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Simplified models of xenon spatial oscillations

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Abstract

In this paper simplified models of the xenon spatial instability, especially axial oscillations, are derived. The nature of the oscillations makes it possible to represent the axis of the core with only two points. This simple model gives a good physical insight into the problem and is shown to be rather accurate, compared with other models.

Both unstable and stable periodic solutions have been predicted with the non-linear model. They have later been verified by digital simulation.

The linear stability is shown to be independent of the control rod, which maintains the criticality. The rod will, however, influence the non-linear behaviour very much.

Zusammenfassung

Vereinfachte Modelle der räumlichen Xenonschwingung

In dieser Arbeit werden vereinfachte Modelle der räumlichen Xenoninstabilität (in großen Leistungsreaktoren) speziell der axialen Schwingungen hergeleitet. Die Natur der Xenonschwingungen macht es möglich, den Reaktor nur mit zwei Punkten darzustellen. Dieses einfache Zweipunktmodell gibt eine gute physikalische Einsicht in das Problem; es hat sich im Vergleich zu anderen Modellen als genauer erwiesen.

Sowohl die stabilen als auch die instabilen periodischen Lösungen sind mit dem Modell vorausgesagt worden; sie wurden später durch eine digitale Simulierung bestätigt.

Es wird gezeigt, daß die lineare Stabilität unabhängig von dem Kontrollstab ist, welcher die Kritikalität aufrechterhält. Der Kontrollstab hat allerdings eine große Bedeutung für die nichtlinearen Lösungen.

EURATOM KEYWORDS

XENON
OSCILLATIONS
STABILITY
REACTOR CORE
REACTOR KINETICS
CONTROL ELEMENTS
NEUTRON FLUX
MOCKUP
NUMERICALS
DIFFERENTIAL EQUATIONS
TRANSIENTS
EIGENVALUES
DIGITAL SYSTEMS

Definition of symbols

Symbol	Explanation	Normal value
$B^2(z, t)$	Material buckling	
$c(z, t)$	Absorption term	
M^2	Migration area	440 cm²
$u(z, t)$	Control term in buckling	
$\alpha(z)$	Temperature coefficient, expressed as reactivity bounded in fuel temperature increase above the moderator at mean flux and infinite gitter	−0,226 %
	Normalization to mean flux density $\bar{\Phi} = 5{,}65 \cdot 10^{13}$, $M^2 = 440$ cm²	$\alpha = -0{,}514$
β	Xenon influence on changes in buckling (−3,2 % on reactivity) at saturation	−0,73
γ_x	Fraction of xenon yield (relative to xenon + iodine yield)	0,05
γ_i	Fraction of iodine yield (relative to xenon + iodine yield)	0,95
$\Phi(z, t)$	Neutron flux density, normalized to mean flux density $\bar{\Phi}$ $\bar{\Phi} = 5{,}65 \cdot 10^{13}$ cm^{-2} s^{-1}	1
λ_x	Xenon disintegration constant	0,0756 h^{-1}
λ_i	Iodine disintegration constant	0,1058 h^{-1}
σ_x	Microscopic xenon cross section	$2{,}29 \cdot 10^{-18}$ cm²

Introduction

The xenon instability problem has been extensively analysed, after spatial oscillations originally were demonstrated in Savannah River in 1955 and in Shippingport in 1958. Since *Ward* made the first analysis in 1956, a large number of papers on the subject have been published. Most of the analysis is based on linearized models [1–4], but non-linear approaches, especially point reactor models, have received attention [5–8]. Digital simulation is an extensively used tool in the studies [9–12]. The behaviour of the oscillations has been described at the same time.

During the very last years, the xenon spatial instability problem has gained new attention [13–15]. This depends on the fact, that the reactors, which are presently being built, will have geometrical dimensions close to the xenon stability limits. The purpose of this paper is to present some simple models, based on finite differences, that have been successfully used in the study of xenon oscillations. These models have given physical insight into the influence of different parameters and non-linearities on the stability. The behaviour of the solutions have been compared with more complex models, using transient studies and eigenvalue calculations. The results are very encouraging, since it has been possible to explain all the different types of linear and non-linear solutions by means of the simple models. The study has been concentrated upon the xenon propagation along the axis of a cylindrical core.

As criticality must be maintained by a control rod or by homogeneous absorption in the core, it is interesting to know how stability is affected by this absorption. It is shown, that the critical height is independent of absorption configuration or rod motion for symmetric, flat, neutron fluxes. For large perturbations, as shown previously [11, 14], the rod motion may induce coupled oscillations, which may amplify the flux tilt, caused by xenon. Both stable and unstable periodic solutions have been found by the approximate models, and they are later verified by digital simulation of more complex models.

Basic equations

Here the neutron balance in the core is described by one group diffusion theory. As the xenon oscillations are very slow, compared with neutron life time, we neglect the delayed neutrons, and assume further, that the changes in neutron flux density and temperature distribution occur instantaneously. The neutron flux density along the core axis then satisfies the following equation:

$$\frac{\partial^2}{\partial z^2}\,\Phi(z,t) + B^2(z,t)\,\Phi(z,t) = 0 \qquad (1)$$

where we have assumed a space independent diffusion constant.

