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# An integrated cost model for metal cutting operations based on engagement time and a cost breakdown approach

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**Abstract:** In all manufacturing processes, it is important to determine the costs and their distribution between different sequential processing steps. A cost equation based directly on the losses during manufacturing, such as rejection rate, stops and waste of workpiece materials, also provides a valuable aid in giving priority to various development activities and investments. The present work concerns how a cost model presented earlier for calculating part costs can be developed to describe part costs as a function of the cutting data and tool life time  $T$  selected. This enables a tool life model to be directly integrated into the cost model by use of tool engagement time. The model presented also takes into account the part costs for scrap incurred in connection with forced tool changes. Examples are also given of how the model developed can be used in the economic evaluation of various cutting tools and workpiece materials.

**Keywords:** cost model; cost breakdown approach; metal cutting; Colding equation; tool life; cutting data.

**Biographical notes:** Jan-Eric Ståhl received his PhD from Lund University Sweden in 1986 in Metal Cutting. He was appointed as an Associate Professor and Full Professor at the Department of Mechanical Engineering, Lund University, Sweden in 1987 and 1990, respectively. He has been working in education and research in the area of production and materials engineering for more than 30 years. He was the Director of Educational Programs at the Faculty of Engineering and Vice Dean at the respective faculty responsible for the industrial connection. He initiated and started up the Swedish Production Academy in 2006.

## 1 Introduction

In discrete production, part costs are inversely related to the firm's ability to compete. Models for computing the costs of a part can be described for different hierarchic levels and in differing detail. Macroeconomic models used at a system level for determining retrospectively what the costs of manufacturing a given component have been can be rather exact. It is much more difficult to predict the costs in advance, particularly when different variables or parameters of relevance are unknown or vary statistically. Precise cost computations are also more difficult to achieve when variables of central importance, such as cycle time, rejection level and downtime rate, are partly dependent upon one another. Comprehensive economic models usually make use of aggregated data, without distinguishing between the value-added and non-value-added time consumed. Economic models can also involve detailed cost computations regarding the processing carried out. Such models, referred to as microeconomic or cost-breakdown models, tend to be concerned directly with the manufacturing process in question. Differences between microeconomic and macroeconomic models in the account they provide of the production of a component have been dealt with earlier by Tipnis et al. (1981). The present author (Ståhl, 2005; Ståhl et al., 2007) has also presented a cost-breakdown model of this at a system level, one that has been employed and implemented by many others, such as Jönsson (2012), who also analysed a number of other cost models described in the literature, the results he arrived at being shown in Table 1. Microeconomic models have also been presented, for example, by Colding (1978), Alberti et al. (1985) and Ravignani and Semeraro (1980). Such models can be used, in connection with metal cutting, to describe the relationship between cutting data, tool lifetime, and processing costs. Models of this sort usually do not include costs of rejections and downtimes, but do include costs of changing the tool or workpiece. In the present study, an integrated cost model including both loss terms (so-called q-parameters) and a complete model of the lifetime of metal-cutting tools is introduced, one that is restricted to cutting operations and concerns primarily turning operations. It is based upon the same principles as those of a model developed by the author earlier (Ståhl, 2005). A comparable model concerning the costs a given surface requirement concerning the  $R_a$ -values would entail has been reported by Schultheiss et al. (2016).

In the present study, the Swedish currency (SEK) is used in all examples, but the model is generic and is independent of the currency unit selected.

**Table 1** A summary of models presented in the literature

Parameters																
	System level															
	Present study	Ståhl, (2016), Ståhl et al., (2007)	Ravignani et al. (1980)	Alberti et al. (1985)	Colding (1978)	Kohat et al. (2000)	Ozbayrak et al. (2004)	Aderoba (1997)	Dhavalie (1990)	Yamashina and Kubo (2002)	Son (1991)	Chidamwong (2003)	Cauchick- Miguei et al. (1996)	Branker et al. (2011)	Noto La Diega et al. (1993)	Needy et al. (1998)
Material	x	x	x	x	x		x	x	x	x	x	x	x	x	x	
Labor	x	x		x	x		x	x	x <sup>1</sup>	x	x	x	x	x	x <sup>1</sup>	x
Machine depreciation						x <sup>1</sup>	x <sup>1</sup>	x	x	x	x	x	x	x		
Machine cost	x	x	x	x	x		x <sup>1</sup>	x	x	x	x	x	x	x		
Floor space	x	x					x <sup>1</sup>	x	x		x					
Utilities (e.g., energy)	x	x					x <sup>1</sup>	x	x <sup>1</sup>	x	x	x	x	x		
Tool cost	x	x	x		x		x	x <sup>1</sup>	x <sup>1</sup>	x	x	x	x	x	x	
Maintenance	x	x				x <sup>1</sup>	x <sup>1</sup>	x <sup>1</sup>	x <sup>1</sup>	x	x	x	x	x		
Repairs	x	x				x	x	x <sup>1</sup>	x <sup>1</sup>		x	x	x	x	x <sup>1</sup>	x
Material handling	x	x				x	x <sup>1</sup>	x <sup>1</sup>	x <sup>1</sup>		x	x		x	x <sup>1</sup>	
Computer							x	x	x <sup>1</sup>							
Inventory	x	x	x		x	x	x		x <sup>1</sup>		x					
Quality: prevention											x	x <sup>2</sup>				
Quality: appraisal						x <sup>1</sup>				x	x	x	x			
Quality: failure (scrap)	x	x			x		x <sup>2</sup>			x	x	x	x			
Reworking	x	x	x				x <sup>2</sup>			x	x	x	x <sup>1</sup>			
Downtime	x	x					x <sup>2</sup>				x	x				
Speed loss	x	x								x		x				
Setup	x	x			x		x			x	x	x	x	x		x
Waiting	x	x														
Idling	x	x					x	x <sup>1</sup>			x	x			x	
Environmental												x		x		
Parameters																
Process level																
Cycle time	x	x	x	x	x											
Tool engagement time	x		x	x	x											
Idling in cycle	x			x	x											
Cutting data	x		x	x	x											
Tool life	x		x	x	x											

Note: <sup>1</sup>Included, but not as a separate parameter. <sup>2</sup>Mentioned as considered, but the equation is not presented in the paper. <sup>3</sup>The cost is expressed as a leasing cost.

Source: As modified by Jönsson (2012)

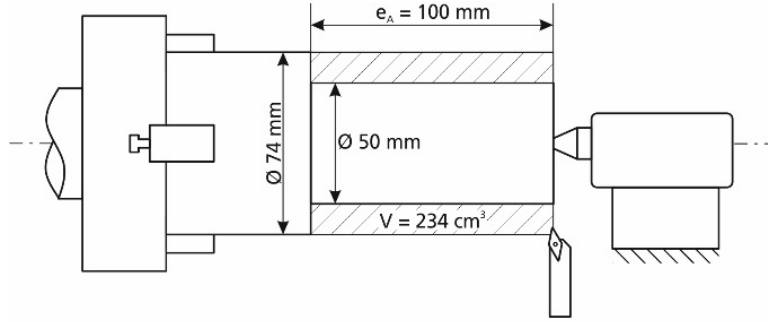
## 2 The metal-cutting process and how the cycle time is produced

A particular cycle time is needed in order to process a component in the manner shown in Figure 1. The cycle time  $t_0$  can be obtained as the sum of the engagement time  $t_e$ , of the remaining time  $t_{rem}$ , the latter concerning the movement of the tool associated with non-value-added time, and of the time needed for change of the workpiece and of the tool  $T_{tct}$ .

