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A COMPARISON BETWEEN TWO DIFFERENT METHOD OF MOMENTS CODES FOR PERIODIC PROBLEMS

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We compare two method of moments codes using RWG elements for two-dimensional periodicity. The test cases are common elements employed in the design of frequency selective surfaces: crossed dipoles, tripoles, and linear dipoles. The last case is also examined with a supporting dielectric substrate. It is shown that the codes agree very well. One of the codes does not implement a layered Green's function, and is therefore much slower when studying the dielectric substrate since a lot of degrees of freedom are spent on discretizing this structure. There is also a larger difference between the codes in this case.

1 Introduction

In satellite reflector antennas and stealth radomes, it is often necessary to employ frequency selective surfaces. These are usually realized as periodic structures, where the period is a substantial fraction of the targeted wavelength. Great ingenuity has been demonstrated in the design of such surfaces, sometimes resulting in very complex geometries. The design of such surfaces rely heavily on computational methods, and in order to reduce the computational complexity it is common to study the model problem of an infinite plane structure, where the calculations can be reduced to a unit cell.

Once the problem is reduced to a unit cell, any computational method which can use periodic boundary conditions can usually be employed to solve the problem. Methods like Finite Differences in the Time Domain (FDTD) or Finite Element Method can typically be used, but may have to spend many degrees of freedom on representing relatively large chunks of free space. This problem can be less problematic for Method of Moments (MoM) approaches, since such codes typically only use degrees of freedom on the surfaces of the geometry. However, MoM is a technically involved procedure, where the biggest

computational effort lies in the computation of the impedance matrix. In addition, for periodic problems it is necessary to compute a periodic Green's function, which further adds to the computational burden.

In spite of these shortcomings, it is very interesting to study MoM-codes for periodic problems, due to the high accuracy that can be achieved and the possibility of using it in hybrid methods [5]. It can also be beneficial when solving problems with many sources, since the impedance matrix is independent of the sources and can be computed once and for all.

In this contribution, we present comparisons between two different MoM codes, both based on Rao-Wilton-Glisson spatial basis functions [1] and able to handle arbitrary lattices. The two codes were developed in Sweden and Italy, and met each other through the EU Network of Excellence "Antenna Center of Excellence", ACE.

2 The two codes

At the heart of the problem lies the computation of the impedance matrix, where the typical element has the form (where the Bloch wave vector $\boldsymbol{k}$ represents the phase shift between neighboring unit cells)

$$Z_{mn}(\boldsymbol{k}) = \int_{T_m}\int_{T_n} f_m(\boldsymbol{x})\widetilde{G}(\boldsymbol{x}-\boldsymbol{y},\boldsymbol{k})f_n(\boldsymbol{y})\,\mathrm{d}\boldsymbol{x}\,\mathrm{d}\boldsymbol{y} \tag{1}$$

with some variations depending on which matrix element is being computed. The basis functions are taken as the Rao-Wilton-Glisson (RWG) functions [1], meaning the functions $f_m(\boldsymbol{x})$ and $f_n(\boldsymbol{y})$ are zero except on the triangles T_m and T_n, where they are linear functions or constants depending on which matrix element is being computed. The Bloch wave vector $\boldsymbol{k}$ is restricted to the reciprocal unit cell U'.

The major computation effort lies in the handling of the periodic Green's function $\widetilde{G}(\boldsymbol{x},\boldsymbol{k})$. Using the Floquet-Bloch representation [2] (the multidimensional version of the Poisson summation formula) this can be expressed as an infinite sum either in spatial domain,

$$\widetilde{G}(\boldsymbol{x},\boldsymbol{k}) = \sum_{\boldsymbol{n}\in\mathbb{Z}^2} \mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot(\boldsymbol{x}+\boldsymbol{x_n})}G(\boldsymbol{x}+\boldsymbol{x_n}) = \sum_{\boldsymbol{n}\in\mathbb{Z}^2} \mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot(\boldsymbol{x}+\boldsymbol{x_n})}\frac{\mathrm{e}^{\mathrm{i}\kappa|\boldsymbol{x}+\boldsymbol{x_n}|}}{4\pi|\boldsymbol{x}+\boldsymbol{x_n}|} \tag{2}$$