The xenon and iodine concentrations X and I respectively satisfy:

$$\frac{\partial X}{\partial t} = -\lambda_x X + \lambda_i I + \gamma_x \sigma_x \Phi - \sigma_x X \Phi \qquad (2)$$

$$\frac{\partial I}{\partial t} = -\lambda_i I + \gamma_i \sigma_x \Phi \qquad (3)$$

The state variables are normalized to the saturation equilibrium value of xenon for an infinite neutron flux density.

We include the top and bottom reflectors in the core and assume that the thermal flux density is zero on the core boundary.

The total power P

$$P(t) = \int_z k(z)\,\Phi(z,t)\,dz \qquad (4)$$

is assumed to be controlled by a stable control system. The variables are written in incremental form:

$$\begin{aligned} \Phi(z,t) &= \Phi^0(z) + \varphi(z,t) \\ X(z,t) &= X^0(z) + \xi(z,t) \\ I(z,t) &= I^0(z) + \eta(z,t) \end{aligned} \qquad (5)$$

where the superscripts mean equilibrium values.

The equations (2) and (3) are rewritten in the form:

$$\frac{\partial \xi}{\partial t} = -\lambda_x \xi + \lambda_i \eta + \gamma_x \sigma_x \varphi - \sigma_x (X^0 \varphi + \Phi^0 \xi + \varphi \cdot \xi) \qquad (6)$$

$$\frac{\partial \eta}{\partial t} = -\lambda_i \eta + \gamma_i \sigma_x \varphi \qquad (7)$$

The buckling term B^2 can be expanded into two parts, one equilibrium part B^{20} and one perturbation part,

$$B^2(z,t) = B^{20}(z) + \alpha(z)\,\varphi(z,t) + \beta\,\xi(z,t) + c(z,t) + u(z,t) \qquad (8)$$

It is assumed that the temperature effects on buckling are proportional to flux variations.

The spatial derivative in (1) is approximated by finite differences, which transforms (1) to N algebraic conditions. The xenon and iodine dynamics (6; 7) is then valid in N space points.

Finite difference axial models

A digital program, called TRAXEN, has been written for the computer CDC 3600 in order to simulate the xenon oscillations. The program is based on the non-linear equations (1; 4; 6; 7; 8). It has been used as a check of the simple models and has also been applied in the Swedish Marviken heavy water reactor study.

It is shown [11], that the rod motion may induce xenon oscillations as a result of large disturbances. The influence of several non-linear parameters, such as flux disturbance, control configuration and temperature feedback, has been studied.

The model is easily linearized. Since the nonlinearities in the model are polynomials, the system equations can be written

$$\dot{x} = Ax + g(x)$$

where the vector function $g(x)$ is continuous at the origin and has the properties

$$g(0) = 0 \qquad \lim_{\|x\| \to 0} \frac{\|g(x)\|}{\|x\|} = 0$$

Hence if $x = 0$ is an asymptotically stable solution to the linearized equation, it is also a stable solution to the non-linear equation.

For the linear equations it is easy to show, that one rod is not sufficient for the control of the oscillations. It is also proven, that stability of a flat flux is the same with rod control as with homogeneous control for small disturbances.

The statement is verified by digital simulation also for other flux shapes.

The linearized xenon and iodine equations are derived from (6) and (7), where the products of the variable increments are neglected:

$$\frac{d\xi_k}{dt} = -\lambda_x \xi_k + \lambda_i \eta_k + \gamma_x \sigma_x \varphi_k - \sigma_x (X_k^0 \varphi_k + \Phi_k^0 \xi_k) \qquad (9)$$

$$\frac{d\eta_k}{dt} = -\lambda_i \eta_k + \gamma_i \sigma_x \varphi_k \qquad k = 1, \ldots, N \qquad (10)$$

The subscript k stands for space point.

The linearized neutron flux diffusion equation in space point k is derived directly from (1), (5) and (8) with $u(z,t) = 0$,

$$\varphi_{k-1} + \varphi_{k+1} + \varphi_k\left[-2 + h^2(B_k^{20} + \alpha\,\Phi_k^0)\right] + h^2 \Phi_k^0 [c_k + \beta\,\xi_k] = 0 \qquad k = 1, \ldots, N \qquad (11)$$

The power condition (4) is simplified to:

$$\sum_{i=1}^{N} \varphi_i = 0 \qquad (12)$$

The terms c_k in (11) represent the lumped action of the control rod in each space point. With one control rod the c_k's are related through

$$f_i(c_1, \ldots, c_N) = 0; \qquad i = 1, \ldots, N\text{-}1 \qquad (13)$$

Now, to express the N variables φ_k in the state variables ξ_k and η_k we have $2N$ equations (11–13) for the $2N$ unknown parameters c_k and φ_k. This simple discussion shows directly that the rod insertion is uniquely determined at each moment, if the total power is separately controlled. Thus the xenon process is not controllable by only one rod.

