Optimum code rate in cellular systems using adaptive modulation

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Optimum Code Rate in Cellular Systems using Adaptive Modulation

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Abstract— **It is essential to find suitable modes (combinations of alphabet sizes and code rates) for transmission in cellular systems. Bad choices result in low spectrum efficiency. An analytical method is proposed to find the modes which give the highest spectrum efficiency when using adaptive modulation. It is demonstrated that data rates considerably higher than 271 and 384 kbps are feasible in GSM and EDGE systems by using adaptive modulation. It is further demonstrated that the spectrum efficency can be improved more than 100% by using three modes in a GSM system.**

I. Introduction

Which code rates should be combined with 4-QAM, 8-PSK, and 16-QAM when using adaptive modulation in a cellular system? The analytical method proposed here estimates the optimum code rates by searching for the maximum spectrum efficiency. The highest average spectrum efficiency is found by assuming optimal coding, optimal channel estimation, and optimal synchronization. By optimal coding is meant a coding arrangement which is optimal in AWGN, however, the channel is modelled as slow and flat Rayleigh fading.

The method trades throughput for cluster size. High throughput demands a high signal to interference ratio (SIR), and thus a large cluster size. The optimum set of code rates combines a reasonable throughput with medium cluster sizes.

The set of channel alphabet sizes is limited to $M = \{4, 8, 16\}$. These three alphabet sizes are combined with three code rates to form three different modes. The modes usually demand different SIR:s, and it is not sure that all modes can operate successfully everywhere in a cell. Adaptive modulation selects the mode with the highest throughput among the modes that can operate successfully.

The proposed method requires specification of the minimum throughput at a desired BER. For example are the minimum throughputs in section III $T_{\min} = 271$ kbps and $T_{\min} = 384$ kbps at BER 10^{-6}. In addition to the slow, flat Rayleigh fading, the channel is also slow fading due to shadowing. The consequence is that signal outages will occur, and the outage probability is specified as a figure of coverage.

The presented method is analytical; no simulations are needed. A cellular system designer may therefore benefit from the method by using it as a prospective-study tool. However, while simulation facilitates a detailed system model, we have reduced the level of detail when modelling the cellular system. What is lost in detail is, hopefully, gained in clarity.

II. Method

A. Method for Finding Spectrum Efficiency

Spectrum efficiency in a cellular system is often defined as the total throughput in a cell-cluster divided by the cluster's total area and bandwidth [1]. This section departs with a study of the throughput in a cellular system, and arrives at a simple expression for spectrum efficiency.

Assume that the total bandwidth assigned to a cell is divided into n_c channels each having a constant bandwidth W Hertz. A user is offered throughput $d_k(t)$ at time t when using channel k, and the momentary total throughput $T_{\text{cell}}(t)$ in a cell is the sum of all user throughputs,

$$T_{\text{cell}}(t) = \sum_{k=1}^{n_c} d_k(t). \tag{1}$$

Hence, $T_{\text{cell}}(t)$ changes as each user's throughput changes.

Each user's throughput depends on the signal to interference ratio (SIR) when adaptive modulation is used. $T_{\text{cell}}(t)$ is therefore a function of the user-SIR:s in a cell. The SIR Γ in a cellular system is a random variable due to shadow fading of the signal. Moreover, the expected SIR is a function of the distance r between base station and mobile station. The probability density function of Γ is thus conditioned on r:

$$f_\Gamma(\Gamma|r). \tag{2}$$

The adaptive modulation scheme allows three different alphabet sizes $M \in \{4, 8, 16\}$. Associated with each alphabet size is a code rate: R_4, R_8, and R_{16}, where all three code rates are in the interval $[0, 1]$. Thus, three sets (modes) of M and R are formed: mode A, mode B, and mode C. The set $\{A,B,C\}$ is a permutation of $\{4\ 8, 16\}$, and it is introduced to give a simple expression for the average throughput $T(r)$ in (4) below. Three data rates are associated with the three modes: d_A, d_B, and d_C. The data rate of mode i, $i \in \{A, B, C\}$, is

$$d_i = R_i \log_2(i). \tag{3}$$

A minimum SIR, which must be fulfilled to sustain the specified BER, is also associated with each mode. These minimum signal to interference ratios are denoted $\mathrm{SIR_A}$, $\mathrm{SIR_B}$, and $\mathrm{SIR_C}$, where $\mathrm{SIR_A} \leq \mathrm{SIR_B} \leq \mathrm{SIR_C}$. If several modes fulfill the demand on BER, the mode with the highest data rate is chosen. This way the system adapts the modulation to a changing SIR.

