



# LUND UNIVERSITY

## Bargaining in Collusive Markets

Andersson, Ola

2006

[Link to publication](#)

*Citation for published version (APA):*

Andersson, O. (2006). *Bargaining in Collusive Markets*. (Working Papers, Department of Economics, Lund University; No. 21). Department of Economics, Lund University.  
[http://swopec.hhs.se/lunewp/abs/lunewp2006\\_021.htm](http://swopec.hhs.se/lunewp/abs/lunewp2006_021.htm)

*Total number of authors:*

1

**General rights**

Unless other specific re-use rights are stated the following general rights apply:

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

Read more about Creative commons licenses: <https://creativecommons.org/licenses/>

**Take down policy**

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

LUND UNIVERSITY

PO Box 117  
221 00 Lund  
+46 46-222 00 00



# Bargaining in Collusive Markets\*

Ola Andersson<sup>†</sup>  
Lund University

November 14, 2006

**Abstract.** In this paper we investigate collusion in an infinitely repeated Bertrand duopoly where firms have different discount factors. In order to study how a collusive agreement is reached we model the equilibrium selection as an alternating-offer bargaining game. The selected equilibrium has several appealing features: First, it is efficient in the sense that it entails immediate agreement on the monopoly price. Second, the equilibrium shows how discount factors affect equilibrium market shares. A comparative statics analysis on equilibrium market shares reveals that changes in discount factors may have ambiguous effects on market shares.

**JEL:** C72, D43, L11, L41

**Keywords:** Bargaining, different discount factors, explicit collusion, market shares

## 1 Introduction

Traditionally, most theoretical investigations of repeated interaction and collusion have focused on what outcomes can be sustained as subgame perfect equilibria (SPE) (Feuerstein 2005). This has led to an "embarrassment of riches" (Tirole 1988) -almost everything is an equilibrium. However, this presumes that firms can easily negotiate and agree on *which* equilibrium to implement. There is a large body of empirical studies on how firms collude

---

\*We thank Hans Carlsson, Antonio Cabrales, Håkan Holm, Robin Rander and seminar participants at the 2006 EARIE conference in Amsterdam and Lund University for valuable comments and suggestions. Financial support from Wallander-Hedelius foundation is gratefully acknowledged.

<sup>†</sup>Department of Economics, Lund University, Box 7082, 220 07 Lund, Sweden, email: ola.andersson@nek.lu.se, Tel. +46 (0)46 222 00 00, Fax. +46 (0)46 222 41 18

and some of them put forward evidence on meetings and communication between colluding firms (e.g. Genosove and Mullin 2001, Howe 1973 and Becker 1971). For instance, Levenstein (1997) finds evidence of colluding firms meeting regularly in order to agree on a collusive agenda. Moreover, in a survey of cartels, Levenstein and Suslow note that:

Bargaining problems were much more likely to undermine collusion than was secret cheating. About one quarter of the cartel episodes ended because of bargaining problems. Bargaining issues affected virtually every industry studied. (Levenstein and Suslow 2002, p. 16)

Despite this observation, most theoretical models of repeated interaction assume that negotiations are frictionless. This enables theorists to glance over the problem of how a collusive strategy can readily be attained. Since, in a perfect bargaining environment we need not model the actual bargaining, we can simply assume that it occurs. We conjecture that, by restricting attention to frictionless bargaining, we ignore problems that actual firms face. If we instead consider imperfect negotiations we are forced to think about how colluding firms negotiate. Moreover, this enables us to analyze what effect firm characteristics may have on negotiations and on their outcomes.

Historically, IO models of repeated interaction have focused on either price or quantity competition. However, in a study of about twenty cartel decisions made by the European Commission, Harrington (2006) notes that almost every cartel coordinated on both issues. This paper tries to overcome this shortcoming by incorporating market share decisions into the strategy space of firms.

In this paper we study firms, with different discount factors, negotiating over how to collude in an infinitely repeated Bertrand game (IRBG). We focus on the role of discount factors because they are important determinants for firms trying to collude. Moreover, the case of allowing discount factors to differ has not received much attention previously.<sup>1</sup>

---

<sup>1</sup>For a thorough discussion why firms might have different discount factors see Harrington (1989).

## 2 Related Literature

In a seminal paper Friedman (1971) studies the set of non-cooperative equilibria in an infinitely repeated game where firms have different discount factors. He finds that, if firms have sufficiently high discount factors, there exist subgame perfect equilibria (SPE) that do not consist of playing a stage game equilibrium in every period. To overcome the problem of multiple SPE Friedman makes an ad hoc assumption that firms choose the equilibrium that give the firm equal temptation to deviate from the equilibrium path. As mentioned before, we believe that this greatly oversimplifies the negotiation problem faced by colluding firms. Moreover, it is not clear that this equilibrium selection criterion is the relevant one for firms. Subsequent game theoretic papers on repeated games have concentrated on the case of equal discount factors.<sup>23</sup>

There have been relatively few attempts to model bargaining between firms in repeated oligopoly interactions. However, Harrington (1989) (H89) investigates bargaining and collusion in an IRBG where firms have different discount factors. In this model collusive prices and market shares are determined by a Nash bargaining solution. H89 finds that there exists a unique equilibrium. Moreover, if firms' discount factors are above 0.5 the equilibrium is symmetric. We find this symmetry property surprising since, if firms are asymmetric why should we expect the equilibrium to be symmetric. The reason for this symmetry property is that H89 restricts attention to stationary strategies and this makes the Nash bargaining solution independent of discount factors.<sup>4</sup>

In this paper we take a similar approach as in H89, however, we model the bargaining as an alternating-offer bargaining game (Rubinstein 1982) where firms take turns to make proposals on how to collude in the subsequent IRBG. We think that this approach will better capture how discount factors affect bargaining and collusion between firms.

---

<sup>2</sup>See for instance Fudenberg and Maskin (1986) and Abreu (1986, 1988)

<sup>3</sup>One exception is Lehrer and Pauzner (1999) who study the set of payoffs that can be implemented as SPE when discount factors goes to one. They find that this set is generally larger than under equal discounting.

<sup>4</sup>By restricting attention to stationary strategies the discounted sum of future profits can be rewritten as  $\frac{1}{(1-\delta_i)}\pi_i$  for some profit  $\pi_i$  of the stage game. Since the Nash bargaining solution is invariant to affine transformations of profits the effect of discount factors is absent.