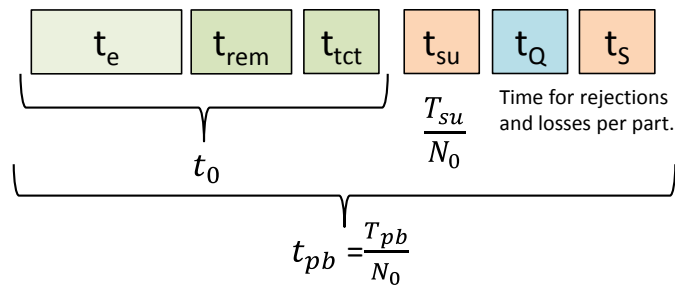
$$t_0 = t_e + t_{rem} + t_{tct} = t_e + t_{rem} + \frac{t_e}{T} \cdot t_{tct} \quad (1)$$

where  $T$  is the tool lifetime selected and  $T_{tct}$  is the time taken for tool change,  $t_{tct}$  being the contribution of the workpiece to the time required for tool change. An alternative way of describing the time needed for tool change is to treat it as being a downtime. It can nevertheless be practical, if the time per workpiece needed for tool change is short, to consider it to be part of the added-value time. The engagement time  $t_e$  is then the added-value time, and the remaining time  $t_{rem}$  can be regarded as the loss in time that occurs in processing, a loss that is unavoidable due to the nature of added-value processing.

**Figure 1** Longitudinal turning in principle



**Figure 2** The individual times that lead to the total manufacturing time per part



Just as with other losses, such as those of rejections  $q_Q$ , this loss can be dealt with and described by use of a loss factor  $q_{rem}$  and be expressed in relation to the value-added engagement time  $t_i$  as shown in equation (2).

$$q_{rem} = \frac{t_{rem}}{t_e + t_{rem}} \quad (2)$$

The loss term  $q_{ict}$  for tool change can be described in a similar way in regard to the engagement time and the additional time that tool change brings about as:

$$q_{ict} = \frac{t_{ict}}{t_e + t_{rem} + t_{ict}} \quad (3)$$

where the average tool change time  $t_{ict}$  per part can be computed as:

$$t_{ict} = \frac{t_e}{T} \cdot T_{ict} \quad (4)$$

The cycle time  $t_0$  can then be expressed, with the help of the loss terms  $q_{rem}$  and  $q_{ict}$ , as:

$$t_0 = \frac{t_e}{(1 - q_{rem}) \cdot (1 - q_{ict})} \quad (5)$$

The additional time  $t_{rem}$  is strongly dependent upon the preparations made, whereas the tool change time  $T_{ict}$  depends more upon the manner of working and the machine characteristics. The production time  $T_{pb}$  for a batch of  $N_0$  parts, as shown in Ståhl (2005), can be computed using equation (6), which takes account of the rejection rate  $q_Q$  and the downtime rate  $q_S$ .

$$T_{pb} = T_{su} + \frac{t_0 \cdot N_0}{(1 - q_Q) \cdot (1 - q_S)} \quad (6)$$

Use of the loss terms in equation (5) enables the cycle times in equation (6) to be expressed, in equation (7), with the help of the engagement time  $t_e$ .

$$T_{pb} = T_{su} + \frac{t_e \cdot N_0}{(1 - q_{rem}) \cdot (1 - q_{ict}) \cdot (1 - q_Q) \cdot (1 - q_S)} \quad (7)$$

The formalism above agrees with the principle developed by the author earlier (Ståhl, 2005; Ståhl et al., 2007) for dealing with losses (q-parameters) associated with the cycle time.

The engagement time  $t_e$  is determined by the cutting data selected ( $v_c, f, a_p$ ) and by the volume of work material  $V$  to be removed. It can be computed using equation (8) or equation (9), where  $e_A$  is the axial distance involved.

$$t_e = \frac{V}{v_c \cdot f \cdot a_p} \quad (8)$$

$$t_{e,A} = \frac{e_A}{f \cdot N} = \frac{e_A \cdot \pi \cdot D \cdot 10^{-3}}{f \cdot v_c} \quad (9)$$

The time per part produced  $t_{pb}$  can be computed by dividing equation (7) by the series length  $N_0$  using equation (10).

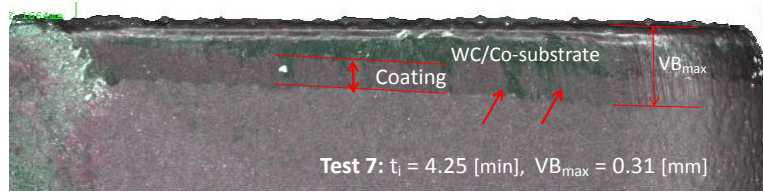
$$t_{pb} = \frac{t_{pb}}{N_0} = \frac{t_{su}}{N_0} + \frac{t_e}{(1-q_{rem}) \cdot (1-q_{lct}) \cdot (1-q_Q) \cdot (1-q_s)} \quad (10)$$

In this case the engagement time  $t_e$  rather than the cycle time  $t_0$  serves as the primary basis for the computations carried out.

### 3 Tool life and tool life models

The lifetime  $T$  of a cutting tool is determined by the tool's characteristics, as well as by the work material, the cutting data and the types of operations involved, as well as by the tool lifetime criterion employed. One tends to distinguish between a wear-based model and a lifetime model. A wear-based model describes how the speed with which the tool is worn down changes as a function of time and of other process data. A tool lifetime model describes the total engagement time  $t_i$  up to a predetermined total tool lifetime, for example such that  $VB = 0.30$  mm under the processing conditions present (cutting data).

**Figure 3** Example of a tool lifetime criterion of  $VB = 0.3$  mm



#### 3.1 Colding equation

The tool lifetime  $T$  can be modelled in a variety of ways. The model most frequently employed is an extension of the Taylor model, an 'extended Taylor'. It involves use of four constants. Colding's equation usually functions somewhat better than an 'Extended Taylor', which in its most usual form has five constants. Colding's equation describes, for an application having a predetermined tool lifetime criterion, the relationship between the cutting speed  $v_c$  and both the equivalent chip thickness  $h_e$  and the tool lifetime  $T$ . It is based on Woxén's (1932) assumption that for a given equivalent chip thickness  $h_e$  the tool lifetime  $T$  is always the same. Colding's (1982) equation is presented here as equation (11).

$$v_c = \exp \left[ K - \frac{(\ln(h_e) - H)^2}{4 \cdot M} - (N_0 - L \cdot \ln(h_e)) \cdot \ln(T) \right] \quad (11)$$

where  $K$ ,  $H$ ,  $M$ ,  $N_0$  and  $L$  are Colding's constants. The equivalent chip thickness  $h_e$  can be computed in terms of Woxén's approximation using equation (12).

$$h_e = \frac{A_w}{l_w} = \frac{a_p \cdot f}{\frac{a_p - r_e(1 - \cos \kappa)}{\sin \kappa} + \kappa \cdot r_e + \frac{f}{2}} \quad (12)$$

Use of the equivalent chip thickness  $h_e$  is advantageous in its combining four separate parameters to form a single one. According to Woxén (1932) and as also shown in many practical cases, a given equivalent chip thickness is associated with a given tool lifetime  $T$ . It is also possible to express Colding's equation in a way such that the tool lifetime  $T$  is obtained as a function of  $v_c$  and  $h_e$  in a given application.

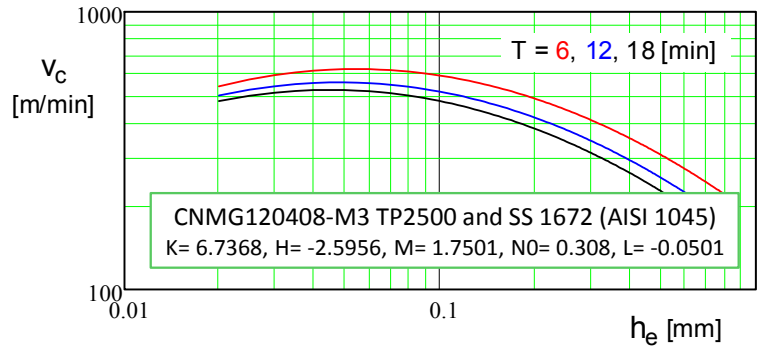
$$T = e - \frac{H^2 - 2 \cdot H \cdot \ln(h_e) + \ln(h_e)^2 - 4 \cdot K \cdot M + 4 \cdot M(v_c)}{4 \cdot M(N_0 - L \cdot \ln(h_e))} \quad (13)$$

The engagement time  $t_e$ , and thus the associated loss terms as well, are dependent upon the parameters  $T$ ,  $v_c$  and  $h_e$ . The additional parameters needed to compute  $h_e$  are the indirect variables, as they are called.