or in spectral domain,

$$\widetilde{G}(\boldsymbol{x},\boldsymbol{k}) = \frac{1}{|U|}\sum_{\boldsymbol{n}\in\mathbb{Z}^2} \mathrm{e}^{\mathrm{i}\boldsymbol{k_n}\cdot\boldsymbol{x}}\frac{\mathrm{e}^{\mathrm{i}k_{x_3\boldsymbol{n}}|z|}}{-2\mathrm{i}k_{x_3\boldsymbol{n}}}, \quad k_{x_3\boldsymbol{n}} = \sqrt{\kappa^2-|\boldsymbol{k}+\boldsymbol{k_n}|^2} \tag{3}$$

where $G(\boldsymbol{x})$ is the free space Green's function and $\kappa = \omega/c$ is the free space wave number. Both representations require many terms to be useful, and are therefore generally not well suited for numerical calculations. Although something like Kummer's transformation (adding and subtracting asympotic behaviors of the sums [3]) can be used, most methods instead employ a suitable combination of the two representations.

During 2004–2005, the development of a periodic method of moments code at Lund University was financed by the project NFFP3+ (Nationellt FlygForskningsProgram).

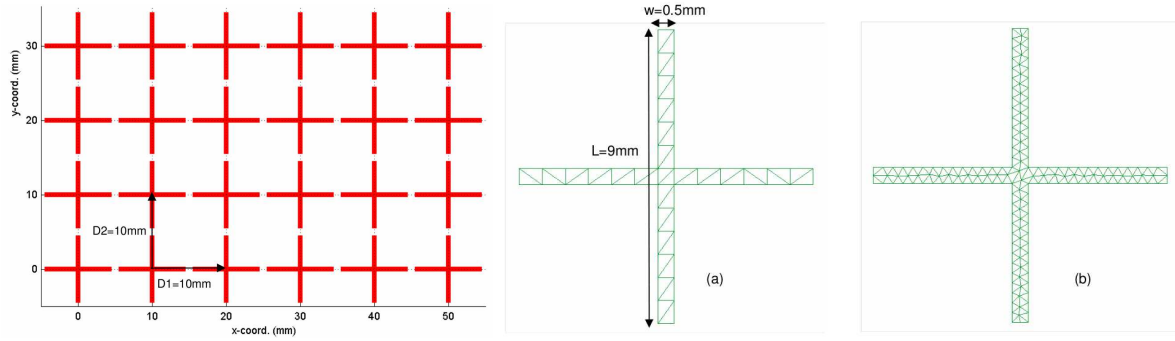

Figure 1: Crossed dipoles in square lattice. Two meshes, one coarse and one fine, are considered.

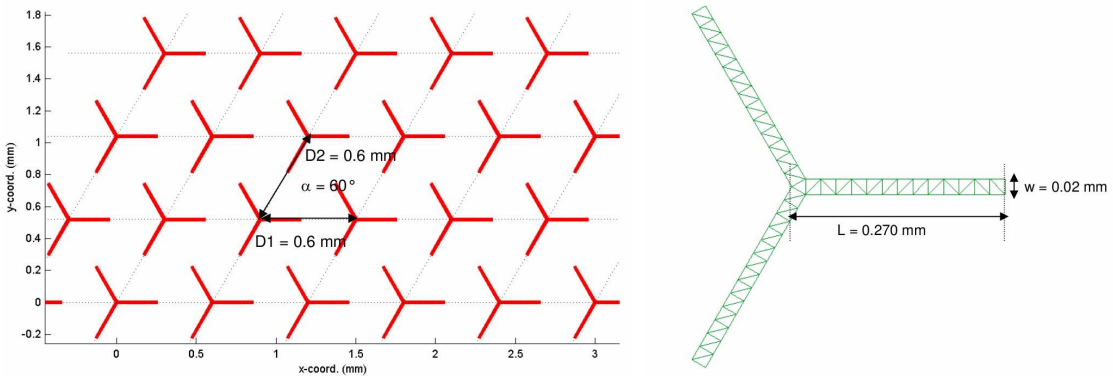

Figure 2: Tripoles in triangular lattice.

This was done as an extension of the GEMS code (General ElectroMagnetic Solvers) developed in Uppsala and KTH and recently commercialized by Efield AB. The periodicity is introduced by extracting the Green's function from the computation of the impedance matrix, and replacing it with a periodic Green's function, leaving the rest of the code basically intact. The computation of the Green's function is based on the Ewald approach [4–7], and is computed on-the-fly.

