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Sjöberg, Daniel; Ramprecht, Jörgen

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WAVES IN FERROMAGNETIC MEDIA

Daniel Sjöberg[1], *Jörgen Ramprecht*[2]

[1]Department of Electrical and Information Technology, Lund University
[2]Division of Electromagnetic Engineering, Royal Institute of Technology
[1]Email: daniel.sjoberg@eit.lth.se, [2]Email: jorgen.ramprecht@ee.kth.se

ABSTRACT

Ferromagnetic media are inherently nonlinear and thus present many interesting features when studying wave propagation. Compared to "usual" dielectric media, effects such as nonreciprocity, hysteresis, and possibility of biasing the material by an external field appear. In very small specimens, such as nanosized particles, the typical self-ordering effects promote such things as spontaneously uniform magnetization and spin waves. In this contribution, we give a brief presentation of magnetic phenomena in the GHz range, and their application to electromagnetic waves, particularly absorbers.

1. INTRODUCTION

In ferromagnetic media, there is a strong coupling between the magnetic moments in neighboring atoms. The precise mechanism of this coupling still remains obscure, but a reasonable phenomenological model is the Landau-Lifshitz-Gilbert equation, first presented in 1935 in a slightly different form [9],

$$\frac{\partial \boldsymbol{M}}{\partial t} = -\gamma\mu_0 \boldsymbol{M} \times \boldsymbol{H}_{\mathrm{eff}} + \alpha \frac{\boldsymbol{M}}{|\boldsymbol{M}|} \times \frac{\partial \boldsymbol{M}}{\partial t} \qquad (1)$$

Note that since the right hand side is orthogonal to $\boldsymbol{M}$, the amplitude of the magnetization is preserved, $|\boldsymbol{M}| = M_{\mathrm{S}}$ where M_{S} is the saturation magnetization of the medium. This equation forms the basis of treating ferromagnetic media in the GHz-regime, where it is usually linearized around a static effective field $\boldsymbol{H}_{\mathrm{eff},0}$. The same qualitative mathematical model applies to ferrites and ferrimagnetic materials; for ferrimagnetic materials a coupled system of LLG equations, one for each sublattice of magnetization, should be applied.

It is not easy to compute the static field, since this entails the strong, nonlinear coupling between different atoms or magnetic domains, and deals with many effects like anisotropy, demagnetization, and magnetostriction. In spite of intensive research, it is still not clear if this is a well posed mathematical problem [2], due to its nonlinear nature. However, since the static magnetization $\boldsymbol{M}_0$ has zero time derivative, the static part of the LLG equation simplifies to $\boldsymbol{0} = -\gamma\mu_0 \boldsymbol{M}_0 \times \boldsymbol{H}_{\mathrm{eff},0}$, which implies $\boldsymbol{H}_{\mathrm{eff},0} = \beta \boldsymbol{M}_0$ for some scalar β.

Assuming the static field is known, the small signal permeability can be written [15]

$$\boldsymbol{\mu} = \begin{pmatrix} \mu & -\mathrm{i}\mu_{\mathrm{g}} & 0 \\ \mathrm{i}\mu_{\mathrm{g}} & \mu & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad (2)$$

where the distinguished axis (the z axis) is along the static field. The different entries have the frequency dependence

$$\mu(\omega) = \frac{\beta - \mathrm{i}\alpha\omega/\omega_{\mathrm{S}}}{(\beta - \mathrm{i}\alpha\omega/\omega_{\mathrm{S}})^2 - (\omega/\omega_{\mathrm{S}})^2} \qquad (3)$$

$$\mu_{\mathrm{g}}(\omega) = \frac{\omega/\omega_{\mathrm{S}}}{(\beta - \mathrm{i}\alpha\omega/\omega_{\mathrm{S}})^2 - (\omega/\omega_{\mathrm{S}})^2} \qquad (4)$$

where β is the bias parameter, and $\omega_{\mathrm{S}} = \gamma\mu_0 M_{\mathrm{S}}$ is the intrinsic precession angular frequency. In the remainder of this paper, we describe some phenomena that arise in dealing with these descriptions of magnetic media.