The tool lifetime in terms of Colding's equation can be expressed indirectly as a function of the variables connected with the equivalent chip thickness, i.e.,  $h_e = h_e(f, a_p, r_e, \kappa)$ . In Figure 4, the tool lifetime  $T$  is shown as a function of the cutting speed  $v_c$  as computed for various feeding levels  $f$  or  $h_e$  levels when the remaining parameters are held constant ( $a_p = 4$  [mm],  $r_e = 0.8$  [mm] och  $\kappa = 95^\circ$ ), which true as well in the additional examples taken up.

Examples of the use of Colding's equation, which describes the relationship between the cutting speed  $v_c$  and the equivalent chip thickness  $h_e$  for a given tool lifetime are shown in Figure 4.

**Figure 4** An example of a graph describing the Colding-plane for a particular application, that of the combination of  $v_c$  and  $h_e$  for a given tool lifetime  $T$



### 3.2 Number of workpieces per edge

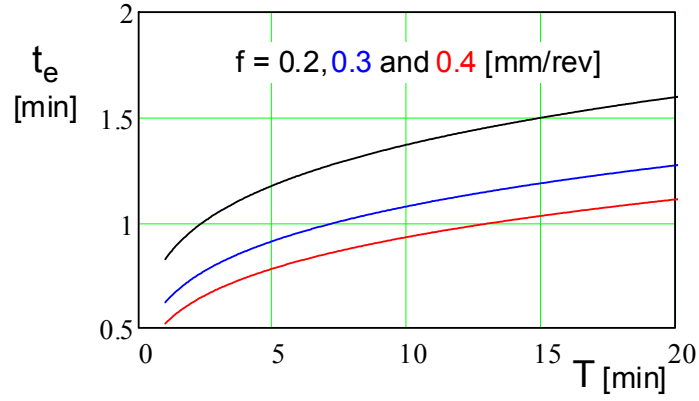
The number of workpieces  $N_{wt}$  that the edge of a cutting tool is able to process can be computed by dividing the lifetime  $T$  of the tool by the total intervention  $t_e$ .

$$N_{wt} = \frac{T}{t_e} \quad (14)$$

One often attempts to select the cutting data in such a way that a change in the cutting tool and in the workpiece take place at the same time, i.e., that the next-lower whole number  $N_{wt}$  for the latter is selected. In Figure 5, the relationship between the intervention

time  $t_e$  in equation (8) and the tool lifetime  $T$  for each of three different feeding rates  $f$  for a given chip volume  $V = 500 \text{ cm}^3$  is shown.

**Figure 5** Examples of the relationship between intervention time  $t_e$  and tool lifetime  $T$  for three different feeding rates  $f$  selected, for a chip removal volume of  $V = 500 \text{ cm}^3$

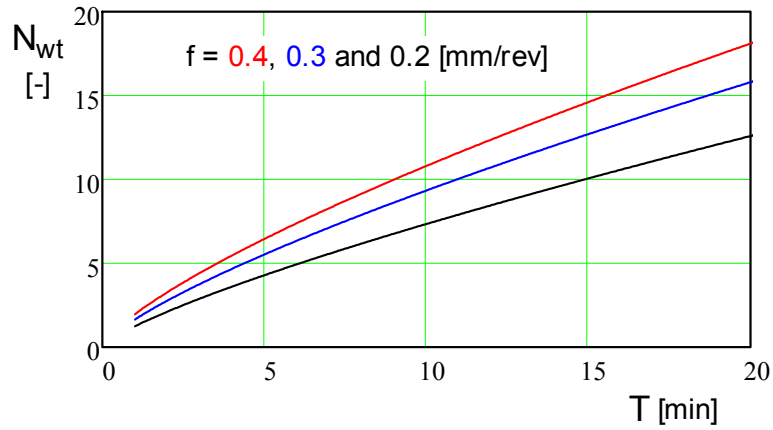


The number of tool changes needed for processing a batch of size  $N_0$  can be computed as:

$$n_{tcb} = \frac{t_e \cdot N_0}{T} \quad (15)$$

where  $N_0$  is the batch size. This is not to be confused with Colding's constant  $N_0$ .

**Figure 6** Examples of the numbers of parts  $N_{wt}$  able to be produced with use of cutting edges of differing tool lifetime  $T$  for each of three different feeding rates  $f$  at  $V = 500 \text{ cm}^3$



### 3.3 Rejections related to tool changes

A non-negligible rejection rate can often be noted in conjunction with tool change. It is not unusual for some 50% of the rejections to occur when the switchover takes place, the remainder of them occurring soon after the new tool has been installed. The reasons for rejection vary, some of the primary reasons being the following:

- Locational errors due to varying degrees of wear, to the cutting forces thus produced, and to difficulties in finding the correct reference position in the coordinate system. Problems of this sort are accentuated in connection with non-stiff fixturing
- Changes in size and form of the cutting tool and the edge radius of it, a problem that is accentuated when little variation in this respect can be tolerated.
- Inadequate routines for tool change.

In a linear model the number of parts rejected in connection with a tool change can be computed as:

$$N_{Qtc} = P_{Qtc} \frac{t_e \cdot N_0}{T} \quad (16)$$

where  $p_{Qtc}$  is the number of parts rejected, or the probability of rejection of a given part, when tool change takes place. For  $p_{Qtc} = 1.0$  a part is rejected at each tool change, whereas for  $p_{Qtc} = 0.25$  a part is rejected when 4 tool changes take place, etc. The value of  $p_{Qtc}$  can thus be greater than 1.0.

**Figure 7** Number of parts in a batch rejected in connection with tool change  $N_{Qtc}$ , shown for differing probabilities  $p_{Qtc}$  as a function of the tool lifetime selected

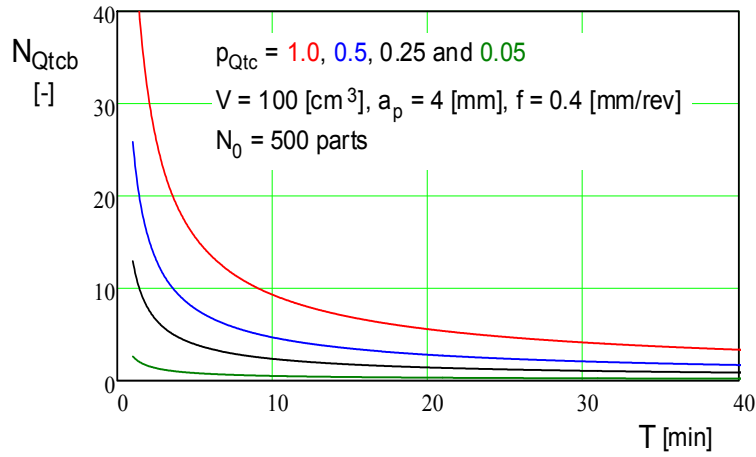
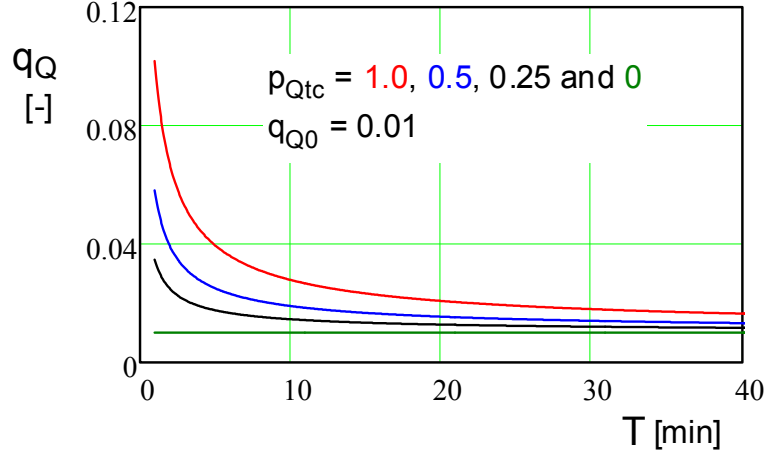


Figure 8 exemplifies how the rejection level  $q_Q$  is affected by the tool lifetime  $T$  selected, shown for different rejection rates in connection with tool change  $p_{Qtc}$ .