### 3 The Infinitely Repeated Bertrand Game

#### 3.1 The Stage Game

Consider an industry with two firms producing a homogenous product using identical constant returns to scale production technology. Without loss of generality we normalize marginal costs to zero. The market demand function  $D(p)$  is assumed to satisfy the following assumptions.

- A1:  $D : \mathbb{R}_+ \rightarrow \mathbb{R}_+$  is a continuous and bounded function.
- A2:  $\exists \bar{p} > 0$  such that  $D(p) = 0$  if and only if  $p \geq \bar{p}$ .
- A3:  $D(p)$  is continuous and strictly decreasing in  $p \forall p \leq \bar{p}$
- A4: There exists a unique industry monopoly price  $p^m$

Firms compete by simultaneously choosing a prices  $p_i \in P = \mathbb{R}_+$   $i = 1, 2$ . Furthermore, define  $p = \min\{p_i, p_j\}$   $i \neq j$  as the market price. The firm that sets the lowest price serves the entire market. A standard assumption in textbook treatments on Bertrand games is that, in case of a price tie, demand is allocated equally among firms. In line with the observations in Harrington (2006) we will let firms allocate the market arbitrarily between them by stating a market share. We also assume that there can be no rationing on consumers. There are several ways to model how market shares get allocated, we choose the following: In addition to choosing a price firms also choose a market share  $s_i \in [0, 1]$ . In case of a price tie and  $s_1 + s_2 = 1$  firms get their quoted share. However, if there is a price tie and  $s_1 + s_2 \neq 1$  firms share the market equally.<sup>5</sup> Formally individual demand equals

$$D_i(p_i, p_j, s_i, s_j) = \begin{cases} 0 & \text{if } p_i > p_j \\ D(p_i) & \text{if } p_i < p_j \\ s_i D(p_i) & \text{if } p_i = p_j \text{ and } s_1 + s_2 = 1 \\ \frac{D(p_i)}{2} & \text{if } p_i = p_j \text{ and } s_1 + s_2 \neq 1 \end{cases} \quad (1)$$

We let  $\pi_i(p_i, p_j, s_i, s_j) = p_i D_i$   $i = 1, 2$  denote the stage game profit of firm  $i$ . Under assumptions A1 - A3 both firms set  $p_i = 0$  and hence earn zero profit in any Nash equilibrium.<sup>6</sup>

---

<sup>5</sup>A similar structure is used in Athey and Bagwell (2001).

<sup>6</sup>Contrary to standard Bertrand games there is actually a continuum of equilibria in this game; one for every possible combination of market shares. However, the important

### 3.2 The Repeated Game

Now consider the stage game, described in the previous section, being repeated an infinite number of times  $t \in \{k, k+1, \dots, \infty\}$  where  $k \in 1, 2, \dots$ . For the moment we can, without loss of generality, let  $k = 1$ . A history  $h(t) \in H(t)$  of the repeated game is the sequence of past price pairs and market shares, hence  $h(k)$  is the "null" history, i.e. the empty sequence, and  $h(t) = \{(p_1, p_2, s_1, s_2)(t)\}_{t=k}^{t-1}$  for  $t \in \{k+1, k+2, \dots, \infty\}$ . A strategy  $\sigma_i$  for firm  $i$  is an infinite sequence of maps  $\sigma_i = \{\sigma_i(t)\}_{t=k}^{\infty}$  where for each  $t$   $\sigma_i(t) : H(t) \rightarrow P \times [0, 1]$ . The objective for each firm is to maximize  $\Pi_i = (1 - \delta_i) \sum_{t=k}^{\infty} \delta_i^{t-1} \pi_i(\sigma_i(t))$  where  $\delta_i \in (0, 1)$  is the firm specific discount factor. We call the firm with the highest discount factor *firm 1* and the firm with the lowest discount factor *firm 2*.

We assume that firms are restricted to use a slightly modified version of the grim trigger strategy presented in Friedman (1971). The strategy specifies that firms start by setting a collusive price and market share and continue to do so until someone deviates. If the market price in the previous period deviated from the collusive price they then set  $p = 0$  forever. We also specify that firms do not alter their market shares in the punishment phase. This is without loss of generality since in the punishment phase both firms earn zero profit. Henceforth we will focus on prices in  $(0, p^m]$  and market shares such that  $s_1 + s_2 = 1$  and thus  $s_1, s_2 \in \Delta$  where  $\Delta$  is the one dimensional simplex.<sup>7</sup> Stated formally, letting  $\bar{p}, \bar{s}_1$  and  $\bar{s}_2$  denote the collusive price and market shares

$$\sigma_i(t) = \begin{cases} (\bar{p}, \bar{s}_i) & \text{if } t = k \text{ or } h(\tau) = (\bar{p}, \bar{p}, \bar{s}_1, \bar{s}_2) \quad \forall \tau \leq t \\ (0, \bar{s}_i) & \text{otherwise} \end{cases}$$

A necessary and sufficient condition for a pair of trigger strategies to be an SPE is that

$$\frac{1}{1 - \delta_i} \bar{p} \bar{s}_i D(\bar{p}) \geq \bar{p} D(\bar{p}) \quad i = 1, 2 \quad (2)$$

---

issue here is that every equilibrium leads to zero profits.

<sup>7</sup> We focus on prices in  $(0, p^m]$  because there is no reason for firms to decide on a higher price since this will impose further restrictions on the collusive strategies. To see this it suffices to note that for  $p \in (p^m, \bar{p})$  (2) simplifies to  $s_i \geq (1 - \delta_i) \frac{p^m D(p^m)}{p D(p)}$ . By definition  $p^m D(p^m) \geq p D(p)$ .

