Interleaver design for turbo codes based on the performance ofiterative decoding

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Interleaver design for turbo codes based on the performance of iterative decoding

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Abstract— **The performance of a turbo code is dependent on two properties of the code: its distance spectrum and its suitability to be iteratively decoded. Both these properties are dependent on the specific interleaver used in the turbo-coding scheme. This paper describes a strategy for interleaver design that includes a criterion for the performance of iterative decoding. This criterion is based upon correlation properties of the extrinsic information, which is used as *a priori* inputs to the constituent decoders. Simulations comparing interleaver choices indicate an improved turbo code performance for interleavers designed with algorithms including a correlation criterion.**

I. Introduction

Since the introduction of turbo codes, quite an effort has been made to find good interleaver designs, e.g. [1], [2], [3], [4], [5]. Most of these efforts have been directed towards finding interleavers which result in codes with better distance spectra. This optimization criterion is the natural choice for codes that are decoded with a maximum likelihood decoding algorithm. However, turbo codes are decoded iteratively, which is suboptimal compared to maximum likelihood decoding. This calls for an additional design criterion in the code construction. The performance degradation of iterative decoding compared to the performance of maximum likelihood decoding is dependent on the particular interleaver used in the turbo encoder. This issue should be included in the design of interleavers for turbo codes.

There are previously proposed interleaver designs that give good iterative decoding performance, for example the S-random interleaver [1]. This is a semi-random interleaver in which permutations resulting in 'short cycles' are avoided. A short cycle occurs when two bits that are close to each other remain close after interleaving. It is concluded in [6] that these short cycles degrade the performance of iterative decoding. The reason for this degradation is that they cause nearby extrinsic inputs to be correlated to each other. Since extrinsic inputs that are close to each other influence the same part of the decoding trellis, it is natural that the performance of the decoder is degraded if these inputs are correlated. In this paper an approximation of the correlation coefficients between the decoder inputs and extrinsic outputs is used as a design criterion in an interleaver design algorithm.

The presentation is based upon the iterative decoding scheme depicted in Fig. 1. Each soft-input/soft-output constituent decoder has three input sequences and three output sequences, see for example [7]. The three inputs are: the received systematic sequence, x; the parity sequence, $y^{(1)}$ or $y^{(2)}$; and the *a priori* probability sequence from the previous decoder (extrinsic input). The three outputs are a weighted version of the received systematic sequence, a weighted version of the extrinsic input, and finally the new extrinsic output, $Le^{(1)}$ or $Le^{(2)}$. The decision variables are formed as the sum of the three outputs from each decoder. The constituent decoders used throughout this investigation employ the maximum *a posteriori* probability (MAP) algorithm as described in [7].

Section II discusses the correlation properties of the extrinsic information, and introduces a useful approximation of these correlation coefficients. These coefficients are then used in the interleaver design in Section III, which also includes a discussion of other design criteria. In Section IV, interleavers designed with the discussed criterion are evaluated in terms of error correcting performance, and compared to other interleaver structures.

II. Correlated extrinsic information

The extrinsic outputs from the first decoder after the first decoding step have been found to be correlated to the systematic inputs in an exponentially decaying fashion. The solid line in Fig. 2a shows empirically found correlation coefficients between extrinsic output 50 and the entire sequence of systematic inputs (105 bits). These coefficients are obtained by averaging $\left(Le_{50} - \overline{Le_{50}}\right)\left(x_i - \overline{x_i}\right)$