The average throughput $T(r)$ experienced by a mobile user on distance r from the base station can now be expressed

$$\begin{aligned} T(r) \quad = \quad & d_A \int_{\mathrm{SIR_A}}^{\mathrm{SIR_B}} f_\Gamma(\Gamma|r)d\Gamma + \\ & d_B \int_{\mathrm{SIR_B}}^{\mathrm{SIR_C}} f_\Gamma(\Gamma|r)d\Gamma + \\ & d_C \int_{\mathrm{SIR_C}}^{\infty} f_\Gamma(\Gamma|r)d\Gamma. \end{aligned} \tag{4}$$

In order to eliminate the conditioning on r, we specify a uniform customer density. The probability density function for r is then

$$f_r(r) = \frac{2r}{R_0^2}. \tag{5}$$

Here, R_0 is the range of the base station. The base station's antenna is assumed to be elevated, and the mobile user can not come closer to it than $R_{\min}$, thus $R_{\min} \leq r \leq R_0$. The average throughput of a randomly picked user is denoted T_{user}, and is calculated as

$$T_{\mathrm{user}} = \int_{R_{\min}}^{R_0} T(r) f_r(r) dr = \frac{2}{R_0^2} \int_{R_{\min}}^{R_0} T(r) r dr. \tag{6}$$

The average throughput of a cell with n_c users (channels) is $T_{\mathrm{cell}} = n_c T_{\mathrm{user}}$. The relation between T_{cell} and $T_{\mathrm{cell}}(t)$ in (1) is $T_{\mathrm{cell}} = \mathrm{E}[T_{\mathrm{cell}}(t)]$, i.e., T_{cell} is the expected value of $T_{\mathrm{cell}}(t)$ with respect to time t. A whole cell cluster with size β has average throughput

$$T_{\mathrm{cluster}} = \beta T_{\mathrm{cell}} = \beta n_c T_{\mathrm{user}}. \tag{7}$$

All cells are assumed to have constant cell area ζ, which is an accurate model of a cellular system with homogeneous terrain and constant user density. The spectrum efficiency in Mbps/MHz/area-unit is now defined as

$$\eta \triangleq \frac{T_{\mathrm{cluster}}}{\zeta \beta W \beta n_c} = \frac{T_{\mathrm{user}}}{\zeta W \beta}. \tag{8}$$

The spectrum efficiency in (8) can be simplified by excluding the cell-area ζ and the bandwidth W because they are independent of the cluster size and each user's data rate. The final expression for η is thus

$$\eta \propto \frac{T_{\mathrm{user}}}{\beta}, \tag{9}$$

which is used in the rest of the paper. Expression (9) does not give absolute values of the spectrum efficiency, however, it is only the change in η between different modes which is of interest here.

The parameter remaining to be calculated in (9) is the cluster size β. A one-to-one relationship (described below) exists between the distribution of Γ and the cluster size. The position in the cell where the SIR is lowest (on the average) is on the cell's edge. Therefore, Γ on the cell's edge must not go below $\mathrm{SIR_A}$ ($\mathrm{SIR_A} \leq \mathrm{SIR_B} \leq \mathrm{SIR_C}$) more often than specified by the outage probability P_{out}. Mathematically,

$$P(\Gamma \leq \mathrm{SIR_A}) = P\left(z \leq \log_e(\mathrm{SIR_A})\right) \leq P_{\mathrm{out}} \tag{10}$$

must be fulfilled on the cell's edge. Here, $z = \log_e(\Gamma)$.

When both wanted signal and all interferers suffer fading with lognormal distribution, the SIR Γ has lognormal distribution and therefore $z = \log_e(\Gamma)$ has normal distribution. The mean and variance of z are m_z and σ_z^2 respectively. Expression (10) can then be rewritten

$$1 - Q\left(\frac{\log_e(\mathrm{SIR_A}) - m_z}{\sigma_z}\right) \leq P_{\mathrm{out}}, \tag{11}$$

where $Q(\cdot)$ is the normalized upper tail probability function for the normal distribution. The cluster size β is a function of m_z, and thereby also of σ_z, and β can be found by table-lookup when m_z and σ_z are known.