The Politecnico di Torino and the University of Siena have developed a code where the periodic Green's function is expressed as a combined sum of spectral terms and spatial terms. This representation is obtained by applying the Poisson summation formula to the asympotic behavior of the spectral representation of the periodic Green's function [8]. This is then evaluated in a set of points and then interpolated in MoM code to generated the MoM matrix for the unit cell.

3 Comparison cases

In a first step, we have compared simple structures such as dipoles, crossed dipoles, and tripoles, in rectangular and skew lattices. More specifically, the test cases are:

1. Crossed dipoles in a quadratic lattice, see Figure 1 for geometry and mesh. This case has been validated against two other codes, MESTIS and Ansoft Designer, for normal incidence.

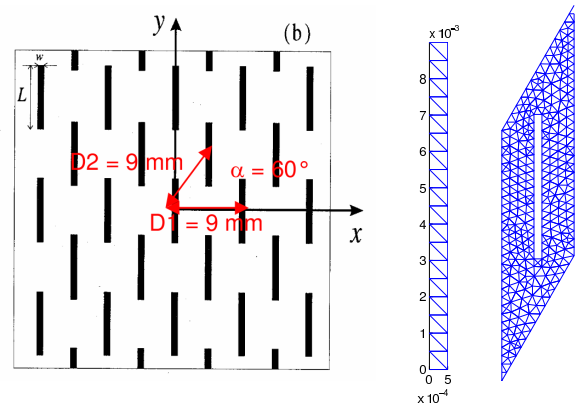

Figure 3: Dipoles in triangular lattice. The mesh for the dipole is in the middle, the mesh for the dielectric substrate in the unit cell on the right (which is shrunk in scale compared to the mesh for the dipole, the dipole fits in the rectangular hole). When using the substrate the number of triangles increase from 36 for the dipole alone, to 1212.

2. Tripoles in a triangular lattice, see Figure 2 for geometry and mesh. This case is taken from [9, p. 269] and has been validated against the results therein.

3. Dipoles in a triangular lattice, see Figure 3 for geometry and mesh. This case is from [9, p. 50], and also covers the inclusion of a thin dielectric substrate supporting the dipoles ($\epsilon_r = 3.0$ and thickness 0.1 mm).

At this stage, none of the geometries extend across the boundary of the unit cell. This will be addressed in further test cases. The parameter of choice for studying these cases was chosen to be the reflection coefficient, due to the common use of these structures as frequency selective reflector surfaces.

In all cases except for the dipoles on the dielectric substrate, the mesh is exactly the same in both codes. For this last case however, the codes are fundamentally different: the Lund code does not implement a layered Green's function, and must therefore mesh the entire dielectric substrate. This costs a lot of degrees of freedom (almost a factor of 100 more than the single dipole), and makes this simulation much slower than the other for the Lund code. Therefore, this case was calculated on the Lund University Linux cluster Lunarc (http://www.lunarc.lu.se), whereas the other cases were done on a simple desktop computer.

4 Results

In this section we present the results of the test cases. We do not present all the data available, but have instead picked a few data sets providing some insight to the similarities and differences between the two different codes.

All the figures are composed in the same way: to the left are the values from the two codes (red for Lund, blue for Torino) for amplitude and phase, and to the right is the relative error in dB-scale using the mean value as reference. In some of the figures, the results agree so well that it is difficult to distinguish the two curves in the figures on the left. Both codes are typically run on a standard desktop computer.

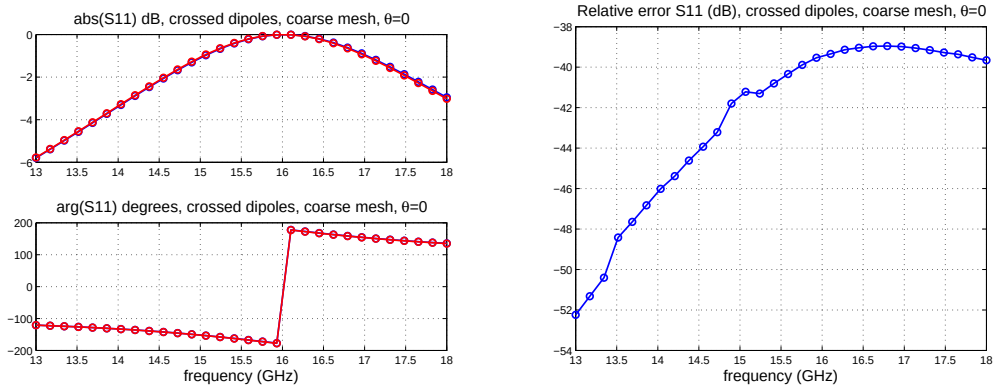

Figure 4: Results for crossed dipoles, coarse mesh.