The inequality simply states that *firm i* must weakly prefer staying in the collusive phase to deviating. The best deviation is setting a price slightly under the collusive price. Inequality (2) simplifies to

$$\bar{s}_i \geq (1 - \delta_i) \quad i = 1, 2 \quad (3)$$

This gives a lower bound on each firm's market share. The bound is decreasing in  $\delta_i$  which means that as a firm gets more patient it requires less market share. The set of SPE allocations  $N(\delta)$  is thus

$$N(\delta) \equiv \{(p, s_1, s_2) \in [0, p^m] \times \Delta \mid s_i \geq (1 - \delta_i), \text{ or } p = 0\} \quad (4)$$

Since the one-shot Nash equilibria are independent of the discount factor  $N(\delta) \neq \emptyset$ . However, for  $N(\delta)$  to include other elements we must have  $\delta_1 + \delta_2 \geq 1$ .<sup>8</sup>

Given  $N(\delta)$  we can now describe the set of payoffs that are sustainable as SPE payoffs.

$$V(\delta) \equiv \{(v_1, v_2) \mid \exists (p, s_1, s_2) \in N(\delta), v_i = \pi_i(p, s) \quad i = 1, 2\} \quad (5)$$

$V(\delta)$  defines an SPE "slice" of the "cake" whose elements can be attained by choosing an appropriate price and market share allocation from  $N(\delta)$ . However, it is one task for firms to realize and agree on the existence of  $V(\delta)$  and quite a different, and potentially more difficult, one to coordinate and agree on which element of  $V(\delta)$  to implement. In this paper we assume that firms meet before the IRBG begins to negotiate over what equilibrium to implement. This approach is also taken in H89 where the bargaining is modelled as a Nash bargaining game. We notice two problems with using the Nash bargaining game: First, due to the Pareto optimality axiom underlying the Nash bargaining solution, monopoly pricing is assumed. Second, as noted in the introduction, the objective function, i.e. the Nash product, is independent of firms' discount factors. In line with Binmore et. al (1986) we think that any asymmetry between firms should be captured in the Nash product and thus be an important determinant in the bargaining process.

---

<sup>8</sup>By summing up inequality 3 for  $i = 1, 2$  and using  $\bar{s}_1 + \bar{s}_2 = 1$  we get the stated result.

## 4 The Bargaining Model

As we saw in the previous section the set of possible collusive agreements that are subgame perfect in the IRBG may be quite large as long as firms are sufficiently patient. To answer the question of which element of  $N(\delta)$  firms will select we model the equilibrium selection process as a generalized alternating-offer bargaining game, i.e. the Rubinstein (Rubinstein 1982) game which was generalized in Binmore (1987), where firms take turns to propose a price and market share allocation. As in H89 the bargaining takes place before firms enter into IRBG.<sup>9</sup> Once a proposal is accepted the bargaining game ends and the IRBG begins, i.e. we do not allow renegotiation. We assume that firms only make proposals that can be implemented as an SPE in the IRBG. That is, the proposals have to be elements of  $N(\delta)$ . We now turn to a formal description of the bargaining process.

We study two different bargaining protocols,  $P1$  and  $P2$ .  $P1$ : firm 1 makes a proposal  $(p, s^1) \in N(\delta)$ , where  $s^i = (s_1^i, s_2^i) \in \Delta$ , at  $t = 1$  and at every subsequent odd  $t$  if no agreement has been reached at an earlier time period. Moreover, firm 1 responds to offers from firm 2 at every even  $t$  if no agreement has been reached at an earlier time period. Consequently firm 2 makes proposals at every even  $t$  and gets to respond to offers at odd  $t$ . In the second protocol  $P2$  the roles of firm 1 and firm 2 are reversed.

A history in the game consists of all previous proposals, thus  $h(1)$  is the "null" history and  $h(t) = \{(p, s^i)(t)\}_{t=1}^{t-1} \in \mathcal{H}(t)$  for  $t \in \{1, 2, \dots, \infty\}$ . A bargaining strategy  $\psi_1$  for firm 1 is an infinite sequence of maps  $\psi_1 = \{\psi_1(t)\}_{t=1}^{\infty}$ . Where, for each  $t$   $\psi_1(t) : \mathcal{H}(t) \rightarrow [0, p^m] \times \Delta$ ,  $\psi_1(t) = (p, s^i)(t) \in N(\delta)$  in all periods where it is 1's turn to make a proposal and  $\psi_1(t) : \mathcal{H}(t) \rightarrow \{Y, N\}$  in periods when it is firm 2's turn to make a proposal. A bargaining strategy  $\psi_2$  for firm 2 defined analogously. There may be unboundedly long bargaining paths where all offers are rejected. All these paths lead to the same zero profit outcome which we denote as the disagreement outcome  $D$ . The outcome of a bargaining strategy pair is a tuple  $d(\psi) = (p, s_1, s_2, k) \in N(\delta) \times \{1, 2, \dots\} \cup D$  where  $k$  is the time period when agreement is reached.

We restrict attention to pairs of strategies  $(\psi, \sigma)$  where  $d(\psi)$  is implemented in the initial phase,  $t = k$ , of  $\sigma$ . This rules out uninteresting equilibria where firms make an agreement in the bargaining game then ignore it

---

<sup>9</sup>This assumption is relaxed in section 5.1, where simultaneous bargaining and price competition is considered.

and play something else in the subsequent IRBG.

#### 4.1 Equilibrium Analysis

In alternating offer bargaining games firm 1 has a *strategic advantage*; because it has the highest discount factor it is less eager, than firm 2, to settle quickly on an agreement. firm 1 can use this to propose an agreement in its own favor. However, due to the structure of strategies in the IRBG firm 2 also has a potential advantage since more market share must be allocated to it in order for the agreement to satisfy the IC constraints in (3). These two effects work in opposite direction and will, as we will see, have a strong influence on the structure of equilibrium agreements. Note also that a first-mover advantage is embedded in the bargaining procedure. We deal with this and present the outcomes under these two protocols separately.

The main objective of this paper is to derive a unique solution to the bargaining game -and thereby to determine what SPE strategy of the IRBG to implement. This is established in *Proposition 1* which will be proved in a sequence of lemmas.