**Figure 8** Examples of different rejection levels  $q_Q$ , account being taken of rejections in connection with tool changes



The value of  $N_{Qtc}$  represents a portion of the traditional  $q_Q$  rejection rate value. If one assumes that  $q_{Q0}$  represents causes of rejection in addition to that of tool change (Ståhl, 2005; Ståhl et al., 2007), the rate of tool rejection as a whole can be described as:

$$q_Q = \frac{N_{Qctb} + \frac{q_{Q0}}{1 - q_{Q0}} \cdot N_0}{N_{Qctb} + \frac{N_0}{1 - q_{Q0}}} \quad (17)$$

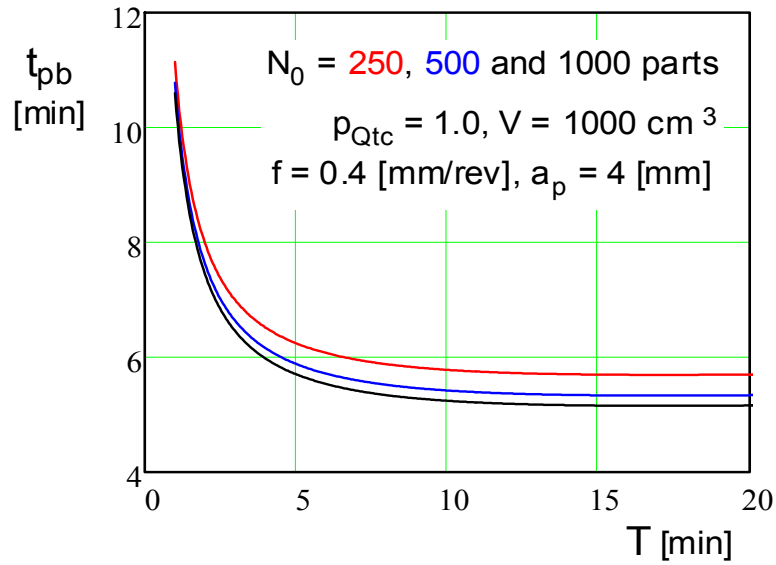
The part rejection rate  $q_Q$  is directly or indirectly dependent upon the number of different parameters as follows:

$$q_Q = q_Q \left( \underbrace{f, a_p, r_e, \kappa}_{h_e}, \underbrace{T, V}_{t_e \rightarrow}, \underbrace{N_0, P_{Qtc}}_{N_{Qtc}} \right) \quad (18)$$

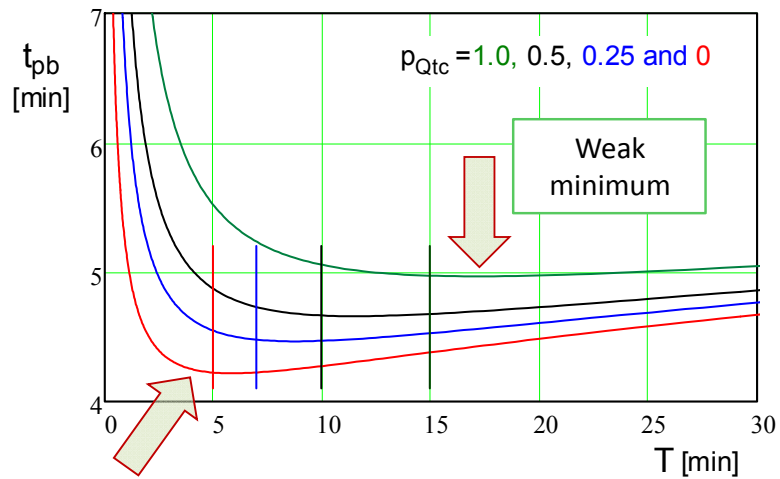
### 3.4 Time for manufacturing a batch of size $N_0$

For a particular rejection rate, as specified in equation (17), the production time per component  $t_{pb}$  can be computed with use of equation (10) shown earlier. Figure 9 illustrates that when rejection occurs in connection with tool change the effect of tool lifetime upon the time needed to produce a component is reduced. The minimal production time per part, or the maximal production rate, is then no longer as extreme compared to the microeconomic models.

**Figure 9** Production time for a component  $t_{pb}$  in batch production involving a series size of  $N_0$ , shown as a function of the tool lifetime  $T$  selected



**Figure 10** The production times  $t_{pb}$  associated with different rejection rates  $p_{Qtc}$  during tool change, which results in the minimal part-manufacturing time being displaced in the direction of higher values of  $T$



#### 4 Integrated part cost model

The costs per part can be described, as shown below, in a manner similar to that involved in the standard model (Ståhl, 2005; Ståhl et al., 2007). The difference is that here the technical cutting arrangements are fully integrated, i.e., that the macro-model takes

account of all the loss terms associated with the cycle time  $t_0$ , account also being taken of relations between the cutting process and the overall rejection rate  $q_Q$ .

The cost of producing a component consists of a variety of different elements:

- tool costs per component  $K_A$
- alongside the cost for the workpiece material  $k_B$ , the costs of workpiece material per rejected part,  $K_{BQ}$
- machine costs during production per component,  $K_{CP}$
- machine costs during downtimes per component,  $K_{CS}$
- Direct costs for personnel per component,  $K_D$ .

#### 4.1 Tool costs per part

Tool costs per component can be computed using equation (19) below.

$$K_A = \frac{K_A}{z} \cdot \frac{t_e}{T \cdot (1 - q_Q)} \quad (19)$$

where  $k_A$  is the tool costs and  $z$  is the number of cutting edges that can be used in the cutting tool,  $z$  not needing to be a whole number, since it can represent an expected average over a given period of time. It is important to also consider the tool cost for manufacturing of rejected parts by included the term for quality utilisation  $(1 - q_Q)$ .

#### 4.2 Costs of workpiece material in rejected parts

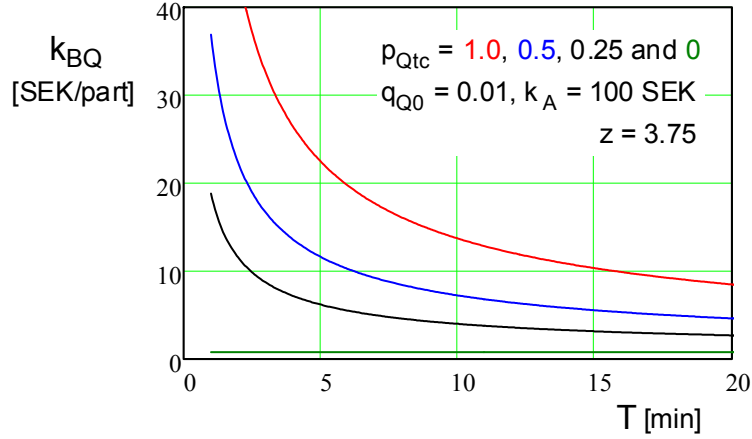
The costs of workpiece material for rejected parts can be computed using equation (20) below.

$$K_{BQ} = \frac{k_B}{(1 - q_Q)(1 - q_B)} - k_B \quad (20)$$

Since  $k_B$  is the cost of work material, subtraction of it enables the rejection costs of it (for  $q_Q \neq 0$ ) and the material waste (for  $q_B \neq 0$ ) to be computed. That approach is appropriate for following changes in added value over a series of production steps or operations. Otherwise, if  $k_B$  is not subtracted, the material costs computed at a later processing stage can turn out to be far too high, so that the precision of the results is lost. If rejection occurs in connection with tool change, the value of  $q_Q$  can be computed by use of the earlier equation (17).

In view of the rejection rate  $q_Q$  being dependent upon the number of tool changes that occur, the rejection costs can be seen to also be dependent upon the tool lifetime  $T$  selected, and in this way to also be indirectly dependent upon the cutting data selected. This relationship is exemplified in Figure 11.