What is left before the method is complete is a relation between the BER and $\{\mathrm{SIR_A}, \mathrm{SIR_B}, \mathrm{SIR_C}\}$. Such a relation is presented in II-D. Then, given the BER, the code rates $\{R_4, R_8, R_{16}\}$, and P_{out}, we can find β and T_{user}. In the investigation below, spectrum efficiency is calculated for the modes that can be created out of $M = \{4, 8, 16\}$ and $R = \{0.02, 0.04, \ldots, 1\}$.

B. Cellular System Model

The cellular system is modelled as a perfectly symmetric grid of equally sized hexagonal cells. As mentioned above, a mobile user never comes closer to the base station than $R_{\min}$. The range of the base stations are R_0, and therefore $R_{\min} \leq r \leq R_0$ where we have chosen $R_{\min} = 0.1 R_0$.

Only cochannel interference is accounted for, and the interfering cochannel base stations can be geometrically arranged in so-called group-tiers. Without describing the geometry in detail, we will present an expression for the positions of the interfering base stations as a function of β. The expression gives the positions of the cochannel base stations illuminating a user when all base station antennas have 120° lobe widths. The expression, together with supplementary information about the parameters in it, renders it possible to repeat the results in this paper.

The position of cochannel base station number n in group-tier p relative to the origin is described by $z(n,p) \in \mathbb{C}$,

$$\begin{aligned} z(n,p) &= \sqrt{\frac{2\pi}{\sqrt{3}}} R_0 \sqrt{\beta} \sqrt{p^2 - p|q| + |q|^2} \times \\ & e^{j\left[\frac{\pi}{3}\lfloor \frac{n}{p} \rfloor + \arcsin\left(\frac{\sqrt{3}}{2}\frac{q}{\sqrt{p^2-p|q|+|q|^2}}\right) - \alpha\right]}, \\ & \begin{cases} n = 3p, \ldots, 5p - \mathrm{mod}(p,2) \\ q = \mathrm{mod}(n,p) - \lfloor \frac{p}{2} \rfloor \end{cases} . \end{aligned} \tag{12}$$

Here, the angle α depends on the actual allocation of carrier frequencies in the cells, and it changes with the cluster size. However, α is assumed constant in this simple model, and is chosen as $\alpha = \pi/6$. The number of cochannel interferers accounted for in this analysis is $\lfloor P(P+3/2) \rfloor = 13$, which corresponds to $p = 1, \ldots, P$, where $P = 3$.

C. Propagation Model

The area mean power at distance r from a transmitter is described by

$$S(r) = S(R_0)\left(\frac{r}{R_0}\right)^{-\gamma}, \tag{13}$$

where the propagation exponent γ is between 3.5 and 4.0 [2]. $S(r)$ is the average over the shadow fading, which, as mentioned before, is assumed to have lognormal distribution. Inspired by [3], Wilkinson's method [4], [5] is used for adding the independent unidentically distributed lognormal random variables that model the cochannel interference. Unidentically distributed lognormal random variables is a consequence of the different distances to the interferers in a cellular system.

The SIR's standard deviation σ_z is often called the dB-spread when measured in dB. To imitate a GSM system as closely as possible, typical GSM-parameters are adopted in order to find a suitable value for σ_z. With outage probability $P_{\mathrm{out}} = 0.05$, $\beta = 9$, and necessary $\mathrm{SIR}_{\mathrm{min}} = 9$ dB in a GSM system [6], the cellular system model above gives that the dB-spread is 5.1 dB for each interferer when $\gamma = 3.5$.

The mobile user is assumed to move slowly enough for the slow fading to be virtually constant over many symbols. The sum of interference over a symbol is modelled as AWGN, which is plausible when the number of interferers is large [7].

D. Channel Model and Coding Arrangement

An average lower bound on the BER for given M and R in slow, flat Rayleigh-fading channel was derived in [8], and is reviewed here for the reader's convenience. A consequence of the slow and flat fading property is that the sampled output y of the matched filter in the receiver is

$$y = ce^{j\phi}x + u, \tag{14}$$

when symbol x is transmitted. Here, $c \in \mathrm{Rayleigh}(\sigma_c^2)$ is the channel's influence on the amplitude, and $\phi \in \mathrm{Rect}(0, 2\pi)$ is a random phase shift introduced by the channel. The random variable u is AWGN. It is assumed that perfect channel side information eliminates $e^{j\phi}$. The power attenuation due to distance is fully taken care of in (13); the second moment of c can be set to 1. This gives that the variance of c is $\sigma_c^2 = 1/2$. The additive noise u and the representation of the transmitted symbol x are complex, i.e. $x, u \in \mathbb{C}$, while $c \in \mathbb{R}$.