**Proposition 1** *For every combination of  $\delta_1$  and  $\delta_2$  such that  $\delta_1 + \delta_2 \geq 1$  there exists a unique SPE  $\bar{\psi}$ . Outcomes,  $d(\bar{\psi})$ , are presented in Table 1.*

**Table 1. SPE Outcomes**

	$\underline{s}_i \leq \delta_i(1 - \delta_j)\lambda$	$\underline{s}_1 \leq \delta_1(1 - \delta_2)\lambda, \underline{s}_2 > \delta_2(1 - \delta_1)\lambda$	$\underline{s}_i > \delta_i(1 - \delta_j)\lambda$
$P1$	$(p^m, (1 - \delta_2)\lambda, \delta_2(1 - \delta_1)\lambda, 1)$	$(p^m, 1 - \underline{s}_2, \underline{s}_2, 1)$	$(p^m, 1 - \underline{s}_2, \underline{s}_2, 1)$
$P2$	$(p^m, \delta_1(1 - \delta_2)\lambda, (1 - \delta_1)\lambda, 1)$	$(p^m, \delta_1(1 - \underline{s}_2), 1 - \delta_1(1 - \underline{s}_2), 1)$	$(p^m, \underline{s}_1, 1 - \underline{s}_1, 1)$

*Lemma 1* will enable us to restrict attention to the monopoly price and *Lemma 2 - 4* will describe the equilibrium proposals for different values of  $\delta_1$  and  $\delta_2$ . *Lemma 1 - 4* and all proofs are given in the appendix. For expositional purpose we define  $\lambda \equiv \frac{1}{1 - \delta_1 \delta_2}$ . Moreover, let  $\underline{s}_i \equiv (1 - \delta_i)$   $i = 1, 2$  as the IC constraints on the market sharing agreements.

Since the set of possible proposals is quite large it will be helpful to exclude those elements that never arise in equilibrium. Intuitively there is no rivalry on the price selection and therefore we should expect that there is no real disagreement about the price. *Lemma 1* confirms this thought and establishes that firms' SPE proposals always entail agreement on the

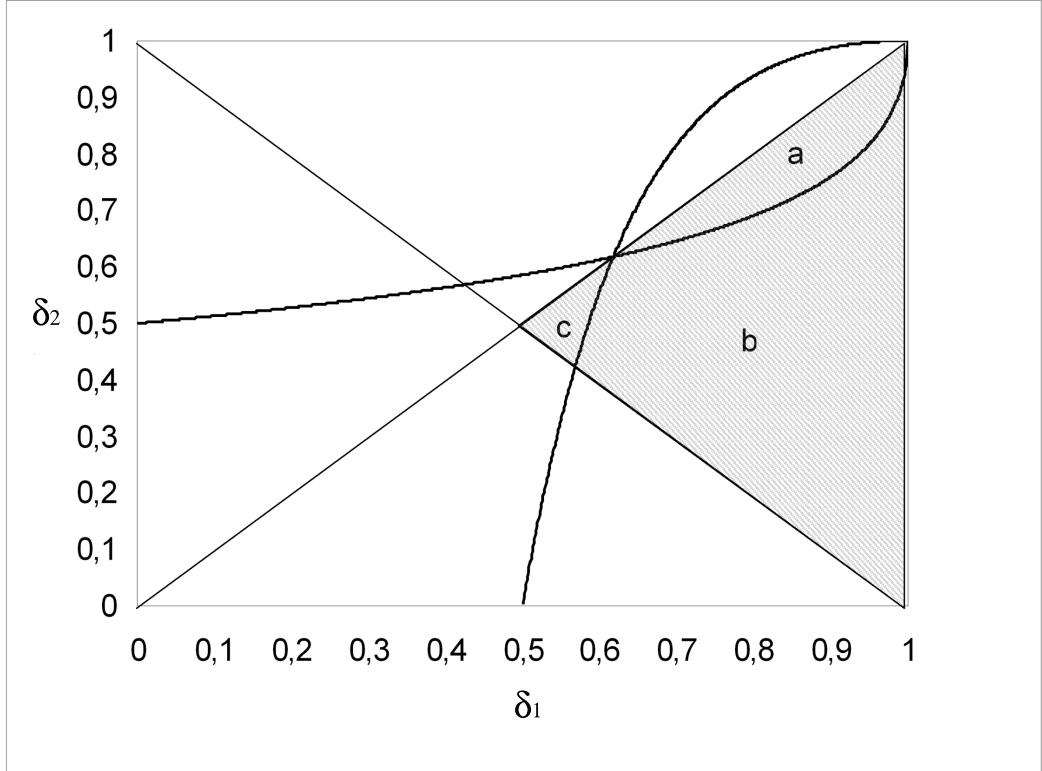


Figure 1: The constraints on discount factors

monopoly price  $p^m$ . This reduces the alternating-offer bargaining game into a problem of proposing market shares and actually shares many features with the familiar Rubinstein bargaining game (Rubinstein 1982).

Let us now turn to describing the equilibrium agreements. From *Table 1* it is evident that the structure of the equilibrium depends on whether  $\delta_i(1 - \delta_j)\lambda \geq \underline{s}_i$  or  $\delta_i(1 - \delta_j)\lambda < \underline{s}_i$   $i = 1, 2$ .

The boundary  $\underline{s}_i = \delta_i(1 - \delta_j)\lambda$  has one real solution,  $\delta_i = \frac{(1 - \sqrt{1 - \delta_j})}{\delta_j}$ . These two constraints are depicted in *Figure 1* together with the two restrictions  $\delta_1 \geq \delta_2$  and  $\delta_1 + \delta_2 \geq 1$ . The shaded region shows where the two restrictions hold. As can be seen in *Figure 1* this region is divided into three areas, *a*, *b* and *c*. We will now turn to a discussion of the equilibrium agreements for pairs of discount factors in the three different areas.

The second column of *Table 1* corresponds to area *a* in *Figure 1*, a

situation where both firms have high and not to different discount factors. Under these conditions *Lemma 2* shows that equilibrium market shares will perfectly reflect firm 1's strategic advantage. In this case the proposing firm's proposal is only constrained by the backward induction constraint that its allocation proposal cannot exceed what the other firm can expect to get in the subsequent subgame. Thus the IC boundaries do not affect the solution.

The third column of *Table 1* corresponds to area *b* in *Figure 1*. *Lemma 3* shows that, in this situation, firm 1 cannot fully use its strategic advantage since it must also take firm 2's IC constraint into consideration when it makes a proposal. In fact firm 1 offers firm 2 exactly what is required to fulfill its IC constraint. It is worth noting that this can happen even if both discount factors are close to one.

The fourth column of *Table 1* corresponds to area *c* in *Figure 1*, a situation where both firms have low discount factors. *Lemma 4* shows that both firms make equilibrium proposals that give the opponent exactly what is required by the corresponding IC constraint. This follows from the fact that both firms are so impatient that it is better for firms to accept an agreement on its IC boundary than to wait for its turn to make a proposal.

