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# FASTER-THAN-NYQUIST MODULATION BASED ON SHORT FINITE PULSES

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## ABSTRACT

We investigate faster-than-Nyquist modulation based on short finite pulses over the AWGN channel. We consider several pulse shapes and compare their information rates for several system setups. We compare the effect of increasing the alphabet size versus of increasing the signaling rate. The outcome is that for these pulses the FTN symbol rate is of greater importance than the alphabet size. Finally we test some concatenated coding schemes where faster-than-Nyquist modulation constitutes the innermost encoder; the outcome is very good.

Key Words: *Coded modulation, Faster Than Nyquist, Concatenated coding, Information rates, Intersymbol interference*

## 1. INTRODUCTION

Consider the modulation problem: Constructing a real valued continuous time signal that carries a discrete-time symbol sequence. Many techniques, both linear and nonlinear, are available in the literature. Examples of nonlinear techniques are frequency shift keying and continuous phase modulation. In this paper we consider the most common, single carrier linear modulation. If the symbol sequence to be transmitted is denoted  $\mathbf{a}$  the transmitted signal equals

$$s(t) = \sum_{i=-\infty}^{\infty} a_i h(t - iT). \quad (1)$$

In (1)  $T$  denotes the symbol interval and  $h(t)$  can be any real-valued continuous pulse. To greatly simplify the decoding it is common to take  $h(t)$  as a  $T$ -orthogonal pulse; that is  $\int h(t)h(t-nT)dt = \delta_n$  where  $\delta_n$  is Dirac's delta function and integration limits are  $(-\infty, \infty)$ .

Mazo [1] pointed out that if the symbol interval is decreased from  $T$  to  $\tau T$ ,  $\tau < 1$ , the signal minimum Euclidean distance can still be preserved. Thus, the asymptotic error performance is unaffected. With  $h(t)$  taken as the sinc pulse, the minimum distance is preserved for  $\tau > .802$ . The transmitted signal now equals

$$s(t) = \sum_{i=-\infty}^{\infty} a_i h(t - i\tau T). \quad (2)$$

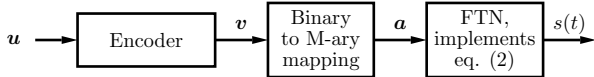
Since the signaling rate is faster-than-Nyquist (FTN), intersymbol interference (ISI) in the receiver cannot be avoided. The earliest papers on FTN signaling [1] – [3] considered only its minimum Euclidean distance. More recently other pulse shapes than the sinc were investigated in [4], as well as some efficient receivers. The possible information rates of FTN were investigated in [5]. The outcome is that FTN has significant potential to improve transmission, both in theory and in practice.

The target of this paper is to investigate nonbinary alphabet sizes for FTN and to investigate the performance of short pulses. We will, however, conclude that the best alphabet is often the smallest possible, a binary one. One's criterion for "best" can change the outcome; if decoding complexity is considered, binary alphabets are not suitable.

Some words on wireless applications are needed. A wireless channel is not constant over time. But if the channel is slowly time-varying, it can be modelled as being constant over a short time duration  $\approx t_{\text{coh}}$  seconds. In order to avoid inter block interference it is then necessary to transmit signals that are limited to time duration  $t_{\text{coh}}$  seconds. Another example is multiuser systems based on timesharing of the channel; each user must only transmit a signal of at most  $t_{\text{coh}}$  seconds, otherwise the signals from different users will interfere at the receiver.

To achieve Shannon's capacity formula,  $W \log(1 + P/N_0W)$  it is necessary, for single carrier systems, to use a sinc pulse. This pulse is however in theory of infinite duration. In practice truncated smoother pulses, such as spectral root raised cosine pulses are used. These still require significant duration, and in order to avoid inter block interference, a large efficiency loss appears. In order to get a reasonable efficiency, the underlying pulse shape must be short, as is the case in this paper.

The rest of the paper is organized as follows. In Section 2 the system model is defined and in Section 3 information rates are defined and computed. In Section 4 we test a coding system based on FTN. Finally we draw some conclusions in Section 5.