2. WAVE PROPAGATION AND RESONANCE FREQUENCY

The small signal permeability (2) represents a gyrotropic medium, which means that waves propagating along the distinguished axis are naturally described in circular polarization, where left and right hand circularly polarized waves have different wave speeds. The physical explanation of this is that the magnetization is connected to the electron orbits around the atoms, and the different circular polarizations either work with or against these orbits.

As can be seen from (3) and (4), the entries μ and μ_{g} in the permeability matrix $\boldsymbol{\mu}$ depend on frequency. Ignoring the losses, the resonance frequency is given by $|\beta\omega_{\mathrm{S}}|$, which demonstrates that the resonance frequency can be controlled by the static magnetic field, since this controls β. This field can be biased by an external magnetic field, $\boldsymbol{H}_0^{\mathrm{e}}$, which can be used to tune the resonance. Indeed, this is the classical ferromagnetic resonance [7, 5], which is a well established method of characterizing ferromagnetic films and particles. In the off-resonance case, it is usually the nonreciprocal nature of the permeability which is used, for instance in classical microwave components based on ferrites such as isolators and circulators [13, 1].

Since the bias parameter β is typically in the order of unity, it is seen that the resonance frequency is in the order of the intrinsic precession angular frequency ω_S, which is proportional to the saturation magnetization M_S. For the classical ferromagnetic substances iron, nickel and cobalt, values of the frequency $\omega_S/2\pi$ are 61 GHz, 51 GHz, and 17 GHz, respectively.

When considering wave propagation in ferromagnetic media, the electric conductivity must be taken into account. For materials like iron, this conductivity is very high and prohibits electromagnetic waves. In microwave applications, usually ceramic ferrites are used due to their low electric losses. The conductivity can be controlled by manufacturing composite materials, where small ferromagnetic particles are embedded in a background matrix material with low conductivity. If the particles are small enough, and possibly coated with a thin isolating layer, the losses due to electric conductivity can be dramatically reduced and wave propagation in the composite is possible. This leads us to the next topic, small particles.

3. NANOPARTICLES

As discussed above, some material properties like electric conductivity can be controlled by considering composite material technology. In order to determine the effective magnetic properties of the bulk composite material, we need to consider the properties of a small ferromagnetic particle. When the particles are small enough, maybe 10–100 nm, it is energetically favourable to form only one magnetic domain in the particle. This is an ideal situation to be treated by the LLG equation, and the result is that a particle can be described by a permeability as in (2), which is uniform throughout the particle [14, 20].

If the matrix material is nonmagnetic, it has the relative permeability $\boldsymbol{\mu}_2 = \mathbf{I}$. If we further assume that all particles have the same direction of magnetization, the following generalized Hashin-Shtrikman formula can be used to estimate the effective permeability of the composite material [10, p. 145]

$$\boldsymbol{\mu}_{\text{eff}} = \mathbf{I} + f_1\mu_2(\boldsymbol{\mu}_1 - \mathbf{I})[\mathbf{I} + (f_2/3)(\boldsymbol{\mu}_1 - \mathbf{I})]^{-1} \quad (5)$$

It is well known that this formula is good for spherical inclusions, but that the exact result depends on the precise microstructure. However, it can be shown that it provides a bound in the complex plane for the diagonal elements of the effective permeability, as demonstrated in Figure 1. By a diagonal element we mean the quadratic form $\mu_{\text{eff}} = \boldsymbol{H}^* \cdot (\boldsymbol{\mu}_{\text{eff}}\boldsymbol{H})/|\boldsymbol{H}|^2$ for any constant vector $\boldsymbol{H}$. With no information on volume fraction, the effective permeability must be inside the dash-dotted lines. If the volume fraction is known, μ_{eff} is restricted by the dashed curves, and if we further know the effective material must be isotropic μ_{eff} is restricted by the solid curves. With more information on the microstructure, tighter bounds can be formulated [10, 3].

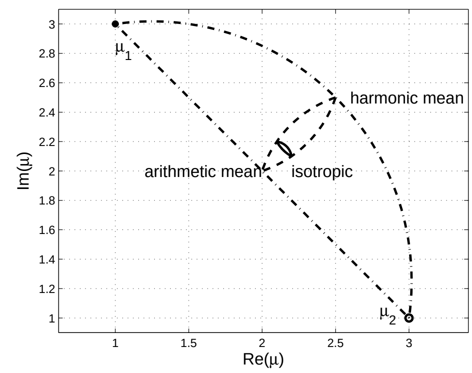

Fig. 1. Bounds in the complex plane for the effective permeability of an arbitrary mixture of component materials μ_1 and μ_2. All curves are parts of circles in the complex plane, where the dashed lines correspond to the Hashin-Shtrikman formula (5).