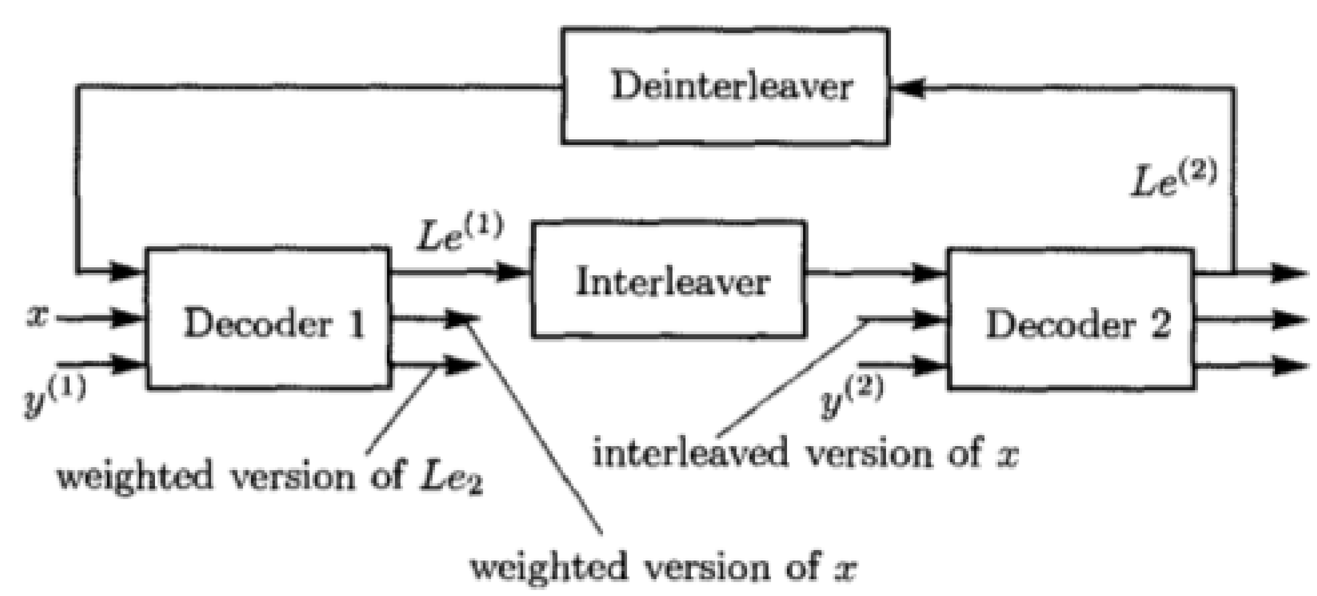

Fig. 1. Structure of the iterative decoder

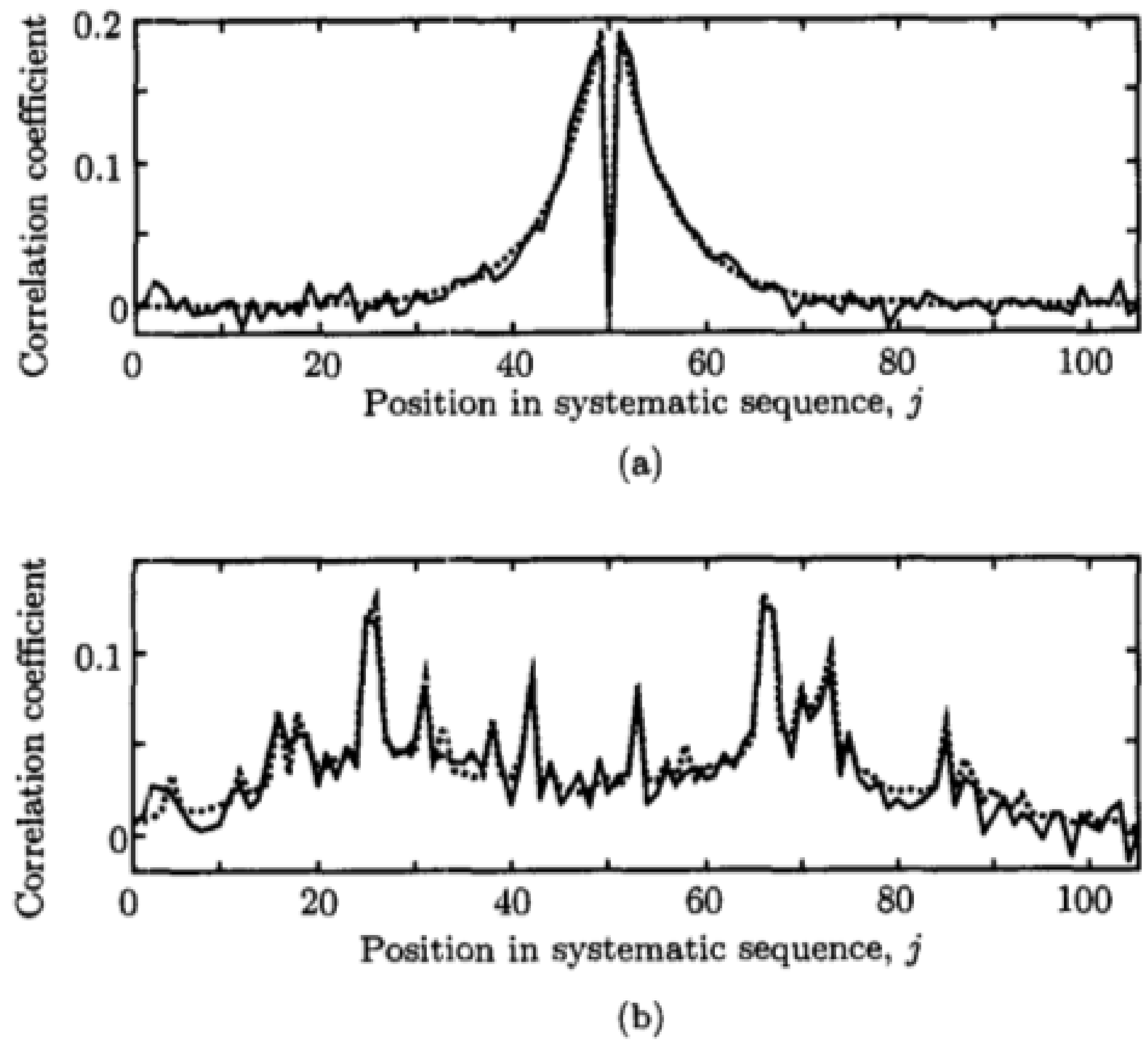

Fig. 2. Correlation coefficients between extrinsic output 50 and the entire systematic sequence, after the first (a) and second (b) decoder. Solid: empirically found coefficients, dotted: approximated coefficients.

over a large number of blocks and normalizing with the variances of Le_{50} and x_i, when repeatedly transmitting the all-zero code word over an AWGN channel. The turbo code uses a 105 bit interleaver and constituent encoders using feedback and parity polynomials 15_{oct} and 17_{oct} respectively. The dotted line shows approximated coefficients, achieved using an exponentially decaying function. Let $\rho^{(n)}_{Le_i,x_j}$ denote the approximated correlation coefficient between extrinsic output i and systematic input j after decoding step n. After the first decoding step we have:

$$\rho^{(1)}_{Le_i,x_j} = \begin{cases} 0 & \text{if } i = j \\ ae^{-c|i-j|} & \text{otherwise} \end{cases} \qquad i,j = 1,2,\ldots,N, \tag{1}$$