**Fig. 1.** System model for the transmitter. The encoder has rate  $R < 1$ .

## 2. SYSTEM MODEL

The system model for the transmitter is illustrated in Figure 1. The transmitter consists of three parts, an encoder, a mapper function and a modulator. The input to the encoder is a sequence of independent, identically distributed (i.i.d.) bits  $\mathbf{u}$ . The encoder encodes  $\mathbf{u}$  into a codeword  $\mathbf{v}$ . The rate of the encoder is  $R < 1$  bits in/bit out. For the moment we assume an arbitrary encoder, but in Section 4 a convolutional code and an interleaver synthesize the encoder. The codeword is passed to the mapper which takes a binary  $k$ -tuple as input and outputs a symbol  $a$  from an alphabet  $\mathcal{A}_M$ . This alphabet has cardinality  $M$  and we assume an MPAM alphabet, that is,  $\mathcal{A}_M = \{\pm(M-1), \pm(M-3), \dots, \pm 1\}$ . The transmitted signal is now formed by (2) and launched over the AWGN channel. The bitrate  $R_b$  of the system equals

$$R_b = \frac{Rk}{\tau T} \text{ bits/s.} \quad (3)$$

The consumed energy per bit is equal to

$$E_b = \frac{E_s}{kR} = \frac{M^2 - 1}{3kR} \int h^2(t) dt, \quad (4)$$

where  $E_s$  is the average energy per symbol.

A crucial parameter of the system is its bandwidth. We assume only a power spectral density (PSD). It is proportional to the Fourier transform of  $h(t)$ , i.e.,

$$\text{PSD} = \frac{E_s}{\tau T} |H(f)|^2 = \frac{kRE_b}{\tau T} |H(f)|^2. \quad (5)$$

It is assumed in (5) that the symbols in the vector  $\mathbf{v}$  are i.i.d. What is normally meant by bandwidth is a single scalar number. A standard way to measure it is by the so-called power bandwidth, which is the frequency in which the PSD holds, say, 99.9 % of the total power. But from our experience, such a measurement is risky; when  $R_b$  increases, the power bandwidth makes less sense and eventually does not reflect the actual signal performance very well. In this paper the aim is to maximize the bit rate  $R_b$  for a fixed PSD and to investigate the influence of other system parameters, such as  $\tau$  and  $\mathcal{A}_M$ .

The optimal sequence estimation decoder filters the received signal  $r(t) = s(t) + n(t)$  with a filter matched to  $h(t)$ ;  $n(t)$  is white Gaussian noise. Since  $h(t)$  is real valued this matched filter equals  $h(-t)$ . The output of the matched filter is sampled at the signaling rate, i.e.,  $1/\tau T$ . The sequence values at the output of the sampling unit are sufficient statistics for estimating  $\mathbf{u}$ .

This sequence can be written as  $\mathbf{r} = \mathbf{a} \star \mathbf{g} + \boldsymbol{\eta}$ , where  $\mathbf{g}$  is the sampled autocorrelation function of  $h(t)$ ; i.e.,

$$g_j = \int h(t)h(t + j\tau T) dt \quad (6)$$

and  $\boldsymbol{\eta}$  is colored Gaussian with correlation sequence  $\mathbf{g}$ . Since the pulse  $h(t)$  is assumed to be finite the ISI  $\mathbf{g}$  is of course finite as well. It is possible to whiten the noise  $\boldsymbol{\eta}$  by a whitening filter. This filter is constructed from  $\mathbf{g}$  by spectral factorization; for details see [6], [7]. The so-called whitened matched filter (Forney) model is obtained and can be expressed as  $\tilde{\mathbf{r}} = \mathbf{a} \star \mathbf{h} + \mathbf{w}$ , where  $\mathbf{h}$  represents causal ISI with the property  $\mathbf{h}[n] \star \mathbf{h}[-n] = \mathbf{g}$  and  $\mathbf{w}$  is white Gaussian noise with variance  $N_0/2$ .

We investigate four different pulse shapes:

- A time raised cosine pulse

$$h_A(t) = 1 - \cos(2\pi t/T), \quad 0 \leq t \leq T \quad (7)$$

- A half cycle sinusoidal pulse

$$h_B(t) = \sin(\pi t/T), \quad 0 \leq t \leq T \quad (8)$$

- A triangular pulse

$$h_C(t) = \begin{cases} t/T, & 0 \leq t \leq T/2 \\ 1 - t/T, & T/2 < t \leq T \end{cases} \quad (9)$$

- An optimal finite Nyquist pulse from [9],

$$h_D(t) = \sum_{i=1}^K c_i \psi_i(t), \quad 0 \leq t \leq 3T \quad (10)$$

The pulse  $h_D(t)$  is much more complicated than the other three. It is constructed by a linear combination of truncated prolate spheroidal wave functions [8] denoted  $\psi_i(t)$ . The coefficients  $\mathbf{c}$  are tabulated in [9]. The pulse is optimal in the sense that it is the Nyquist pulse of length  $3T$  that maximizes the spectral power concentration in the frequency band  $[-1/T, 1/T]$  Hz.

