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International Asset Pricing and the Benefits from World Market Diversification

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Abstract

This paper extends previous tests of the conditional CAPM using different asymmetric and non-diagonal multivariate GARCH-M specifications for eight large national markets and the world market simultaneously. To solve the well-known problems associated with the likelihood functions of multivariate GARCH models, maximization is performed using simulated annealing, a Markov Chain Monte Carlo stochastic optimization method. We find that a model with double asymmetric effects and a time-varying price of world covariance risk supports all tested asset-pricing restrictions and that the previously often employed symmetric diagonal specification is overwhelmingly rejected. The evidence suggests that investors from all countries could expect statistically significant benefits from international diversification but that gains are considerable larger for investors with smaller home markets than for US and Japanese investors.

Keywords: international asset pricing; portfolio diversification; asymmetric and non-diagonal multivariate GARCH; simulated annealing

1 Introduction

International diversification as a stock market investment strategy to create portfolios with a higher expected return for a given level of risk, has been extensively discussed since the early works of Grubel (1968), Levy and Sarnat (1970) and Solnik (1974). The documented gains from international diversification are often attributed to lower correlations between national stock markets than within a stock market. A strong candidate for this lower correlation is that the industry structure differs between countries, which in turn implies differing investment opportunities, see e.g. Griffin and Karolyi (1998). On the other hand, market liberalizations and globalization in general during the last two decades have created larger cross-border trade in stocks and thereby cross-country correlations could be expected to have increased as argued in Longin and Solnik (1995) and Boucrelle, Solnik and Yann (1996). Potentially, this means that diversification services from investing abroad have decreased over time.

Many studies take the perspective of the US investor when discussing international diversification. Lin, Engle and Ito (1994) argue that cross-country correlations seem to increase when the US market turns downwards. From this it follows that US investors benefit the least from international diversification when they need it the most. Errunza, Hogan and Hung (1999) argue that because many foreign securities are listed on the US market, home made international diversification is an attractive alternative for US investors. Despite the arguments against international diversification, many recent papers find it economically worthwhile to pursue portfolio strategies involving international stock holdings, among them Chang, Eun and Kolodny (1995), Solnik (1997) and Griffin and Karolyi (1998).

In this paper we follow De Santis and Gérard (1997) and use the conditional CAPM as a vehicle to investigate benefits from international diversification for US as well as non-US investors over the last three decades 1970:1-1999:12. This approach allow us to investigate ex ante benefits from world market diversification taking both time-varying first and second conditional moments into account. The models are estimated simultaneously for the eight largest stock markets in the MSCI world market index. The extension to non-US investors contributes to the literature by examining the following arguments that applies to smaller countries.[1] First, for smaller-country investors we might expect a larger gain from international diversification because a smaller

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[1]The "size" of a country is here interpreted in terms of the market capitalization of its stock market.

home market offers more limited investment opportunities, at least in some industries. This also means that the kind of home made international diversification discussed in Errunza, Hogan and Hung (1999) should be more difficult to implement. Second, market turbulence on smaller stock markets may not be contagious internationally, which implies that investing abroad may offer a larger protection against home market downturns for smaller-country investors.

A standard econometric setup to estimate the conditional CAPM is the quasi-maximum likelihood (QML) theory applied to the multivariate symmetric diagonal GARCH-M. This econometric specification allows for a time-varying correlation structure between national stock market returns, which is potentially important when time-variations in diversification benefits are investigated. However, if conditional variances and covariances are higher during stock market downturns, as is argued in Engle and Ng (1993), Glosten, Jagannathan and Runkle (1993) and Kroner and Ng (1998), the econometric specification should allow for asymmetric effects in variances and covariances. Further, a diagonality restriction excludes cross-country volatility spillover effects, which could be questioned in a context where an underlying assumption is that stock markets are interrelated. A distinct contribution of our work is that we extend the work of De Santis and Gérard (1997) and estimate and test the conditional CAPM based on both multivariate asymmetric and multivariate non-diagonal conditional covariance parameterizations for a relatively large cross-section of assets.

One explanation for the scarcity of more flexible multivariate specifications in the literature is probably that the associated likelihood functions are known to be notoriously difficult to maximize. For this reason, we suggest the use of simulated annealing (SA) to maximize the likelihood functions for multivariate GARCH-M models. Simulated annealing is a stochastic search algorithm that uses the ideas of Markov Chain Monte Carlo (MCMC) to find the QML estimates. Because the SA algorithm is derivative free, is in principle insensitive to starting values and is able to escape from truly local optima, the problems present when using more conventional optimization algorithms are clearly mitigated.

The remainder of the paper is organized as follows. Section 2 provides a brief recapitulation of the theoretical background, the conditional CAPM in an international context. Section 3 contains a discussion of the empirical model and the different empirical specifications employed. Section 4 provides a description of the data. Section 5 presents the empirical results on asset pricing and international diversification. Section 6 summarizes and concludes.

2   The Conditional CAPM

The asset pricing equation of the conditional version of the classical one-period CAPM of Sharpe (1964) can be written

$$E\left[R_{it}|\mathbf{\Omega}_{t-1}\right] - R_{ft} = \beta_{iM,t-1}\left(E\left[R_{Mt}|\mathbf{\Omega}_{t-1}\right] - R_{ft}\right) \; ; \forall i \tag{1}$$

where

$$\beta_{iM,t-1} \equiv \frac{Cov\left(R_{it}, R_{Mt}|\mathbf{\Omega}_{t-1}\right)}{Var\left(R_{Mt}|\mathbf{\Omega}_{t-1}\right)} \tag{2}$$

is the possibly time-varying beta for asset i with the market, M. Further, R_{it} is the nominal return on asset i between time $(t-1)$ and t, R_{ft} is the return on a risk-free asset and R_{Mt} is the return on the market portfolio. All expectations are taken with respect to the market-wide set of information $\mathbf{\Omega}_{t-1}$ available to investors at that time. The basic intuition behind the model is that assets with positive correlations with the market will carry a positive risk-premium above the risk-free return and the beta measures the size of this premium. In contrast, because an asset with negative correlation serves as a hedge against the market, the model suggests that investors will hold this asset even if the expected return is below the risk-free rate of return. The conditional nature of the model implies that portfolio investments at each time $(t-1)$ are made conditional on $\mathbf{\Omega}_{t-1}$.

If the time-varying price of market covariance risk is defined by

$$\delta_{t-1} \equiv \frac{E\left[R_{Mt}|\mathbf{\Omega}_{t-1}\right] - R_{ft}}{Var\left(R_{Mt}|\mathbf{\Omega}_{t-1}\right)} \tag{3}$$

the asset pricing equation can be rewritten as

$$E\left[R_{it}|\mathbf{\Omega}_{t-1}\right] - R_{ft} = \delta_{t-1}Cov(R_{it}, R_{Mt}|\mathbf{\Omega}_{t-1}) \; ; \forall i. \tag{4}$$

This latter formulation of the conditional CAPM is extensively used in empirical work. The standard starting point in the international asset pricing literature is 1970, and hence there is an implicit assumption that international capital markets could be considered at least approximately integrated from the 1970s and onwards. Even though the model originally is derived for a closed economy it can be reinterpreted for international asset pricing purposes if R_{it} is interpreted as, for example, the return on the market index for country i and R_{Mt} as the return

on the world market portfolio. Alternatively, the model could be considered as a special case of the international asset pricing model of Adler and Dumas (1983) in which currency risk is not significantly priced.

2.1 Implications for international diversification

Next, we turn to implications for international diversification. We assume that the CAPM asset pricing restrictions hold, i.e. that the world market portfolio is conditionally efficient, and analyze world market diversification in the classical Markowitz (1952) mean-variance framework.[2] The idea is to create two equally risky portfolios, one internationally diversified I and one purely domestic i. The conditional CAPM then provides us with expressions for the expected returns of the two portfolios and the difference in expected returns can be interpreted as the expected benefit from international diversification. Hence, in general, the expected benefit from international diversification can be written

$$E\left[R_{It} - R_{it}|\mathbf{\Omega}_{t-1}\right] \tag{5}$$

where $R_{It} = \theta_{t-1}R_{Mt} + \left(1 - \theta_{t-1}\right)R_{ft}$ is the return on an internationally diversified portfolio with the same level of conditional volatility as a domestic stock market portfolio with return R_{it}. The expected excess returns according to the conditional CAPM for the two portfolios are

$$E\left[R_{It}|\mathbf{\Omega}_{t-1}\right] - R_{ft} = \delta_{t-1}cov\left(\theta_{t-1}R_{Mt}, R_{Mt}|\mathbf{\Omega}_{t-1}\right) = \delta_{t-1}\theta_{t-1}var\left(R_{Mt}|\mathbf{\Omega}_{t-1}\right) \tag{6}$$

$$E\left[R_{it}|\mathbf{\Omega}_{t-1}\right] - R_{ft} = \delta_{t-1}cov\left(R_{it}, R_{Mt}|\mathbf{\Omega}_{t-1}\right) \tag{7}$$

where the weight $\theta_{t-1} > 0$ is given by the two equations

$$var\left(R_{it}|\mathbf{\Omega}_{t-1}\right) = var\left(R_{It}|\mathbf{\Omega}_{t-1}\right) \tag{8}$$

$$var\left(R_{It}|\mathbf{\Omega}_{t-1}\right) = \theta_{t-1}^2 var\left(R_{Mt}|\mathbf{\Omega}_{t-1}\right) \tag{9}$$

which is equivalent to $\theta_{t-1}^2 = var\left(R_{it}|\mathbf{\Omega}_{t-1}\right)/var\left(R_{Mt}|\mathbf{\Omega}_{t-1}\right)$. Combining equations (6) and (7), the expected gain from world market diversification for the domestic investor according to

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[2] The asset pricing restrictions implied by the conditional CAPM are tested in Section 5.2.

the conditional CAPM is

$$E\left[R_{It} - R_{it}|\mathbf{\Omega}_{t-1}\right] = \delta_{t-1}\left[\theta_{t-1}var\left(R_{Mt}|\mathbf{\Omega}_{t-1}\right) - cov\left(R_{it}, R_{Mt}|\mathbf{\Omega}_{t-1}\right)\right]. \tag{10}$$

Some intuition for equation (10) can be obtained if the special case $\theta_{t-1} = 1$ is considered. Then

$$E\left[R_{It} - R_{it}|\mathbf{\Omega}_{t-1}\right] = \delta_{t-1}\left[var\left(R_{it}|\mathbf{\Omega}_{t-1}\right) - cov\left(R_{it}, R_{Mt}|\mathbf{\Omega}_{t-1}\right)\right] \tag{11}$$

or alternatively, using the conditional correlation between the domestic return and the world market return

$$\rho_{iM,t-1} = \frac{cov\left(R_{it}, R_{Mt}|\mathbf{\Omega}_{t-1}\right)}{\sqrt{var\left(R_{it}|\mathbf{\Omega}_{t-1}\right)var\left(R_{Mt}|\mathbf{\Omega}_{t-1}\right)}} \tag{12}$$

equation (10) can be rewritten as

$$E\left[R_{It} - R_{it}|\mathbf{\Omega}_{t-1}\right] = \delta_{t-1}\left(1 - \rho_{iM,t-1}\right)var\left(R_{it}|\mathbf{\Omega}_{t-1}\right). \tag{13}$$

Equation (11) highlights that the benefit from international diversification is increasing in the amount of idiosyncratic risk $var\left(R_{it}|\mathbf{\Omega}_{t-1}\right) - cov\left(R_{it}, R_{Mt}|\mathbf{\Omega}_{t-1}\right)$. According to the conditional CAPM, only a higher systematic risk is rewarded a higher expected return because idiosyncratic risk could be diversified away. Equation (13) illustrates that the diversification benefit is decreasing in the level of correlation with the world market portfolio as well as the fact that there is no gain from diversification if $\rho_{iM,t-1} = 1$. We also see that the expected benefit from international diversification is increasing in the price of world covariance risk δ_{t-1}. Unfortunately, the corresponding expression derived in DeSantis and Gerard (1997) is flawed, and hence their conclusions on diversification benefits are invalid.[3]

3   Empirical Specification

An empirical formulation of the conditional CAPM in an economy with N risky assets is

$$R_{it} - R_{ft} = \delta_{t-1}h_{iMt} + \varepsilon_{it} \; ; \; \varepsilon_{it}|\mathbf{\Omega}_{t-1} \sim N\left(0, h_{iit}\right) \; ; \; i = 1, ..., N \tag{14}$$

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[3]De Santis and Gerard incorrectly claim that the expected return on the international portfolio is equal to $\delta_{t-1}\theta_{t-1}^2 var\left(R_{Mt}|\mathbf{\Omega}_{t-1}\right)$. Hence, because $\theta_{t-1} > 1$ with few exceptions, they systematically overstate the value of international diversification.

where h_{iit} is the idiosyncratic conditional variance of asset i and h_{iMt} is the conditional covariance of asset i with the market portfolio M. The asset pricing model itself does not impose any restrictions on the dynamics of conditional second moments or the evolution over time of the price of covariance risk, two issues that are discussed below.

3.1 Conditional variance and covariance dynamics

Given the success of GARCH-processes to capture important aspects of the conditional variance we parameterize the conditional covariance matrix as a multivariate GARCH(1,1) process and allow the conditional mean to be a linear function of the conditional covariances.[4] Particularly, we adopt a parameterization that allows for time-varying correlations between stock markets. The GARCH framework used to model the time-varying correlation structure between countries is potentially important when focus is on changes of diversification benefits over time. The time-varying pattern of cross-country correlations is examined in Longin and Solnik (1995), and they reject the hypothesis of constant conditional correlations.[5]

Because we work with a cross section of eight countries together with the world market, the number of parameters in the GARCH processes is clearly quite large. De Santis and Gérard (1997) overcomes this difficulty by assuming diagonality, i.e. they exclude spillover effects between different countries' variance processes. To further reduce the dimension of the parameter space they use the parsimonious representation of Ding and Engle (1994) in which the number of parameters is growing linearly in the size of the cross-section. The cost for this is that they are not able to incorporate asymmetric variance and covariance effects. We extend the analysis in De Santis and Gérard (1997) and allow for either asymmetric variance and covariance effects (asymmetric specifications) or cross-country variance and covariance effects (non-diagonal specifications). Table I summarizes the variance and covariance specifications, which are explained in more detail in the next subsection.

