathrm{col}(q(t),v(t)) = 0$.

A2. The form x^*Q_0x with $x = \mathrm{col}(0,v)$ is negative definite in v.

A3. For any $\omega \in (0,\infty)$ and for any vector $q \in \mathbb{C}^{n_q}$ if $R(i\omega)\,\mathrm{col}(q,0) = 0$ then $(q^*,0)Q_0\,\mathrm{col}(q,0) > 0$.

The previous analysis can be summarized in the following statement.

Theorem 1 ([3]) *Assume the Assumptions A1–A3 hold and the open loop transfer functions from u to y and from v to y are strictly proper. Then*

1. *The minimal value of γ for which there exist a γ-contractive stabilizing controller is equal to the maximal value of γ for which the system (2), (3) has an N-vector solution $\Phi(z)$ with $h = 0$. Denote this value by γ_{opt}.*

2. *Let $\gamma > \gamma_{opt}$. Then the set of all stabilizing γ-contractive controllers is defined by the equation (6) with $\|D\|_\infty < 1$. The function $\Phi(z)$ is computed from (2), (3) under the condition $h^TQ_\gamma h = J = \mathrm{diag}\{I_{N-n-k}, -I_k\}$ with $k = \dim v$; n is the number of equations and N is the sum of the dimensions of y, u, v.*

Assertion 1 is standard for all solutions to the $\mathcal{H}^\infty$ control problem. Singularity of the linear equations system indicates that of the corresponding algebraic Riccati equation in the state-space approach [1].

3 Delayed systems

The scalar multi-delayed control system is described by

$$R(z) = \begin{pmatrix} a(z) & -b(z) & -c(z) \end{pmatrix} = \sum_{k=0}^{m} \begin{pmatrix} a_k(z) & -b_k(z) & -c_k(z) \end{pmatrix} e^{-k\tau z}$$

and $Q = \mathrm{diag}(1,1,-1)$.

The one-column function $\Phi(z)$ in this case is

$$\Phi(z) = h + Q^{-1}[R^*(z)X(z)]_{0-} = \begin{pmatrix} h_y \\ h_u \\ h_v \end{pmatrix} + \left[\sum_{k=0}^{m} \begin{pmatrix} a_k(-z) \\ -b_k(-z) \\ c_k(-z) \end{pmatrix} e^{k\tau z} X(z) \right]_{0-}$$

where the function $X(z) = [X]_{0-}(z)$ and the vector $h = (h_y, h_u, h_v)^T$ are to be determined from the basic equation $R(z)\Phi(z) = 0$. This equation takes the form

$$\begin{aligned} 0 = & \sum_{k=0}^{m} a_k(z)e^{-k\tau z}(h_y + [\sum_{k=0}^{m} a_k(-z)e^{k\tau z}X(z)]_{0-} - \\ & - \sum_{k=0}^{m} b_k(z)e^{-k\tau z}(h_u - [\sum_{k=0}^{m} b_k(-z)e^{k\tau z}X(z)]_{0-} - \\ & - \sum_{k=0}^{m} c_k(z)e^{-k\tau z}(h_v + [\sum_{k=0}^{m} c_k(-z)e^{k\tau z}X(z)]_{0-} \end{aligned}$$

The leading term in the right hand side of this equation as $z \to \infty$ is $a_0(z)h_y$. Therefore it holds $h_y = 0$.

In the sequel it will be shown that the function $X(z)$ can be found in the form

$$X(z) = \frac{r(z)}{\Pi(z)}, \tag{7}$$

where the function $r(z)$ is the Fourier transforms of a generalized function $\rho(t)$ which has the support set in the interval $[0, m\tau]$. The function $\Pi(z)$ is a result of the outer factorization of the function

$$H(z) = a(z)a(-z) + b(z)b(-z) - c(z)c(-z)$$

that is

$$\Pi(-z)\Pi(z) = H(z) \tag{8}$$

and all zeros of $\Pi(z)$ have negative real parts.

Consider the last problem of factorization. If such a factor $\Pi(z)$ does not exist then $H(z)$ has zeros on the imaginary axis. It is easy to prove with sinusoidal input $v(t)$ that in this case the standard $\mathcal{H}^\infty$ control problem has no solutions. Therefore assume the factorization exists.