## 3. INFORMATION RATES

In this section we find the possible information rates of the system for the different pulses under constrained input alphabets. We also investigate optimal alphabet sizes to use for different throughputs.

### 3.1. Information Rates

Here we assume that the symbols  $a_k$  are drawn from a finite alphabet. We still assume that the symbols are i.i.d. This corresponds to Shannon's random coding approach. For practical encoders the assumption implies a restriction; but this is not very serious because most of the standard encoders generate uncorrelated

and equally likely output symbols, especially if there is an interleaver involved in the encoder.

The difference between capacity and information rate is that capacity involves a maximization procedure over the input symbol distribution. When this distribution is fixed, there is no maximization and the result is denoted as information rate. This information rate, in bits per channel use, is defined as

$$I_{cu} = \lim_{N \rightarrow \infty} \frac{1}{N} \mathcal{I}(\mathbf{r}; \mathbf{v}) = \lim_{N \rightarrow \infty} \frac{1}{N} \mathcal{I}(\tilde{\mathbf{r}}; \mathbf{v}), \quad (11)$$

where  $\mathcal{I}(\cdot, \cdot)$  is the mutual information operator and  $N$  is the blocklength. It is assumed that the encoder block in Figure 1 has a rate  $R$  that satisfies  $kR \leq I_{cu}$ . Then there exists a code such that the error rate tends to zero as the blocklength grows. Here we are actually more interested in the information rate in bits per  $T$  seconds; that is

$$I = \frac{I_{cu}}{\tau} \text{ bits/T s} \quad (12)$$

Equation (11) equals

$$\lim_{N \rightarrow \infty} \frac{1}{N} \mathcal{I}(\tilde{\mathbf{r}}; \mathbf{v}) = \lim_{N \rightarrow \infty} \frac{1}{N} [\mathcal{H}(\tilde{\mathbf{r}}) - \mathcal{H}(\mathbf{w})], \quad (13)$$

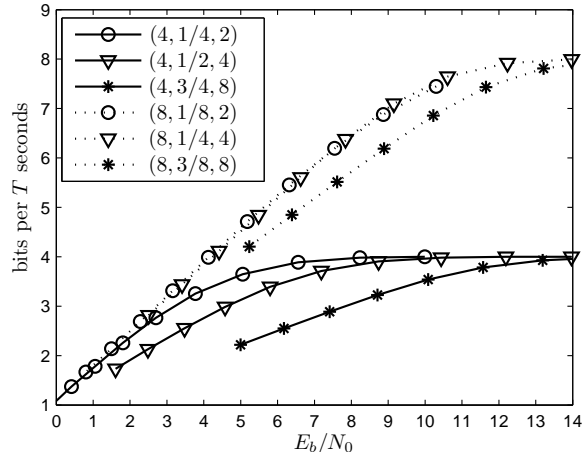
where  $\mathcal{H}(\mathbf{x})$  is the differential entropy function. The last term of (13) can be directly evaluated and equals

$$\lim_{N \rightarrow \infty} \frac{1}{N} \mathcal{H}(\mathbf{w}) = \frac{1}{2} \log_2(\pi e N_0). \quad (14)$$

The remaining term to compute is  $\mathcal{H}(\tilde{\mathbf{r}})$ ; this is, however, a major difficulty in the case of ISI. Upper and lower bounds are given by Shamaï and others in a long series of papers; see [10] and the extensive reference list therein. That paper also conjectures a lower bound to the information rate which is remarkably tight and easy to compute, but no proof is given. Recently, there have been simulation based methods proposed to compute  $\mathcal{H}(\tilde{\mathbf{r}})$  [11]. A brief review of one is given next.