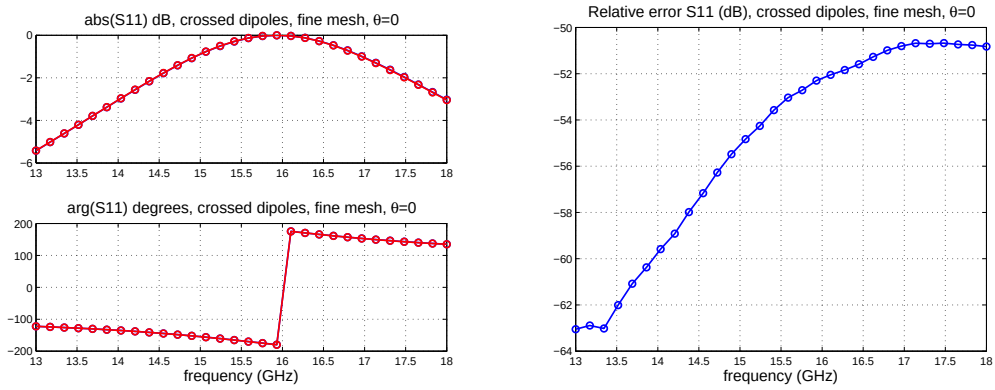

Figure 5: Results for crossed dipoles, fine mesh.

4.1 Crossed dipoles

The results for test case 1 are given in Figures 4 and 5. It can be seen that the methods agree very well, and seem to agree even better when the mesh is refined. Typically, the codes agree within -40 dB for the coarse mesh, and -50 dB for the fine mesh.

4.2 Tripoles

For test case 2, the tripoles, we give the results in Figures 6 and 7 for TE and TM polarization and 45° incidence. Typically, the codes agree within -40 dB.

4.3 Dipoles

The results for dipoles without a substrate are given in Figure 8. It is seen that the results agree within -40 dB except for high frequencies, where the difference is more pronounced.

For oblique incidence on dipoles on a substrate, the results are given in Figures 10 and 11. We see that even though the codes agree reasonably well, the differences are much larger in these cases than in test cases 1 and 2. The errors seem to be in the same range

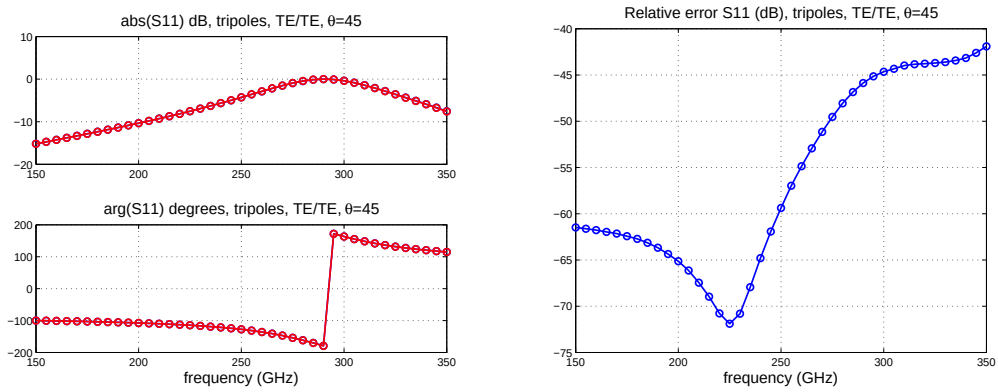

Figure 6: Results for tripoles, TE polarization, 45° incidence.