**Figure 11** Examples of the rejection costs  $k_{BQ}$  in connection with tool change being a function of the tool lifetime  $T$  selected, shown for different rejection rates  $p_{Qtc}$



#### 4.3 Machine costs per part

Machine costs per component can be computed using equation (21) below, provided the machine costs per hour  $k_{CP}$  are known.

$$K_{CP} = \frac{k_{CP}}{60} \cdot \frac{t_e}{(1 - q_{rem}) \cdot (1 - q_{tct}) \cdot (1 - q_Q)} \quad (21)$$

where  $k_{CP}$  are the machine costs per hour during production, expressed as SEK/hr, which can be computed with use of the annuity method (Ståhl, 2005; Ståhl et al., 2007).

#### 4.4 Costs per part of time losses

The downtime costs per purchased and approved part can be computed using equation (22). If the hourly machine costs during the downtime  $K_{CS}$  are known,  $k_{cs}$  can be computed through use of an annuity method (Ståhl, 2005; Ståhl et al., 2007).

$$K_{cs} = \frac{k_{cs}}{60} \cdot \left( \frac{t_e \cdot q_s}{(1 - q_{rem}) \cdot (1 - q_{tct}) \cdot (1 - q_Q) \cdot (1 - q_s)} + \frac{T_{su}}{N_0} \right) \quad (22)$$

#### 4.5 Personnel costs per part

Direct costs for personnel can be computed using equation (23) below. It is assumed here that the operator who adjusts the machine is the same one who normally drives it.

$$K_D = \frac{K_D \cdot n_{op}}{60} \cdot \left( \frac{t_e}{(1 - q_{rem}) \cdot (1 - q_{tct}) \cdot (1 - q_Q) \cdot (1 - q_s)} + \frac{T_{su}}{N_0} \right) \quad (23)$$

where  $k_D$  is the salary costs in SEK/hr and  $n_{op}$  is the number of operators connected with the production segment in question. Dividing the result obtained by 60 is done to harmonise salary  $k_D$  per hr. with the cycle time  $t_0$  in minutes.

#### 4.6 Total direct costs per part

Summing the results for equation (19)–equation (23) enables the direct costs per component to be computed using equation (24).

$$k = K_A + K_B + K_{CP} + K_{CS} + K_D \quad (24)$$

One can note that a large number of parameters or variables are involved in computing the part costs as a whole using equation (24). There are more than 35 parameters altogether that influence in a direct or indirect way the costs of manufacturing a component. The list below presents the most important parameters. The bottom row presents the parameters that are indirectly involved.

In Figure 12, one can note that an increase in the probability of rejection  $P_{Qct}$  during tool change leads to a reduction in the clarity of what tool lifetime is optimal and to an ever higher value for the tool lifetime  $T$  appearing best.

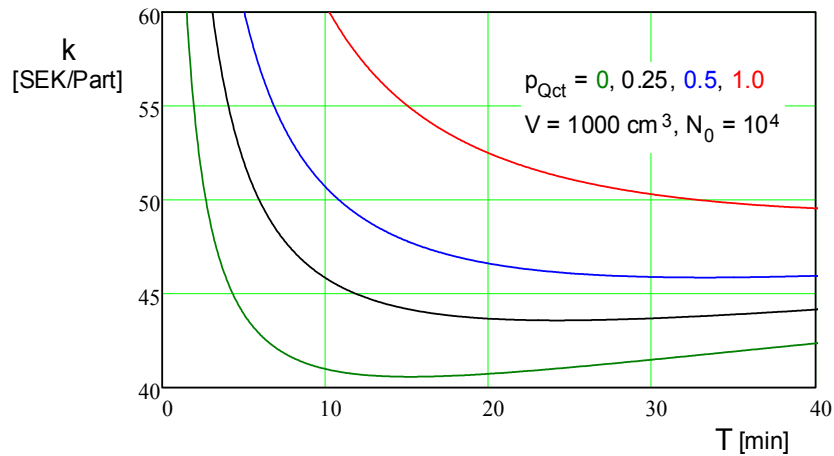
If both the production costs  $k$  in terms of equation (24) and the production time  $t_{pb}$  in terms of equation (10) are known, cost graph Hägglund (2013) can be constructed in line with Figure 14. Hägglund's original graph takes account of the q-parameters  $q_{rem}$  and  $q_{lct}$  but not of loss terms for rejections or downtimes.

In Figure 14, a linear rise in equipment and personnel costs (blue line) can be noted, that can be computed using equation (25).

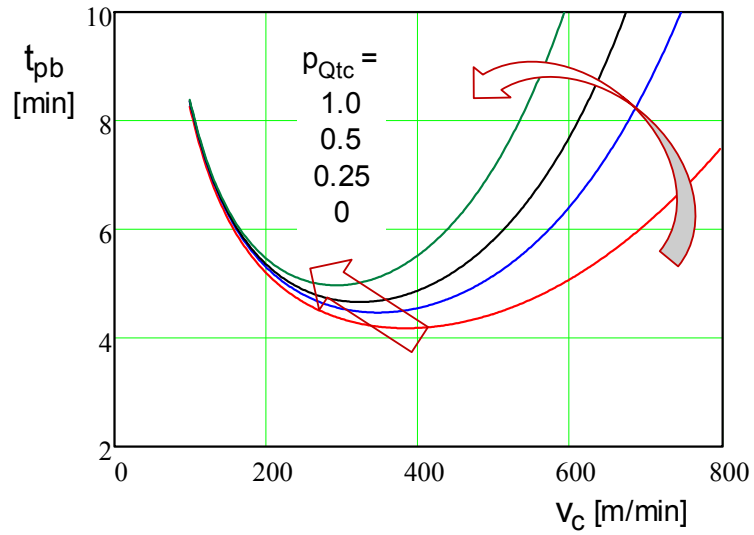
$$k_c = \frac{(k_{cp} \cdot (1 - q_s) + k_{cs} \cdot q_s + n_{op} \cdot k_D) \cdot t}{60} \quad (25)$$

Figure 15 illustrates how costs of producing a component vary with cutting speed for different values of  $p_{Qtc}$  during tool change.

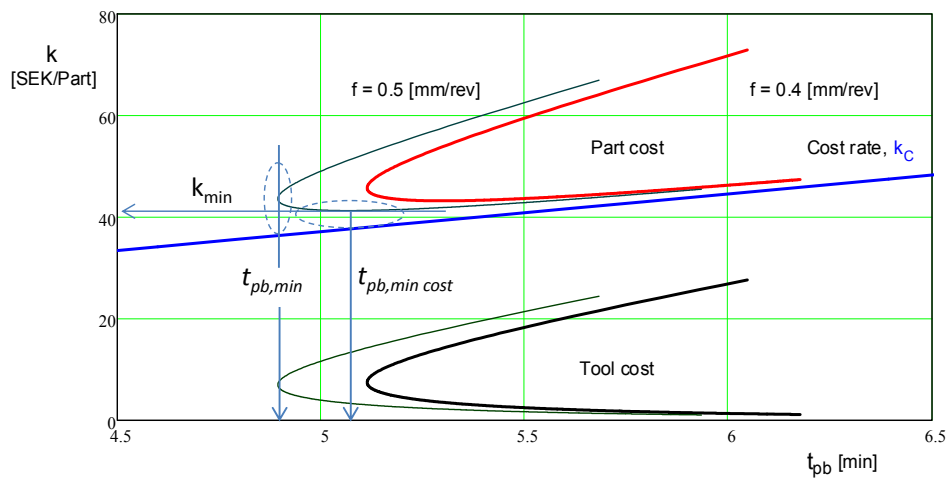
**Figure 12** Examples of the part costs  $k$  involved in SEK/part for different values of  $p_{Qct}$ , shown as a function of tool lifetime  $T$