First generate a long sequence  $\tilde{\mathbf{r}}$  and run a forward pass of the BCJR algorithm. The output of this algorithm is  $p(\tilde{\mathbf{r}})$ . Due to the Shannon-Breiman-McMillan Theorem  $-\frac{1}{N} \log_2 p(\tilde{\mathbf{r}})$  converges with probability one to the entropy rate  $\frac{1}{N} \mathcal{H}(\tilde{\mathbf{r}})$ . When actually performing this, one computes  $-\frac{1}{N} \log_2 p(\tilde{\mathbf{r}})$  as  $\frac{1}{N} \sum \log_2 \lambda_k$ , where  $\lambda_k$  are the scale factors that bring the sum of state metrics to 1 at each trellis depth. In our simulations the blocklength is  $N = 3 \times 10^7$ .

We now turn to the numerical results. Three different alphabet sizes,  $M \in \{2, 4, 8\}$ , are considered. We start with results for the pulses  $h_A(t)$ ,  $h_B(t)$  and  $h_C(t)$ . The information rates turn out to be monotonically increasing with decreasing  $\tau$ . That is, increasing the symbol rate is always beneficial, for all three pulses. Moreover, we found that the best pulse out of the three is always the time raised cosine pulse,  $h_A(t)$ .



**Fig. 2.** Information rates for several system setups. The pulse shape used is  $h_A(t)$ . The legend gives  $(I_{\max}, \tau, M)$ .

### 3.2. Alphabet Size Investigation

In order to compare different system setups fairly, we demand that they offer the same maximal throughput. By this we mean that the information rates should tend to the same value for both setups as  $\text{SNR} \rightarrow \infty$ . The maximal throughput has the asymptote

$$I_{\max} = \lim_{N_0 \rightarrow 0} I = \frac{\log_2(|\mathcal{A}_M|)}{\tau} = \frac{\log_2(M)}{\tau} \text{ bits/T s}. \quad (15)$$

This implies that if a binary system has symbol time  $\tau T$  then a quaternary and an octal system must have symbol times  $2\tau T$  and  $3\tau T$  respectively.

In Figure 2 we show the resulting information rates for  $h_A(t)$  with all three alphabets. The solid set of curves corresponds to  $I_{\max} = 4$ , that is,  $\tau$  is taken as  $1/4$ ,  $2/4$  and  $3/4$  for 2-, 4- and 8PAM respectively. The dashed set corresponds to  $I_{\max} = 8$ , that is,  $\tau$  is taken as  $1/8$ ,  $2/8$  and  $3/8$  for 2-, 4- and 8PAM. From the figure it is clear that for  $I_{\max} = 4$  the highest information rate occurs for 2PAM. But for  $I_{\max} = 8$  we see that roughly the same information rates are obtained for 2PAM and 4PAM; the curve for 8PAM is far below. The outcome is virtually identical for  $h_B(t)$  and  $h_C(t)$ . From this we draw the conclusion that it is more important to increase the signaling rate than it is to increase the signaling alphabet. However, there seems to be some kind of threshold: for  $\tau$  below the threshold it is not important whether the signaling alphabet is increased or the signaling rate is increased. In Figure 2 the threshold appears to occur at about  $\tau = 1/4$ .

So far we have only considered a few specific pulses. We next investigate a more general case. Assume any pulse with duration  $T$  and  $\tau = 1/2$ . For input alphabet 2PAM, these signal systems have  $I_{\max} = 2$ . We compare against orthogonal systems with  $\tau = 1$  and a 4PAM alphabet. For a given transmitter power the in-

formation rate of the FTN system is denoted  $I = 2I_{cu}$ . The information rate of the orthogonal pulse system is denoted  $I_4$ . We will show that  $I > I_4$ . This implies that FTN is always superior to Nyquist for these setups.