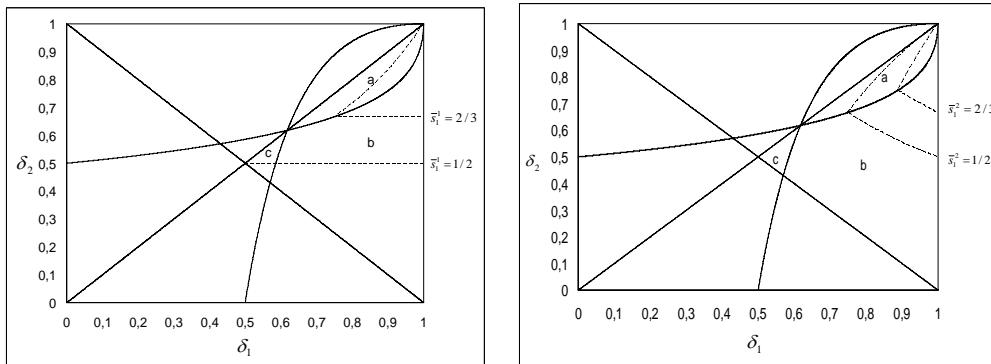


Figure 2: Level curves of  $\bar{s}_1^1$  under  $P1$     Figure 3: Level curves of  $\bar{s}_1^2$  under  $P2$

By plotting some level curves of the equilibrium market share functions of firm 1, some further insights can be gained. For this purpose, let  $\bar{s}_i^j$  be the equilibrium market share for firm  $i$  under bargaining protocol  $P_j$ . In figures 2 and 3 we have complemented *Figure 1* with two level curves of  $\bar{s}_1^j$ . By focusing on the level curve  $\bar{s}_1^1 = 1/2$  in *Figure 2* it is easily seen that, for combinations of  $\delta_1$  and  $\delta_2$  in area *a*, firm 1 is always endowed with an

equilibrium market share above 1/2. However, for combinations of  $\delta_1$  and  $\delta_2$  in area  $b$  and  $c$  this is not always true. Shifting attention to  $\bar{s}_1^2 = 1/2$  in *Figure 3* one can deduce that, for combinations of  $\delta_1$  and  $\delta_2$  in area  $c$ , firm 1 always has an equilibrium market share below 1/2. Moreover, this is not true for combinations of  $\delta_1$  and  $\delta_2$  in area  $a$  and  $b$ .

#### 4.1.1 Discussion

*Proposition 1* shows that there exist a unique equilibrium which, moreover, has several interesting properties. First, the equilibrium agreement is efficient in the sense that it entails immediate agreement on the monopoly price. Second, discount factors have a strong influence on the structure of equilibrium agreements. On one hand, firm 2 requires a larger market share to make the proposal meet the IC constraint. On the other hand firm 2 is more eager to settle quickly, and is thus more willing to accept less favorable proposals. Interestingly these two effects work in opposite directions. *Proposition 1* shows that these two effects have an intricate relation, and as we consider different combinations of  $\delta_1$  and  $\delta_2$  we see how they affect equilibrium agreements. Generally we find that agreements are asymmetric, thus we do not find the strong symmetry properties reported in H89.

## 4.2 Comparative Statics

Since we have derived a unique solution it also makes sense to calculate some comparative statics. We divide the analysis into three parts accordingly to the three areas  $a$ ,  $b$  and  $c$  in *Figure 1*. The reader can also get some intuition to the comparative statics analysis by studying figures 4.1 and 4.1.

Area  $a$  in *Figure 1* depicts a situation where firms' discount factors are high and not too different. By taking the partial derivative of the equilibrium proposals, presented in column two of *Table 1*, with respect to  $\delta_1$  and  $\delta_2$  we get the following, where  $\bar{s}_i^j$  is the equilibrium market share for firm  $i$  under bargaining protocol  $Pj$ .

$$\frac{\partial \bar{s}_i^i}{\partial \delta_i} > 0 \quad \frac{\partial \bar{s}_i^i}{\partial \delta_j} < 0 \quad (6)$$

Thus, a relative increase in  $\delta_1$  will lead to larger share of the market for firm 1. The reason is that firm 1 has increased its strategic advantage and

that none of the IC constraints are binding. We get the opposite effect when considering the effect of a relative increase in  $\delta_2$  on equilibrium shares; an increase in firm 2's discount factor will lower firm 1's strategic advantage and thus increase firm 2's equilibrium share.

Focusing on area  $b$  in *Figure 1* we get

$$\frac{\partial \bar{s}_1^1}{\partial \delta_1} = 0 \quad \frac{\partial \bar{s}_1^1}{\partial \delta_2} > 0 \quad (7)$$

$$\frac{\partial \bar{s}_2^2}{\partial \delta_2} < 0 \quad \frac{\partial \bar{s}_2^2}{\partial \delta_1} < 0 \quad (8)$$

The partial derivatives in (7) show that only an increase in  $\delta_2$  will change firm 1's equilibrium proposal: firm 1 cannot use its increased strategic advantage since it is already offering firm 2 a share on firm 2's IC constraint, which is unaltered. However, (7) show that a marginal increase in  $\delta_2$  will increase firm 1's equilibrium share since he can now offer firm 2 a lower share that still meets firm 2's IC constraint. It is worth noting that this will happen even though *firm 1's strategic advantage* has decreased.

To the contrary, partial derivatives (8) show that increases in both discount factors will lower firm 2's equilibrium offer. There are two effects at work here: First, a marginal increase in firm 2's own discount factor lowers firm 1's strategic advantage. However, it will also lower firm 2's own IC constraint. Since this second effect will dominate the first, an increase in  $\delta_2$  will actually make firm 2 keep less market share for itself. Second, a marginal increase in firm 1's discount factor will also lower firm 2's proposed equilibrium share because firm 1 increases its strategic advantage and since firm 2's IC constraint does not bind in (8).

*Lemma 4* corresponds to a situation where both firms have low and not too different discount factors. This is depicted as area  $c$  in *Figure 1*.

$$\frac{\partial \bar{s}_i^i}{\partial \delta_i} = 0 \quad \frac{\partial \bar{s}_i^i}{\partial \delta_j} > 0 \quad (9)$$

The partial derivatives in (9) show that a marginal increase in *firm i's own discount factor* will not have an marginal effect on its equilibrium proposal because it is already proposing *firm j* a share on its IC constraint. An increase in *firm j's discount factor* will increase *firm i's market share* since *firm i* will make a new proposal on the IC constraint of *firm j*.