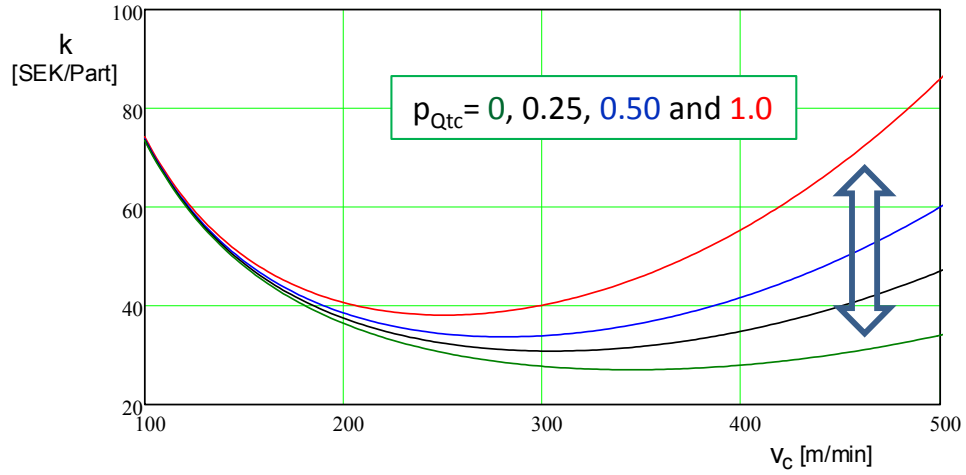
**Figure 13** Production time  $t_{pb}$  shown as a function of cutting speed  $v_c$  for different rejection rates  $p_{Qtc}$  during tool changes, where the point of minimum  $t_{pb}$  value is displaced toward lower  $v_c$  values as  $P_{Qtc}$  increases



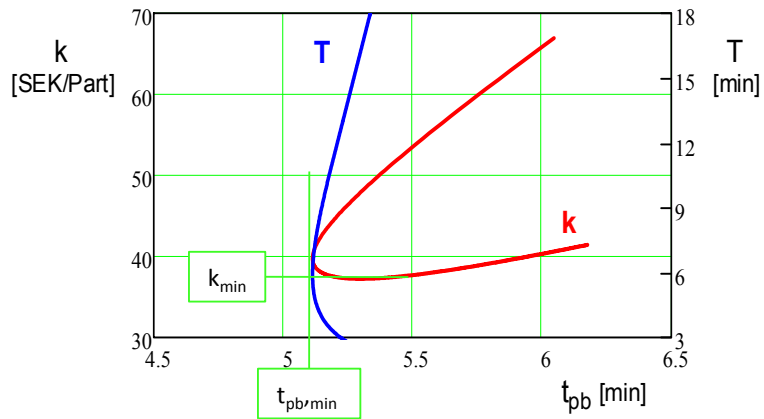
**Figure 14** Part costs and tool costs shown as a function of production time  $t_{pb}$ , increasing the feeding rate, if conditions permit, leading to shorter production times and lower costs



**Figure 15** Examples of part costs  $k$  shown as a function of cutting speed  $v_c$  for different rejection rates  $p_{Qtc}$  during tool change



**Figure 16** Part costs  $k$  and tool lifetime  $T$  shown as a function of production time  $t_{pb}$

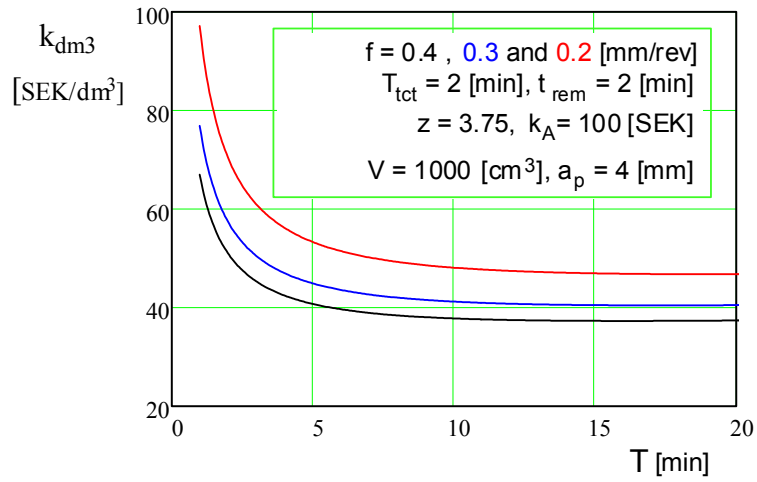


## 5 Machining costs per cubic centimetre of workpiece material

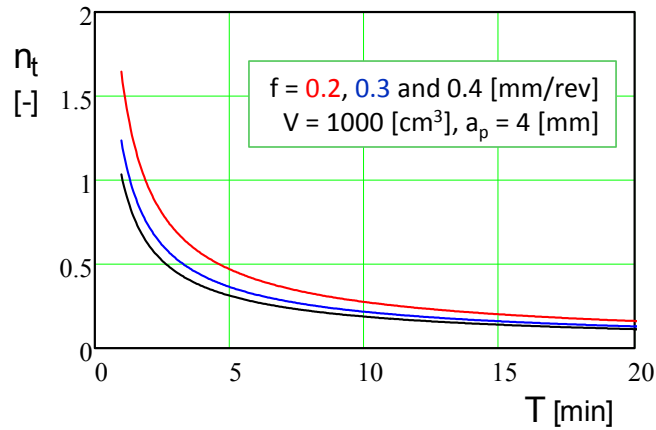
Dividing the part costs  $k$  by  $V$ , the volume of material removed, enables the material removal costs  $kcm^3$  to be computed in terms of SEK/cm<sup>3</sup> using equation (26).

$$kcm^3 = \frac{k}{V} \quad (26)$$

**Figure 17** Material removal costs  $k_{dm3}$  expressed as SEK/dm<sup>3</sup> for  $V = 1.0 \text{ dm}^3$ , shown as a function of tool lifetime  $T$  for each of three different feeding rates



**Figure 18** The number of cutting tools (cutting edges) needed for the removal of  $V = 1.0 \text{ dm}^3$ , shown as a function of the tool lifetime  $T$  selected



The number of cutting edges or the parts of these consumed in the removal of a given volume of material can be computed using equation (27). The number of components that can be produced per cutting edge  $N_{wt}$  can be computed using equation (14).

$$n_t = \frac{1}{N_{wt}} = \frac{t_e}{T} \quad (27)$$

Results obtained by use of equation (26) and equation (27) are exemplified in Figure 17 and Figure 18, respectively.

## 6 Relations of part costs to variations in machinability

Cutting data that appear promising do not always result in the tool lifetime  $T$  that was aimed at. This can lead to a loss in tempo  $q_p$ , involving a discrepancy between the nominal or recommended data and the actual cutting data used in the application at hand.

### 6.1 Differentiation of the Colding equation

One possibility for describing such a situation is to differentiate Colding's equation with regard to the relatively strong constant  $K$  that the equation contains. This enables one to create a new and more adequate tool lifetime model.

Such a differentiation of Colding's model can be based on use of equation (28).

$$\Delta T = \frac{dT}{dK} \cdot \Delta K \quad (28)$$

where  $\Delta K$  is the change in Colding's constant  $K$  corresponding to the error in the tool lifetime  $\Delta T$ . If the tool lifetime turns out to be 3 minutes shorter than was expected, one lets  $\Delta T = -3$  minutes.

Rewriting equation (28) results in  $\Delta K$  having the value of

$$\Delta K = \frac{\Delta T}{\frac{dT}{dK}} \quad (29)$$

A derivation of Colding's equation (13) with regard to  $K$  results in:

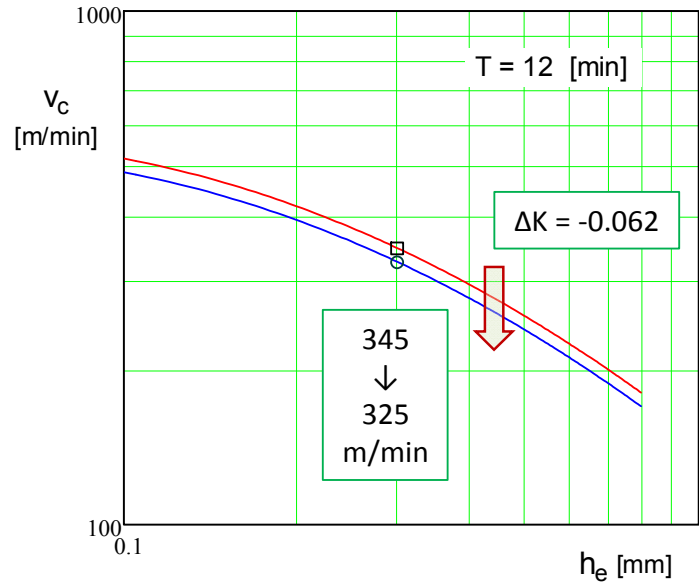
$$\frac{dT}{dK} = e - \frac{H^2 - 2 \cdot H \cdot \ln(h_e) + \ln(h_e)^2 - 4 \cdot K \cdot M + 4 \cdot M \cdot \ln(v_c)}{\frac{4 \cdot M (N0 - L \cdot \ln(h_e))}{N0 - L \cdot \ln(h_e)}} \quad (30)$$