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[4]Higher order GARCH-processes could also be estimated, but most empirical studies suggest that a GARCH(1,1)-process is sufficient. An examination of higher order GARCH-processes is left for future multivariate studies.

[5]This evidence rules out the constant conditional correlation GARCH model of Bollerslev (1990).

Table I:
Conditional variance and covariance specifications.

	Description	#Covariance parameters
Model 1	Symmetric diagonal	63
Model 2	Asymmetric diagonal Sign effect	72
Model 3	Asymmetric diagonal Size effect	72
Model 4	Asymmetric diagonal Sign and size effects	81
Model 5	Symmetric non-diagonal	117

Briefly, the asymmetric specification with sign effect allows conditional variance and conditional covariance to react differently to negative and positive shocks, i.e. different signs of shocks. Similarly, the asymmetric specification with size effect allows conditional variance and conditional covariance to react differently to different absolute sizes of shocks, i.e. to shocks larger in absolute value than some threshold value. Finally, the non-diagonal specification allows for cross-country spillover effects in conditional variances. i.e. that shocks in one country market affect the conditional variance in another market.

We estimate three different specifications of the price of risk for each of the conditional covariance specifications. Table II summarizes our conditional mean specifications, which are discussed in more detail below. The price of world covariance risk is specified as a linear function of the information variables lagged one period.

Table II:
Conditional mean specifications.

	Description	#Mean parameters
Model A	Constant price of covariance risk	1
Model B	Time varying price of covariance risk Informations variables: a constant, excess return on the world market, US time premium and US default premium	4
Model C	Time varying price of covariance risk Informations variables: a constant, excess return on the world market, US time premium, US default premium, US inflation and growth in US industrial production	6

3.1.1 The BEKK conditional covariance matrix

Engle and Kroner (1995) propose a covariance matrix parameterization according to (the BEKK model)

$$\mathbf{H}_t = \mathbf{\Phi} + \mathbf{A}'\varepsilon_{t-1}\varepsilon'_{t-1}\mathbf{A} + \mathbf{B}'\mathbf{H}_{t-1}\mathbf{B} \tag{15}$$

where ε_{t-1} is the $N \times 1$ vector of lagged model residuals, $\mathbf{\Phi}$ is a symmetric $N \times N$ parameter matrix and $\mathbf{A}$ and $\mathbf{B}$ are $N \times N$ parameter matrices. To impose positive definiteness on the conditional covariance matrix we let $\mathbf{\Phi} = \mathbf{C}'\mathbf{C}$ and restrict $\mathbf{C}$ to be lower triangular. We do not impose symmetry on $\mathbf{A}$ and $\mathbf{B}$ because we do not want to preclude asymmetric spillovers between variance processes for different countries. For example, it could be reasonable to assume that there are US market effects on the variance of smaller stock markets, but not the other way around. To limit the number of parameters to estimate, most studies either uses a small cross-section of assets or places strong restrictions on the coefficient matrices. For example, the restriction that $\mathbf{A}$ and $\mathbf{B}$ are diagonal means that the conditional variance of an asset depends only on the asset's own past squared residual and the asset's own past conditional variance.[6]

To allow for cross-effects between the variances of international asset markets is a very intuitive assumption in a context where the basic idea is that markets are interrelated. The diagonality assumption is particularly disturbing for smaller countries because the conditional variance of smaller-country stock market returns is more likely to be affected not only by past own market effects. Instead, we try to use economically motivated restrictions. More precisely, we assume that the variance processes of a group of larger countries are not affected by smaller markets. If the countries are ordered by group this means that we assume that $a_{ij}, b_{ij} \neq 0$ if country i belongs to a group of larger countries than does country j, otherwise $a_{ij}, b_{ij} = 0$. The ordering of the country markets and the world market can be found in Table III.

This grouping means for example that we allow for volatility spillover effects from Germany, the UK and Japan to France, Canada, Netherlands and Switzerland, but not the other way around. The impact on the world market variance is therefore restricted to world market effects only, while there are spillover effects from the world market to all country markets.

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[6] Appendix A provides explicit illustrations of the different conditional covariance specifications in the bivariate case.

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Table III:

Country grouping by size.

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Group 1: France, Canada, Netherlands and Switzerland
Group 2: Germany, the UK and Japan
Group 3: the US
Group 4: the World

3.1.2 The BEKK conditional covariance matrix with asymmetric effects

One additional benefits of our approach is the possibility to incorporate general asymmetric responses of variances and covariances to positive and negative innovations in returns. The importance of these so called "asymmetric effects" is thoroughly discussed in Engle and Ng (1993), Glosten, Jagannathan and Runkle (1993) and Kroner and Ng (1998). These authors find that the effect on conditional variance is larger for negative shocks than for positive shocks.[7] The asymmetric effect is often termed "leverage effect". This is because it involves the explanation that the volatility of a firm's equity increases as a consequence of an increased financial leverage when the firm's value decreases (because of bad news).[8] Here we use the term "asymmetric sign effect" to distinguish it from the "asymmetric absolute size effect" discussed below. Both these asymmetric parameterizations imply a non-linear relation between the conditional variance and the squared own lagged residual.

Kroner and Ng (1998) add a term to the BEKK covariance matrix process to capture asymmetric responses of the conditional variances and covariances to return innovations. In their spirit, a general modified BEKK covariance matrix can be specified according to

$$\mathbf{H}_t = \mathbf{C}'\mathbf{C} + \mathbf{A}'\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}'_{t-1}\mathbf{A} + \mathbf{B}'\mathbf{H}_{t-1}\mathbf{B} + \mathbf{G}'\boldsymbol{\xi}_{t-1}\boldsymbol{\xi}'_{t-1}\mathbf{G} + \mathbf{S}'\boldsymbol{\eta}_{t-1}\boldsymbol{\eta}'_{t-1}\mathbf{S} \tag{16}$$

where $\mathbf{G}$ and $\mathbf{S}$ are $N \times N$ parameter matrix and

$$\xi_{it} = \varepsilon_{it}I_{\xi it} \text{ where } I_{\xi it} = 1 \Leftrightarrow \varepsilon_{it} < 0 \text{ otherwise } I_{\xi it} = 0 \tag{17}$$

$$\eta_{it} = \varepsilon_{it}I_{\eta it} \text{ where } I_{\eta it} = 1 \Leftrightarrow |\varepsilon_{it}| > \sqrt{h_{iit}} \text{ otherwise } I_{\eta it} = 0 \tag{18}$$

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[7]Kroner and Ng (1998) uses a bivariate specification to detect both asymmetric variance and covariance effects.

[8]This mechanism is formalized by Christie (1982) and is also discussed in detail in Cho and Engle (1999).

The first asymmetric model estimated is an asymmetric diagonal sign effects model, i.e. a specification with $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{G}$ diagonal and $\mathbf{S} = \mathbf{0}$. Kroner and Ng (1998) suggest a similar multivariate extension of the univariate Glosten, Jagannathan and Runkle (1993) model of asymmetric conditional variance. We also propose another parameterization, the asymmetric absolute size effects model of conditional covariance corresponding to $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{S}$ diagonal and $\mathbf{G} = \mathbf{0}$. This parameterization implies that the conditional variance is higher when the squared own lagged residual is larger than its conditional expectation. In contrast to the previous specification, this specification is by definition symmetric with respect to the sign of the own lagged residual. The final asymmetric diagonal specification is then the combination of the above two models, i.e. a model that allows for both asymmetric sign effects and asymmetric absolute size effects, corresponding to $\mathbf{A}$, $\mathbf{B}$, $\mathbf{G}$ and $\mathbf{S}$ diagonal.

3.2   Conditional mean dynamics

We follow many previous researchers and use the conditional CAPM with constant price of covariance risk, i.e. $\delta_{t-1} = \delta$; $\forall t$, as the basic model:

$$R_{it} - R_{ft} = \delta h_{iMt} + \varepsilon_{it} \; ; \; \varepsilon_{it}|\mathbf{\Omega}_{t-1} \sim N\left(0, h_{iit}\right) \tag{19}$$

where δ is the common price of covariance risk for all markets, see for example Ng (1991) and Chan, Karolyi and Stultz (1992) and De Santis and Gérard (1997). However, the assumption of a constant price of world covariance risk over time could be too restrictive. The conditional CAPM is only partial equilibrium model and does not identify the underlying variables that affect the possibly time-varying price of conditional covariance risk. This implies that all specifications of the price of risk, including the constant price of risk specification, could be criticized for a lack of theoretical foundation.

The fundamental problem from an asset pricing perspective is that the conditional CAPM with a constant (positive) price of risk is usually rejected by the data. These rejections could be driven by the fact that in some periods realized return is a bad proxy for expected return, even if realized return is a reasonable proxy over longer periods of time. The increased flexibility provided by a time-varying price of risk allows the conditional CAPM to better accommodate such periods and, as a consequence, the model is not rejected. But this does not come without a

cost; in some periods the estimated price of covariance risk is inevitably negative. An additional problem is that the estimated price of risk is probably a very noisy estimate of the true price of risk and hence it is not surprising if the estimate occasionally is negative.

An estimated negative price of covariance risk is equivalent to a negative expected excess return on the world market portfolio, i.e. that the expected return on the world market is lower than the risk-free rate of return, see equation (3). In other words, in periods when the price of risk is negative, the world market portfolio is not conditionally efficient. This is evidence against the theoretical model only if we believe that expected excess return sometimes is negative in equilibrium. The competing interpretation is that estimated negative expected excess returns simply reflect that the econometric model adopts to negative realized returns. A common suggestion in the literature is to impose the additional restriction $\delta_{t-1} > 0$ during estimation. However, this auxiliary restriction appears to assume the difficulty away rather than solve it.[9] Our conclusion from the above discussion is to be cautious to evaluate the conditional CAPM and its implications on a period by period basis and instead consider averages over longer periods of time. Of course, this suggestion mirrors the contents of the rational expectations assumption underlying the empirical formulation of the conditional CAPM; given information, expectations are correct on average, but not necessarily correct period by period.

The idea of defining the time-varying price of risk as a deterministic function of lagged economic variables is previously used in the literature by for example Carrieri (2001), DeSantis and Gerard (1997,1998) and Campbell (1991). We believe that there is a trade-off between the number of economic instruments used and a risk for overfitting. The time-varying price of covariance risk does in the most unfavorable scenario reflect changes in realized returns and not changes in the underlying true price of covariance risk. As a consequence, too many information variables could create an unrealistically volatile price of risk. Despite this problem, we estimate two alternatives to the constant price of risk model. Our first alternative is to model the time-varying price of covariance risk as a linear function of a constant, the lagged world market excess

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[9]For example, one of the asset pricing tests in De Santis and Gerard (1997) rejects the conditional CAPM. They argue that this is a consequence of the positivity restriction or that the rejection "... is indeed driven by the inability of the CAPM to accommodate negative expected returns".

return, the US time premium $(TIME_{t-1})$ and the US default premium (DEF_{t-1}):

$$\delta_{t-1} = \delta_0 + \delta_1(R_{M,t-1} - R_{f,t-1}) + \delta_2 TIME_{t-1} + \delta_3 DEF_{t-1}. \tag{20}$$

In our second alternative, we extend the first alternative to include the lagged inflation $(INFL_{t-1})$ and the lagged change in industrial production (IP_{t-1}):

$$\delta_{t-1} = \delta_0 + \delta_1(R_{M,t-1} - R_{f,t-1}) + \delta_2 TIME_{t-1} + \delta_3 DEF_{t-1} + \delta_4 INFL_{t-1} + \delta_5 IP_{t-1}. \tag{21}$$

Because the conditional CAPM is not able to accommodate a negative expected excess return on an asset with positive correlation with the market portfolio, one alternative is that there are missing global risk-factors in the asset pricing equation. These factors could for example be a conditional skewness factor in the spirit of Harvey and Siddique (2000) or exchange rate risk factors as in Carrieri (2001) and De Santis and Gerard (1998). Another interpretation is that there are missing priced country specific factors and that a model of partial segmentation would be more appropriate, a hypothesis which is tested in Section 5.2.

3.3   The log-likelihood function

The likelihood functions of large scale multivariate GARCH-M models are extremely difficult to maximize with conventional gradient based optimization methods. Basically, there are three related problems involved. First, the likelihood function probably has multiple local optima. Second, the number of combinations of reasonable starting values increases exponentially in the number of parameters to be estimated. Third, convergence properties are in general poor. As an alternative, we maximize the likelihood function by means of simulated annealing, a stochastic optimization method in the MCMC family.[10] The SA algorithm is derivative free, is in principle independent of the starting values chosen and is able to escape from truly local optima. Thus, the difficulties involved with conventional optimization are at least alleviated. The main drawback

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[10] Relevant references to Simulated Annealing include Corona A., Marchesi M., C. Martini and S. Ridella (1987), Goffe W., G. Ferrier and J. Rogers (1994) and Fishman G. (1996), pages 384-406. The program used in this paper is a C++ implementation of the algorithm in Goffe et. al. (1994).

of simulated annealing is that it is quite time-consuming.[11]

If the conditional distribution of returns given $\mathbf{\Omega}_{t-1}$ is assumed multivariate Normal distributed, the global log-likelihood can be written

$$\ln L\left(\boldsymbol{\phi}\right) = -\frac{TM}{2}\ln\left(2\pi\right) - \frac{1}{2}\sum_{t=1}^{T}\ln\left[\det\left[\mathbf{H}_{t}\left(\boldsymbol{\phi}\right)\right]\right] - \frac{1}{2}\sum_{t=1}^{T}\boldsymbol{\varepsilon}'_{t}\left(\boldsymbol{\phi}\right)\mathbf{H}_{t}^{-1}\left(\boldsymbol{\phi}\right)\boldsymbol{\varepsilon}_{t}\left(\boldsymbol{\phi}\right) \qquad (22)$$

where $\boldsymbol{\phi}$ is the vector of unknown parameters in the empirical model. Estimation is performed conditional on the first observation. The iterations are started with the initial residual vector equal to zero and the initial covariance matrix equal to the sample covariance matrix. For estimation and inference the quasi-maximum likelihood (QML) approach is used, see Bollerslev and Wooldridge (1992) and Gouriéroux (1997), pages 43-52 and 117-120. Under certain regularity conditions, the QML estimator $\hat{\boldsymbol{\phi}}_{QML}$ is consistent and asymptotically Normal even if the normality assumption is violated and

$$\sqrt{T}\left(\hat{\boldsymbol{\phi}}_{QML} - \boldsymbol{\phi}_{0}\right) \xrightarrow{d} N\left(\mathbf{0}, \mathbf{P}_{0}^{-1}\mathbf{Q}_{0}\mathbf{P}_{0}^{-1}\right) \qquad (23)$$

where $\boldsymbol{\phi}_{0}$ is the true parameter vector. In practice, to calculate robust test statistics, the negative of the expected Hessian $\mathbf{P}_{0}$ and the expected outer-product matrix $\mathbf{Q}_{0}$ are approximated by replacing the true parameter vector with the QML estimator.[12]

4   The Data

We use monthly MSCI stock market index total returns, i.e. returns including dividends, for the time period 1970:1-1999:12 from France (FRA), Canada (CAN), Netherlands (NRL), Switzerland (SWI), Germany (GER), United Kingdom (UK), Japan (JPN), United States (US) and the world market (WLD).[13] Returns from all countries are recalculated in the French Franc,

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[11] The processor time required to estimate the models in this paper is approximately 24 hours on a Pentium III 800 MHz.