For small single-domain particles, additional effects arise due to the exchange forces between the magnetic spins in each atom. The exchange forces are modelled by a contribution in the effective magnetic field $\boldsymbol{H}_{\text{eff}}$ proportional to $\nabla^2\boldsymbol{M}$. Since variations on a small spatial scale give rise to large spatial derivatives, this contribution becomes more important the smaller the particle is. Including this effect in the LLG equation (1) makes the principal part similar to the Schrödinger equation, which is a hyperbolic equation supporting wave solutions. Thus, disturbances in the distribution of the magnetization in the particle may then propagate as a wave [6]. This is a wave involving only magnetic interaction, and is called a spin wave. This contributes in its turn to additional resonances in the particle, but does not change the general form of the permeability tensor in (2). However, the frequency dependence in (3) and (4) must be changed [14].

When the size of the particles shrink even further, say below 10 nm which is the typical thickness of a domain wall, the exchange forces become so strong that it is no longer energetically favorable for the particle to form a single domain (except for the highly idealized case of exactly ellipsoidal particles with no lattice defects). We then reach the super-paramagnetic region, where the internal magnetic field inside the particle is relatively weak and the (unbiased) net magnetization drops drastically. However, the particle's response to an external magnetic field may still be very strong, resulting in almost uniform alignment of the spins in the particle.

Nanosized particles have found interesting use in so called ferro-fluids [12]. These are colloidal suspensions of nanoparticles coated with a thin layer, usually consisting of organic molecules, to keep the par-

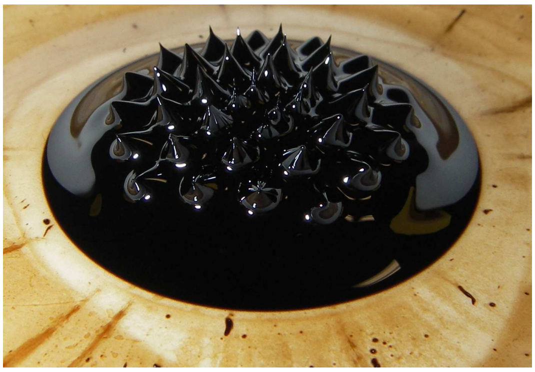

Fig. 2. Ferrofluid over a neodymium magnet. Photo by Steve Jurvetson [4].

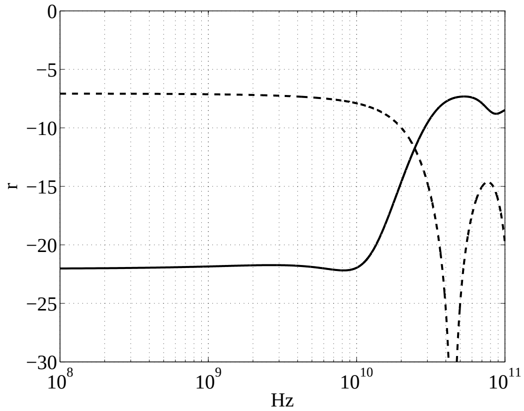

Fig. 3. Plots in dB-scale of $|r_{\mathrm{co}}|^2$ (solid line) and $|r_{\mathrm{cross}}|^2$ (dashed line) for a 1 mm thick ferromagnetic absorber with $\beta = 0$ [15].

ticles from coagulating. Since the particles are free to move around in the liquid, the ferrofluid's response to magnetic fields is very strong, even for small field strengths. In order to minimize the magnetic energy, the surface of the liquid may form interesting patterns as in Figure 2.

Ferrofluids do not support electromagnetic waves, but their strong response to magnetic fields and coupling to hydrodynamics has manifested in the interdisciplinary field of ferrohydrodynamics [18]. Ferrofluids find applications as rotary shaft seals in hard drives, cooling of loudspeakers, and medicine.