### 4.3 Identical Discount Factors

It is interesting to investigate the special case of identical discount factors, i.e.  $\delta_1 = \delta_2 = \delta$  and thus  $\underline{s}_1 = \underline{s}_2 = \underline{s}$ .<sup>10</sup> If discount factors are equal the only *Lemma 2* and *4* are valid since the conditions in *Lemma 3* are not fulfilled. By solving the equation  $\delta = \frac{(1-\sqrt{1-\delta})}{\delta}$  we get a critical discount level that divide the problem into two parts corresponding to area  $a$  and  $c$  in *Figure 1*. The solution equals  $\frac{\sqrt{5}}{2} - \frac{1}{2} \approx 0.61$ .

**Table 2. SPE Outcomes**

	$\delta \geq \frac{\sqrt{5}}{2} - \frac{1}{2}$	$\delta < \frac{\sqrt{5}}{2} - \frac{1}{2}$
<i>P1</i>	$(p^m, \frac{1}{(1+\delta)}, \frac{\delta}{(1+\delta)}, 1)$	$(p^m, (1-\underline{s}, \underline{s})$
<i>P2</i>	$(p^m, \frac{\delta}{(1+\delta)}, \frac{1}{(1+\delta)}, 1)$	$(p^m, \underline{s}, 1-\underline{s})$

*Table 2*, which is a corollary to *Proposition 1*, shows that if firms' discount factors are less than  $\frac{\sqrt{5}}{2} - \frac{1}{2}$  then firms cannot fully use their first mover advantage since they also have to make proposals that fulfill the IC constraint of the responding firm. It is also easy to see that, in the limit when firms are very patient they propose an equal share of the market.

## 5 Extensions

### 5.1 Simultaneous Bargaining and Competition

It is not unreasonable to assume that negotiations and competition may take place within the same stage game. Busch and Wenn (1996) study a game where firms play a Rubinstein alternating-offer game and, in case of disagreement after a proposal, play a disagreement game. They find that there may exist multiple equilibria for certain specifications of the disagreement game. However, for our purposes, they show that the equilibrium proposed in this paper is still unique. To see this assume that firms play the Rubinstein game and after a rejected proposal play the one shot Bertrand game. As soon as a proposal is accepted the bargaining ends and firms play the IRBG. Without loosing the general properties of the results we can also assume equal discount factors. It is easy to verify that the equilibrium proposals in *Proposition 1* and playing the Nash equilibrium in every disagreement game are SPE. Busch and Wenn argue that in order to get a firm,

---

<sup>10</sup> Note that if discount factors are equal firm 1 has no strategic advantage.

say firm 2, to play a non Nash equilibrium of the disagreement game firm 1 must promise to compensate this in its subsequent proposal. However, it must also be profitable for firm 1 to make such a compensation. Busch and Wenn derive the necessary and sufficient uniqueness condition  $w_1 = w_2 = 0$ , where  $w_1$  and  $w_2$  are defined as follows

$$w_1 = \max_{(p_1, p_2, s_1, s_2) \in [0, p^m]^2 \times \Delta} \left\{ \pi_1(p_1, p_2, s_1, s_2) - \left[ \max_{(p_2^*, s_2^*) \in [0, p^m] \times [0, 1]} \pi_2(p_2^*, p_1, s_2^*, s_1) - \pi_2(p_2, p_1, s_2, s_1) \right] \right\}$$

$$w_2 = \max_{(p_1, p_2, s_1, s_2) \in [0, p^m]^2 \times \Delta} \left\{ \pi_2(p_1, p_2, s_1, s_2) - \left[ \max_{(p_1^*, s_1^*) \in [0, p^m] \times [0, 1]} \pi_1(p_1^*, p_2, s_1^*, s_2) - \pi_1(p_1, p_2, s_1, s_2) \right] \right\}$$

It is easy to see that  $w_1 = w_2 = 0$  thus the equilibrium proposed in this paper is still unique.

## 6 Conclusion

Empirical evidence put forward by Levenstein and Suslow (2002) suggests that the most difficult task for firms trying to collude is to bargain and agree on what collusive strategy to implement. In this paper we study bargaining and collusion in an IRBG where firms have different discount factors. We model the bargaining as an alternating-offer bargaining game where firms take turns to propose a collusive price and market share. In this setting there are two effects from discount factors: In alternating offer bargaining games the most patient firm has a strategic advantage; since it has the highest discount factor it is less eager than the less patient firm to settle quickly on an agreement. The more patient firm can use this to propose an agreement that favors it. However, due to the structure of strategies in the IRBG the less patient firm also has a potential advantage since more market share must be allocated to it in order for the agreement to satisfy the IC constraints. Interestingly these two effects work in opposite direction and it is hard to ex ante determine which effect will dominate the other. The main contribution of the paper is that we derive a unique SPE in the bargaining game -and thereby determining which strategy of the Bertrand game firms will implement. The equilibrium has several appealing features: First, it is efficient in the sense that it entails immediate agreement on the monopoly

price. Second, the equilibrium gives clarity to how discount factors affect equilibrium market shares. We also perform a comparative statics analysis on equilibrium market shares. This analysis reveals that changes in discount factors may have ambiguous effects on market shares.

The results in this paper reveal some of the complexities faced by firms trying to collude. Moreover it points to the importance of not only studying the set of possible collusive strategies but also to study how a collusive strategy can be chosen.

## 7 Appendix

### 7.1 Lemma 1:

**Lemma 1** *If a bargaining strategy  $\bar{\psi}$  constitutes an SPE then  $\bar{p}(t) = p^m$  for every  $t$  where firm  $i$  is the proposer.*

*Lemma 1* is a corollary of *Proposition 1-3* in Binmore (1987) and its proof is therefore omitted.