[12] See Bollerslev and Wooldridge (1992) for a detailed discussion in the context of multivariate GARCH models.

[13] The country weights in the MSCI world market portfolio are (1999): France (2.4%), Canada (2.5%), Netherlands (2.6%), Switzerland (3.2%), Germany (4.5%), the UK (9.9%), Japan (15.7%) and the US (47.4%). Together these countries constitute 88.2% of the world market portfolio.

Canadian Dollar, Dutch Gulden, Swiss Franc, German Mark, UK Pound, Japanese Yen and the US dollar. All total return indices are obtained from the EcoWin database. Summary statistics for the US-dollar denominated returns are contained in Table IV.

Table IV:
Summary statistics for US dollar denominated monthly returns.

	FRA	CAN	NRL	SWI	GER	UK	JPN	US	WLD
Mean (% per year)	12.89	10.30	15.46	13.20	12.42	12.97	13.51	12.57	12.06
Std (% per year)	22.90	19.16	17.70	18.89	20.41	23.07	22.60	15.30	14.28
Skewness	−0.349	−0.782	−0.433	−0.312	−0.426	0.476	−0.042	−0.641	−0.663
Kurtosis	4.59	6.36	5.17	4.50	4.32	8.76	3.57	6.20	5.24
Min (%)	−26.37 (May-81)	−24.89 (Mar-80)	−19.60 (Nov-73)	−19.40 (Oct-87)	−19.40 (Sep-90)	−24.25 (Oct-87)	−21.54 (Mar-90)	−23.85 (Oct-87)	−18.59 (Oct-87)
Max (%)	23.77 (Jan-75)	16.53 (Aug-82)	22.85 (Jan-75)	21.98 (Jan-75)	18.42 (Dec-89)	44.74 (Jan-75)	21.72 (Oct-90)	16.38 (Oct-74)	13.73 (Jan-75)
Q(12)	12.39 [0.414]	11.87 [0.456]	15.69 [0.206]	10.14 [0.603]	17.12 [0.145]	14.52 [0.269]	24.76 [0.016]	8.70 [0.728]	14.60 [0.264]

Note: Q(12) is the Ljung-Box test for up to 12:th order serial correlation.

Skewness is mostly negative and kurtosis is above three for all countries, indicating skewed unconditional distributions for stock returns with thicker tails and higher peaks than the Normal distribution. The "worst month" for four of the countries, including the world market, is related to the stock market crash October 1987 and the "best month" is January 1975 for five of the countries. Based on Ljung-Box tests for up to 12:th order autocorrelation, with the exception of Japan, the returns show no significant autocorrelation. Table V contains simple correlations for the US-dollar denominated excess returns.

Table V:
Correlation matrix for US dollar denominated monthly returns.

	FRA	CAN	NRL	SWI	GER	UK	JPN	US	WLD
FRA	1	0.449	0.622	0.617	0.621	0.547	0.392	0.454	0.634
CAN		1	0.547	0.467	0.360	0.516	0.295	0.717	0.723
NRL			1	0.722	0.691	0.642	0.431	0.581	0.742
SWI				1	0.692	0.557	0.426	0.504	0.679
GER					1	0.440	0.371	0.396	0.587
UK						1	0.367	0.512	0.685
JPN							1	0.277	0.678
US								1	0.839
WLD									1

15

Not surprisingly, the highest correlation, 0.839, is between the US returns and the world market returns. More surprisingly is perhaps that the lowest correlation, 0.277, is between the US and Japan. We may also note that Germany is the country with the lowest correlation with the world market, 0.587.

Summary statistics for the different interest rates used to calculate excess returns for each of the currencies, can be found in Table VI. For France, Canada, the UK, Japan and the US, the 3 month treasury bill rate is used for the whole sample period. For Netherlands, Switzerland and Germany there is no treasury bill available at the beginning of the sample and instead the discount rate is used as a proxy for the risk-free rate.[14] The average interest rates ranges from a low 3.91% per year for Switzerland to a high 9.42% per year for the UK. The high positive skewness for the US is probably a consequence of a number of positive outliers during the monetary experiment 1979-1982. In contrast to the stock returns, kurtosis is below three for most of the interest rates, indicating thinner tails and a less peaked unconditional distribution when compared to the Normal distribution.

Table VI:
Summary statistics for risk-free interest rates.

	FRA	CAN	NRL	SWI	GER	UK	JPN	US
Mean (% per year)	8.69	8.18	6.05	3.91	5.53	9.42	5.34	6.66
Std (% per year)	3.28	3.54	2.21	2.14	1.98	3.06	3.19	2.65
Skewness	0.176	0.777	0.631	0.673	0.628	0.280	0.249	1.276
Kurtosis	2.83	3.57	2.81	2.73	2.80	2.09	2.88	4.73

Summary statistics including correlations for the macroeconomic information variables used to model the time-varying price of covariance risk can be found in Table VII. The instruments used are a constant, the lagged excess world stock market return, the lagged US time premium measured as the difference in yields between a 10 year treasury bond and a three month treasury bill, the lagged US default premium measured as the difference in yields between a 10 year treasury bond and Moody's Baa rated bond, the US inflation measured as the growth of consumer

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[14]The treasury bill data is from the EcoWin database or the IFS CD-ROM (series 60C). The discount rate data is from the IFS CD-ROM. The discount rate is used for the time periods 1970:1-1978:2 (Netherlands), 1970:1-1980:1 (Switzerland) and 1970:1-1975:6 (Germany).

prices and the growth in US industrial production. The correlations are relatively low, which indicates that the macroeconomic variables contain nonredundant information.

Table VII:
Summary statistics for macroeconomic information variables.

	TIME	DEF	INFL	IP
Mean	1.616	1.946	0.416	0.238
Std	1.272	0.575	0.336	0.815
Skewness	-0.456	0.619	0.886	-0.778
Kurtosis	2.75	3.70	4.00	6.79
Correlations	TIME	DEF	INFL	IP
TIME	1	0.314	-0.436	0.135
DEF		1	-0.256	-0.210
INFL			1	-0.100
IP				1

5 Empirical Evidence

5.1 Conditional CAPM

To select the most appropriate model specification for US dollar denominated returns among the $5 \cdot 3 = 15$ models summarized in Tables I and II, we perform a number of LR-tests. First, we test the different covariance specifications against each other. Second, for each covariance specification, we carry out LR-tests of the different mean specifications against each other. These tests give us the preferred model specification. Later, we will assume that the most appropriate model specification for the US dollar denominated returns is also the most appropriate specification for returns denominated in other currencies.

We find that the symmetric diagonal models (Models 1A, B, and C) are overwhelmingly rejected by all other conditional covariance specifications, see Table VIII. That is, our evidence suggest that the symmetric diagonal multivariate GARCH(1,1) specification is not flexible enough to capture the conditional variance and covariance dynamics of stock returns. Both asymmetric and non-diagonal specifications are clearly superior, at least from a statistical perspective. From an economic perspective the implication is that asymmetric effects as well as volatility spillovers play significant roles in our understanding of variability and covariability of international stock returns. This is an important result for future work, because nearly all existing

conditional asset pricing studies, including Carrieri (2001), DeSantis and Gerard (1997,1998) and Chan, Karolyi and Stultz (1992), that employ the multivariate GARCH model, either use the symmetric diagonal specification or use a very limited cross-section of assets.

We also find that the asymmetric sign effect model in the spirit of Glosten, Jagannathan and Runkle (1993) and Kroner and Ng (1998), is superior to the asymmetric absolute size effect model.[15] In other words, it seems more important to allow conditional variances and covariances to respond asymmetrically to negative and positive shocks, than to small and large shocks (in absolute value). However, the models that include *both* sign and size effects (Models 4A, B and C) easily reject the models that allow for only one of the asymmetric effects (Models 2A, B and C and Models 3A, B and C). This result suggests that the dichotomy of negative and positive shocks is not completely adequate for conditional variance and covariance modeling; there are still systematic characteristics of conditional second moments of stock returns that are not captured by such a model.

Table VIII:
Likelihood ratio tests of covariance specification.

H_0	H_1	LogL0	LogL1	χ^2	dgf	p-value
Model 1A	Model 2A	−8492.01	−8454.45	75.12	9	0.000
Model 1B	Model 2B	−8483.52	−8445.23	76.57	9	0.000
Model 1C	Model 2C	−8479.13	−8443.58	71.09	9	0.000
Model 1A	Model 3A	−8492.01	−8462.83	58.37	9	0.000
Model 1B	Model 3B	−8483.52	−8454.34	58.35	9	0.000
Model 1C	Model 3C	−8479.13	−8448.40	61.46	9	0.000
Model 1A	Model 4A	−8492.01	−8439.15	105.71	18	0.000
Model 1B	Model 4B	−8483.52	−8431.15	104.73	18	0.000
Model 1C	Model 4C	−8479.13	−8429.71	98.84	18	0.000
Model 1A	Model 5A	−8492.01	−8439.58	104.86	54	0.000
Model 1B	Model 5B	−8483.52	−8422.12	122.80	54	0.000
Model 1C	Model 5C	−8479.13	−8418.30	121.66	54	0.000
Model 2A	Model 4A	−8454.45	−8439.15	30.60	9	0.000
Model 2B	Model 4B	−8445.23	−8431.15	28.16	9	0.001
Model 2C	Model 4C	−8443.58	−8429.71	27.74	9	0.001
Model 3A	Model 4A	−8462.83	−8439.15	47.36	9	0.000
Model 3B	Model 4B	−8454.34	−8431.15	46.39	9	0.000
Model 3C	Model 4C	−8448.40	−8429.71	37.37	9	0.000

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[15] These two models have the same number of parameters and hence the likelihood values can be compared directly. The likelihood values can be found in Table VII.

The asymmetric models and the non-diagonal model are not nested and we are not able to perform a formal test of these specifications. However, because the non-diagonal model contains 36 additional parameters compared to the preferred asymmetric specification, the relatively small increase in likelihood value, points in favor of the more parsimonious asymmetric specification.[16] Hence, we take as our preferred covariance specification the asymmetric specification with both sign and size effects. To be able to perform a formal test an interesting extension of this study is to simultaneously allow for both asymmetric effects and volatility spillovers.

Turning to the alternative specifications of the market price of risk, we find that both specifications with a time-varying market price of risk outperform the model with a constant market price of risk, regardless of the particular covariance specification employed, see Table IX. Interestingly, for the two models that allows for asymmetric sign effects (Models 2B and C and Models 4B and C), we can not reject specification B of the conditional mean, while for the models that do not allow for asymmetric sign effects (Models 1B and C, Models 3B and C and Models 5B and C), specification B is rejected. This result suggests a trade-off between conditional covariance modeling and conditional mean modeling in the sense that a rich conditional covariance parametrization seems to reduce the need for a complex conditional mean parameterization. In turn this may indicate that the need for a complex time-varying conditional mean dynamics at least partly is a consequence of a not-good-enough conditional covariance dynamics and not of an actually time-varying price of risk.

Estimates of individual coefficients for the preferred model can be found in Table X. The ARCH-coefficients (the matrix $\mathbf{A}$) and GARCH-coefficients (the matrix $\mathbf{B}$) are highly significant for all countries. This is in line with previous results in the literature. Our new evidence is on asymmetric variance and covariance effects. The significant coefficients in the matrix $\mathbf{G}$ indicate that conditional variance is higher for negative shocks for France, Netherlands, Switzerland and the UK. Similarly, the significant coefficient in the matrix $\mathbf{S}$ indicate that conditional variance is higher for shocks large in absolute value for France, Canada and Germany. The asymmetric effects in conditional covariances could be thought of as a consequence of common negative shocks or common large shocks, i.e. when shocks to two countries both are negative or large in

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[16]In fact, penalized likelihood criteria for model selection, e.g. Akaike, Schwarz-Bayes and Hannan-Quinn, select the asymmetric specification.

Table IX:
Likelihood ratio tests of price of risk specifications.

H_0	H_1	LogL0	LogL1	χ^2	dgf	p-value
Model 1A	Model 1B	-8492.01	-8483.52	16.99	3	0.000
Model 1A	Model 1C	-8492.01	-8479.13	25.76	5	0.000
Model 1B	Model 1C	-8483.52	-8479.13	8.77	2	0.012
Model 2A	Model 2B	-8454.45	-8445.23	18.44	3	0.000
Model 2A	Model 2C	-8454.45	-8443.58	21.74	5	0.000
Model 2B	Model 2C	-8445.23	-8443.58	3.29	2	0.193
Model 3A	Model 3B	-8462.83	-8454.34	16.97	3	0.000
Model 3A	Model 3C	-8462.83	-8448.40	28.85	5	0.000
Model 3B	Model 3C	-8454.34	-8448.40	11.89	2	0.003
Model 4A	Model 4B	-8439.15	-8431.15	16.01	3	0.001
Model 4A	Model 4C	-8439.15	-8429.71	18.88	5	0.002
Model 4B	Model 4C	-8431.15	-8429.71	2.87	2	0.238
Model 5A	Model 5B	-8439.58	-8422.12	34.93	3	0.000
Model 5A	Model 5C	-8439.58	-8418.30	42.57	5	0.000
Model 5B	Model 5C	-8422.12	-8418.30	7.65	2	0.022

absolute value. The significant parameters in the matrix **G** all have the same sign (positive), which implies that conditional covariances between France, Netherlands, Switzerland and the UK are higher after common negative shocks. Similarly, the significant parameters in the matrix **S** are all negative, which implies that conditional covariances between France, Canada and Germany increase after common large positive or negative shocks. France is the only country with both significant sign effects and significant absolute size effects in conditional variance and Japan is the only country for which asymmetric effects seem completely unimportant. This, of course, explains why the symmetric diagonal model is so strongly rejected by the asymmetric models.