Use of the differentiation that equation (30) provides enables  $\Delta K$  to be computed. Inserting it in equation (13) yields the following:

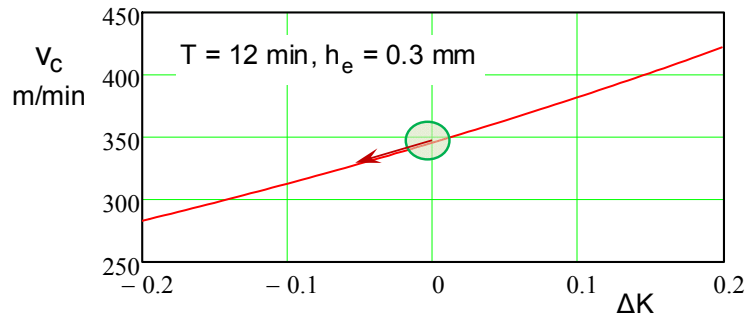
$$T = e - \frac{H^2 - 2 \cdot H \cdot \ln(h_e) + \ln(h_e)^2 - 4 \cdot (K + \Delta K) \cdot M + 4 \cdot M \cdot \ln(v_c)}{4 \cdot M \cdot (N0 - L \cdot \ln(h_e))} \quad (31)$$

In the present case, in which  $\Delta T = -3$  minutes and  $dT/dK = 48.67$ , one obtains  $\Delta K = -0.062$ . This results in a reduction in the cutting speed  $v_c$  of abt. 20 m/min in connection with obtaining a tool lifetime of  $T = 12$  minutes. Changes in machinability, described as  $\Delta K$ , are reported in Figure 19.

**Figure 19** Adjusting the cutting speed by  $\Delta K$  in order to obtain a tool lifetime of 12 minutes



**Figure 20** An example of the relationship between the cutting speed and  $\Delta K$  used for obtaining a tool lifetime of  $T = 12$  min



## 6.2 Relations of part costs to variations in machinability

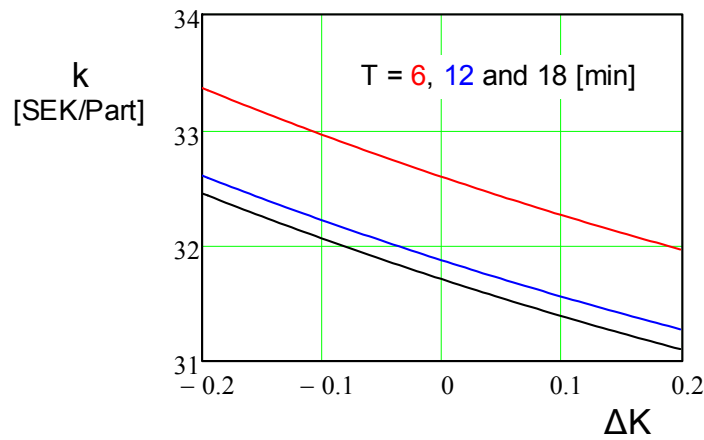
Manufacturing costs are affected by variations in machinability. Calibration of the lifetime of a tool by use of particular cutting data can be achieved by introducing the parameter  $\Delta K$  through use of equation (29). The example above, linked to Figure 19 and Figure 20, involved correcting the value of the cutting speed  $v_c$ , which is done quite frequently in cases in which there is wear without such further complications as tool failure or damage to the cutting edge. It is also possible to correct other constants in Colding's equation, such as the H-constant or  $N0$ .

Faced with a case like that described above, there are two differing experimentally anchored approaches that can be used, the one involving changes in cutting speed and the

other changes in the equivalent chip thickness  $h_e$  (primarily through changes in the feeding rate  $f$ ) with the aim of achieving a particular tool lifetime. It could be appropriate here to use  $\Delta K$  in combination with  $\Delta H$ , for example.

Measuring machinability while taking account of the tool lifetime though use of the parameter  $\Delta K$  enables  $k$ , the costs per component, to be computed. At the same time, a cost increase  $\Delta k$  due to a reduction in machinability would be independent of the tool lifetime  $T$  selected, since  $T$  is a variable in Colding's equation and both  $K$  and  $K + \Delta K$  are constants. Figure 21 exemplifies how the costs of producing a component can vary as a function of changes in machinability,  $\Delta K$ .

**Figure 21** Examples of part costs shown as a function of  $\Delta K$



## 7 Relations of part costs to tool costs and tool performance

The effects of a wide variety of parameters and variables, some of them included in Figure 22 or in the symbol list below, can be studied by use of the cost model described here.

The effects of tool costs on the cost of producing a given component are exemplified in Figure 23 and Figure 24.

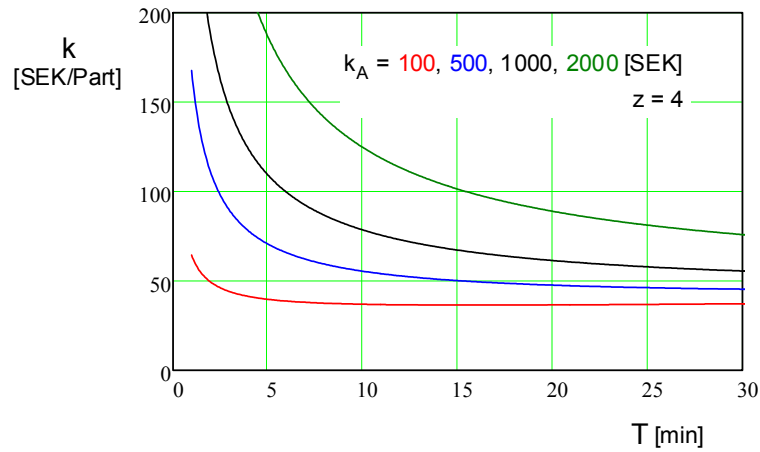
**Figure 22** Parameters and variables that contribute to the costs  $k$  of producing a particular component

$$k(f, a_p, r_\varepsilon, \kappa, T, \Delta K, V, N_0, p_{Qtr}, q_S, T_{su}, K_0, p, n, T_{plan}, k_{Az}, z)$$

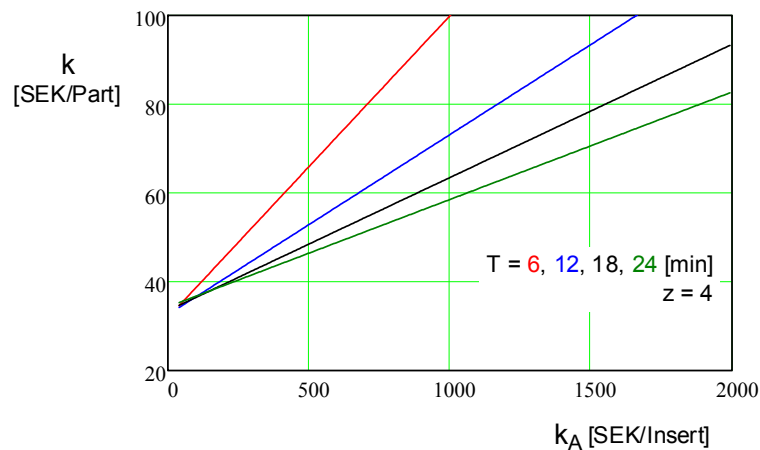
$$k(0.4, 4, 0.8, 95, T, \Delta K, 10^3, 10^3, p_{Qtr}, 0.10, 180, K_0, p, n, T_{plan}, k_{Az}, z)$$

Note: The machinability of the work material  $\Delta K$  and the tool costs  $k_{Az}$ , both of which are marked here, are particularly important.