For a flat equilibrium flux, a suitable state variable representation is achieved by adding all the xenon and iodine equations,

$$\frac{d}{dt}\left(\sum_1^N \xi_k\right) = (-\lambda_x - \sigma_x \Phi^0)\left(\sum_1^N \xi_k\right) + \lambda_i \sum_1^N \eta_k \qquad (14)$$

$$\frac{d}{dt}\left(\sum_1^N \eta_k\right) = -\lambda_i\left(\sum_1^N \eta_k\right) \qquad (15)$$

The state vector is chosen

$$x^T = \left(\xi_1 \eta_1 \xi_2 \eta_2 \ldots \xi_{N\text{-}1} \eta_{N\text{-}1} \left(\sum_k \xi_k\right) \left(\sum_k \eta_k\right)\right) \qquad (16)$$

The dynamics is represented by a $2N$ order system,

$$\frac{dx}{dt} = A\,x$$

We can partition the $2N \cdot 2N$ order matrix A,

$$A = \begin{bmatrix} A^0 & B & \\ 0 & a_1 & a_{12} \\ & 0 & a_2 \end{bmatrix} \qquad (17)$$

The scalars a_1 and a_2 are always negative (14), (15), and the matrix A^0 is of order $(2N\text{-}2) \cdot (2N\text{-}2)$.

The flat flux has a simple equilibrium buckling:

$$\left(\frac{H}{N+1}\right)^2 B_k^{2\,0} = \begin{cases} 0 & 2 \leq k \leq N\text{-}1 \\ 1 & k = 1;\ k = N \end{cases} \qquad (18)$$

We assume two different kinds of absorption terms. In the first case one control rod is inserted from space point 1 to k–1 in equilibrium, and we get:

$$\begin{aligned} c_1 &= c_2 = \ldots = c_{k\text{-}1} = 0 \\ c_k &= c \\ c_{k+1} &= \ldots = c_N = 0 \end{aligned} \qquad (19)$$

as the rod absorption in the $(k-1)$ first points is included in the equilibrium buckling. The rod oscillation occurs in point k.

In the second case homogeneous control we get

$$c_1 = c_2 = \ldots = c_N = c \qquad (20)$$

It is proven [16], that the eigenvalues of matrix A^0 (17) are independent of the control method. We get also the same eigenvalues of A^0 by neglecting c and one state variable, e.g.

$$x_{2N\text{-}1} = \sum_k \xi_k$$

can be removed.

Simulations have verified, that this statement also holds for other flux shapes.

Two point axial models

Non-linear equations

In order to get a simple model of the xenon instability we make an approximation of the stationary diffussion equation, using only two finite space points. The neutron equations then are no more than two algebraic conditions. These conditions are combined with the four xenon and iodine differential equations.

The flux equations (1; 8; 11; 12) are simplified to:

$$\alpha_1 \varphi_1^2 + \varphi_1(\beta\xi_1 + c_1 + u_1 - g_1) + + \Phi_1^0(c_1 + u_1 + \beta\xi_1) = 0 \qquad (21)$$

$$\alpha_2 \varphi_1^2 + \varphi_1(-\beta\xi_2 - c_2 - u_2 + g_2) + + \Phi_2^0(c_2 + u_2 + \beta\xi_2) = 0 \qquad (22)$$

where

$$\begin{aligned} g_1 &= \frac{27}{H^2} - (B_1^{2\,0} + \alpha_1 \Phi_1^0) \\ g_2 &= \frac{27}{H^2} - (B_2^{2\,0} + \alpha_2 \Phi_2^0) \end{aligned} \qquad (23)$$

and H is the core height.

Condition (13) has the simple form

$$c_1 = c;\ c_2 = 0 \qquad (24)$$

for rod control, and

$$c_1 = c_2 = c \qquad (25)$$

for homogeneous control.

If the state vector, defined in (16), is used, the xenon and iodine equations are transformed to:

$$\frac{dx_1}{dt} = (-\lambda_x - \sigma_x \Phi_1^0)\, x_1 + \lambda_i x_2 + + \sigma_x(\gamma_x - X_1^0)\, \varphi_1 - \sigma_x \varphi_1 x_1 \qquad (26)$$

$$\frac{dx_2}{dt} = -\lambda_i x_2 + \gamma_i \sigma_x \varphi_1 \qquad (27)$$

$$\frac{dx_3}{dt} = \sigma_x(\Phi_2^0 - \Phi_1^0)\, x_1 + (-\lambda_x - \sigma_x \Phi_2^0)\, x_3 + + \lambda_i x_4 + \sigma_x(X_2^0 - X_1^0)\, \varphi_1 + \sigma_x \varphi_1[x_3 - 2x_1] \qquad (28)$$

$$\frac{dx_4}{dt} = -\lambda_i x_4 \qquad (29)$$

The xenon process is thus described by (21; 22; 26–29 and 24 or 25).

Linearization of the model

A symmetric two space point model is a special case of the flat flux (18). The dynamic equations are very attractive, because the fourth order system can be easily partitioned into two second order systems (17) and two eigenvalues are always negative, independent of the core parameters.