### 7.2 Lemma 2 - 4

The proofs in this section follow the structure of Binmore, Shaked and Sutton (1989). From *Lemma 1* we know that firms will always propose the monopoly price, hence from now on we set  $p = p^m$ . This, together with the fact that firm profit is monotonically increasing in market share, allows us to restrict attention to market shares in the proofs of the propositions. To show uniqueness we need a couple of definitions. We denote a subgame, when it is *firm  $i$ 's* turn to make a proposal, as  $G_i$ . Also let  $M_i$  be the supremum SPE market share in  $G_i$  and let  $m_i$  be the corresponding infimum of  $G_i$ . We now state four conditions that the SPE must fulfill.

$$m_1 \geq 1 - \max\{\delta_2 M_2, \underline{s}_2\} \quad (10)$$

$$1 - M_1 \geq \max\{\delta_2 m_2, \underline{s}_2\} \quad (11)$$

$$m_2 \geq 1 - \max\{\delta_1 M_1, \underline{s}_1\} \quad (12)$$

$$1 - M_2 \geq \max\{\delta_1 m_1, \underline{s}_1\} \quad (13)$$

Inequality (10) states that the least share firm 1 can expect in any SPE must be weakly better than one minus the most that firm 2 can expect in the subsequent subgame. However, by assumption it must also fulfill the IC constraint  $\underline{s}_2$ . (11) states that the largest share that firm 1 can expect does not exceed one minus the discounted minimum share that firm 2 can expect in the subsequent subgame. Moreover,  $M_1$  cannot be larger than one minus the IC constraint of firm 2. Inequality (12) and (13) are explained analogously.

### 7.2.1 Lemma 2

**Lemma 2** If  $\underline{s}_i \leq \delta_i(1 - \delta_j)\lambda$   $i = 1, 2$   $i \neq j$  then there exists a unique SPE where:

Firm 1 proposes:  $\bar{\psi}_1(t) = (p^m, 1 - \delta_2\lambda, \delta_2(1 - \delta_1)\lambda)$  and accepts all agreements where  $\pi_1(\psi_2(t)) \geq \delta_1\pi_1(\bar{\psi}_1(t + 1))$ .

Firm 2 proposes:  $\bar{\psi}_2(t) = (p^m, \delta_1(1 - \delta_2)\lambda, 1 - \delta_1\lambda)$  and accepts all agreements where  $\pi_2(\psi_1(t)) \geq \delta_2\pi_2(\bar{\psi}_2(t + 1))$ .

**Proof.** The proof of *Lemma 2* exactly follow the more general proof of *Proposition 3.4* in Osborne and Rubinstein (1990). ■

### 7.2.2 Lemma 3

**Lemma 3** If  $\underline{s}_1 \leq \delta_1(1 - \delta_2)\lambda$  and  $\underline{s}_2 > \delta_2(1 - \delta_1)\lambda$  then there exists a unique SPE where:

Firm 1 proposes:  $\bar{\psi}_1(t) = (p^m, 1 - \underline{s}_2, \underline{s}_2)$  and accepts all agreements where  $\pi_1(\psi_2(t)) \geq \delta_1\pi_1(\bar{\psi}_1(t + 1))$ .

Firm 2 proposes:  $\bar{\psi}_2(t) = (p^m, \delta_1(1 - \underline{s}_2), 1 - \delta_1(1 - \underline{s}_2))$  and accepts all agreements where  $\pi_2(\psi_1(t)) \geq \delta_2\pi_2(\bar{\psi}_2(t + 1))$ .

**Proof.** The proof of *Lemma 3* is a simple generalization of the one given in Binmore, Shaked and Sutton (1989) and will therefore be omitted.<sup>11</sup> ■

### 7.2.3 Lemma 4

**Lemma 4** If  $\underline{s}_i > \delta_i(1 - \delta_j)\lambda$   $i = 1, 2$   $i \neq j$  then there exists a unique SPE where:

Firm 1 proposes:  $\bar{\psi}_1(t) = (p^m, 1 - \underline{s}_2, \underline{s}_2)$  and accepts all agreements where  $\pi_1(\psi_2(t)) \geq \delta_1\pi_1(\bar{\psi}_1(t + 1))$ .

Firm 2 proposes:  $\bar{\psi}_2(t) = (p^m, \underline{s}_1, 1 - \underline{s}_1)$  and accepts all agreements where  $\pi_2(\psi_1(t)) \geq \delta_2\pi_2(\bar{\psi}_2(t + 1))$ .

**Proof.** We first establish that the strategy in *Lemma 4* is subgame perfect and we then establish uniqueness.

First note that  $(p^m, \bar{s}^1)$  and  $(p^m, \bar{s}^2)$  in *Lemma 4* are IC. Now consider a subgame  $G_1$  at time period  $t$  and assume that firm 2 sticks to the strategy in *Lemma 4*. If firm 1 proposes  $\bar{s}^1$  there will be immediate agreement and

---

<sup>11</sup>Will be provided by the author upon request

firm 1 gets  $1 - \underline{s}_2$ . Any other strategy of firm 1 will lead to either agreement on  $\underline{s}_1$  in  $\tau \geq t + 1$  or on  $\underline{s}_1^1 \leq 1 - \underline{s}_2$  in  $\tau \geq t$ . The first strategy is not an improvement. To see this note that by assumption  $\underline{s}_1 + \underline{s}_2 \leq 1$  which implies that  $1 - \underline{s}_2 > \delta_1 \underline{s}_1$ . The second strategy is not an improvement for obvious reasons. Now consider instead that firm 1 sticks to the strategy in *Lemma 4*. If firm 2 accepts  $\bar{s}^1$ , it receives  $\underline{s}_2$ , any other strategy will lead to either agreement on  $\underline{s}_2^2 \leq 1 - \underline{s}_1$  in  $\tau \geq t + 1$  or on  $\underline{s}_2$  in  $\tau > t$ . The second is not an improvement for the obvious reasons. To see that the first is not an improvement note that we then must have  $\underline{s}_2 \geq \delta_2(1 - \underline{s}_1)$ . This can be rewritten as  $1 \geq \delta_2 \lambda$ . By assumption,  $\underline{s}_1 > \delta_1(\underline{s}_2) \lambda$  which can be rewritten as  $\frac{\underline{s}_1}{\underline{s}_2} > \delta_1 \lambda$ . Now it suffices to note that  $1 \geq \frac{\underline{s}_1}{\underline{s}_2}$  and  $\delta_1 \lambda \geq \delta_2 \lambda$ . Thus,  $1 \geq \delta_2 \lambda$ . Analogous arguments apply in  $G_2$ . This implies that the strategy is subgame perfect.