where N is the size of the interleaver, and the constants a and c adjust the amplitude and the exponential decay rate respectively. The values of a and c in Fig. 2 are 0.23 and 0.18 respectively. The reason $\rho^{(1)}_{Le_i,x_i} = 0$ is the implementation of the constituent decoders; the decoder output '*weighted version of* x' contains all the dependency between the decoder outputs at time i and the systematic input at time i, and therefore the correlation between the extrinsic output i and systematic input i is zero.

The interleaver does not affect the correlation coefficients after the first decoding step, $\rho^{(1)}_{Le_i,x_j}$. However, investigating the same correlation coefficients after the second decoding step reveals that the interleaver plays an important role. Such correlation coefficients are shown in Fig. 2b, both empirically found (solid) and approximated (dotted). The approximation is achieved by an

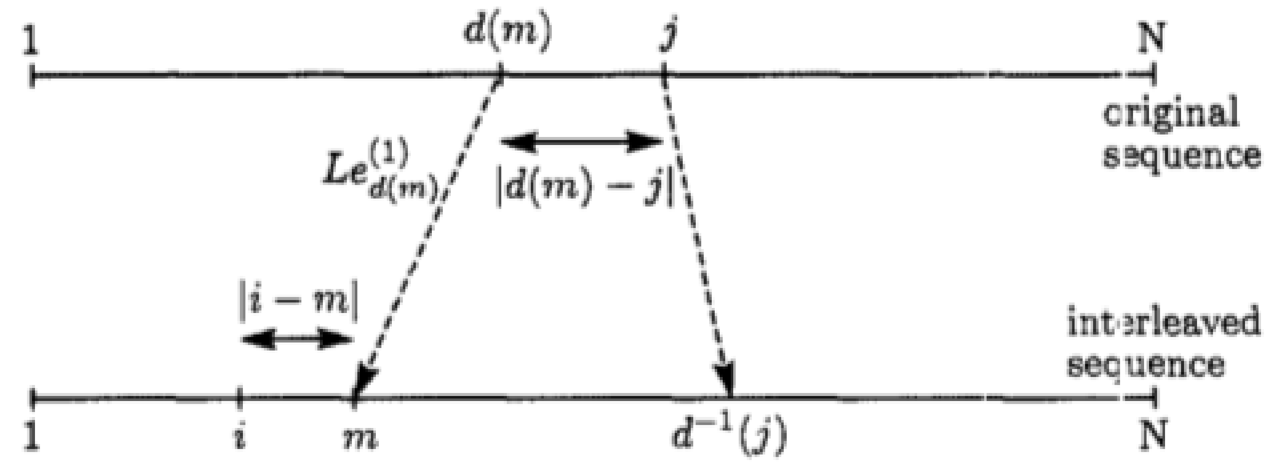

Fig. 3. Illustration of how element $d(m)$ of the interleaver contributes to the correlation between extrinsic output i from the second decoder and systematic input j.