The system is described by the state equations.

$$\frac{dx}{dt} = Ax + Bu$$

$$y = Cx + Du$$

where the measurement variable

$$y = \varphi_1$$

and the control vector u is found from (21; 22). Here the model is used first to study the stability conditions, then to find out the transient response to a reactivity input. The input is assumed to be symmetric, i.e.

$$u_1 = -u_2 = u$$

in (21; 22).

Then we easily find the expressions for the matrices A, B, C and D for the symmetric case, i.e. α, g, Φ^0 and X^0 are space independent in (21–23). Assuming homogeneous control (25) we find:

$$A = \begin{bmatrix} -\lambda_x - \sigma_x \Phi^0\left[1+(X^0-\gamma_x)\frac{\beta}{g}\right] & \lambda_i & \frac{\sigma_x \beta \Phi^0}{2g}(X_0-\gamma_x) & 0 \\ \gamma_i \sigma_x \frac{\beta\Phi^0}{g} & -\lambda_i & -\gamma_i \sigma_x \frac{\beta\Phi^0}{2g} & 0 \\ 0 & 0 & -\lambda_x - \sigma_x \Phi^0 & \lambda_i \\ 0 & 0 & 0 & -\lambda_i \end{bmatrix}$$

$$B = \frac{\sigma_x \Phi^0}{g}(\gamma_x - X^0 \quad \gamma_i \quad 0 \quad 0)^T$$

$$C = \frac{\beta\Phi^0}{2g}(2 \quad 0 \quad -1 \quad 0)$$

$$D = \frac{\Phi^0}{g} \qquad (30)$$

Only two states, x_1 and x_2, are both controllable and observable, and the transfer function

$$G(s) = \frac{Y(s)}{U(s)} = C(sI - A)^{-1} B + D \qquad (31)$$

is consequently of second order.

In the rod control case (24) we get some minor changes of the system equations.

We get this new A matrix by multiplying the elements a_{13} and a_{23} of the matrix in (30) by two. The B and D matrices are unchanged, while the element −1 in C is changed to −2.

We can simply check, that the eigenvalues of the A matrices are equal in the two cases, as we proved previously. *Thus the stability of the xenon process can be analysed by means of a second order submatrix, which is independent of the control configuration.*

As only two states are both controllable and observable, the *transfer function (31) is also independent of control configuration. Thus, the transient amplitudes are equal in the two cases.*

Linear analysis

The two point model

The eigenvalues of the second order A submatrix (30) determine the stability of the xenon process. Since such parameters as core height, temperature coefficient and mean flux are included in the eigenvalues it is easy to study the influence on stability or on transient response amplitude. We get two stability conditions

$$a_{11} + a_{22} < 0 \text{ or}$$

$$\lambda_x + \sigma_x \Phi^0 + \frac{\sigma_x}{g}(X^0 - \gamma_x)\Phi^0 \beta + \lambda_i > 0 \qquad (32)$$

and

$$a_{11} a_{22} - a_{12} a_{21} > 0 \quad \text{or}$$

$$\lambda_i \left\{ -\lambda_x - \sigma_x \Phi^0 - \frac{\sigma_x}{g}(X^0 - \gamma_x)\Phi^0 \beta + \gamma_i \sigma_x \frac{\Phi^0 \beta}{g} \right\} < 0 \qquad (33)$$

where a_{ij} are coefficients of the matrix A (30).

As $\gamma_i + \gamma_x = 1$ by definition and $\beta \leq 0$ and $X^0 \leq 1$, condition (33) is always satisfied, and stability is determined only by (32).

Condition (33) also implies, that the two eigenvalues are always situated on the same side of the imaginary axis.

From (33) we also find, that the eigenvalues are complex at the stability limit. The period is:

$$T = \frac{2\pi}{\sqrt{a_{11} a_{22} - a_{12} a_{21}}} = \frac{2\pi}{\left[\lambda_i\left(\lambda_x + \sigma_x \Phi^0 + \frac{\sigma_x (X^0 - 1)\Phi^0 \beta}{g}\right)\right]^{1/2}} \qquad (34)$$

It decreases asymptotically as

$$\frac{1}{\sqrt{\Phi^0}}$$

for increasing flux level.

The influence of core height on the two eigenvalues is shown in Fig. 1. For decreasing core dimension the eigenvalues converge towards

$$s_1 = -\lambda_x - \sigma_x \Phi^0$$
$$s_2 = -\lambda_i$$

that is, they are coincident with the other two eigenvalues, (14; 15). For increasing core height one eigenvalue diverges towards infinity.

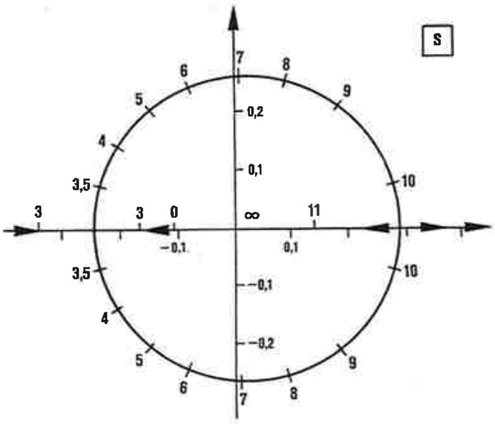

Fig. 1: The locus of the two most significant eigenvalues of the linearized symmetric two point model (30). The parameter is core height in m

With typical core data—from the Marviken reactor—the period time (34) at the critical height $H = 6{,}93$ m is found to be

$$T = 23{,}8 \text{ h}$$

As known previously a negative temperature coefficient has a stabilizing effect on the xenon oscillations. This is illustrated in Fig. 2.