Although perfect channel side information is available, the Rayleigh fading channel degrades the performance compared to an AWGN channel with constant SIR. An expression for the channel capacity $C_{\mathrm{AWGN}}(c)$ of a two-dimensional signal constellation with equiprobable channel symbols in AWGN is presented in [9]. The average channel capacity C_{Ray} of the Rayleigh fading channel is found by averaging $C_{\mathrm{AWGN}}(c)$ over all possible values of c, $c \in [0, \infty)$, [10]

$$C_{\mathrm{Ray}} = \int_0^\infty f_C(c) C_{\mathrm{AWGN}}(c)\, dc. \tag{15}$$

Here, $f_C(c)$ is the density function of the Rayleigh distribution.

The average capacity is not a bound but an estimate of the capacity of a Rayleigh fading channel. However, the number of transmitted bits per channel access approaches C_{Ray} if the code words' length are allowed to approach infinity. Very long code words require very long decoding delays, and the decoding delay is thus unconstrained. So far, it has been assumed that the Rayleigh fading channel is memoryless. A virtually memoryless channel can be obtained by introducing large interleavers.

The discrete-input/discrete-output system from encoder to decoder is regarded as a binary symmetric channel (BSC) [11]. The capacity of a BSC, C_{BSC}, is a function of the BER, while C_{Ray} in (15) is a function of the SIR. The capacity of the Rayleigh fading channel lies within $0 \leq C_{\mathrm{Ray}} \leq R\log_2(M)$, and due to the data processing theorem [9],

$$C_{\mathrm{BSC}} R \log_2(M) \leq C_{\mathrm{Ray}}. \tag{16}$$

Equality in (16) gives a relationship between a specific BER through C_{BSC} and the appropriate average SIR-level through C_{Ray}. The minimum SIR sustaining a given BER with data rate $R\log_2(M)$ can thus be found.

III. Results

The proposed method is demonstrated in this section, and the results are divided into four cases. What separates the cases is the minimum required throughput $T_{\min}$, and if adaptive modulation is allowed or not. The first case requires $T_{\min} = 271$ kbps, but only 4-QAM is permitted. This case is comparable to GSM. In Case 2, the demand on throughput is also $T_{\min} = 271$ kbps, but adaptive modulation is permitted to find what can be gained in spectrum efficiency. In the third and fourth cases, the minimum throughput is 384 kbps. The third case is when 8-PSK is the only modulation method, while adaptive modulation is permitted in the fourth case.

The results apply to some extent to the evolution of GSM called EDGE. In EDGE, 8-PSK is proposed for supplying the users with 384 kbps-links. Throughout the demonstration, β is constrained to $7 \leq \beta \leq 9$ to imitate operational GSM-systems, and this makes it theoretically possible to deploy the parameters proposed here in an existing system. The channel separation in all four cases is 200 kHz, and the required BER is 10^{-6}.

Case 1: $T_{\min} = 271$ kbps and 4-QAM.
The highest spectrum efficiency is obtained at code rate $R_4 = 0.78$ when using 4-QAM to offer 271 kbps-links to the users. The performance of 4-QAM is intended to be a "standard case" used for comparison with Cases 2, 3, and 4 below.

Case 2: $T_{\min} = 271$ kbps and adaptive modulation.
At least 271 kbps is required everywhere in the cell, and adaptive modulation using 4-QAM, 8-PSK, and 16-QAM is applied. The highest spectrum efficiency is obtained when the code rates are $R_4 = 0.78$, $R_8 = 0.82$, and $R_{16} = 0.92$. The spectrum efficiency gain compared to Case 1 (non-adaptive modulation) is 111%.
Employing only two alphabet sizes instead of all three results in inferior spectrum efficiency. The alphabet-combination coming closest to the performance of $M = \{4, 8, 16\}$ is $M = \{4, 16\}$. The spectrum efficiency gain when using $M = \{4, 16\}$ is 102% compared to Case 1.
The optimum code rates above give the average throughput depicted in Fig. 1. Note that a required throughput of 271 kbps everywhere in the cell implies a considerably higher average throughput. The whole cell is offered an average throughput larger than 500 kbps according to Fig. 1.
The throughput is larger than the minimum throughput level at least 95% of the time. This explains why the average throughput in Fig. 1 can be lower than the minimum level. The minimum level is equal to the highest attainable data rate 736 kbps in 20% of the cell. Lower data rates are used a fraction of time equal to $P_{\text{out}} = 0.05$, and the average throughput must hence be lower than the minimum level. The result from Fig. 1 is that almost 50% of the cell is offered a minimum of nearly 500 kbps.