equal weight averaging of the correlation stemming from the systematic and extrinsic inputs:

$$\rho^{(2)}_{Le_i,x_j} = \frac{1}{2}\overbrace{ae^{-c|d^{-1}(j)-i|}}^{\text{systematic input}} + \frac{1}{2}\overbrace{\sum_{\substack{m=1\\m\neq i}}^{N} a^2 e^{-c(|d(m)-j|+|i-m|)}}^{\text{extrinsic input}}. \tag{2}$$

The interleaver is present in the form of $d(m)$; input position $d(m)$ in the original sequence is interleaved to position m by the interleaver, as illustrated in. Fig. 3. The approximation is intuitively justified by the following reasoning: The new extrinsic output from the second decoder at position i is correlated to the extrinsic input at position m, and this correlation is exponentially decreasing with the distance between i and m, i.e. $|i-m|$. Further, the extrinsic input at position m stems from output $d(m)$ from the first decoder, and it is thus correlated to the systematic bit j according to (1), i.e. exponentially decreasing with $|d(m)-j|$. The total correlation between extrinsic output i from the second decoder and systematic input j, contributed from extrinsic output $d(m)$ from the first decoder, is therefore approximated as $ae^{-c(|i-m|)}ae^{-c(|d(m)-j|)}$. The systematic input part of (2) is the same as in the case of the first decoder, except that the systematic inputs now are interleaved.

The effect of short cycles on the correlation coefficients $\rho^{(2)}_{Le_i,x}$ is apparent: they will make the extrinsic outputs in the vicinity of the destination of a cycle excessively correlated to the part of the systematic sequence corresponding to the origin of the cycle. Such a situation is depicted in Fig. 4a, where the extrinsic outputs in the vicinity of i become highly correlated to the systematic sequence at the vicinity of j. These cycles, which require only one interleaving to complete, are in the following called *primary cycles*. It is also possible to form cycles which require more than one interleaving/deinterleaving to complete. These cycles are referred to as *secondary cycles*, and exemplified in Fig. 4b. Ordinary block interleavers form a very large amount of these secondary cycles. Pseudo-random and S-random interleavers are plausible to form some such cycles, but not in an amount that significantly degrades the