Since  $\tau = 1/2$  and  $h(t)$  has duration  $T$ , the ISI  $\mathbf{h}$  consists of two taps,  $h_0$  and  $h_1$ . When normalized to unit energy, these can be written as  $h_0 = (1 + s^2)^{-1/2}$  and  $h_1 = s(1 + s^2)^{-1/2}$ , where  $0 \leq |s| \leq 1$  determines the amount of ISI present. The conjectured lower bound [10] on  $I_{cu}$  reads

$$I_{cu} \geq \mathcal{C}_b(\chi), \quad (16)$$

where  $\mathcal{C}_b$  is the capacity of the binary input memoryless scalar Gaussian channel and  $\chi$  is a function of  $s$  and  $N_0$ . It is easy to see that  $\mathcal{C}_b(\chi)$  is minimized for any  $N_0$  if  $s = 1$ , that is, when  $h_0 = h_1 = 1/\sqrt{2}$ . This is the system with the least lower bound on the information rate, and if we can show that this worst system is still better than  $I_4$ , we have the proof. We have not shown this analytically; instead we have evaluated  $2\mathcal{C}_b(\chi)$  and  $I_4$  by numerical computation and observed that  $2\mathcal{C}_b(\chi) > I_4$  for all reasonable  $N_0$ . Future research will seek a formal proof.

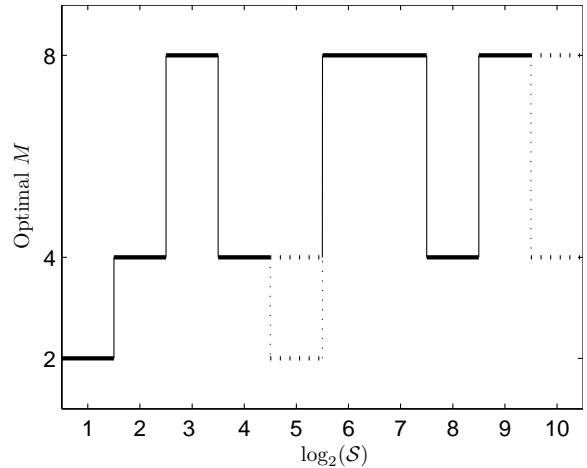
We next investigate the information rates versus decoding complexity. For modern, turbo like, coding structures, iterative decoding is usually employed. This will essentially involve two BCJR decoders, one for the encoder and one for the FTN modulator. The decoding complexity is measured by the number of states in a full complexity BCJR algorithm. If the object is to maximize the information rate of the modulator with respect to the decoding complexity, the situation is somewhat different from that in Figure 2. Different combinations of signaling rates and alphabets give rise to different state complexities. Denote the state complexity  $\mathcal{S}$ ; then

$$\mathcal{S} = M^{L_h - 1},$$

where  $L_h$  is the support of the ISI response  $\mathbf{h}$ . For pulse shapes of duration  $T$ , we have  $L_h = \lceil 1/\tau \rceil$ , and consequently

$$\mathcal{S} = M^{\lceil 1/\tau \rceil - 1}$$

Assume that only  $\mathcal{S} = 8$  can be accepted at the receiver side. This complexity is obtained for 2PAM with  $\tau = 1/4$ . For 4PAM,  $\mathcal{S} = 8$  cannot be reached; the largest state complexity smaller than 8 that can be reached is  $\mathcal{S} = 4$ , which is obtained for  $\tau = 1/2$ . For 8PAM,  $\tau = 1/2$  gives  $\mathcal{S} = 8$ . Out of these three combinations, it turns out to be optimal to use 8PAM for essentially all SNRs. In Figure 3 we plot the optimal alphabet size versus the state complexity; complexities range up to  $\mathcal{S} = 1024$ . For some complexities there is hardly any difference between some of the alphabets, and in that case the lines are dotted. As can be seen the optimal alphabet is a somewhat complicated function; it is mostly affected by the fact that not all complexities can be reached for all alphabets.



**Fig. 3.** Optimal alphabet size (y-axis) as a function of decoding complexity  $\mathcal{S}$ . The dotted levels indicate that two alphabets perform equally well.