In the non-diagonal model, 22 out of the 54 off-diagonal elements are significant and there are clearly volatility spillovers from larger countries to smaller countries, see Table XI.[17] The parameters at the main diagonal in **A** and **B** are highly significant and of similar (absolute) magnitude as in the preferred asymmetric model, with Switzerland as the interesting exception. For Switzerland, the diagonal element in the matrix **B** is clearly insignificant and instead there are highly significant volatility spillovers from Germany to Switzerland. Another anticipated spillover is from the US to Canada, while the strong influence of Japan on France, Canada and

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[17]To save some space, estimates of the 45 parameters in the intercept matrix **C** are not reported for this model.

Netherlands is perhaps somewhat more surprising. There is also a significant impact of the UK on all of the smaller European markets.

<div align="center">

Table X:
QML estimates of Model 4B.

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$$r_{it} = \delta_{t-1}h_{iMt} + \varepsilon_{it} \; ; \; \varepsilon_{it}|\mathbf{\Omega}_{t-1} \sim N(0, h_{iit})$$

$$\delta_{t-1} = \delta_0 + \delta_1(R_{M,t-1} - R_{f,t-1}) + \delta_2 TIME_{t-1} + \delta_3 DEF_{t-1}$$

$$\mathbf{H}_t = \mathbf{C}'\mathbf{C} + \mathbf{A}'\varepsilon_{t-1}\varepsilon'_{t-1}\mathbf{A} + \mathbf{B}'\mathbf{H}_{t-1}\mathbf{B} + \mathbf{G}'\xi_{t-1}\xi'_{t-1}\mathbf{G} + \mathbf{S}'\eta_{t-1}\eta'_{t-1}\mathbf{S}$$

$$\xi_{it} = \varepsilon_{it}I_{\xi it} \text{ where } I_{\xi it} = 1 \Leftrightarrow \varepsilon_{it} < 0 \text{ otherwise } I_{\xi it} = 0$$

$$\eta_{it} = \varepsilon_{it}I_{\eta it} \text{ where } I_{\eta it} = 1 \Leftrightarrow |\varepsilon_{it}| > \sqrt{h_{iit}} \text{ otherwise } I_{\eta it} = 0$$

δ_0	δ_1	δ_2	δ_3
−10.965** [0.005]	−0.027 [0.901]	0.117 [0.904]	6.731** [0.000]

	$c_{.1}$	$c_{.2}$	$c_{.3}$	$c_{.4}$	$c_{.5}$	$c_{.6}$	$c_{.7}$	$c_{.8}$	$c_{.9}$
$c_{1.}$	0.007 [>0.999]								
$c_{2.}$	0.026 [>0.999]	−0.015 [>0.999]							
$c_{3.}$	−0.654 [0.910]	0.589 [0.922]	−0.095 [0.916]						
$c_{4.}$	0.073 [0.731]	0.246 [0.206]	0.029 [0.833]	−0.992** [0.000]					
$c_{5.}$	0.175 [0.519]	0.488* [0.028]	−0.124 [0.573]	−0.090 [0.755]	−0.609** [0.003]				
$c_{6.}$	−0.280 [0.082]	−0.099 [0.464]	−0.337* [0.023]	−0.481* [0.028]	−0.465** [0.005]	0.658** [0.000]			
$c_{7.}$	−0.376* [0.012]	−0.258* [0.035]	−0.256** [0.005]	−0.386** [0.007]	−0.516** [0.000]	−0.572** [0.000]	0.670** [0.000]		
$c_{8.}$	−0.360** [0.008]	0.249* [0.044]	−0.271** [0.002]	−0.390** [0.008]	−0.450** [0.001]	−0.310* [0.030]	−0.559** [0.000]	0.410** [0.000]	
$c_{9.}$	1.018** [0.000]	1.200** [0.000]	0.938** [0.000]	1.243** [0.000]	0.893** [0.000]	1.201** [0.000]	1.044** [0.000]	0.986** [0.000]	1.039** [0.000]

a_{FRA}	a_{CAN}	a_{NRL}	a_{SWI}	a_{GER}	a_{UK}	a_{JPN}	a_{US}	a_{WLD}
0.207** [0.000]	0.171** [0.000]	0.113** [0.000]	0.153** [0.000]	0.185** [0.000]	0.198** [0.000]	0.259** [0.000]	0.265** [0.000]	0.243** [0.000]
b_{FRA}	b_{CAN}	b_{NRL}	b_{SWI}	b_{GER}	b_{UK}	b_{JPN}	b_{US}	b_{WLD}
0.936** [0.000]	0.941** [0.000]	0.963** [0.000]	0.930** [0.000]	0.950** [0.000]	0.929** [0.000]	0.941** [0.000]	0.940** [0.000]	0.939** [0.000]
g_{FRA}	g_{CAN}	g_{NRL}	g_{SWI}	g_{GER}	g_{UK}	g_{JPN}	g_{US}	g_{WLD}
0.181* [0.017]	0.069 [0.134]	0.165** [0.000]	0.127* [0.018]	0.057 [0.325]	0.251** [0.000]	0.038 [0.404]	0.064 [0.077]	0.084** [0.005]
s_{FRA}	s_{CAN}	s_{NRL}	s_{SWI}	s_{GER}	s_{UK}	s_{JPN}	s_{US}	s_{WLD}
−0.174** [0.000]	−0.130** [0.000]	0.040 [0.192]	−0.045 [0.247]	−0.125** [0.000]	0.070 [0.103]	−0.016 [0.686]	−0.016 [0.542]	−0.022 [0.277]

Notes: The δ-parameters reported in the table are multiplied by 100.
** and * denote parameters statistically significant at the 1% and 5% levels, respectively.
Robust p-values are in parentheses.

Finally, there are significant influences from the world market on both most of the smaller countries and on the US, Japan and the UK. We could also note that there are no significant volatility spillovers to the German market, not even from the world market. Taken together, our evidence suggests that volatility on smaller markets are sensitive to news on the larger markets, while spillovers to larger markets are restricted to influences from the world market.[18]

A graph of the time-varying price of world covariance risk for the preferred asymmetric model (Model 4B) and for the preferred non-diagonal model (Model 5C), can be found in Figure 1. As can be seen, the two additional information variables in the second model clearly increase the volatility of the price of covariance risk. Another difference that can be observed is that during the period 1994-1997, the price of risk for the asymmetric model is about zero average, while it is clearly positive on average for the non-diagonal specification. Otherwise, the two graphs show very similar dynamics and the correlation between the two different prices of risk is a high 0.817. Both are highly significant on average and the mean of the time-varying price of risk for the asymmetric model is equal to 2.310 and for the non-diagonal model the mean is equal to 3.281, both with Newey-West corrected p-values of less than 0.001.

Conditional correlations between the US market and the world market are graphed in Figure 2. Conditional correlations vary over time and ranges from 0.709 to 0.954 for the asymmetric model (Model 4B) and from 0.547 to 0.940 for the non-diagonal model (Model 5C). The correlation between the two correlation time-series is a high 0.949, but there is a slight tendency for the non-diagonal model to produce lower correlations during the second half of the sample, especially 1990-1993. The most interesting pattern found is that there is a clear upward trend in correlations from 1993 and onwards. However, the increased correlations are not higher than in the 1970s and the first half of the 1980s. The lowest average correlations for both models can be found between 1985 and 1993. Interestingly, it can be seen that there are sudden increases in conditional correlation following the stock market crash in 1987 and, to less extent, the Asian crisis in 1998. This observations support the arguments in the literature, see e.g. Lin, Engle and Ito (1994), that US market declines are contagious internationally. The higher correlations imply less diversification benefits for US investors during these periods. However, the expressions for

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[18] The latter result is probably, at least to some extent, an effect of the high correlation between the US market and the world market.

Table XI:
QML estimates of Model 5C.

$$r_{it} = \delta_{t-1}h_{iMt} + \varepsilon_{it} \; ; \; \varepsilon_{it}|\mathbf{\Omega}_{t-1} \sim N\left(0, h_{iit}\right)$$

$$\delta_{t-1} = \delta_0 + \delta_1(R_{M,t-1} - R_{f,t-1}) + \delta_2 TIME_{t-1} + \delta_3 DEF_{t-1} + \delta_4 INFL_{t-1} + \delta_5 IP_{t-1}$$

$$\mathbf{H}_t = \mathbf{C}'\mathbf{C} + \mathbf{A}'\varepsilon_{t-1}\varepsilon'_{t-1}\mathbf{A} + \mathbf{B}'\mathbf{H}_{t-1}\mathbf{B}$$

	δ_0	δ_1	δ_2	δ_3	δ_4	δ_5
	−6.058 [0.274]	−0.176 [0.598]	0.053 [0.964]	7.540** [0.002]	−12.325** [0.010]	−0.842 [0.602]

	$a_{.FRA}$	$a_{.CAN}$	$a_{.NRL}$	$a_{.SWI}$	$a_{.GER}$	$a_{.UK}$	$a_{.JPN}$	$a_{.US}$	$a_{.WLD}$
$a_{FRA.}$	0.082** [0.009]								
$a_{CAN.}$		0.232** [0.000]							
$a_{NRL.}$			−0.084** [0.007]						
$a_{SWI.}$				−0.166* [0.016]					
$a_{GER.}$	0.077* [0.040]	−0.043 [0.181]	0.043 [0.195]	0.184** [0.006]	0.250** [0.000]				
$a_{UK.}$	−0.040 [0.278]	0.035 [0.265]	0.086** [0.000]	0.081 [0.125]		0.322** [0.000]			
$a_{JPN.}$	−0.150** [0.001]	0.071 [0.154]	−0.084* [0.033]	0.012 [0.904]			0.400** [0.000]		
$a_{US.}$	−0.121 [0.271]	0.205* [0.039]	−0.033 [0.674]	0.229 [0.306]	0.128 [0.105]	0.033 [0.646]	0.034 [0.631]	0.305** [0.000]	
$a_{WLD.}$	0.387* [0.035]	−0.339* [0.047]	0.161 [0.239]	−0.220 [0.579]	−0.150 [0.199]	−0.198* [0.032]	−0.317** [0.001]	−0.128* [0.017]	0.163** [0.000]

	$b_{.FRA}$	$b_{.CAN}$	$b_{.NRL}$	$b_{.SWI}$	$b_{.GER}$	$b_{.UK}$	$b_{.JPN}$	$b_{.US}$	$b_{.WLD}$
$b_{FRA.}$	0.984** [0.000]								
$b_{CAN.}$		0.925** [0.000]							
$b_{NRL.}$			0.954** [0.000]						
$b_{SWI.}$				−0.056 [0.860]					
$b_{GER.}$	−0.019 [0.374]	0.033 [0.184]	0.024 [0.321]	0.412** [0.008]	0.914** [0.000]				
$b_{UK.}$	0.041* [0.019]	−0.009 [0.408]	−0.010 [0.233]	0.156* [0.029]		0.924** [0.000]			
$b_{JPN.}$	0.071** [0.000]	−0.048* [0.045]	0.054* [0.012]	0.026 [0.791]			0.901** [0.000]		
$b_{US.}$	0.083* [0.038]	−0.097* [0.018]	0.062 [0.093]	−0.012 [0.945]	−0.016 [0.581]	−0.023 [0.325]	−0.030 [0.361]	0.944** [0.000]	
$b_{WLD.}$	−0.244** [0.000]	0.138 [0.112]	−0.167* [0.018]	0.294 [0.379]	0.027 [0.586]	0.019 [0.683]	0.137* [0.028]	0.006 [0.848]	0.952** [0.000]

Notes: The δ-parameters reported in the table are multiplied by 100.
** and * denote parameters statistically significant at the 1% and 5% levels, respectively.
Robust p-values are in parentheses. Estimates of the intercept matrix $\mathbf{C}$ not reported.

the expected gain (11) and (13), show that this conclusion may be premature because expected diversification benefits also depends on the amount of non-diversifiable risk and the price of world covariance risk.

The longer periods with clearly negative prices of risk are for both specifications associated with the oil crisis in 1973-1974 and the monetary experiment 1979-1982. It is argued in Boudoukh, Richardson and Smith (1993) that these are periods with high expected inflation and a downward sloping yield curve associated with negative US risk premia. De Santis and Gérard (1997) find a corresponding pattern for international stock markets which is similar to our findings.

5.2   Asset pricing tests

We test our preferred model against three relatively standard alternative specifications based on the asset pricing model restrictions. The first alternative specification is a partial segmentation model, where the expected return is allowed to depend on a country specific constant and the country specific (idiosyncratic) conditional variance:

$$R_{it} - R_{ft} = \alpha_i + \delta_{t-1} h_{iMt} + \gamma_i h_{iit} + \varepsilon_{it}. \tag{24}$$

The CAPM implies that the country specific components are not priced, i.e. that $\alpha_i = 0$ and $\gamma_i = 0$ for all countries i. The second alternative specification is also a partial segmentation model, but where instead the price of covariance risk is allowed to vary across countries:

$$R_{it} - R_{ft} = \delta_{i,t-1} h_{iMt} + \varepsilon_{it}. \tag{25}$$

According to the CAPM, the price of covariance risk is equal across countries, i.e. $\delta_{i,t-1} = \delta_{t-1}$ for all countries i. The third alternative specification examines if the information variables used have explanatory power in the conditional mean equations:

$$R_{it} - R_{ft} = \lambda_{i0} + \lambda_{i1}(R_{M,t-1} - R_{f,t-1}) + \lambda_{i2} TIME_{t-1} + \lambda_{i3} DEF_{t-1} + \delta_{t-1} h_{iMt} + \varepsilon_{it}. \tag{26}$$

Under the hypothesis that the CAPM is correctly specified the conditional covariance risk alone explains conditional mean dynamics and none of the information variables should have any additional explanatory power.