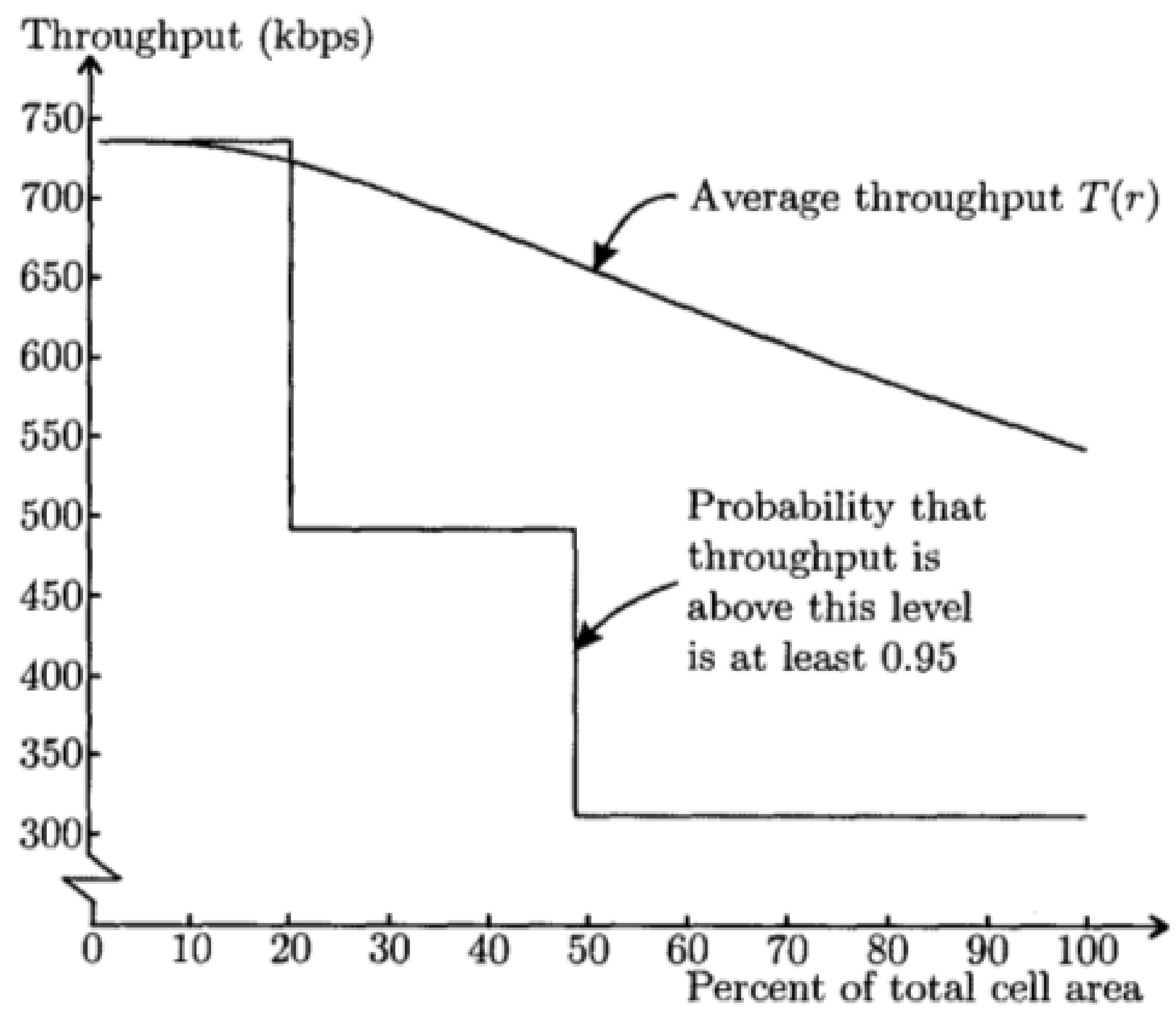

Fig. 1. Average throughput $T(r)$ if the requirement is at least 271 kbps everywhere in the cell. The minimum throughput limit is also drawn. The probability that the throughput goes below the minimum limit is less than or equal to 0.05.