**Figure 23** Examples of the effects of tool costs  $k_A$  on part costs, shown as a function of  $T$



**Figure 24** Examples of the costs of producing a given component, shown as a function of the tool costs  $k_A$  and the tool lifetime  $T$



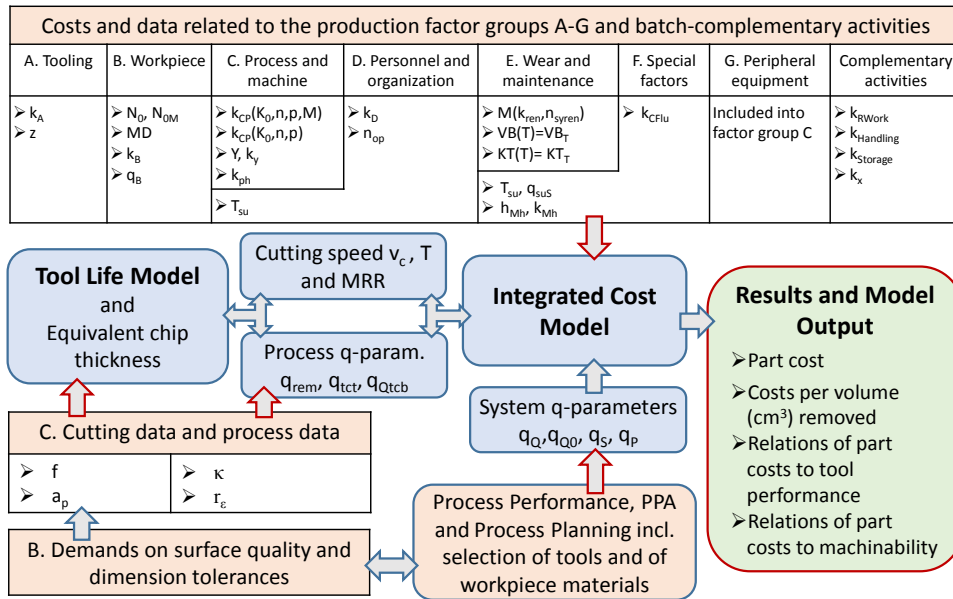
Just as the machinability of a work material can be calibrated in the form of  $\Delta K$ , calibration can also be carried out in connection with tool change or a change in supplier. Calibration of the type  $\Delta K$  can be sensible to perform when two tools of the same type, or designed for applications of the same type, are involved. The approximations this involves are based on the principle that it is simply the  $K$ -constant that distinguishes two tools of the same type and that a given work material does not vary in terms of machinability.

If two tools differ in cost, this can be thought to be attributable at least in part to their performance, e.g., in terms of tool lifetime  $T$ , with use of the same cutting data in both cases, or in the form of metal removal rate (MRR) at a given tool lifetime level.

## 8 Summary and conclusions

A cost model of cutting processes that was developed in which process and system parameters are integrated is reported on here. The model's overall structure is described in Figure 25. The model stems from a model developed by the author earlier Ståhl (2005) referred to here as the standard model. A positive characteristic of the standard model, one retained in the present model, is its taking account of important loss terms (q-terms) concerned with rejections ( $q_Q$ ), time losses ( $q_S$ ), and material waste ( $q_B$ ). The fact that tool lifetime  $T$  could be included in both models has also meant that appropriate models for relating it to cutting tool data and to tool lifetime criteria could be employed. Basing the present model on the engagement time  $t_e$  of the cutting tool, rather than on the cycle time  $t_0$ , enables a direct relationship to be established between process parameters (cutting data), cycle time  $t_0$ , and tool lifetime  $T$ . To be able in the present model to express cycle time  $t_0$  with the help of engagement time, two new q-terms were defined and introduced:  $q_{rem}$ , or time losses associated with non-value-added time being included in cycle time, and  $q_{tct}$ , or time losses brought about by tool changes. In the present case, Colding's equation was used to describe the relationship between tool lifetime  $T$  and cutting data in the form of cutting speed  $v_c$  and equivalent chip thickness  $h_e$ .

**Figure 25** Structure of the model developed for the analysis of integrated manufacturing costs



Other tool lifetime models, such as some variant of the Extended Taylor model, can also be included in the present model. Johansson et al. (2016) have shown that a model of the latter type can provide comparable results when Colding's equation is employed. Using Colding's equation to estimate tool lifetime also makes it possible to assess costs of varying degrees of machinability by differentiating Colding's equation and adjusting the constant  $K$  to  $K + \Delta K$ .

The model as described encompasses a large number of parameters and variables, making it possible to carry out a wide variety of analyses in addition to those reported on here. These include the following:

- computing the costs per cm<sup>3</sup> of the material removed, i.e., assessing a work material's machinability as expressed in economic terms
- analysing the balance between a tool's cost and its performance (cost-performance ratio)
- computing the costs associated with limited tool utilisation

The present model is implemented in Mathcad and can be developed further, e.g., by use of a more advanced model for describing the tool lifetime, one taking account of further tool lifetime criteria. A user-friendly interface should be developed too so as to increase the model's use and industrial application.

## Acknowledgements

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## Nomenclature

<i>Symbol, meaning and units</i>		
$a_p$	Depth of cut	mm
$e_A$	Axial tool travel distance	mm
$D$	Workpiece diameter	mm
$f$	Feed	mm/rev
$h_e$	Equivalent chip thickness	mm
$h_{eW}$	Woxén equivalent chip thickness	mm
$H$	Colding constant	-
$k$	Part cost	SEK/unit
$k_B$	Material cost per part	SEK/unit
$K_B$	Material cost per part including material waste	SEK/unit
$K_{BQ}$	Material cost per part for material waste	SEK/unit
$k_{cm^3}$	Cost per chip volume	SEK/cm <sup>3</sup>
$k_{CP}$	Hourly cost of machines during production	SEK/hr
$K_{CP}$	Machine cost per part during production	SEK/part
$k_{CS}$	Hourly cost of machines during downtime and setup times	SEK/hr
$K_{CS}$	Machine cost per part during disturbances	SEK/part
$k_D$	Average personnel cost per operator	SEK/hr
$K_D$	Average personnel cost per part	SEK
$K$	Colding constant	-
$K_0$	Equipment original investment	SEK
$K_A$	Tool cost per part	SEK/part
$k_{Az}$	Tool cost per insert	SEK/unit
$L$	Colding constant	-
$M$	Colding constant	-
$n$	Technical life time for equipment	year
$n_{op}$	Number of operators	unit
$nt$	Portion of an edge to produce a part	-
$N$	Total amount of parts required to be able to produce $N_0$ parts	unit
$N_{Qctb}$	Number of rejections related to tool changes per batch of $N_0$ parts	unit
$N_0$	Colding constant	-
$N_0$	Nominal batch size	unit
$N_{0M}$	Nominal manufactured batch size	unit
$N_{wt}$	Number of workpieces per cutting edge	unit
$p$	Cost rate for capital (interest level)	%
$p_{Qtc}$	Average portion of rejected parts during tool changes	-
$q_B$	Material waste rate	-
$q_Q$	Scrap rate	-
$q_{Q0}$	Total scrap rate except rejections related to tool changes	-

**Nomenclature (continued)**

<i>Symbol, meaning and units</i>		
$q_{Qtc}$	Scrap rate related to tool change	-
$q_P$	Production-rate loss	-
$q_S$	Downtime rate	-
$q_{rem}$	Time loss factor within $t_0$	-
$q_{tct}$	Time loss factor related to tool change	-
$r_c$	Tool nose radius	mm
$t$	Time in general	min
$T_0$	Ideal cycle time	min
$t_e$	Engagement time	min
$t_{rem}$	Average process idle time within $t_0$	min
$t_{tct}$	Average tool change time per part	min
$T$	Tool life time	min
$T_{su}$	Setup time for a batch	min
$T_{tct}$	Average tool change time	min
$T_{pb}$	Production time for a batch with $N_0$ parts	min
$T_{plan}$	Planned production time per year	hr
$v_c$	Cutting speed	m/min
$V$	Workpiece volume to be removed	cm <sup>3</sup>
$V_B$	Tool flank wear	mm
$z$	Average number of edges per insert	unit
<i>Greek symbol</i>		
$\kappa$	Major cutting edge angel	o