The results from these tests are reported in Table XII. None of the asset pricing restrictions are rejected at conventional levels of significance. The p-value for the hypothesis that country specific components contribute to risk premia is 0.161 and the p-value for the hypothesis that the price of covariance risk is equal across countries is 0.373. Finally, the hypothesis that the information variables add explanatory power in the conditional mean equation is rejected with a p-value of 0.151. Our results from the asset pricing tests agree with the results in DeSantis and Gérard (1997) based on similar alternative asset pricing equations. The main difference is that they test a version of the third specification in which the λ-coefficients are restricted to be equal across countries. Using a different GMM based approach without explicitly modeling the dynamics of second moments, Harvey (1991) also finds support for the conditional CAPM at the international level. Taken together, we find strong empirical support for our specification of the conditional CAPM applied to international stock markets.

Table XII:
Asset pricing tests based on LR-statistics.

Null hypothesis	χ^2	dgf
Are the prices of risk of country specific components equal to zero?		
	22.65 [0.161]	17
Are the prices of covariance risk equal across countries?		
	33.96 [0.373]	32
Do the information variables have additional explanatory power?		
	44.70 [0.151]	36

Notes: p-values in parentheses. dgf = degrees of freedom.

5.3 World market diversification

To calculate expected benefits from world market diversification from the perspective of investors from the different countries, we reestimate the preferred asset pricing model (Model 4B) using excess returns expressed in the corresponding local currency. For example, to evaluate diversification gains for the Japanese investor, returns are calculated in excess of the Japanese risk-free rate and are expressed in Japanese YEN. The time-series averages of the expected gains from international diversification, calculated separately for the full sample 1970-1999 and for the sub-periods 1970-1984 and 1985-1999, can be found in Table XIII. The corresponding time-series

are plotted in Figures 2-9, together with conditional betas and conditional correlations with the world market.

The evidence clearly suggest that there are statistically significant ex ante benefits from world market diversification for investors for all countries. This conclusion is essentially unaffected by a split of the sample into the sub-periods 1970-1984 and 1985-1999. Further, there is only a slight tendency for the magnitude of diversification benefits to decrease over time in the sense that the confidence intervals for the time periods 1970-1984 and 1985-1999 are in general overlapping. One partial explanation for this result is that there is no apparent tendency for conditional correlations with the world market to change over during the three decades the sample covers. In other words, we can find only weak support for the view that globalization and market liberalizations have decreased the expected benefits from international diversification over the last thirty years.

The average expected benefit for the US investor is 0.73% per year over the full sample, 0.55% per year for the period 1970-1984 and 0.91% per year for the period 1985-1999. The conditional correlation of the US stock market with the world market is very high, 0.880 on average, which according to equation (13) drives down diversification benefits for the US investor. We could also note that the US beta with the world market increases dramatically towards the end of the bear market in July-September 1974 (-32%) caused by a severe business cycle downturn and as a consequence of the stock market crash in October-November 1987 (-33%). There is however an interesting difference; in 1974 the beta increases because of a higher domestic volatility relative to the world market, not because of an increased conditional correlation, while in 1987 there is clearly an associated increase in conditional correlation.[19] The protection provided by international diversification is limited in both instances. The increased correlation after the 1987 crash is clearly a symptom of international contagion, but even if the bear market of 1974 does not increase the correlation with the world market, the correlation is still too high for international diversification to provide a good protection.

The corresponding average expected benefits for the Japanese investor are 0.98%, 1.68% and 0.28% per year, respectively. The numbers for the US are all statistically significant while

---

[19]By definition $\beta_{iMt} = \rho_{iMt}\sqrt{h_{iit}/h_{MMt}}$ and hence an increase in beta is a result of either an increased conditional correlation ρ_{iMt} or a higher home market conditional variance relative to the world market conditional variance h_{iit}/h_{MMt}, or both.

26

the average expected gain for the Japanese investor for the period 1985-1999 (0.28%) is not significantly different from zero. The relatively low diversification benefits for the Japanese investor is not primarily driven by a high world market correlation, 0.649 on average, but instead of a combination of a lower price of conditional covariance risk and an appreciating currency. The world price of conditional covariance risk is lower on average simply because excess returns are lower when recalculated in Japanese YEN, i.e. when returns are seen from the perspective of the Japanese investor. The dramatic decrease in both beta and correlation in 1974 is caused by the fact that in October 1974 the return on the Japanese market were -9%, while the US market simultaneously rebounded from the previous losses and offered an return of $+16\%$. This of course made it very profitable to invest outside Japan during this particular period.

Table XIII:
Average expected gains from international diversification (% per year).

At each time t the expected gain is calculated according to equation (10):

$$E_{t-1}\left[R_{It} - R_{it}\right] = \delta_{t-1}\left[\theta_{t-1}var\left(R_{Mt}|\mathbf{\Omega}_{t-1}\right) - cov\left(R_{it}, R_{Mt}|\mathbf{\Omega}_{t-1}\right)\right].$$

	1970:1-1999:12	1970:1-1984:12	1985:12-1999:12
FRA	4.46 [2.84,6.08]	5.12 [2.53,7.71]	3.79 [2.25,5.34]
CAN	2.57 [1.93,3.21]	2.69 [1.66,3.73]	2.44 [1.85,3.03]
NRL	2.66 [1.72,3.60]	2.93 [1.40,4.46]	2.39 [1.52,3.26]
SWI	9.60 [7.44,11.77]	10.10 [6.69,13.50]	9.11 [6.88,11.34]
GER	3.53 [2.22,4.83]	3.49 [1.56,5.43]	3.56 [2.07,5.05]
UK	5.66 [3.25,8.06]	6.93 [2.80,11.06]	4.38 [2.69,6.08]
JPN	0.98 [0.17,1.79]	1.68 [0.48,2.88]	0.28 [−0.59,1.16]
US	0.73 [0.44,1.01]	0.55 [0.16,0.94]	0.91 [0.56,1.26]

Note: Newey-West corrected 95% confidence intervals for the time-series means are in parentheses.

For investors from other countries, the ex ante diversification benefits are clearly much larger, both over the full sample and the two sub-periods. With the possible exception of UK, this is not too surprising given our previous discussion. For investors with smaller home markets it is not possible to invest in all industries or at least investment opportunities are more limited

and hence correlations with the world market should be lower. Even if this is certainly true for Germany and Switzerland, with average conditional correlations of only 0.529 and 0.247, respectively, when returns are recalculated in DM and Swiss franc, the average correlation for Netherlands and Canada are 0.753 and 0.672, respectively. Hence, both the large significant ex ante gains from international diversification for the German and Swiss investors, 3.53% and 9.60% per year, respectively, and the lower gains for the Dutch and Canadian investors, 2.66% and 2.57% per year, could partly be explained by the levels of average correlation with the world market. The average conditional correlation for the UK and France are moderately high and equal to 0.629 and 0.604, respectively. Still, the expected diversification gains are equal to 5.66% and 4.46% per year, respectively, and are higher than for all other countries (except Switzerland). For the UK, the high average diversification benefits is mostly a consequence of the political and economic turbulence in the mid-seventies. At this time, the UK suffered from deep economic problems leading to high interest rates, high inflation and high unemployment rates. In France, the socialist party won the elections in the early 1980s, and together with the communist party they pursued a political program that included nationalization of some large companies and in general could be seen as a strong statement against the free market economy. In France, the consequence was a dramatically decreased correlation with the world market, see Figure 2. In the UK, the country specific volatility increased dramatically relative to the world market volatility, see Figure 8. These episodes severely affected the stock markets and had no direct international counterpart and hence international diversification offered an extremely good protection against this home market turbulence. The average expected gain for the UK investor during the years 1974-1976 were 1.70% per month and 1.40% per month for the French investor during the years 1981-1982. Taken together, this evidence is broadly in line with our a priori intuition that diversification benefits should be larger for investors with smaller home markets, either because of lower correlations with the world market or as a consequence of a higher country specific volatility that is not contagious internationally. This above discussion underlines the lesson from equations (10), (11) and (13) that both correlations and the level of country specific risk in relation to world market risk should be taken into account when evaluating benefits from international diversification.

As long as investors are not able to predict bear markets, it is the average gains from international diversification over longer periods that should encourage investors to diversify,

regardless of potentially lower benefits over shorter periods of time. Given the above discussion, the evidence clearly suggest that investors from all countries should diversify internationally although expected benefits are sometimes small or even negative and certainly differs between investors from different countries.

6   Summary and Conclusions

This paper estimates and tests the conditional CAPM simultaneously for the eight largest national stock markets and the world market portfolio. The conditional nature of the estimated CAPM together with the fully specified first and second moments allow us to investigate ex ante gains from international diversification. The conditional second moments are modeled using five different multivariate GARCH specification; a diagonal symmetric model, three different diagonal asymmetric models and a non-diagonal symmetric model. The conditional first moment is defined as a deterministic linear function of up to six economic information variables. We derive an expression for the expected gain from diversification to the world market that highlights the importance of the amount of non-systematic risk, the conditional correlation and the price of world covariance risk. To calculate the expected benefit from international diversification implied by the conditional CAPM from the perspective of investors in each of the eight countries, we recalculate excess returns denominated in each of domestic currencies and reestimate the preferred model for each of the return series. To overcome the problems associated with the likelihood functions of multivariate GARCH models, we suggest the use simulated annealing. Simulated annealing is derivative-free global optimization algorithm in the Markov Chain Monte Carlo family, that by construction is able to escape from truly local maxima and, in addition, is in principle insensitive to the starting values chosen.

The previous asset-pricing literature using a similar approach often use a very small cross-section of assets or place strong parameter restrictions on the estimated multivariate GARCH process. We extend the current literature testing the conditional CAPM based on both asymmetric and non-diagonal conditional covariance specifications for a relatively large cross-section. We find that the previously often employed diagonal symmetric parametrization is easily rejected against all other specifications. The preferred model is an asymmetric specification with asymmetric effects both with respect to the sign and the absolute size of the shock. This evi-

dence clearly suggest that asymmetric effects should not be neglected when modeling variances and covariances of international stock returns with GARCH models. Interestingly, the diagonal symmetric models are also rejected against the non-diagonal specifications. This evidence imply that spillover effects in variances plays an important role understanding the second moments dynamics of international stock returns. The preferred asset pricing model could not be rejected against any of the alternative models and hence in line with DeSantis and Gerard (1997) and Harvey (1991), we find relatively strong support for the conditional CAPM applied at the country level.

It is in general argued that cross-country correlations have increased during the recent decades as result of market liberalizations and increased globalization in general. This should mean less diversification benefits from investing abroad. We find very limited support for this view. Instead, investors from all investigated countries could expect statistically significant benefits from world market diversification. This conclusion is in principle unaffected by a split of the sample into the sub-periods 1970-1984 and 1985-1999. Because investment opportunities are more limited on smaller stock markets one could expect lower correlations. Also, country specific events on smaller market are more likely not to be contagious internationally. This should mean larger gain from international diversification for smaller-country investors. Our evidence support these arguments. The smallest gains from world market diversification are for the US and Japan, on average 0.73% and 0.98% per year, respectively. The largest gains are found for Switzerland and the UK, 9.60% and 5.66% per year, respectively. For Switzerland this is mostly a consequence of a low correlation with the world market and for the UK it is because of a high level of idiosyncratic risk during certain episodes. These results highlight the importance of taking both correlation and the level of idiosyncratic risk into account when evaluating benefits from international diversification.

References

Adler M. and B. Dumas (1983): International Portfolio Choice and Corporation Finance: A Synthesis", *Journal of Finance*, 38, 925-984.

Bollerslev T. (1990): "Modeling the Coherence in Short-Run Nominal Exchange Rates: A Multivariate Generalized ARCH Model", *Review of Economics and Statistics*, 31, 498-505.

Bollerslev T. and J. Wooldridge (1992): "Quasi-Maximum Likelihood Estimation and Inference in Dynamic Models with Time-Varying Covariances, *Econometric Reviews*, 11, 143-172.

Boucrelle C., Solnik B. and L. Yann (1996): "International Market Correlation and Volatility", *Financial Analyst Journal*, 52, 17-34.

Carrieri F. (2001): "The Effects of Liberalisation on Market and Currency Risk in the European Union", *European Financial Management*, 7, 259-290.

Chan K., A. Karolyi and R. Stulz (1992): "Global Financial Markets and the Risk Premium on US Equity", *Journal of Financial Economics*, 32, 137-168.

Chang E., E. Eun and R. Kolodny (1995): "International Diversification through Closed-End Country Funds", *Journal of Banking and Finance*, 19, 1237-1263.

Cho Y. and R. Engle (1999): Time-Varying Betas and Asymmetric Effects of News: Empirical Analysis of Blue Chip Stocks", Working Paper 7330, NBER.

Christie A. (1982): "The Stochastic Behavior of Common Stock Variance: Value, Leverage and Interest Rate Effects", *Journal of Financial Economics*, 10, 407-432.

Corona A., Marchesi M., C. Martini and S. Ridella (1987): "Minimizing Multimodal Functions of Continuous Variables with the 'Simulated Annealing' Algorithm", *ACM Transactions on Mathematical Software*, 13, 262-280.

De Santis G. and B. Gérard (1997): "International Asset Pricing and Portfolio Diversification with Time-Varying Risk", *Journal of Finance*, 52, 1881-1912.

De Santis G. and B. Gérard (1998): "How Big is the Premium for Currency Risk", *Journal of Financial Economics*, 49, 375-412.

Ding Z. and R Engle (1994): "Large Scale Covariance Matrix Modeling, Estimation and Testing", Working Paper, University of California, San Diego.

Dumas B. (1994): "Partial Equilibrium versus General Equilibrium Models of the International Capital Market", in Van Der Ploeg F. (ed.): *The Handbook of International Macroeconomics*, Cambridge Ma. Basil Blackwell.

Engle R. and V. Ng (1993): "Measuring and Testing the Impact of News on Volatility", *Journal of Finance*, 48, 1749-1778.

Engle R. and K. Kroner (1995): "Multivariate Simultaneous Generalized ARCH", *Econometric Theory*, 11, 122-150.

Errunza V., K. Hogan and M. Hung (1999): "Can the Gains from International Diversification be Achieved Without Trading Abroad", *Journal of Finance*, 54, 2075-2107.

Fishman G. (1996): *Monte Carlo Concepts, Algorithms and Applications*, Springer-Verlag.

Glosten L., R. Jagannathan and D. Runkle (1993): "Relationship Between the Expected Value and the Volatility of the Nominal Excess Return on Stocks", *Journal of Finance*, 48, 1779-1801.

Goffe W., G. Ferrier and J. Rogers (1994): "Global Optimization of Statistical Functions with Simulated Annealing", *Journal of Econometrics*, 60, 65-99.

Gouriéroux C. (1997): *ARCH Models and Financial Applications*, Springer-Verlag.