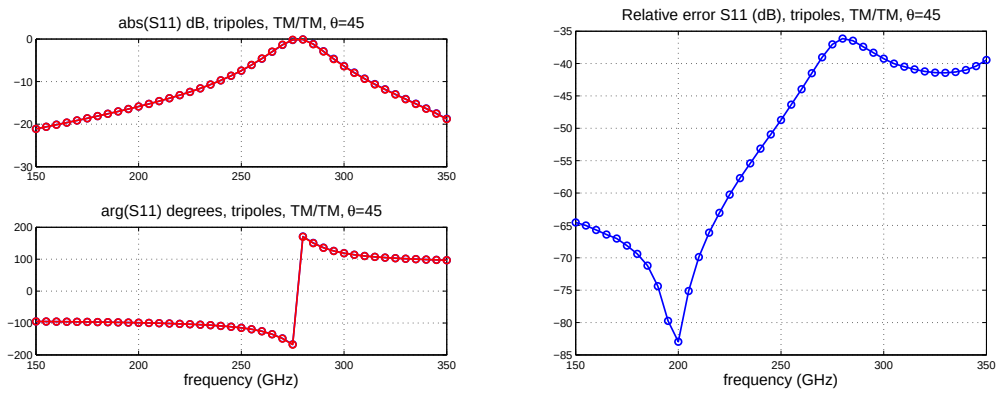

Figure 7: Results for tripoles, TM polarization, 45° incidence.

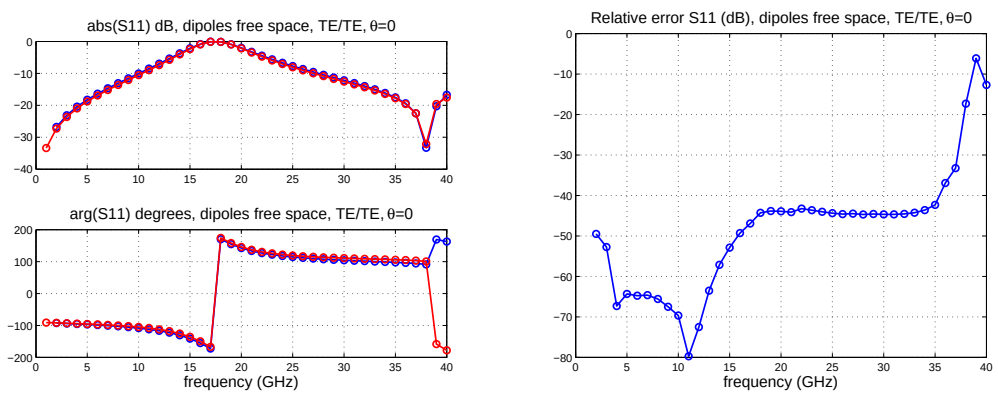

Figure 8: Results for dipoles, normal incidence.

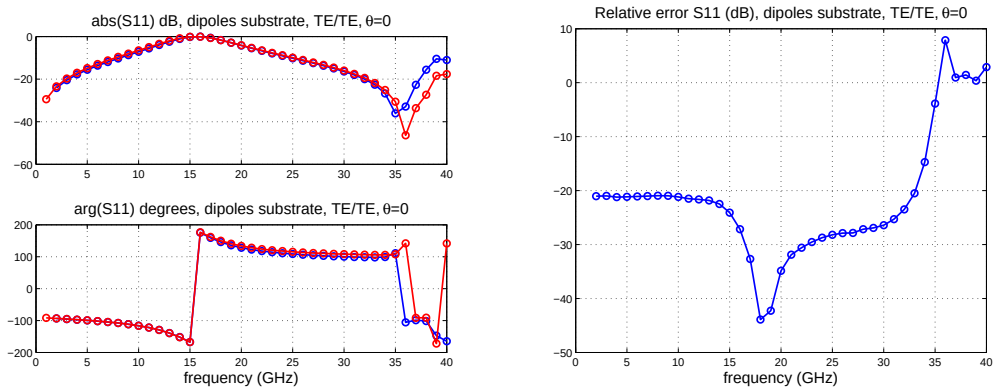

Figure 9: Results for dipoles on substrate, normal incidence.

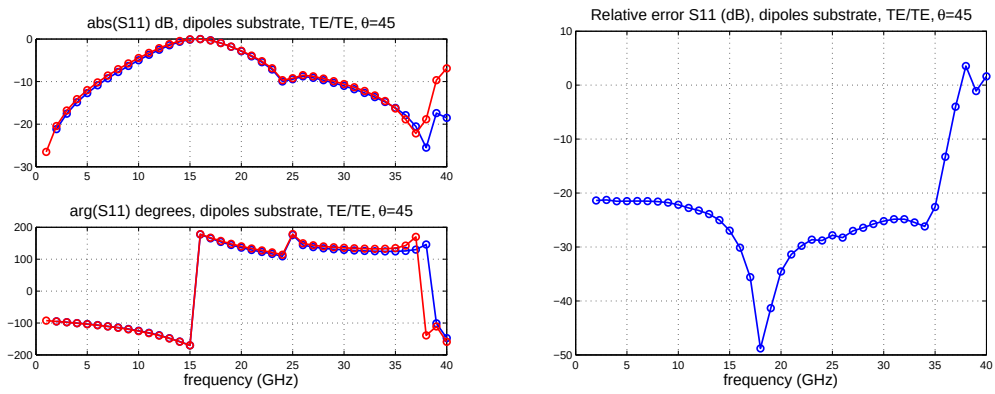

Figure 10: Results for dipoles on substrate, TE polarization, 45° incidence.