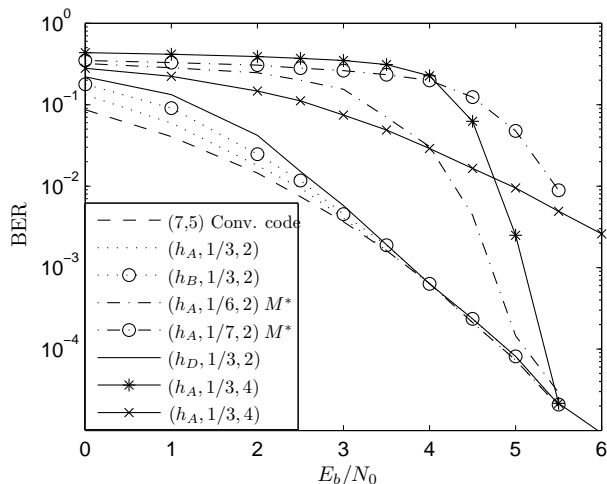
#### 4. CODING SYSTEMS

In this section practical coding setups are tested. The system to be investigated is the one in Figure 1, and the encoder consists of a convolutional code and an interleaver. A block of  $K$  information bits is first encoded by a rate  $1/2$  convolutional code; in total there are  $2K$  code symbols. The output of the encoder is fed to an interleaver with blocksize  $2K$ . The symbol vector  $\mathbf{v}$  is formed by mapping the output of the interleaver on an  $M$ PAM alphabet. Finally, the transmitted signal  $s(t)$  is constructed according to (2).

In this paper we only investigate the (7,5) convolutional code and we set the blocksize  $K = 1024$ . Decoding is done with standard turbo equalization [12]. The performance of the system can never be better than the performance of the underlying convolutional code [13]. However, for FTN this performance is obtained at a considerably higher bit rate. By studying the EXIT charts [14] of the system the convergence threshold can be determined. The system will converge to the outer code performance as soon as there is an open convergence tunnel between the EXIT curves for the FTN system and the outer code. However, such an EXIT chart analysis has not been performed; instead we test the performance of the system by actual receiver tests.

If a full complexity decoder is used as component decoder for the inner code (the ISI), we are limited to rather small values for  $I_{\max}$  for complexity reasons. We have therefore also tested a reduced complexity MAP equalizer called the  $M^*$ -BCJR algorithm.<sup>1</sup> This recently proposed [15] algorithm has shown very good performance on ISI channels; it is an extended version of the  $M$ -BCJR from [16]. The algorithm only retains  $M$  out of the  $\mathcal{S}$  states at each trellis depth, but rather

<sup>1</sup>Note that the alphabet size  $M$  and the  $M^*$  in the algorithm are unrelated.



**Fig. 4.** Receiver tests for several system setups. The legend gives (pulse shape,  $\tau$ ,  $M$ ). Systems must perform above the dashed (7,5) line.

Good Mapping      Bad Mapping

3 × 01	3 × 10
1 × 10	1 × 11
-1 × 11	-1 × 01
-3 × 00	-3 × 00

**Fig. 5.** The mappings used for 4PAM.

than eliminating the other states they are merged into the  $M$  survivor states. This keeps the number of +1s and -1s on the remaining trellis branches in balance.

In Figure 4 we show the performance of several signal systems. The performance of binary systems with  $\tau = 1/3$  and  $h_A(t)$ ,  $h_B(t)$  and  $h_D(t)$  are shown. It is seen that  $h_A(t)$  is slightly better than  $h_B(t)$  which in turn is better than  $h_D(t)$ , but all of them operate very close to the underlying (7,5) code. Also shown is the performance of  $h_A(t)$  with  $\tau = 1/6$  and  $1/7$ . Decoding was performed with the  $M^*$ -BCJR algorithm; the number of kept states at every depth was 8. It is seen that the system with  $\tau = 1/7$  performs poorly. Finally two quaternary systems are tested; system parameters are  $h_A(t)$  and  $\tau = 1/3$ . The difference between the two is in the binary to quaternary mapping. It is seen that the mapping is of great importance. The actual mappings are shown in Figure 5; the “Bad mapping” is a Gray mapping.

The conclusion of Figure 4 and of this section is that the underlying convolutional code performance can be maintained for many of the above mentioned systems even though their bit rates are significantly higher.

## 5. CONCLUSIONS

In this paper we have considered faster-than-Nyquist modulation based on short pulses. We investigated the optimal alphabet size, with the outcome that if a low decoding complexity is the aim, then the alphabet size

should vary with the allowed complexity. But when transmission power is limited, the optimal alphabet is often binary. Moreover, we have demonstrated that the effect on the information rate of increasing the signaling rate is much stronger than increasing the signaling alphabet. We also tested some concatenated coding schemes and showed that bit rates at least six times as high as conventional can be supported by the (7,5) convolutional code. Comparisons with Shannon limits and orthogonal modulation were performed as well. Overall, the FTN method with short, finite pulses is just as promising as earlier papers have shown it to be with infinite-support pulses.

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