Griffin J. and A. Karolyi (1998): "Another Look at the Role of the Industrial Structure of Markets for International Diversification Strategies", *Journal of Financial Economics*, 50, 352-373.

Grubel H. (1968): "Internationally Diversified Portfolios: Welfare Gains and Capital Flows", *American Economic Review*, 58, 1299-1314.

Harvey C. (1991): "The World Price of Covariance Risk", *Journal of Finance*, 46, 111-158.

Harvey C. and A. Siddique (2000): "Conditional Skewness in Asset Pricing Tests", *Journal of Finance*, 55, 1263-1295.

Kroner K. and V. Ng (1998): "Modeling Asymmetric Comovements of Asset Returns", *Review of Financial Studies*, 11, 817-844.

Levy H. and M. Sarnat (1970): "International Diversification for Investment Portfolios", *American Economic Review*, 60, 668-675.

Lin W., R. Engle and T. Ito (1994): "Do Bulls and Bears Move Across Borders? International Transmission of Stock Returns and Volatility", *Review of Financial Studies*, 7, 507-538.

Longin F. and B. Solnik (1995): "Is the Correlation in International Equity Returns Constant: 1970-1990?", *Journal of International Money and Finance*, 14, 3-26.

Markowitz H. (1952): "Portfolio Selection", *Journal of Finance*, 7, 77-91.

Ng L. (1991): "Tests of the CAPM with Time-Varying Covariances: A Multivariate GARCH Approach", *Journal of Finance*, 46, 1507-1521.

Sharpe W. (1964): "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk", *Journal of Finance*, 19, 425-442.

Solnik B. (1997): "Going Global with Equities", *Financial Analyst Journal*, 53, 95-96.

Solnik B. (1974): "Why Not Diversify Internationally?", *Financial Analyst Journal*, 30, 48-54.

Appendix A

In this appendix, we provide explicit illustrations of the different conditional covariance specifications employed. For clarity, we use a simple bivariate model. The elements of the unrestricted BEKK covariance matrix are in the 2×2 case

$$
\begin{aligned}
h_{11t} &= \left(c_{11}^2 + c_{12}^2\right) + a_{11}^2 \varepsilon_{1,t-1}^2 + 2a_{11}a_{21}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_{21}^2 \varepsilon_{2,t-1}^2 + \qquad (27) \\
&\quad b_{11}^2 h_{11,t-1} + 2b_{11}b_{21}h_{12,t-1} + b_{21}^2 h_{22,t-1} \\
h_{22t} &= c_{22}^2 + a_{12}^2 \varepsilon_{1,t-1}^2 + 2a_{12}a_{22}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_{22}^2 \varepsilon_{2,t-1}^2 + \qquad (28) \\
&\quad b_{12}^2 h_{11,t-1} + 2b_{12}b_{22}h_{12,t-1} + b_{22}^2 h_{22,t-1}
\end{aligned}
$$

and

$$
\begin{aligned}
h_{12t} &= c_{12}c_{22} + a_{11}a_{12}\varepsilon_{1,t-1}^2 + a_{12}a_{21}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_{11}a_{22}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_{21}a_{22}\varepsilon_{2,t-1}^2 + (29) \\
&\quad b_{11}b_{12}h_{11,t-1} + b_{12}b_{21}h_{12,t-1} + b_{11}b_{22}h_{12,t-1} + b_{21}b_{22}h_{22,t-1}
\end{aligned}
$$

and an identical element for $h_{21,t}$. The diagonality and symmetry restriction, $a_{12} = a_{21} = 0$ and $b_{12} = b_{21} = 0$, implies

$$
\begin{aligned}
h_{11t} &= \left(c_{11}^2 + c_{12}^2\right) + a_{11}^2 \varepsilon_{1,t-1}^2 + b_{11}^2 h_{11,t-1} \qquad &(30) \\
h_{22t} &= c_{22}^2 + a_{22}^2 \varepsilon_{2,t-1}^2 + b_{22}^2 h_{22,t-1} \qquad &(31)
\end{aligned}
$$

and

$$
h_{12t} = c_{12}c_{22} + a_{11}a_{22}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + b_{11}b_{22}h_{12,t-1}. \qquad (32)
$$

As can be seen, the diagonality assumption implies that there are no spillover effects between variance processes of different countries.

Our non-diagonal alternative implies that if country 1 is the smaller country, we assume that $a_{12} = 0$, but allow for $a_{21} \neq 0$, and assume that $b_{12} = 0$, but allow for $b_{21} \neq 0$. Then, our model of conditional covariance is in the 2×2 case

$$
\begin{aligned}
h_{11t} &= \left(c_{11}^2 + c_{12}^2\right) + a_{11}^2 \varepsilon_{1,t-1}^2 + 2a_{11}a_{21}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_{21}^2 \varepsilon_{2,t-1}^2 + && (33) \\
&\quad b_{11}^2 h_{11,t-1} + 2b_{11}b_{21}h_{12,t-1} + b_{21}^2 h_{22,t-1} \\
h_{22t} &= c_{22}^2 + a_{22}^2 \varepsilon_{2,t-1}^2 + b_{22}^2 h_{22,t-1} && (34)
\end{aligned}
$$

and

$$
h_{12t} = c_{12}c_{22} + a_{11}a_{22}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_{21}a_{22}\varepsilon_{2,t-1}^2 + b_{11}b_{22}h_{12,t-1} + b_{21}b_{22}h_{22,t-1} \qquad (35)
$$

and we see that $\varepsilon_{2,t-1}$ and $h_{22,t-1}$ potentially do affect h_{11t} but also that $\varepsilon_{1,t-1}$ and $h_{11,t-1}$ do not affect h_{22t}.

Turning to the asymmetric diagonal specifications, the BEKK conditional covariance matrix augmented with asymmetric sign and asymmetric absolute size effects is in the 2×2 case

$$
\begin{aligned}
h_{11t} &= \left(c_{11}^2 + c_{12}^2\right) + a_{11}^2 \varepsilon_{1,t-1}^2 + b_{11}^2 h_{11,t-1} + g_{11}^2 \xi_{1,t-1}^2 + s_{11}^2 \eta_{1,t-1}^2 && (36) \\
h_{22t} &= c_{22}^2 + a_{22}^2 \varepsilon_{2,t-1}^2 + b_{22}^2 h_{22,t-1} + g_{22}^2 \xi_{2,t-1}^2 + s_{22}^2 \eta_{2,t-1}^2 && (37)
\end{aligned}
$$

and

$$
h_{12t} = c_{12}c_{22} + a_{11}a_{22}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + b_{11}b_{22}h_{12,t-1} + g_{11}g_{22}\xi_{1,t-1}\xi_{2,t-1} + s_{11}s_{22}\eta_{1,t-1}\eta_{2,t-1}. \qquad (38)
$$

Finally, the models that allow for only one asymmetric effect are obviously restricted versions of the above model, i.e. either $s_{11} = s_{22} = 0$ or $g_{11} = g_{22} = 0$.
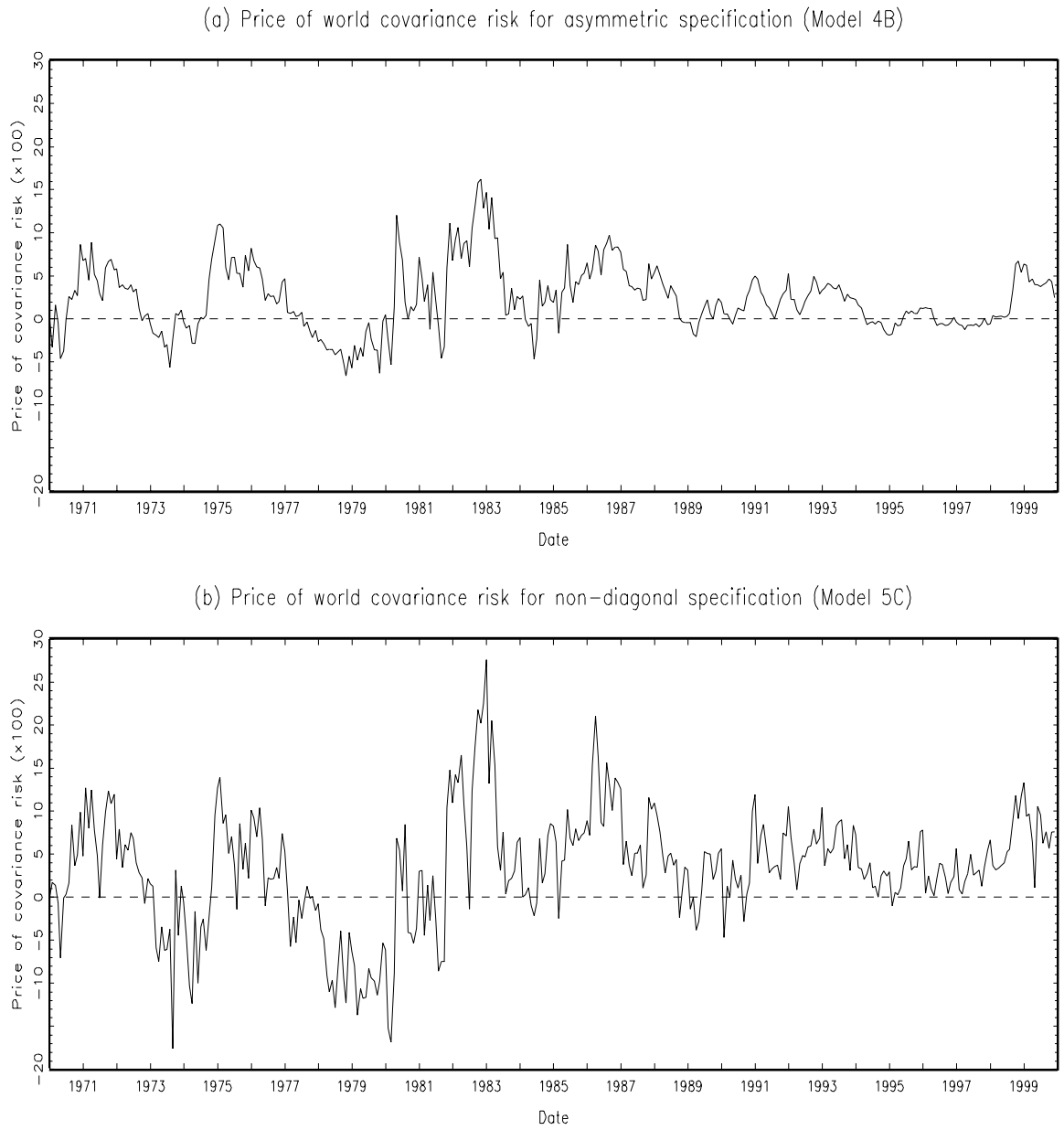
(a) Price of world covariance risk for asymmetric specification (Model 4B)

(b) Price of world covariance risk for non-diagonal specification (Model 5C)

Figure 1: (a) Price of world covariance risk for asymmetric moldel (Model 4B). (b) Price of world covariance risk for non-diagonal model (Model 5C). Excess returns are US-dollar denominated.
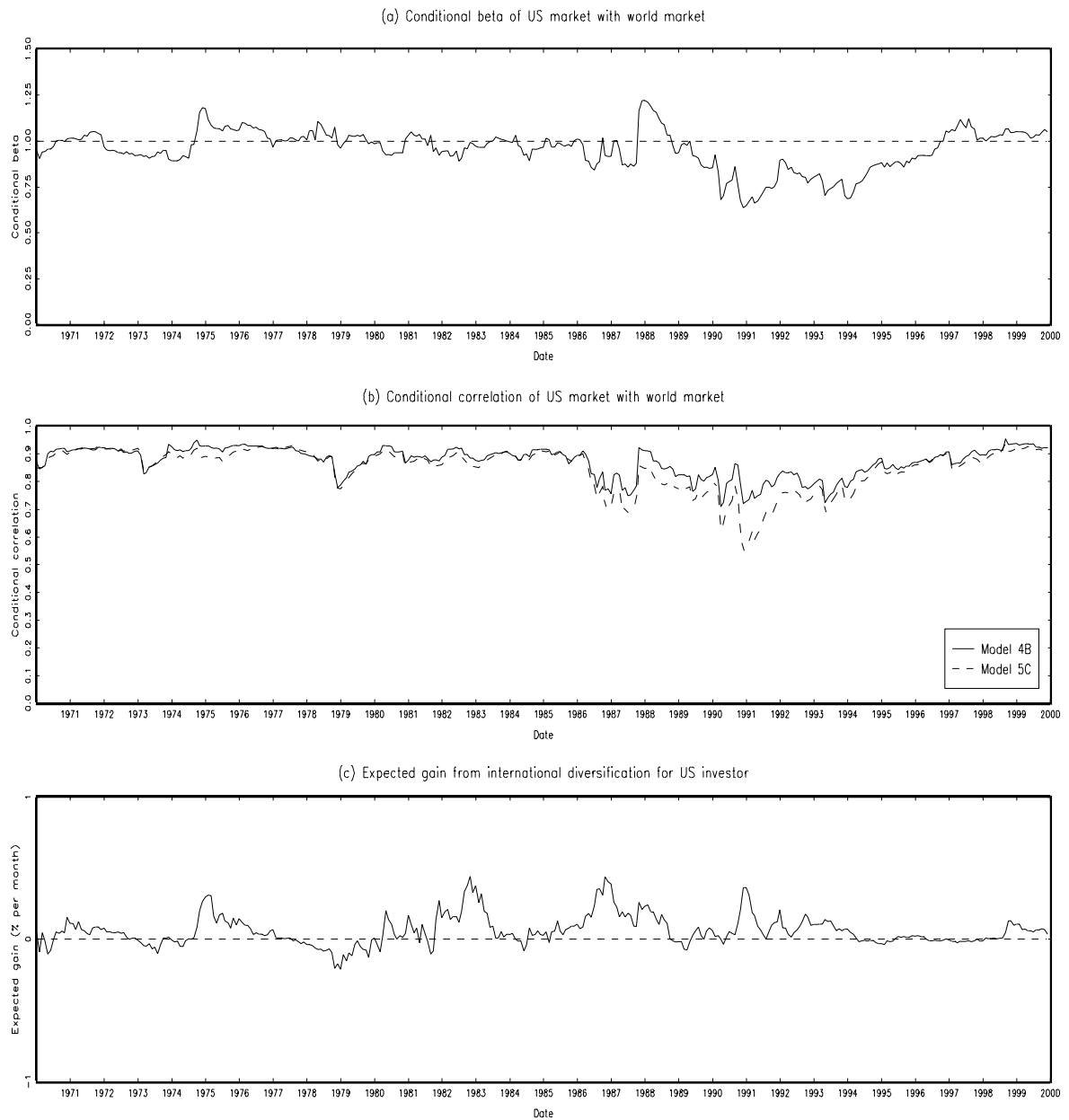
36