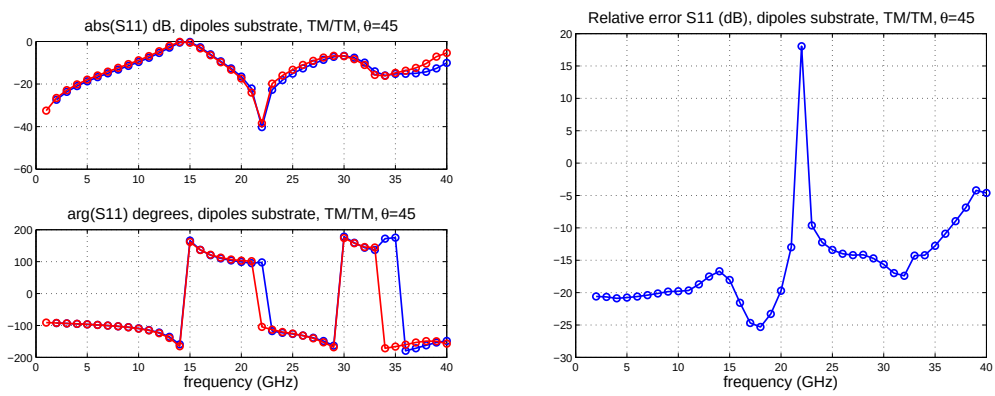

Figure 11: Results for dipoles on substrate, TM polarization, 45° incidence.

when comparing with a commercial code such as Ansoft Designer, which makes it difficult to say that one code is better than another. The larger difference between the codes for the substrate case is probably due to the fact that the Lund code has to discretize the substrate according to the right part of Figure 3, not only at one surface but at two surfaces, which are very close to each other (only 0.1 mm).

5 Conclusions

Two different MoM codes for periodic problems have been compared. They implement the periodic boundary conditions in similar but still different ways, and it is seen that the results agree very well. The Lund code is fast for small problems, but is slowed down severely when used for thin dielectric sheets.

6 Acknowledgments

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References

[1] S. M. Rao, D. R. Wilton, and A. W. Glisson. Electromagnetic scattering by surfaces of arbitrary shape. *IEEE Trans. Antennas Propagat.*, 30(3):409–418, 1982.

[2] Daniel Sjöberg. Homogenization of dispersive material parameters for Maxwell's equations using a singular value decomposition. *Multiscale Modeling and Simulation*, 4(3):760–789, 2005.

[3] C. M. Linton. The Green's function for the two-dimensional Helmholtz equation in periodic domains. *Journal of Engineering Mathematics*, 33:377–402, 1998.

[4] P. P. Ewald. Die Berechnung optischer und elektrostatischer Gitterpotentiale. *Ann. Phys.*, 64:253–287, 1921.

[5] T. F. Eibert, J. L. Volakis, D. R. Wilton, and D. R. Jackson. Hybrid FE/BI modeling of 3-D doubly periodic structures utilizing triangular prismatic elements and an MPIE formulation accelerated by the Ewald transformation. *IEEE Trans. Antennas Propagat.*, 47(5):843–850, May 1999.

[6] Ulf Carlberg, Per-Simon Kildal, and Jan Carlsson. Study of antennas in reverberation chamber using method of moments with cavity Green's function calculated by Ewald summation. *IEEE Trans. Antennas Propagat.*, 47(4):805–814, November 2005.

[7] F. Capolino, D. R. Wilton, and W. A. Johnson. Efficient computation of the 3D Green's function for the Helmholtz operator for a linear array of point sources using the Ewald method. *Journal of Computational Physics*, 223:250–261, 2007.

[8] F. Vipiana, A. Polemi, G. Vecchi, and S. Maci. Hybrid spatial-spectral analysis of periodic structures. In *Antennas and Propagation Society International Symposium*, pages 2867–2870. IEEE, 2006.

[9] J. C. Vardaxoglou. *Frequency Selective Surfaces (Analysis and Design).* Research Studies Press, 1997.