Case 3: $T_{\min} = 384$ kbps and 8-PSK.
The enhanced GSM-system known as EDGE is planned to offer 384 kbps per channel using 8-PSK. It is necessary to switch from 4-QAM to 8-PSK to avoid excessively large cluster sizes when demanding 384 kbps and using non-adaptive modulation.
Optimum spectrum efficiency using 8-PSK is obtained when the code rate is $R_8 = 0.66$. The spectrum efficiency for this case is 12% higher than in Case 1. The reason for such a small difference is partly that R_4 in Case 1 is larger than R_8, and partly that a larger β ($\beta \approx 8$) is needed for 8-PSK than for 4-QAM in Case 1 ($\beta \approx 7$).

Case 4: $T_{\min} = 384$ kbps and adaptive modulation.
The same requirement on throughput as in Case 3, but adaptive modulation using 4-QAM, 8-PSK, and 16-QAM is applied. However, 4-QAM does not contribute very much at such a high throughput. The highest spectrum efficiency is thus obtained by using only 8-PSK and 16-QAM at code rates $R_8 = 0.66$ and $R_{16} = 0.90$. The gain in spectrum efficiency compared to non-adaptive modulation using 8-PSK is 66%.
Fig. 2 depicts the average throughput versus the cell-area it is offered in when using the optimum code rates above. Note that $T_{\min} = 384$ kbps implies a considerably higher average throughput $T(r)$ than 384 kbps. The whole cell is offered an average throughput of more than 500 kbps according to Fig. 2. The reason for the average throughput to be below the minimum throughput is explained in Case 2.
The spectrum efficiency of the system designed for $T_{\min} =$