(a) Conditional beta of US market with world market

(b) Conditional correlation of US market with world market

(c) Expected gain from international diversification for US investor

Figure 2: (a) Conditional beta of US market with world market. (b) Conditional correlation of US market with world market. (c) Expected gain from international diversification for US investor (% per month).
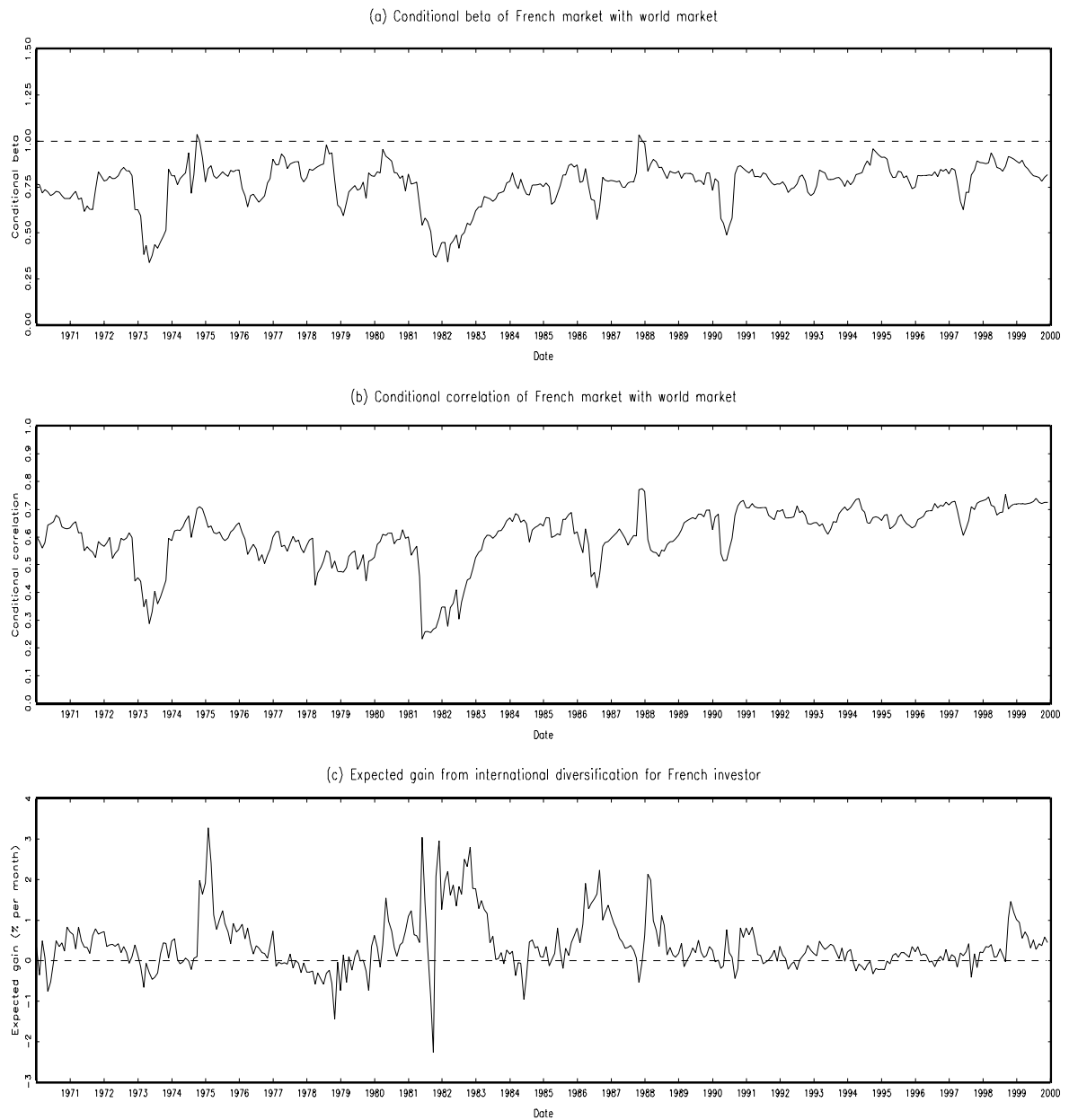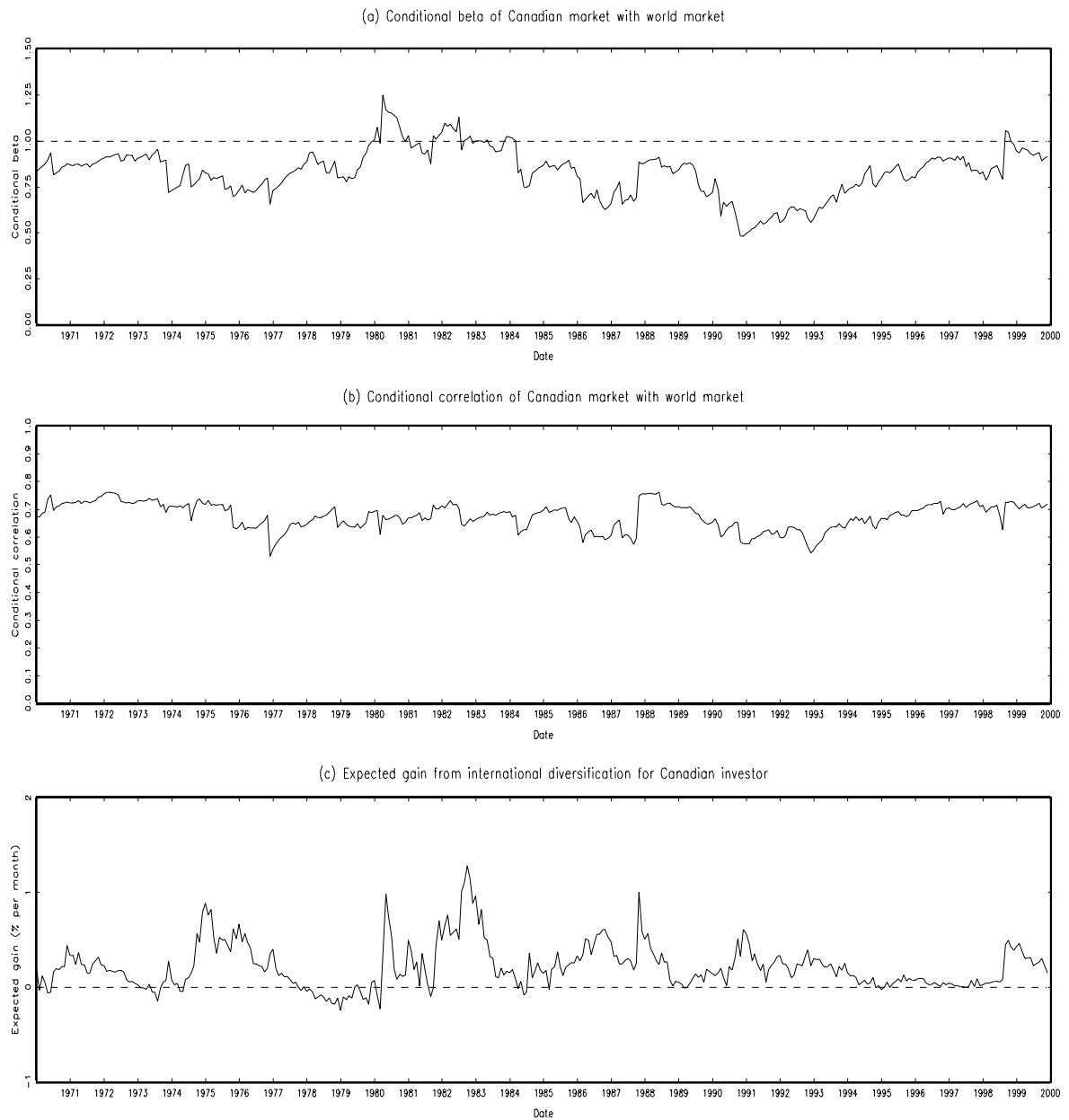
Figure 3: (a) Conditional beta of French market with world market. (b) Conditional correlation of French market with world market. (c) Expected gain from international diversification for French investor (% per month).

Figure 4: (a) Conditional beta of Canadian market with world market. (b) Conditional correlation of Canadian market with world market. (c) Expected gain from international diversification for Canadian investor (% per month).
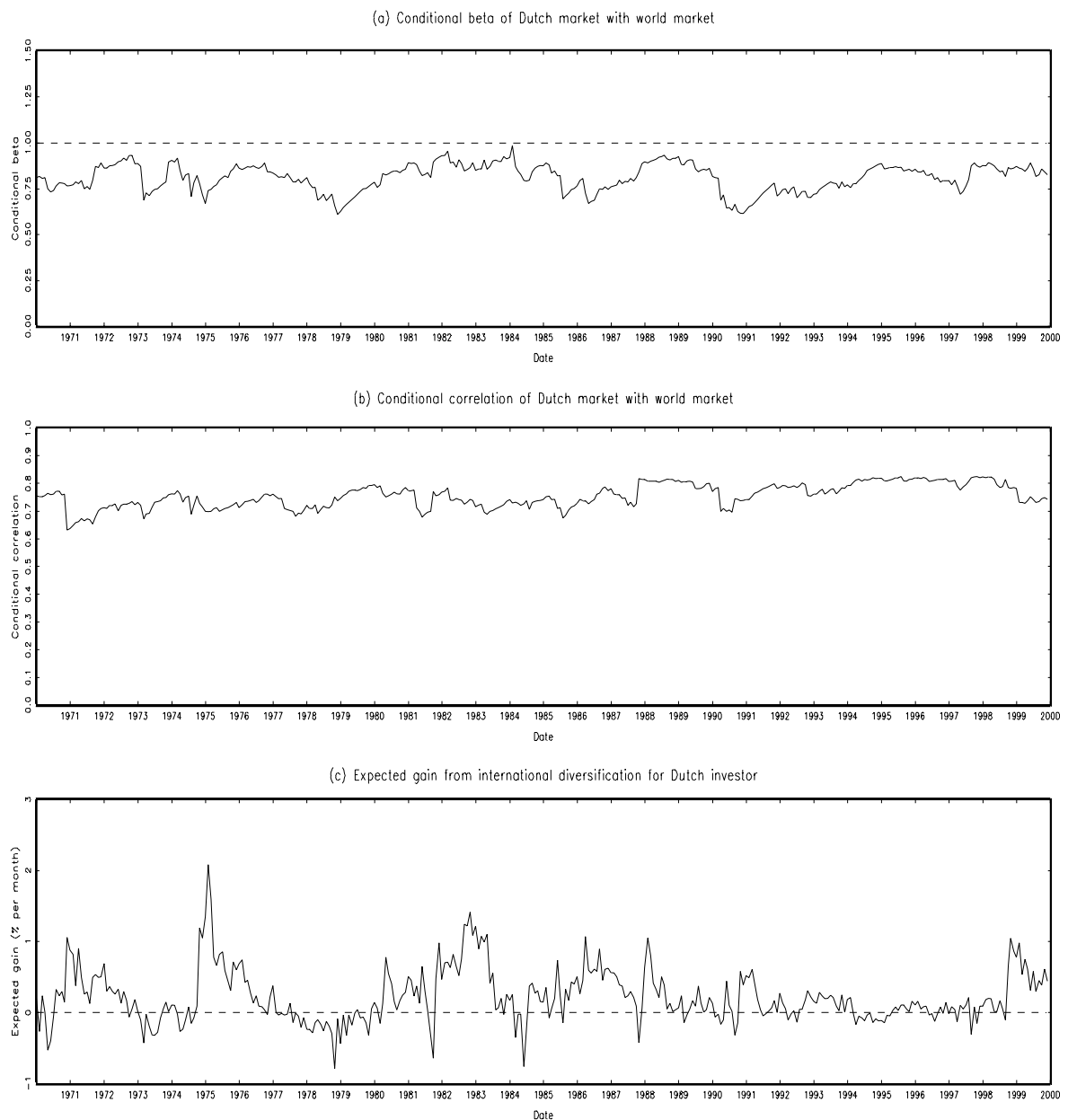
Figure 5: (a) Conditional beta of Dutch market with world market. (b) Conditional correlation of Dutch market with world market. (c) Expected gain from international diversification for Dutch investor (% per month).
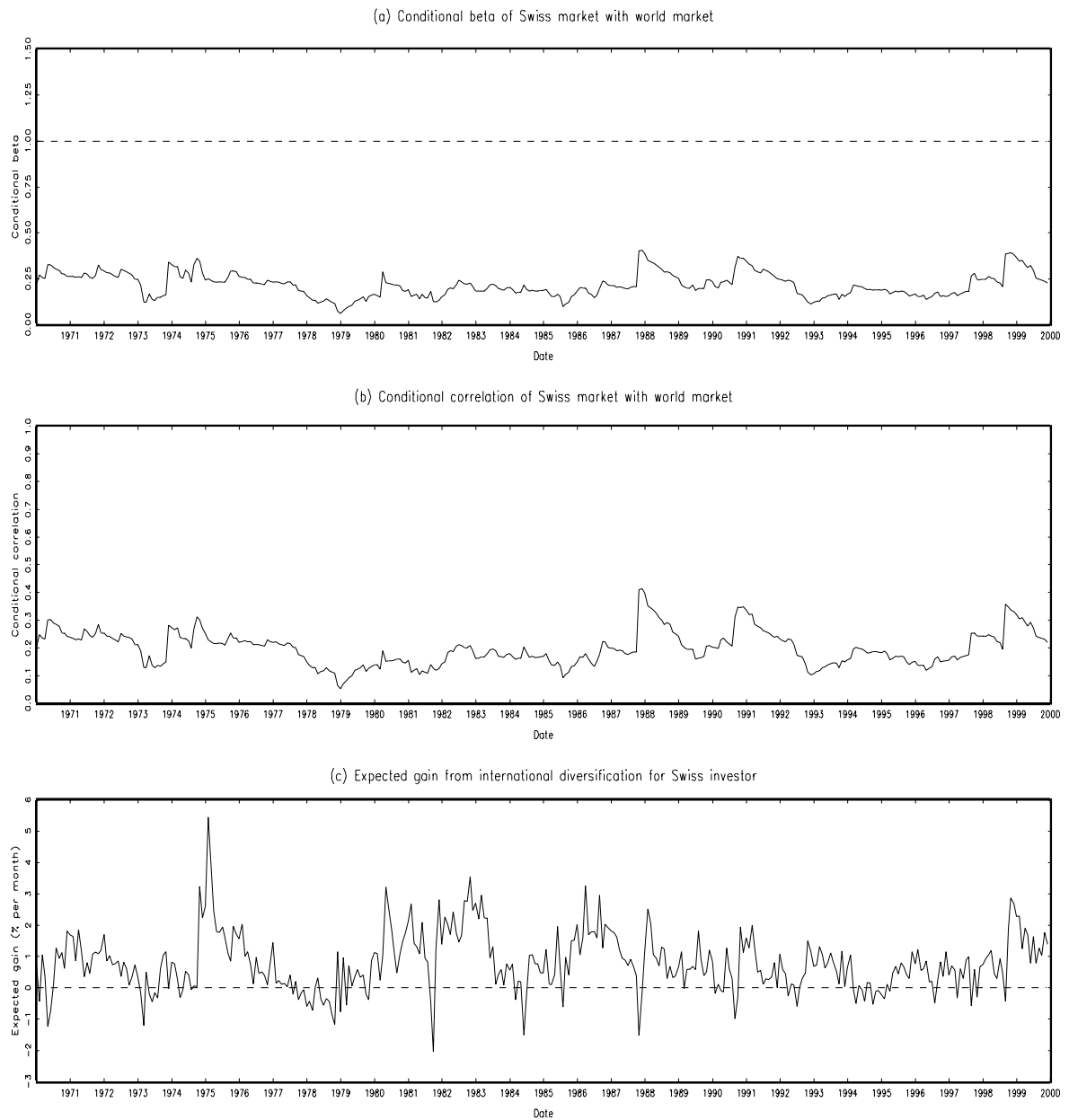
40