According to the Wiener-Paley theorem for generalized functions the function $H(z)$ is the Fourier transform of a function which has a support set in the interval $[-m\tau, m\tau]$. Therefore $\Pi(z)$ is the Fourier transform of a generalized function $\phi(t)$ with a support set in $[0, m\tau]$. The generalized function $\phi(t)$ can be found in the form

$$\phi(t) = \sum_{k=0}^{m} g_k(p)\delta(t - k\tau) + \psi(t) \tag{9}$$

where $\delta(t)$ is the δ–function of Dirac; $p = d/dt$; $g_k(\cdot)$ are polynomials, $0 \le k \le m$; and $\psi(t)$ is a function with the support set in $[0, m\tau]$. Denote by $\Psi(z)$ the Fourier transform of $\psi(t)$.

A numerical technique for the parameter computation was presented in [3] for the single-delayed system. However, the solution can be found for the general case following the same way which leads to the integral-differential quadratic equation on the interval $[0, m\tau]$ with a complete set of boundary conditions. A numerical example is presented in the next section. Assume all the coefficients of g_k, $0 \le k \le m$ and the function ψ are computed.

Consider the basic equation $R(z)\Phi(z) = 0$ with $\Phi = h + Q^{-1}[R^*(z)X(z)]_{0-}$. It holds

$$\begin{aligned} 0 &= R(z)\Phi(z) = \\ &= R(z)h + R(z)Q^{-1}(R^*(z)X(z) - [R^*(z)X(z)]_+). \end{aligned}$$

Notice that $R(z)Q^{-1}R^*(z) = \Pi^*(z)\Pi(z)$ and the function $[R^*(z)X(z)]_+$ is the Fourier transform of a generalized function with a support set in $[-m\tau, 0]$. Therefore, the support set of the inverse Fourier transform $\zeta(t)$ of the function

$$\Pi^*(z)\Pi(z)X(z) = -R(z)h + R(z)Q^{-1}[R^*(z)X(z)]_+$$

is included in $[-m\tau, m\tau]$.

Consider the function $r(z) = \Pi(z)X(z)$ and its inverse Fourier transform $\rho(t)$. The function $\rho(t)$ satisfies the equation

$$\Pi(-d/dt)\rho(t) = \zeta(t)$$

and $\zeta(t) = 0$ for $t > m\tau$. All roots of the function $\Pi(-z)$ have positive real parts by the definition of the spectral factor $\Pi(z)$. Therefore $\rho(t) = 0$ for $t > m\tau$.

Thus, the inverse Fourier transform $\xi(t)$ of the function $X(z)$ satisfies

$$\Pi(d/dt)\xi(t) = 0, \qquad t > m\tau. \tag{10}$$

The function $\xi(t)$ is determined from this equation whenever it is computed on the interval $[0, m\tau]$. The Fourier transform of ξ is $X(z) = r(z)/\Pi(z)$.

The inverse Fourier transform of the basic equation $R(z)\Phi(z) = 0$ gives a linear system of integral-differential equations on the interval $[0, m\tau]$.

All above can be summarized as follows.

- To solve the problem it is sufficient to find the function X or, equivalently, the functions r and Π in (7).
- The function Π is a result of the spectral factorization (8). It is determined by the polynomials g_k and the function ψ in (9). They can be found from an integral-differential quadratic equation (8) on $[0, m\tau]$.
- The inverse Fourier transform $\xi(t)$ of the function X on $[0, m\tau]$ can be found from the linear integral-differential equation $R(z)\Phi(z) = 0$.
- The function $\xi(t)$ on $[m\tau, +\infty)$ can be found from (10).

4 Numerical example

The plant is described by the equation

$$\begin{aligned}\dot{y}(t) + A_0 y(t) + A_1 y(t-\tau) + A_2 y(t-2\tau) + A_3 y(t-3\tau) = \\ = B_0 u(t) + B_1 u(t-\tau) + C_0 v(t) + C_1 v(t-\tau).\end{aligned}$$

The quadratic form in the $\mathcal{H}^\infty$ control problem is $F(y,u,v) = |y|^2 + |u|^2 - |v|^2$. The results of numerical computations will be illustrated for the values $A_0 = 0.5$, $A_1 = 0.6$, $A_2 = 0.2$, $A_3 = 0.3$, $B_0 = 0.8$, $B_1 = 0.9$, $C_0 = 0.1$, $C_1 = 0.4$, $\tau = 0.4$.