If the eigenvalues are complex, the transients include a damped sine component. The condition for complex eigenvalues is simply found (30),

$$(a_{11} + a_{22})^2 - 4 a_{11} a_{22} + 4 a_{12} a_{21} < 0$$

It is already known, that oscillating transient responses do not occur for low flux density levels. In the reactor with $H = 6{,}93$ m, oscillating convergent transients occur for

$$0{,}003 < \Phi^0 < 1{,}0$$

or

$$1{,}6 \cdot 10^{13} < \Phi^0 < 5{,}65 \cdot 10^{13} \text{ cm}^{-2} \text{ s}^{-1}$$

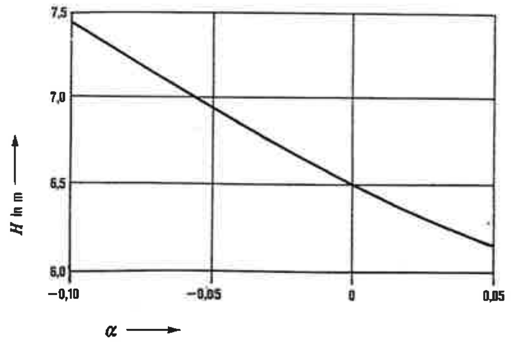

Fig. 2: Critical height as function of the temperature coefficient α for the linear symmetric two point model

Comparison between the linear models

The critical core height has been determined as a function of the number of space points for two standardized neutron flux densities, a flat flux (18) and a sinusoidal flux density. The latter is defined as a flux density having constant equilibrium buckling at all space points.

$$B_k^{20} = \frac{(N+1)^2\, 2}{H^2}\left[1 - \cos\frac{\pi}{N+1}\right] \qquad k = 1, \ldots, N$$

where H is core height.

It converges to $B^{20} = \dfrac{\pi^2}{H^2}$

for infinite N. The flux density then approaches a sine curve. The mean flux density is defined

$$\overline{\Phi} = \frac{1}{N}\sum_{1}^{N} \Phi_k^0$$

for the flat flux density, and

$$\overline{\Phi} = \frac{1}{N+1}\sum_{1}^{N} \Phi_k^0$$

for the sinusoidal flux density.

We assume homogeneous control of the core. Fig. 3 shows the critical height as a function of the number of space points for the two flux shapes. We verify the previously known result that the critical height is greater for the sinusoidal flux. The stability limit for a certain flux form converges

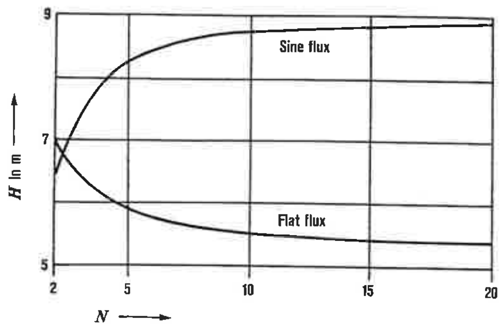

Fig. 3: Critical core height for multipoint models as function of the number of meshpoints of the core (N) for two different symmetric flux shapes

nearly exponentially to a constant value, which depends on the form factor

$$\psi = \frac{\Phi_{max}}{\Phi_{mean}}$$

Call the critical height for N space points $H(N)$. If $H(N)$ is extrapolated from 2, 3 and 4 space points, the result is $H(\infty) = 5{,}55$ m and $\hat{H}(20) = 5{,}55$ m for the flat flux. $H(20)$ has been found to be 5,38 m, so the error is 3,2 %.

A similar extrapolation of the sinusoidal flux curve gives $\hat{H}(\infty) = \hat{H}(20) = 8{,}68$ m, while the computed value of $H(20)$ is 8,89 m. The difference is 2,4 %.

The period of the flat flux oscillation is estimated surprisingly accurately by the two point model. It is found to be 23,81 hours, quite the same with five figures accuracy as the twenty point model. When mean flux is increased five times, the period T (34) decreases to about half. The difference to a ten point flat flux model is less than 10^{-3} hours.

The sinusoidal flux, however, is not so well described by the two point model. This is easily understood, since the curvature cannot be shown by only two points. Even three points will give a much more accurate description as the centre point represents the top of the curve better. The period is:

20,56 hrs for 2 points

22,55 hrs for 3 points

23,09 hrs for 10 points.

The critical height of the three point sinusoidal flux (Fig. 3) is also much better than the height, found with the two point model. The error is 50 % smaller.

The temperature coefficient is another interesting parameter (Fig. 2).

It is a well-known fact that the critical height decreases for increasing temperature coefficient. The ratio of the relative changes of the critical height and the temperature coefficient α is calculated. It is here called Q.