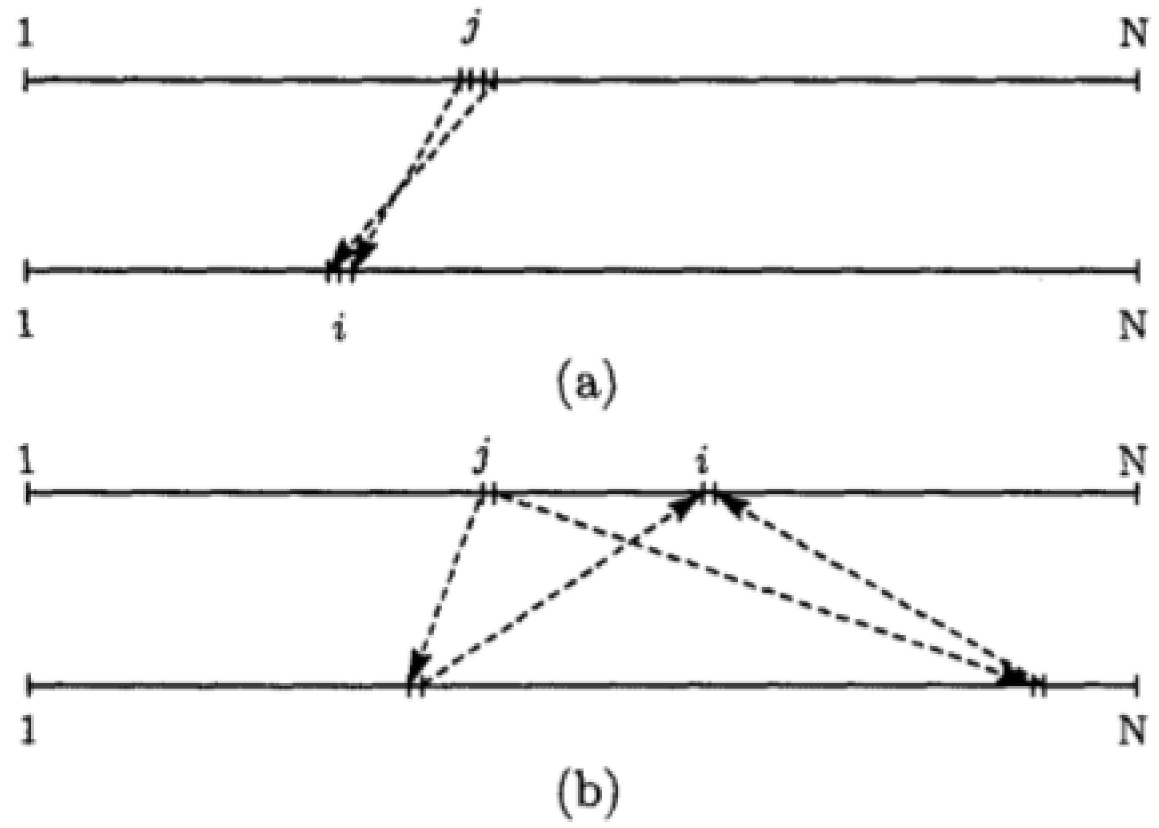

Fig. 4. Example of (a) short primary cycles, and (b) short secondary cycles.

performance of the iterative decoder. The same is true for the interleavers designed with the correlation criterion proposed in this paper.

III. Interleaver design

There are two major criteria in the interleaver design: the distance spectrum of the code and the degree of correlation between extrinsic inputs. Fortunately, these two criteria do not oppose each other. In fact, the S-random interleaver proposed in [1] is motivated with distance spectra arguments. Similarly, the correlation criterion described in Section III-A below results in turbo codes with good distance spectra. However, the main focus of this criterion is on the correlation properties between extrinsic inputs.

The basic procedure of the interleaver design is to assign the elements $d(k), k \in \{1, 2, \ldots, N\}$, element by element until the whole interleaver is defined. Depending on the type of interleaver to be designed, and on the chosen design criteria, various restrictions on the eligible choices for each $d(k)$ may apply. Subsection III-A describes the fundamental design criterion, while subsections III-B to III-D discuss optional restrictions on the eligible choices for each $d(k)$.

A. Minimizing correlation between extrinsic inputs

The best performance of the iterative decoder is achieved if all the extrinsic inputs are totally uncorrelated. Since this is not possible, we will instead make *nearby* extrinsic inputs as uncorrelated to each other as possible. This is achieved by forcing the extrinsic outputs from the *second* decoder to be as uniformly correlated to the systematic sequence as possible, since correlated extrinsic inputs result in extrinsic outputs that are excessively correlated to some part of the systematic sequence. Thus, for each new interleaver element to be defined, say $d(k)$, the correlation coefficients $\rho^{(2)}_{Le_k,x_j}$ resulting from the already defined elements are approximated using (2). The element $d(k)$ is then chosen as the position to which the correlation up to this point is the lowest. The correlation coefficients need only to be calculated for positions j which has not already been assigned. For these positions $d^{-1}(j)$ is not yet defined, and hence the first term of (2), $ae^{-c|d^{-1}(j)-i|}$, does not exist. As a consequence, the minimization is performed as

$$d(k) = \arg\min_{j \in \mathcal{L}} \sum e^{-c(|d(m)-j|+|i-m|)}, \qquad (3)$$

where the summation is carried out over all the already defined interleaver elements $d(m)$. The set $\mathcal{L}$ is the permissible positions to choose from. It contains all positions not already chosen, except for further restrictions due to other design criteria, such as the distance spectrum and trellis termination restrictions, discussed in the following subsections.

The value of the constant c depends on the number of memory elements in the constituent encoders. However, the dependency is rather weak and small variations in the value of c does not significantly influence the performance of the designed interleaver. We use the empirically found value for encoders with three memory elements, which is $c = 0.18$.

The minimization of (3) element by element as described, is a suboptimal algorithm aiming to minimize

$$C(\underline{d}) = \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} e^{-c(|d(i)-d(j)|+|i-j|)}, \qquad (4)$$

where $\underline{d}$ is the entire vector of interleaver elements. Towards the end of the design, it might be impossible to avoid unfortunate mappings with regard to correlation aspects. In these cases it is possible to revise one or more of the earlier defined entries which restrict the choice of the current entry, hereby finding a solution with a lower value on the overall minimization criterion (4).

Designing interleavers with the correlation criterion (3) effectively avoids the short primary cycles exemplified in Fig. 4a. It is possible to expand (3) so that also secondary cycles are avoided. However, these secondary cycles appear in very small numbers in unstructured interleavers, and therefore their influence on the performance of iterative decoding is very small. Simulation results indicate no detectable performance gain associated with incorporating secondary cycles in the design criterion.