In this case

$$\begin{aligned} a(z) &= z + A_0 + A_1 e^{-\tau z} + A_2 e^{-2\tau z} + A_3 e^{-3\tau z}, \\ b(z) &= B_0 + B_1 e^{-\tau z}, \\ c(z) &= C_0 + C_1 e^{-\tau z}. \end{aligned}$$

The computations consist of a factorization followed by a solution to the linear system.

Factorization: The factor $\Pi(z)$ is sought in the form

$$\Pi(z) = z + p_0 + p_1 e^{-\tau z} + p_2 e^{-2\tau z} + p_3 e^{-3\tau z} + \Psi(z)$$

where $\Psi(z)$ is the Fourier transform of $\psi(t)$ and the support set of this function is in $[0, 3\tau]$. The vector function

$$F(t) = \begin{pmatrix} \psi(t) \\ \psi(t+\tau) \\ \psi(t+2\tau) \end{pmatrix} = \begin{pmatrix} F_0(t) \\ F_1(t) \\ F_2(t) \end{pmatrix}, \qquad 0 \le t < \tau,$$

satisfies the quadratic equation

$$\begin{aligned}\dot{F}(t) = \\ = \begin{pmatrix} p_0 & A_1 & A_2 \\ 0 & p_0 & A_1 \\ 0 & 0 & p_0 \end{pmatrix} F(t) + \begin{pmatrix} A_1 & A_2 & A_3 \\ A_2 & A_3 & 0 \\ A_3 & 0 & 0 \end{pmatrix} F(\tau - t) + \\ + \int_t^\tau \begin{pmatrix} F_0(s) & F_1(s) & F_2(s) \\ F_1(s) & F_2(s) & 0 \\ F_2(s) & 0 & 0 \end{pmatrix} F(s-t)\, ds + \\ + \int_0^t \begin{pmatrix} F_1(s) & F_2(s) & 0 \\ F_2(s) & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} F(s+\tau-t)\, ds.\end{aligned}$$

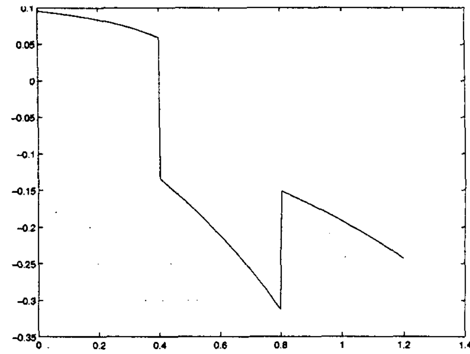

Figure 1: The function $\psi(t)$, $0 \le t \le 3\tau$.

The boundary conditions give $p_1 = A_1$, $p_2 = A_2$, $p_3 = A_3$ and the equations for the limit boundary conditions:

$$\begin{aligned} p_0^2 - 2F_0(0) &= A_0^2 + B_0^2 + B_1^2 - C_0^2 - C_1^2, \\ F_0(\tau) - F_1(0) &= B_0 B_1 - C_0 C_1 + (A_0 - p_0) A_1, \\ F_1(\tau) - F_2(0) &= (A_0 - p_0) A_2, \\ F_2(\tau) &= (A_0 - p_0) A_3. \end{aligned}$$

It is easy to see from these equations that the function $\psi(t)$ has jumps in the points $t = \tau$ and $t = 2\tau$.

The result of simulation is given on Fig. 1.

Linear equation: The basic linear equation $R(z)\Phi(z) = 0$ is transformed to the linear integral-differential equation for the function $\xi(t)$ on the interval $[0, 3\tau]$. It is the second-order differential equation with convolution and with mixed boundary conditions with values in the points $t = 0$, $t = \tau$, $t = 2\tau$ and $t = 3\tau$. The function $\xi(t)$ is continuous but can have jumps of the derivative.