For $-0{,}05 < \alpha < 0$

was found $Q = -0{,}0622$

for all flat fluxes from two through ten meshpoints.

For the sinusoidal flux was found

$Q = -0{,}081$ for the two point model,

$Q = -0{,}069$ for three meshpoints and

$Q = -0{,}068$ for ten meshpoints.

Thus, we find also here, that a three point model gives a rather accurate description of the sinusoidal flux.

Finally some results of critical height calculations with different models are compared, viz. the TRAXEN model, the linear models presented here and a modal expansion model based on clean reactor modes, presented by other authors [2, 17]. The flux shapes are shown in Fig. 4 and the results are listed in Table 1.

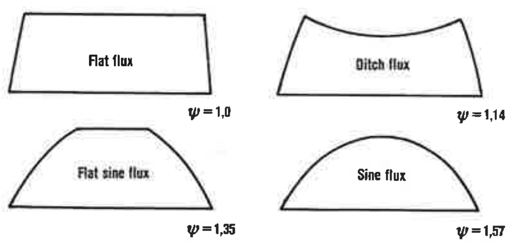

Fig. 4: Different flux shapes used in the calculations

Table 1: Critical heights in m for some neutron flux densities calculated with different models. ($\overline{\Phi} = 1$, $\alpha = -0{,}0514$)

Tab. 1: Kritische Kerndimensionen in m für einige Neutronenflußdichten

Form factor	TRAXEN (20 points)	Modal expansion	Linear model (20 points)
1,0	5,36	5,26	5,38
1,14	5,15	5,20	—
1,35	7,50	7,60	—
1,57	8,89	8,82	8,89

Transient amplitude

As the flux deviation, caused by xenon, must be limited of technological reasons we are interested not only in stability but also in the amplitude of the transients. Fig. 5 shows the maximum flux deviation during a transient for the two point model (30) and the TRAXEN model.

The latter has simulated a symmetric flux with form factor 1,29 approximated by 20 space points [11]. The disturbance consisted of a stepwise movement of 100 pcm reactivity from one core half to the other. The critical heights are 7,25 m (TRAXEN) and 6,93 m (two point model) respectively. The difference between the simulations and the two point model is all the time within 10 %. Around the critical heights the difference is only 2,6 %.

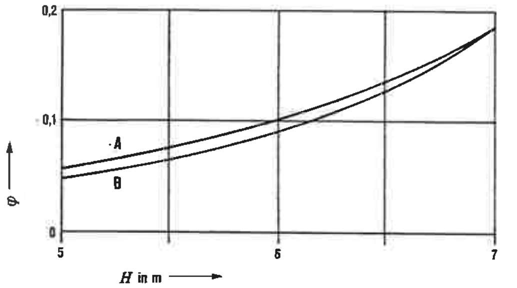

Fig. 5: The maximum amplitude of the flux deviation for different core heights after a 100 pcm reactivity step disturbance. Comparison is made between simulations with TRAXEN (A) and analytical results with a two point model (B)

Nonlinear solutions

General behaviour

The simplified models are also useful for non-linear analysis. It is easy to study the essential influence of the different non-linear terms. Both stable and unstable periodic solutions have been found with the two point model. The nature of the solutions depends very much on the control rod configuration. The qualitative behaviour of the non-linear two point model has been verified by the TRAXEN digital simulations [11]. The amplitudes of the two models are quite different, when the control consists of a rod. Later we explain, why this difference occurs.

Even for the two point model it will be cumbersome to analyse the equations analytically. Therefore a digital program has been written to simulate the two cases rod control and homogeneous control of the two point model.

We have shown, that the control configuration does not affect the stability limit in the linear case. However, it is very important at large disturbances. As the "rod" is acting only in one point, the periodic solutions are very unsymmetric.

Both the eigenvalues of the linear system and the non-linear character of the solutions depend on the temperature coefficient.

Rod control

Fig. 6 explains the behaviour of the system. It shows a qualitative phase plane of the system with two different temperature coefficients, and in every column the core height is decreasing from A to E and F to K respectively.

Some important conclusions can be drawn. At small core heights (E and K) all trajectories are stable and no periodic solutions can be found. When the core size increases, it is possible to get unstable solutions for rather small disturbances (D). However, if the temperature coefficient is negative enough, the unstable limit cycle will disappear (J). The

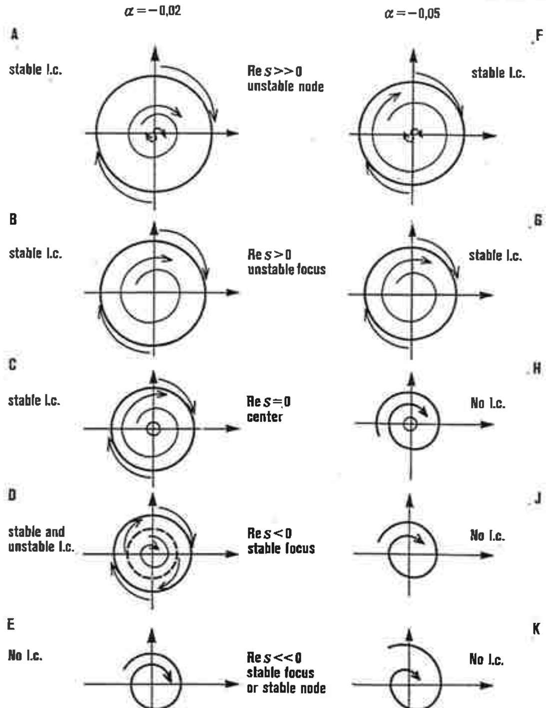

Fig. 6: Qualitative phase planes of the non-linear two point model of a symmetric flux, with rod control, and different temperature coefficients α and core heights. Re s = real part of the greatest eigenvalue of the linearized model