B. Distance spectrum

The lower parts of the distance spectrum of a turbo code correspond primarily to information sequences of low Hamming weight, typically 2, 3 and 4. These low weight code words are produced whenever a low weight input sequence that produces a low weight parity word in the first constituent encoder is interleaved into a sequence that again produces a low weight parity word in the second constituent encoder. To design an interleaver which avoids all such unfortunate mappings is computationally a very complex task, and can probably only be carried out for small interleavers. However, avoiding the worst interleaver mappings of weight-2 input sequences only is not computationally demanding. These interleaver mappings are avoided in the design process by removing the corresponding positions from the set $\mathcal{L}$ in (3). This restriction does not result in a significant performance improvement, for error-rates that we have simulated. However, performance differences might occur at lower error-rates, and therefore we suggest including the criterion in the design.

The interleaver edge effect might, if certain care is not taken, result in severe degradation of the distance spectrum of a turbo code. The likelihood of this degradation is greatly reduced if the trellis of the first encoder is terminated, and if the corresponding tail bits are included in the sequence entering the interleaver. Furthermore, the interleaver edge effects are easiest to avoid if the interleaver design is performed in reverse order, i.e. $k = N, N-1, N-2, \ldots, 1$. This give the largest degree of freedom when assigning the endmost positions in the interleaver, which are the positions likely to result in degrading edge effects. For further information on interleaver edge effects, refer to [8].

C. Self-terminating interleavers

Both the constituent encoders in a turbo code can be terminated in a predefined state by designing a *self-terminating* interleaver [2], [9]. Such an interleaver ensures that the final state of the second encoder is identical to the final state of the first encoder; it is thus sufficient to add tail bits to the information sequence so that the first encoder is terminated in the zero state. The following restriction ensures that the designed interleaver is self-terminating:

$$d(m) \bmod L = m \bmod L, m = 1, 2, \ldots, N,$$

where L is the period of the feedback polynomial ($L \leq 2^\nu - 1$, where ν is the number of memory elements in the encoder). The correlation criterion (3) can be used to design a self-terminating interleaver by restricting the set of permissible positions to choose from, $\mathcal{L}$, so that

$$\mathcal{L} \subseteq \{k + nL | 1 \leq k + nL \leq N, n = 0, \pm 1, \pm 2, \ldots\},$$

where k is the interleaver element being designed, i.e. the same k as in (3). We have not found that self-terminating interleavers that terminate both encoders perform better than interleavers without the above constraint, terminating only the first constituent encoder [8].

D. Symmetric interleavers

In some situations it is desirable to design symmetric interleavers, i.e. interleavers with identical interleaving and deinterleaving rules. For a symmetrical interleaver, $d(k) = l \Leftrightarrow d(l) = k$. If a symmetrical interleaver is to be designed, an extra term, $(1 - \delta_{k-j})e^{-2c|k-j|}$, is added to (3) before the minimization is performed (δ_n is the Kronecker delta-function). The extra term accounts for the correlation resulting from $d(l) = k$, which appears as soon as $d(k) = l$ is chosen. We found no detectable influence on the error correcting performance due to the symmetric interleaver property. It is included here because symmetric interleavers, combined with other structures, result in reduced memory requirement for the interleaver vector storage [10].

IV. Performance Evaluation

We evaluated the interleaver design criterion (3) by comparing bit- and frame-error rate performances obtained with correlation designed interleavers to the performances obtained with various other types of interleavers. These types include for example pseudo-random, S-random [1], block helical simile [2] and ordinary block interleavers. The used turbo codes are rate-1/3 codes using feedback and parity polynomial 15_{oct} and 17_{oct} respectively, and four interleaver sizes: 200, 500, 1000, and 4096 bits.

The correlation designed interleavers whose performances are presented in this section are symmetric but not self-terminating. Furthermore, unfortunate mappings of input weight-2 sequences, as discussed in SectionIII-B, were specifically avoided. Regarding trellis termination, the first constituent encoder is terminated while the second constituent encoder is left in an unknown state (the β-values in the backward recursion of the MAP-algorithm are initiated with the final α-values from the forward recursion).

The best performing interleavers were the ones designed with the correlation criterion. The next best were the S-random interleavers, a result we expected since the two interleaver structures are quite similar. For the largest interleaver size, 4096 bits, the performance difference between the S-random and correlation designed interleavers were insignificant (further, these simulations were not statistically satisfying in time for publication). The frame-error rates for the four best performing codes with interleaver size 500 bits are shown in Fig. 5.