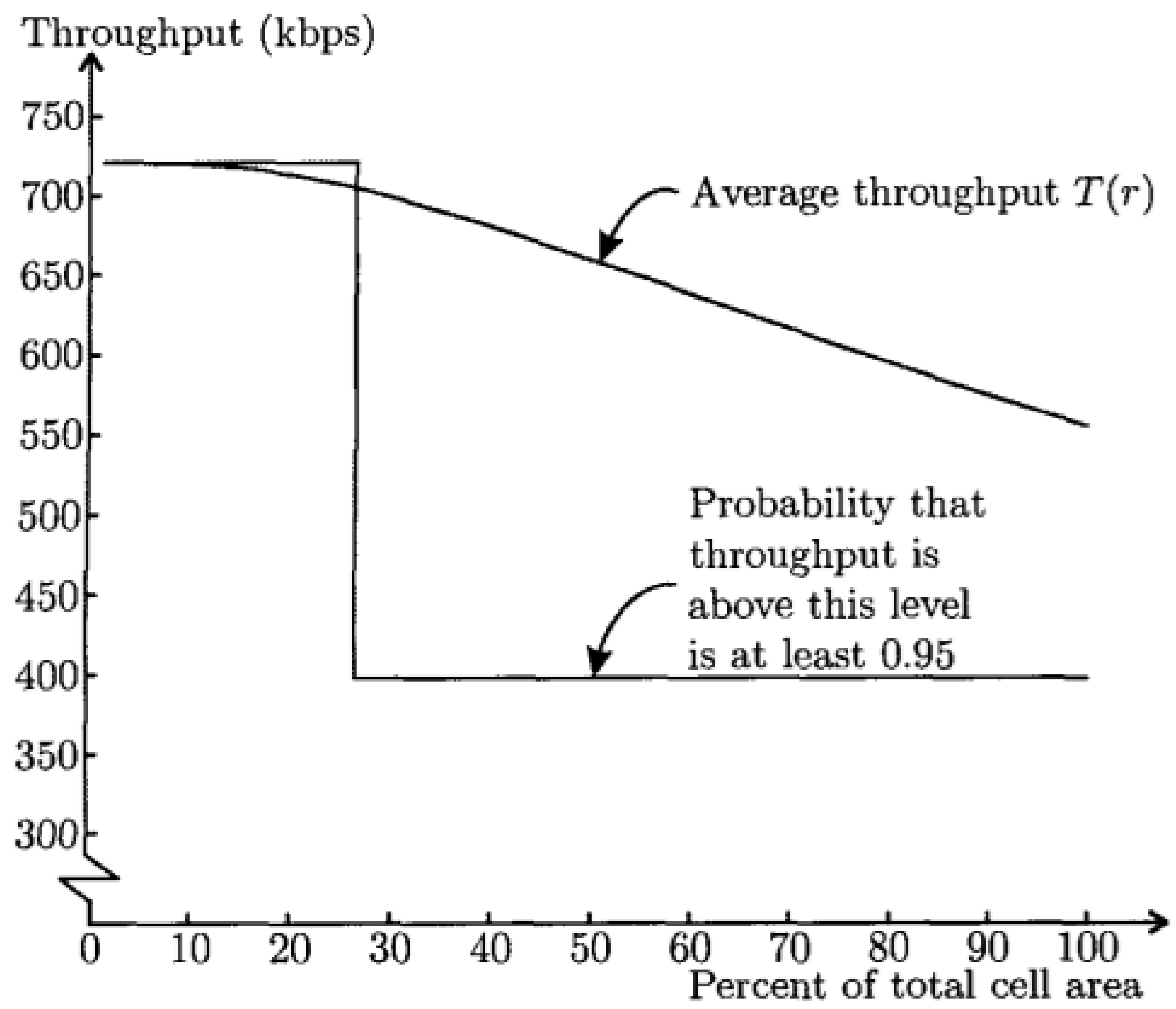

Fig. 2. Average throughput $T(r)$ if the requirement is at least 384 kbps everywhere in the cell. The minimum throughput limit is also drawn. The probability that the throughput goes below the minimum limit is less than or equal to 0.05.

384 kbps is smaller than for the system in Case 2 where the requirement was $T_{\min} = 271$ kbps. The reason for this is that a low $T_{\min}$ allows the available alphabet sizes to be used in a more efficient way. Three modes can cover the signal's variations better than only two modes can.

IV. Conclusion

The main contribution of this paper is a new method which analytically finds the optimum code rates in a cellular system where adaptive modulation is used. Requirements on both quality (BER) and user throughput can be specified. A system designer may use the proposed method to make a pilot-study of a new system design. The advantage with a completely analytical method is that tedious simulations are avoided.

The general conclusion of the demonstration is that adaptive modulation gives substantial improvement in spectrum efficiency. The investigation was made with EDGE in mind, and the goal was to find how large improvements that can be attained by introducing adaptive modulation. In a cellular system with minimum throughput 271 kbps per channel, the spectrum efficiency is increased 111% by allowing 4, 8, or 16 signal alternatives instead of only 4. If the minimum throughput is increased to 384 kbps, a spectrum efficiency gain of 66% is attainable if alphabet sizes 8 and 16 are used instead of only alphabet size 8.

However, $T_{\min} = 271$ kbps results in higher spectrum efficiency than $T_{\min} = 384$ kbps does. The reason is that 4-QAM supports 384 kbps only if the code rate is very high. Therefore, 8-PSK and a medium code rate must be used for transmission on the cell's edge. The code rate used for 16-QAM must be quite high to offer high throughput close to the base station. This leaves a gap for medium SIR:s, although the SIR-levels in a large part of the cell is in the medium region. The problem is solved if a larger alphabet than $M = 16$ is introduced. Then, R_{16} may be decreased to match the medium SIR:s. Therefore, we believe that a substantial number of modes is required to attain high spectrum efficiency in a cellular system with adaptive modulation.

No radio resources (channels or time slots) are reserved for channel estimation although perfect channel side information is assumed. This approximation is good only for slowly varying channels or high data rates. Moreover, it may be necessary to extend the decoding delay to infinity in order to obtain the capacity C_{Ray} in section II-D. Perfect channel side information and perfect synchronization are somewhat unrealistic assumptions.

The cellular system model contains simplifications, e.g., all interferers transmit simultaneously and without interruption. Such simplifications may not be allowed when modelling a real system. Even if the downlink is accurately modelled, applying the method to the uplink may result in significant approximations. Despite these approximations, we believe that the method and the results presented in this paper contributes to a better understanding of adaptive modulation in cellular systems.

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