amplitude of the unstable limit cycle decreases as the core height increases, and it approaches zero at the critical height (C). It will never occur for the more negative α (H).

There are also stable limit cycles, and they occur for less negative temperature coefficients even below the critical height (D). For more negative α they occur only over the critical height (G).

The amplitude of the limit cycle increases with core height and decreases when α gets more negative. The period time is between 24 and 25 hours.

The trajectories are very unsymmetric, a result which has not been verified by other models. This discrepancy is explained later. The symmetry is better for more negative α, as the large amplitudes are damped by the temperature feedback. One example of stable limit cycle is shown in Fig. 7, which corresponds to Fig. 6 (G).

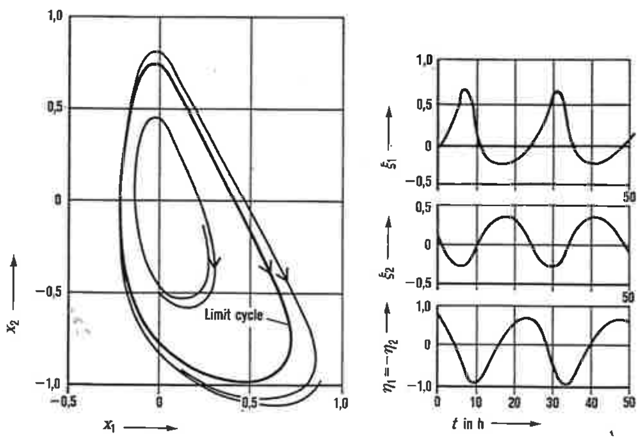

Fig. 7: Projection of the state space and the limit cycle as function of time of the non-linear two point model with rod control. The singular point at the origin is unstable and the trajectories from origin diverge towards a stable limit cycle. The variables are defined in (5) and (16).
$\alpha = -0{,}05$ $H = 6{,}97$ m ($H_{crit} = 6{,}92$ m)
Compare with Fig. 6 (G)

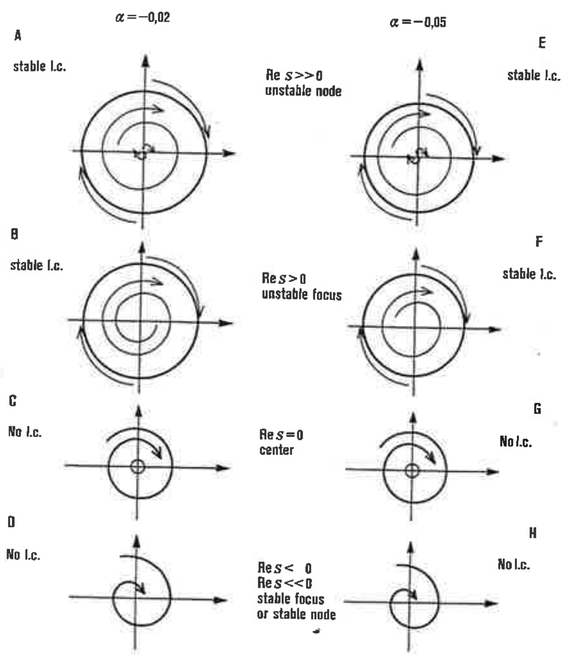

Fig. 8: Qualitative phase planes of the non-linear two point model of a symmetric flux, with homogeneous control, and different temperature coefficients α and core heights. Re s = real part of greatest eigenvalue of the linearized model

Homogeneous control

The critical heights are not changed by the control, as we have proved previously.

Fig. 8 shows that no unstable limit cycle will occur, contrary to the rod control case. The amplitudes of the stable limit cycles are smaller, and the shape of the trajectories is more regular. $x_3 = \xi_1 + \xi_2$ is small all the time and $x_4 = \eta_1 + \eta_2$ converges exponentially to zero. Thus the xenon and iodine deviations in point one are directed opposite the deviations in point two.

Fig. 9 shows a stable limit cycle, corresponding to case 8 (F). The period time is 24 h. The amplitude of the limit cycle is very sensitive to changes in core height. It decreases to 60 % for a 3 cm decrease in core size and is zero at the stability limit, a 2 cm further decrease.