Lower bounds on the required SNR as a function of interleaver size, code rate, and probability of a decoding error is given in [11]. Fig. 6 illustrates these lower bounds as a function of the interleaver size for rate-1/3 codes and a frame-error rate of 10^{-4}. Also shown are the required SNRs for three of the compared interleavers: pseudo-random, S-random ($S = \{9, 14, 17 \text{ and } 42\}$), and cor-

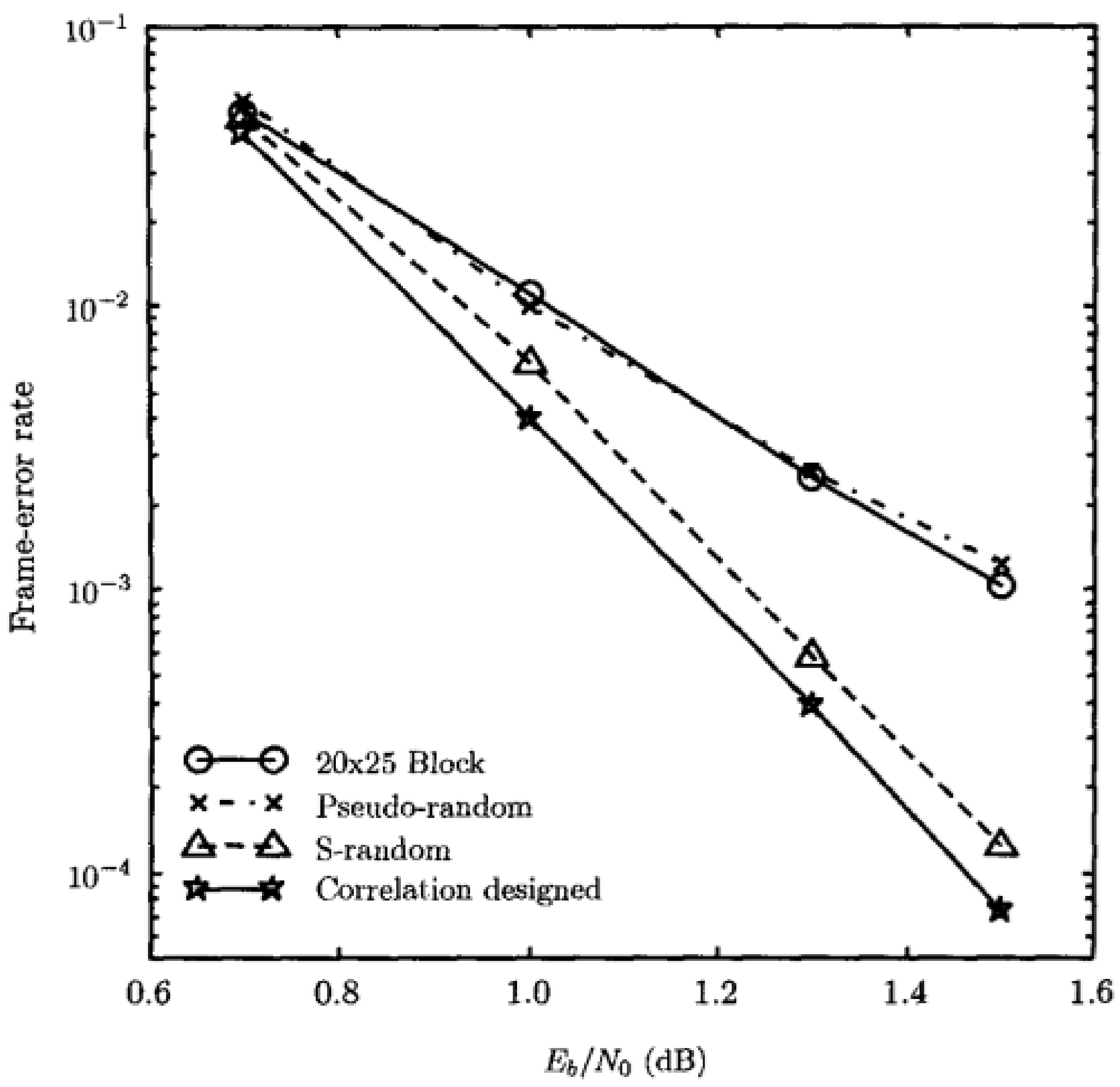

Fig. 5. Simulated frame-error rate performances of rate-1/3 turbo codes with interleavers of size 500 bits.