Figure 6: (a) Conditional beta of Swiss market with world market. (b) Conditional correlation of Swiss market with world market. (c) Expected gain from international diversification for Swiss investor (% per month).
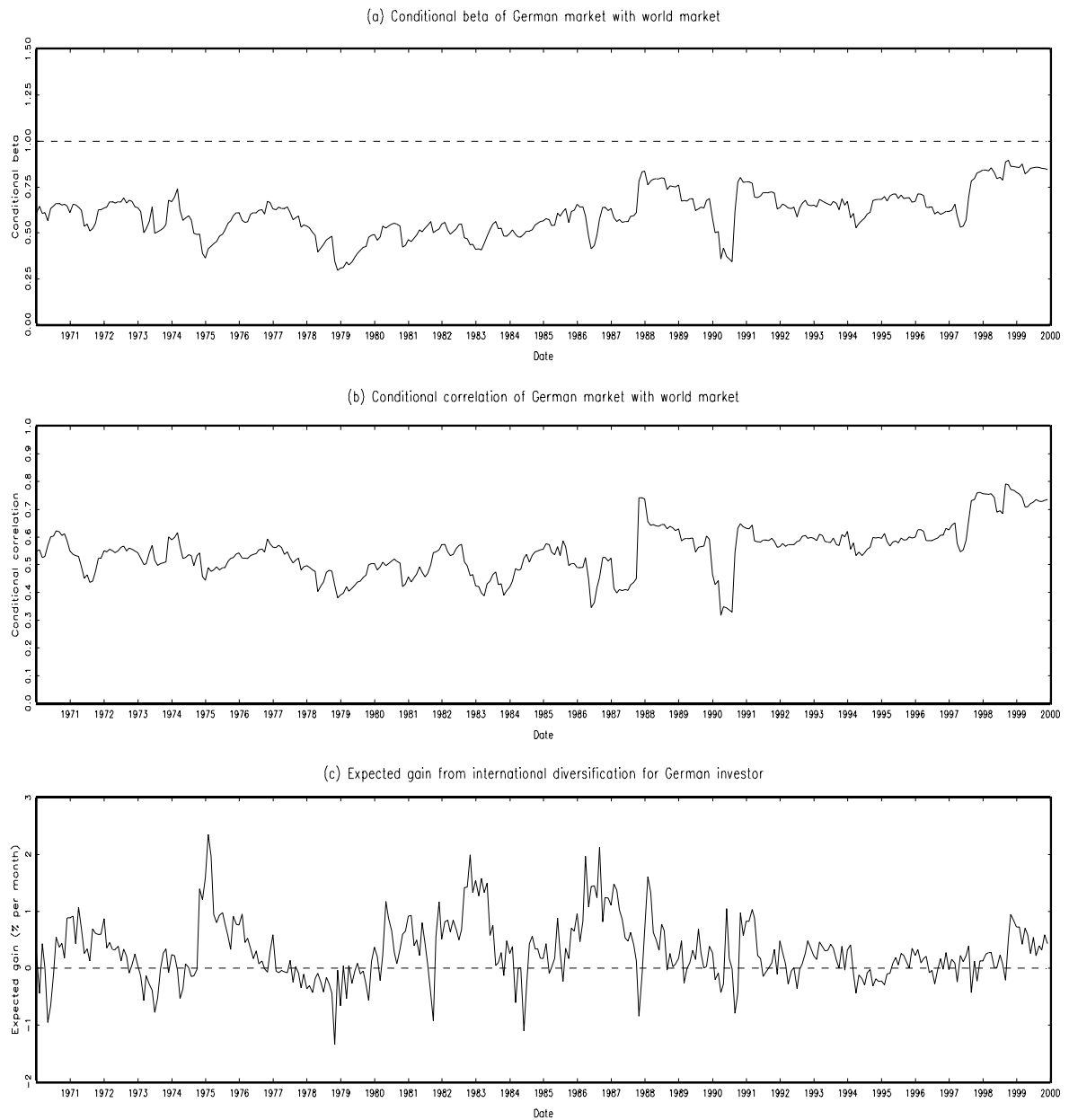
41

(a) Conditional beta of German market with world market

(b) Conditional correlation of German market with world market

(c) Expected gain from international diversification for German investor

Figure 7: (a) Conditional beta of German market with world market. (b) Conditional correlation of German market with world market. (c) Expected gain from international diversification for German investor (% per month).
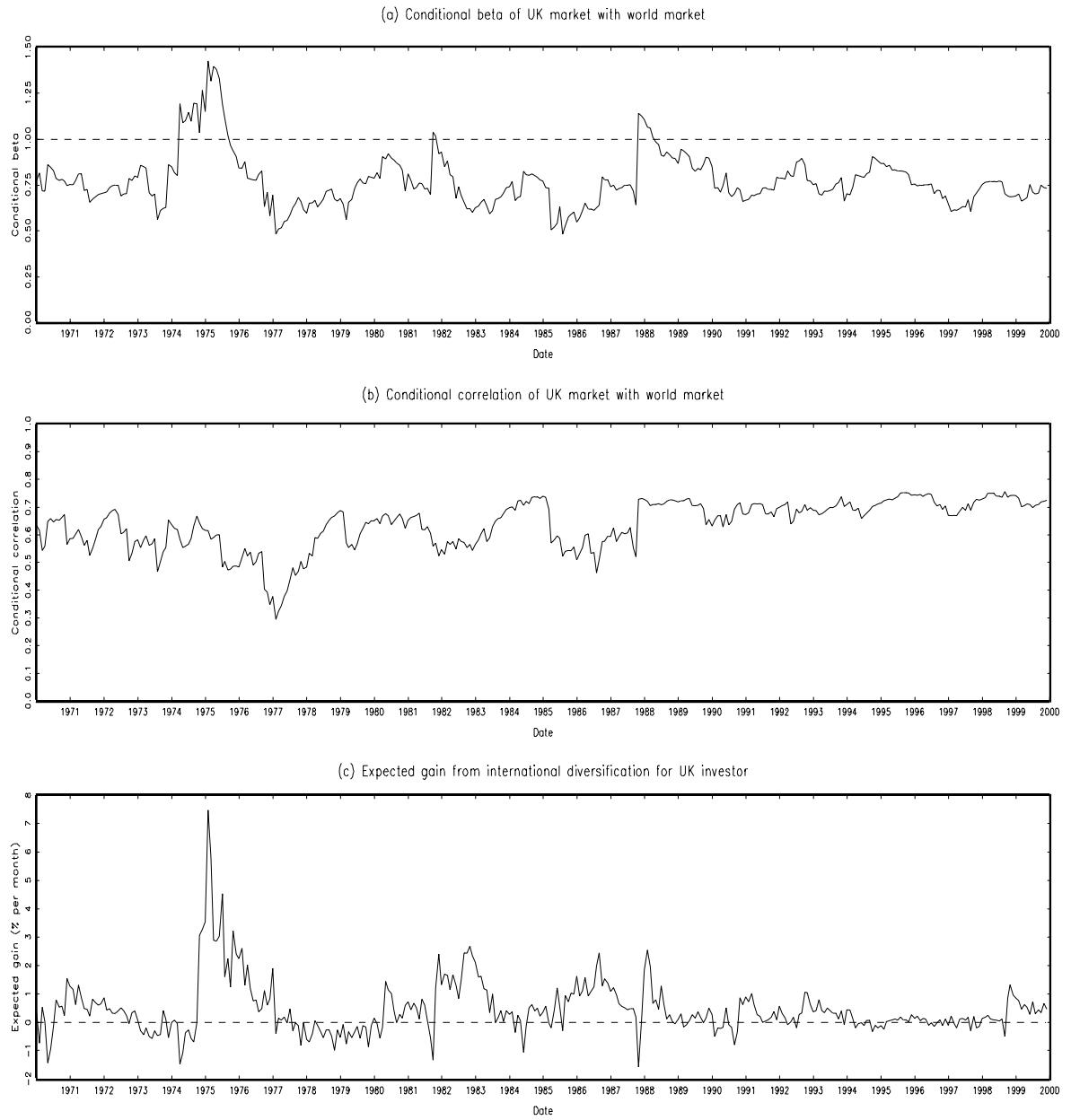
Figure 8: (a) Conditional beta of UK market with world market. (b) Conditional correlation of UK market with world market. (c) Expected gain from international diversification for UK investor (% per month).
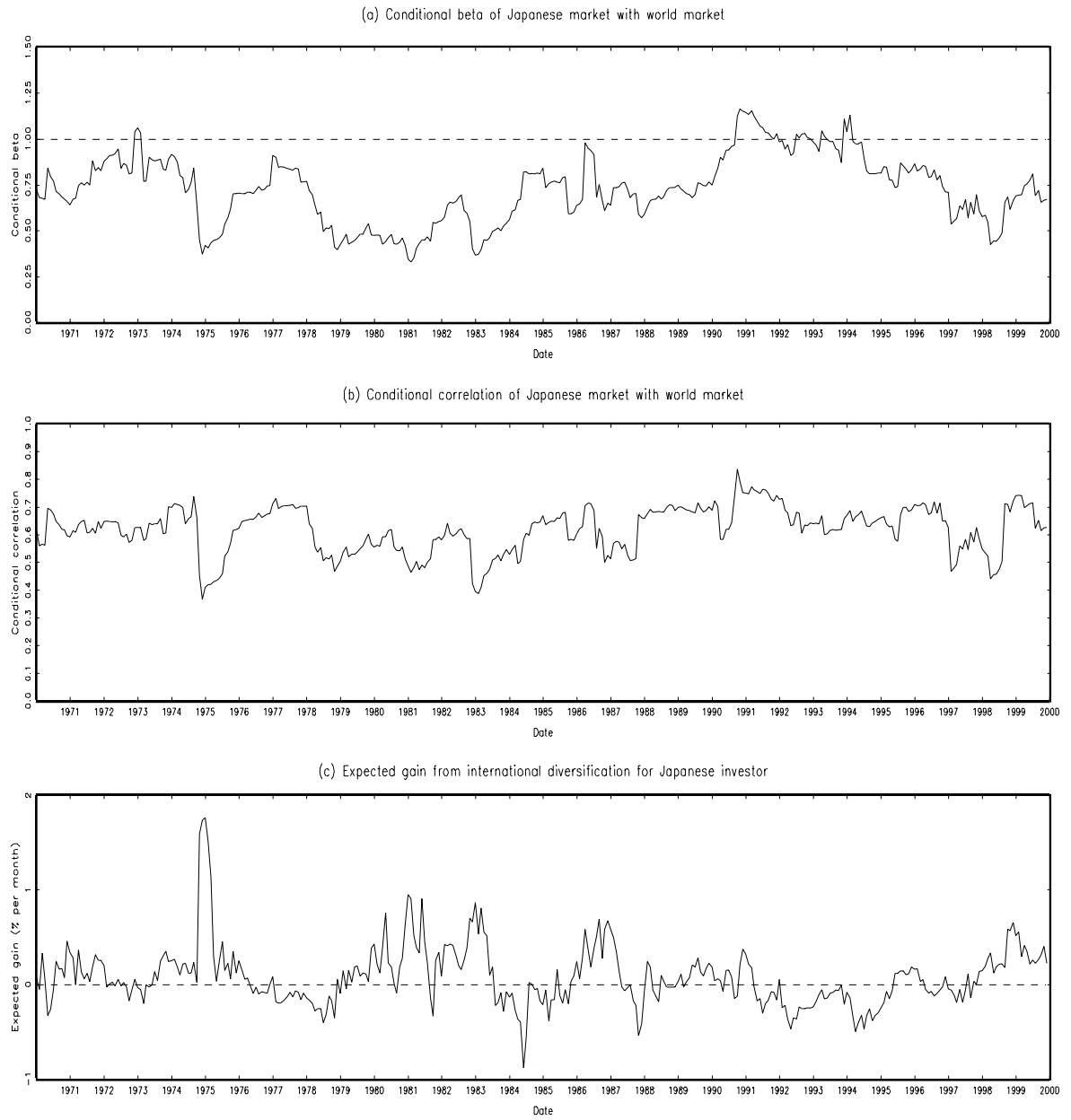
Figure 9: (a) Conditional beta of Japanese market with world market. (b) Conditional correlation of Japanese market with world market. (c) Expected gain from international diversification for Japanese investor (% per month).