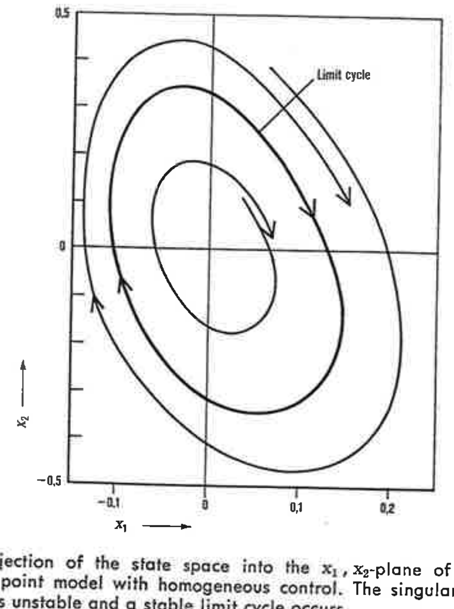

Fig. 9: Projection of the state space into the x_1, x_2-plane of the non-linear two point model with homogeneous control. The singular point at the origin is unstable and a stable limit cycle occurs.
$\alpha = -0{,}05$ $H = 6{,}97$ m ($H_{crit} = 6{,}92$ m)
Compare with Fig. 8 (F)

Stable periodic solutions, but with larger amplitudes, occur even for positive temperature coefficients. This result contradicts the previous results [5], where nonoscillating unstable trajectories were found. Similar nonoscillating solutions occur for space independent models [11], but they have not been verified by any other refined model.

Comparison with more complex models

The qualitative performance of the non-linear two point model has been verified by digital simulation of the non-linear TRAXEN model [11]. The amplitudes of the limit cycles differ between the models, a fact which can be explained.

In order to be able to compare the calculations, we should use the same core parameters. As the critical heights of the two models differ 28 % (Fig. 3), we can only compare the order of magnitude.

In general the unstable limit cycles are smaller and the stable limit cycles larger in the two point model, compared with the TRAXEN model.

The rod configuration is the most important cause of difference. In the two point model the "rod" is acting in one point (24), which means that the absorption is uniformly "distributed" along half the core. As the oscillations are mainly described as first overtone variations, this configuration has a maximum damping or amplifying effect on the amplitudes, and Fig. 7 shows clearly, that the transient is damped once and amplified once during a cycle. These effects are due to the rod.

When a symmetric flux is disturbed, the absorption in the core must be increased [11]. This will make the flux in the "rod point" decrease. Now, if the disturbance has the same direction as the absorption increase, the rod causes an amplification. Thus it is easier to get unstable solutions and the stable limit cycles have a larger amplitude.

In the TRAXEN model the rod arrangement is different, as the rod is inserted and withdrawn during an oscillation. The amplitude of this movement depends on the absorption along the rod and on the disturbance amplitude. The amplifying or damping effect gets smaller, and consequently also the amplitudes of the stable limit cycles. For the same reason it is more difficult to get unstable limit cycles with the TRAXEN model. For ditch fluxes (Fig. 4) the unstable limit cycles are verified. A ditch flux, some 10 cm below the critical height, was disturbed by 100 pcm moved from upper to lower core. This disturbance caused unstable oscillations [11].

In order to get a rod configuration more like the two point model we must use a rod, always inserted into half the core with a variable absorption. The rod configuration problem is subjected a more detailed discussion in a technical report [11].

For homogeneous control it is also possible to verify the different kinds of solutions, shown in Fig. 8.

The amplitudes of the periodic solutions are of the same order of magnitude in the two models. This is quite natural, as no rod can influence on the result.

Fig. 10 shows a numerical example of a flat flux, $H = 5{,}40$ m, $\alpha = -0{,}0514$, 4 cm over the critical height. A stable periodic solution occurs. Large disturbances result in stable trajectories, while small ones cause unstable solutions.

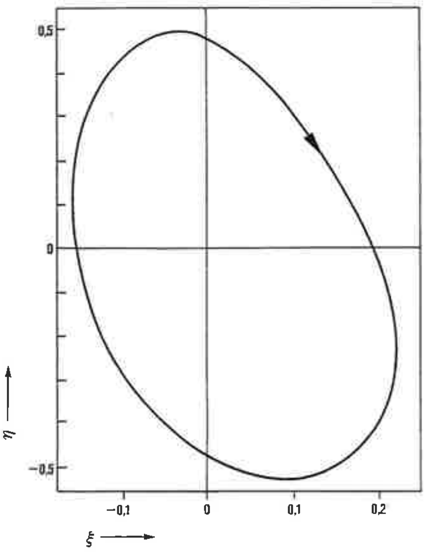

Fig. 10: A limit cycle, calculated with the TRAXEN program for a 20 point non-linear reactor model with $H = 5{,}40$ m ($H_{crit} = 5{,}36$ m), where the variables are the maximum xenon and iodine deviations. Control is homogeneous and flux shape is flat.

The limit cycle is stable. A disturbance of more than 400 pcm reactivity, moved from one core half to the other during two hours, will bring the trajectories outside the limit cycle

Conclusions

Digital simulation is a bad tool in the examination of the principal behaviour of xenon spatial oscillations. The simplified models have been valuable in the preliminary studies of different parameter influences. The models have predicted nonlinear behaviour, such as periodic solutions. Because of these principal examinations it has been possible to come through the digital simulations much faster.

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