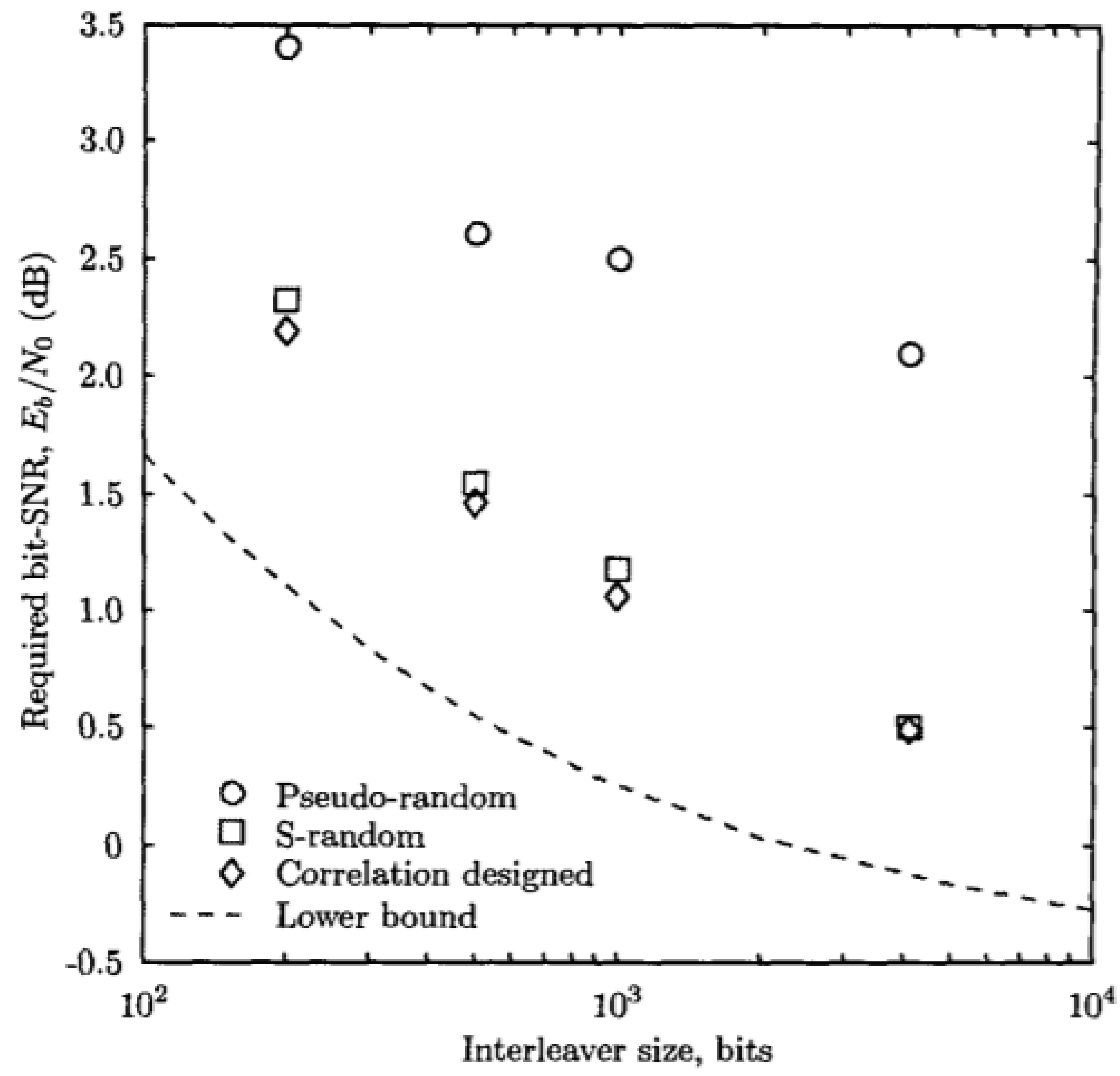

Fig. 6. Required SNR for a frame-error rate of 10^{-4} for various interleaver sizes, and different types of interleavers. The dashed line is a theoretical lower bound for an unconstrained input and unconstrained output AWGN channel.

relation designed. The correlation designed interleavers perform approximately 0.1 dB better than the S-random interleavers, except for the 4096 bits interleavers. The pseudo-random interleavers perform significantly worse than the S-random and correlation designed interleavers.

V. Conclusions

An interleaver design criterion for turbo codes, to be used in an iterative decoding environment, is presented. In contrast to other design approaches, which are often based upon distance spectrum criteria, the main focus of the proposed criterion is the quality of the extrinsic information. Simulations have shown that this methodology produces good interleavers that perform better than those designed using a variety of other techniques. Since the design criterion is of low computational complexity, it is suitable also for large interleaver sizes. Custom made interleavers designed with the correlation criterion can be obtained from the authors.

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