We now turn to proving uniqueness, and to do this we have to consider three cases.

(i) Assume that  $\delta_1 m_1 < \underline{s}_1 \leq \delta_1 M_1$  and  $\underline{s}_2 > \delta_2 M_2$ . This leads to the following conditions.

$$m_1 \geq 1 - \underline{s}_2, \quad (14)$$

$$1 - M_1 \geq \underline{s}_2 \quad (15)$$

$$m_2 \geq 1 - \delta_1 M_1 \quad (16)$$

$$1 - M_2 \geq \underline{s}_1 \quad (17)$$

By (14) and (15) we conclude that  $m_1 \geq 1 - \underline{s}_2$  and  $1 - \underline{s}_2 \geq M_1$ . Thus, since  $M_1 \geq m_1$  we have that  $M_1 = m_1 = 1 - \underline{s}_2$ . But, by assumption we have that  $\delta_1 m_1 < \delta_1 M_1$ . A contradiction.

(ii) Assume that  $\delta_i m_i < \underline{s}_i \leq \delta_i M_i$ . This leads to the following conditions.

$$m_1 \geq 1 - \delta_2 M_2, \quad (18)$$

$$1 - M_1 \geq \underline{s}_2 \quad (19)$$

$$m_2 \geq 1 - \delta_1 M_1 \quad (20)$$

$$1 - M_2 \geq \underline{s}_1 \quad (21)$$

Using (18) and (19) we get  $m_1 \geq 1 - \underline{s}_2 \geq M_1$  thus  $m_1 = M_1$  but this contradicts that  $\delta_1 m_1 < \delta_1 M_1$ .

(iii) Assume that  $\underline{s}_i > \delta_1 M_i$  for  $i = 1, 2$ . According to (10)-(13) this implies that

$$m_1 \geq 1 - \underline{s}_2, \quad (22)$$

$$1 - M_1 \geq \underline{s}_2 \quad (23)$$

$$m_2 \geq 1 - \underline{s}_1 \quad (24)$$

$$1 - M_2 \geq \underline{s}_1 \quad (25)$$

Trivial calculations yield that  $m_i = M_i = (1 - \underline{s}_j)$  for  $i = 1, 2$   $i \neq j$ . ■

*Lemma 2-4* consider every possible combination of discount factors satisfying  $\underline{s}_1 + \underline{s}_2 \leq 1$  and  $\delta_1 \geq \delta_2$ .<sup>12</sup> It is easy to see that the strategies presented in *Lemma 2-4* imply immediate agreement and together with *Lemma 1* we have thus proved *Proposition 1*.

## References

Abreu D., 1986, Extremal Equilibria of Oligopolistic Super games, *Journal of Economic Theory* 39, 191-228.

Abreu D., 1988, On the theory of Infinitely Repeated Games with Discounting, *Econometrica* 56, 383-396.

Athey S. and K. Bagwell, 2001, Optimal collusion with private information, *RAND Journal of Economics* 32, 428-465.

---

<sup>12</sup>  $\underline{s}_1 > \delta_1(1 - \delta_2)\lambda$  and  $\underline{s}_2 \leq \delta_2(1 - \delta_1)\lambda$  cannot hold simultaneously as this leads to the contradicting condition  $\delta_2 > \delta_1$ .

Becker W., 1971, American Wholesale Hardware Trade Associations 1870-1900, *Business History Review* 45, 179-200.

Binmore K., 1987, Perfect Equilibrium in Bargaining Models, in Binmore K. and P. Dasgupta (eds) *The Economics of Bargaining*, Basil Blackwell, ch. 5, 77-105.

Binmore K., A. Rubinstein and A. Wolinsky, 1986, The Nash Bargaining Solution in Economic Modelling, *Rand Journal of Economics* 17, 176-188.

Binmore K., A. Shaked and J. Sutton, 1989, An Outside Option Experiment, *Quarterly Journal of Economics* 104, 753-770.

Busch L. and Q. Wenn, 1986, Perfect Equilibria in a Negotiation Model, *Econometrica* 54, 533-556.

Feuerstein S., 2005, Collusion in Industrial Economics A Survey, *Journal of Industry, Competition and Trade* 5:3/4 163-198.

Fudenberg D. and E. Maskin, 1986, The Folk Theorem in repeated games with discounting or with imperfect public information, *Econometrica* 54, 533-556.

Friedman J., 1971, A non-cooperative Equilibrium for Supergames, *The Review of Economic Studies* 38, 1-12.

Genesove D. and W. Mullin, 2001, Rules, Communication and Collusion: Narrative Evidence from the Sugar Institute Case, *The American Economic Review*, 379-398.

Harrington J. E. Jr., 1989, Collusion among asymmetric firms: The Case of Different Discount Factors, *International Journal of Industrial Organization* 7, 289-307.

Harrington J. E. Jr., 1991, The Determination if Price and Output Quotas in a Heterogenous Cartel, *International Economic Review* 4, 767-792.

Harrington J. E. Jr., 2006, How Do Cartels Operate?, *Foundations and Trends in Microeconomics* 2, 1-105.

Howe M., 1973, A Study of Trade Association Price Fixing, *Journal of Industrial Economics* 21, 236-256.

Levenstein M., 1997, Price Wars and the Stability of Collusion: A Study of the Pre-World War I Bromine Industry, *Journal of Industrial Economics* 45, 117-138.

Levenstein M. and V. Suslow, 2002, What determines cartel success, *University of Michigan Business School Working Paper 02-001*.

Lehrer E. and A. Pauzner, 1999, Repeated Games with Differential Time Preferences, *Econometrica*, vol. 67(2), p 393-412.

Nash J. F., 1950, The bargaining problem. *Econometrica* 28, 155–162.

Osborne M. and A. Rubinstein, 1990, *Bargaining and Markets*, Academic Press Inc, San Diego, California.

Rubinstein A. and A. Wolinsky, 1990, Decentralized Trading, Strategic Behavior and the Walrasian Outcome, *Review of Economic Studies* 57, 63-78.

Rubinstein A., 1982, Perfect Equilibrium in a Bargaining Model, *Econometrica* 50, 97-110.

Ståhl I., 1972, *Bargaining Theory*, Stockholm: Economics Research Institute, Stockholm School of Economics.

Tirole, J., (1988), *The Theory of Industrial Organization*, MIT Press, Cambridge.