4. ABSORBERS

A major application of magnetic media in electromagnetism is as absorbers for electromagnetic waves [8]. Since the loss mechanisms are magnetic, they are most efficient close to a metal surface, where usually the magnetic field is large and the electric field is small. Since the magnetic absorbers can be placed close to the metal, there is no need for the typical quarter-wavelength-spacing necessary for absorbers based on electric losses. This means that the bandwidth of the absorber is determined primarily by the magnetic material properties. Indeed, this is clearly seen in the paper [19], where it is shown that the following relation holds for *any* absorber constructed by lamination:

$$\int_0^\infty \ln\left(\frac{1}{|\Gamma(\lambda)|}\right) \mathrm{d}\lambda \leq 2\pi^2 \sum_i \mu_{\mathrm{s},i} d_i \qquad (6)$$

where Γ is the reflection coefficient, $\mu_{\mathrm{s},i}$ is the static relative permeability of layer i, and d_i is the thickness of the corresponding thickness. This demonstrates that the available bandwidth of an absorber is directly linked to its static magnetic properties.

In particular, if a slab with thickness d could be found such that it had the magnetic material properties

$$\omega\mu_0\mu'' d = \eta_0 \quad \text{and} \quad \mu'' \gg \mu' \qquad (7)$$

where $\eta_0 = 377\,\Omega$ is the intrinsic wave impedance of vacuum, a normal incidence absorber with very wide bandwidth could be constructed [8, p. 337], which can be termed the magnetic Salisbury screen. Unfortunately, these demands are not satisfied by any material found in nature. But the first demand, which essentially requires the imaginary part of the permeability to depend on frequency as $\mu'' \sim 1/\omega$, can actually be obtained from (3) and (4) by setting $\beta = 0$ [15, 16]. Since the second demand, $\mu'' \gg \mu'$ cannot easily be met, the bandwidth is not as big as theory predicts, but an example of what can be achieved is shown in Figure 3.

The condition $\beta = 0$ implies a certain strength and direction of the external magnetic field $\boldsymbol{H}_0^{\mathrm{e}}$ must be applied, and depending on configuration this can be a very strong field, in the order of the saturation magnetization. Also, the performance of the absorber is very sensitive to the bias condition $\beta = 0$, making this an unrealistic design at the present understanding. It is possible to calculate the weight per surface area necessary to construct this kind of absorbent, and typically for iron this results in around 20 kg per square meter [17], which means only non-mobile applications can afford the additional weight from the absorber. However, it may still be a valuable method to deal with low-frequency scattering.

The real usefulness of magnetic materials in absorbers is as surface wave absorbers. The typical surface wave propagates along a metal surface, with the magnetic field parallel to the surface and the electric field in the normal direction. In contrast to the normal incidence case, which is discussed above, the electromagnetic wave can now be expected to propagate a relatively long stretch in the material, making it possible to absorb the wave using only small losses in the material. To reduce surface waves is of importance not only in stealth technology, but also for improving the performance of array antennas where a surface wave may severely influence the impedance characteristics of the antenna elements [11].

5. CONCLUSIONS

In this paper we have discussed the characteristics of ferromagnetic materials and their influence on the propagation of electromagnetic waves. The Landau-Lifshitz-Gilbert equation (1) has been used as the basic model for the magnetic behavior. When considering propagation of weak electromagnetic waves, this equation can be linearized and the result is the classical gyrotropic small signal permeability shown in (2). This permeability can be biased by a static magnetic field. In addition to classical microwave components such as isolators and circulators, this can be used to make a very broad band absorber, although it requires a strong and precise bias.

Many unsolved problems remain for ferromagnetic materials, in particular for multidomain structures. As mentioned in the Introduction, the pure mathematical problem of well-posedness, *i.e.*, existence, uniqueness, and continuity of the solution with respect to data, are not solved [2]. It is also very challenging to perform numerical investigations of the LLG equation, due to the nonlinearity and the many scales and phenomena that each need to be accurately modeled. The long physical experience of the phenomenological model speaks in its favor in idealized cases, but it is still a major problem how to attack large, real life problems in a rigorous manner.

Ferromagnetic materials find use in many different circumstances, and it is fascinating that they remain such an intriguing and difficult area. In particular, when the magnetic properties couple strongly to mechanical properties as in ferrofluids, new and exciting fields of research are opened which require the expertise from many classical subjects.

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