Let $\xi(t)$ be the inverse Fourier transform of the function $X(z)$ and denote

$$\Xi(t) = \begin{pmatrix} \xi(t) \\ \xi(t+\tau) \\ \xi(t+2\tau) \end{pmatrix} = \begin{pmatrix} \Xi_0(t) \\ \Xi_1(t) \\ \Xi_2(t) \end{pmatrix}, \quad 0 \le t \le \tau.$$

Then the basic equation can be written in the form

$$\begin{aligned}\ddot{\Xi}(t) = M_1 \dot{\Xi}(t) + M_2 \Xi(t) + \Xi_2(\tau) \bar{F}(\tau - t) - \\ - \int_0^t G_1(s) E \Xi(s+\tau-t)\, ds - \int_t^\tau G_2(s-t) E \Xi(s)\, ds\end{aligned}$$

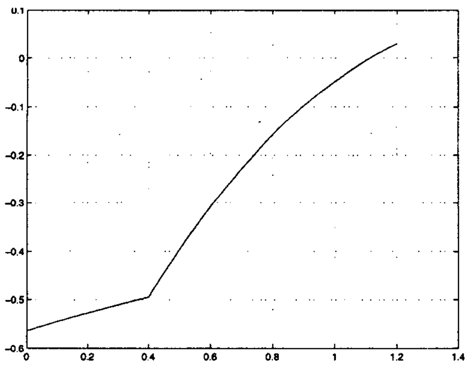

Figure 2: The function $\xi(t)$, $0 \le t \le 3\tau$.

where $\bar{F}(t) = (F_2(t), F_1(t), F_0(t))^T$,

$$
\begin{aligned}
E &= \begin{pmatrix} A_3 & A_2 & A_1 \\ 0 & A_3 & A_2 \\ 0 & 0 & A_3 \end{pmatrix}, \\
M_1 &= \begin{pmatrix} 0 & A_1 & A_2 \\ -A_1 & 0 & A_1 \\ -A_2 & -A_1 & 0 \end{pmatrix}, \\
M_2 &= \begin{pmatrix} D_{00} & D_{01} & D_{02} \\ D_{01} & D_{00}+D_{10} & D_{01}+D_{11} \\ D_{02} & D_{01}+D_{11} & D_{00}+D_{10}+D_{20} \end{pmatrix}, \\
D_{00} &= A_0^2 + B_0^2 - C_0^2, \\
D_{10} &= A_1^2 + B_1^2 - C_1^2, \\
D_{20} &= A_2^2, \\
D_{01} &= A_0A_1 + B_0B_1 - C_0C_1, \\
D_{11} &= A_1A_2, \\
D_{02} &= A_0A_2, \\
G_1(t) &= \begin{pmatrix} F_2(t) & 0 & 0 \\ F_1(t) & F_2(t) & 0 \\ F_0(t) & F_1(t) & F_2(t) \end{pmatrix}, \\
G_2(t) &= \begin{pmatrix} 0 & 0 & 0 \\ F_2(t) & 0 & 0 \\ F_1(t) & F_2(t) & 0 \end{pmatrix},
\end{aligned}
$$

with the boundary conditions

$$
\begin{aligned}
\dot{\Xi}_0(0) &= A_0\Xi_0(0) + A_1\Xi_1(0) + A_2\Xi_2(0) + A_3\Xi_2(\tau) - \\
&\quad - B_0h_u - C_0h_v, \\
\dot{\Xi}_1(0) &= \dot{\Xi}_0(\tau) - B_1h_u - C_1h_v, \\
\dot{\Xi}_2(0) &= \dot{\Xi}_1(\tau), \\
\dot{\Xi}_2(\tau) &= -(p_0\Xi_2(\tau) + A_1\Xi_2(0) + A_2\Xi_1(0) + A_3\Xi_0(0)), \\
\Xi_1(0) &= \Xi_0(\tau), \\
\Xi_2(0) &= \Xi_1(\tau).
\end{aligned}
$$

The result of simulation for the case $h = (0, -1, 0)^T$ is given on Fig. 2.

The central controller and the set of all stabilizing contracting controllers can be obtained by Theorem 1.

5 Conclusions

In this paper, it has been shown that the solution to $\mathcal{H}_\infty$ control problem for a linear system with multiple delays can be obtained through a spectral factorization (or quadratic integral-differential equation) followed by solving a linear integral-differential equation. Both equations can be solved numerically. The approach is based on the explicit operator representation of the solution (2) and has been already applied to single delay systems in [3]. However, in multiple delay case we must assume more general structure of the function ϕ in (9) that might have jumps at the delay time instants. The solution obtained is explicit and complete.

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