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Andersson, Fredrik

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Fredrik Andersson [*][†]

Lund University and the Research Institute of Industrial Economics

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Abstract

The make-or-buy decision is analyzed in a simple framework combining contractual in-completeness with the existence of imperfect but contractible performance measures. Contractual incompleteness gives rise to two regimes, identified with make and buy. The performance measure on which comprehensive contracts can be written is imperfect in the sense of being subject to *manipulation*. The main result is that the impact – or "externality" – of manipulation on true performance is key; a positive (negative) such externality favors make (buy).

JEL Classification: D23, L22, L24

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1 Introduction

The distinction between "make" and "buy" may seem too obvious to attract attention by laymen. While this may reflect reasonable judgment, the scientific endeavor of illuminating the distinction and understanding the forces that determine real-world make-or-buy decisions is a fundamental one.

[*]*Correspondence to*: Fredrik Andersson, Department of Economics, Lund University, P.O. Box 7082, S-220 07 LUND, SWEDEN. Phone +46-46-222 86 76, fax +46-46-222 46 13, email: fredrik.andersson@nek.lu.se.

In this paper, we will try to synthesize two existing approaches in order to obtain a parsimonious framework within which the distinction between make and buy arises endogenously. The approach springs from two strands of literature. First, it builds on the assumption that there are limits to contracting, thus adhering to the literature on incomplete contracts by assuming that assets – and the residual revenue streams generated by assets – cannot be subject to elaborate sharing contracts. Secondly, it maintains that there are substantive contingencies that can sustain incentive contracts; in this we build on comprehensive contracting theory in focusing on the incentives created by such contracts.

A simple example   Consider the standard story of a person, "the principal," who owns an item for sale and wishes to have someone else, "the agent," sell it. The principal's concern is the net revenue from the sale, and absent direct enforcement of the agent's effort, the payment is the sole basis for remuneration of the agent.

The principal may or may not be able to verify the payment with certainty. Even if it is verifiable, however, it may still be that, for instance, the agent has sold the item to a friend at an unduly low price; this would be an instance of manipulation in our sense. In light of such possibilities, the principal may opt for either of two alternatives: (i) to insulate the agent as far as possible from interest in the sale and ascertain that the ultimate transaction takes place between the principal and the buyer, i.e. to make the agent an intermediary with minimal interest in the transaction; or, (ii) to sell the item to the agent, thereby aligning the agent's further incentives completely with the principal's own original incentives. In this example, the tenet of standard principal-agent theory is upset by the contractual incompleteness; instead of trading off incentives and risk sharing by a suitable revenue-sharing formula, the principal will provide *either weak or strong incentives* in order to minimize the scope for manipulation of the basis for such a formula. Applied to this example, the results below show that the risk-sharing-vs.-incentives tradeoff remains in so far that a more risk-averse agent faces weaker incentives, but the incentives will vary discontinuously with the degree of risk aversion.

Basic approach   The undertaking attempted in this paper is to develop a simple principal-agent framework within which the distinction between "employing" an agent (*make*), and hiring an "independent" agent (*buy*) arises endogenously and can be parameterized in such a way that a direct comparison can be made. Specifically, the "true outcome" of the project undertaken belongs to the owner of an underlying *asset*, contracting on which is plagued by incomplete-contracting limitations. At the same time, there exists a performance measure derived from the

"true outcome" but subject to manipulation by the agent; this measure can be the basis for a comprehensive contract.

Having set out and justified this famework, we go on to ask the question how variation among activities in terms of manipulability of the performance measure can guide the make-or-buy decision. The conclusion is that a negative "externality" of manipulation on the productive outcome favors contracting with an independent agent, whereas a positive externality favors employing the agent. The basic intuition is, somewhat crudely, that each regime imposes a constraint on the relative price between productive effort and manipulation; independence combines strong incentives for productive effort with weak manipulation incentives, whereas employment generates moderate incentives in both regards.

Background   Leading sources of insight into the make-or-buy decision are transaction-cost economics, and the property-rights approach.[1] A key conclusion is that activities involving two parties for whom *specific investments* in a relationship are important are more likely to be integrated by one of the parties; the reason is that the party making a specific investment under non-integration is more likely to be subject to "hold-up" by the other party, undermining incentives to make specific investments.[2]

The key assumption within the property-rights approach is that contractual incompleteness (due ultimately to unforeseen contingencies) makes all contracts *renegotiable*. This makes the parties' payoffs depend on renegotiation bargaining, where the *outside options* affect the outcome; parties therefore spend resources investing in their outside options. Ownership of assets – ownership giving *residual control rights* – determines the payoffs in disagreement (i.e. the outside options), as well as the sensitivity of those payoffs to investments. Under plausible assumptions, this creates incentives to overinvest in outside options and to underinvest in relationship-specific capital. The allocation of ownership of assets can be used to ameliorate the underinvestment problem.

While the property-rights approach is conceptually convincing, it suffers from the weakness that contracts are reduced to merely benchmarks for renegotiation; this effectively rules out the use of explicit incentive schemes that are clearly often important in practice.[3] The importance of

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[1] Key contributions within transaction-cost economics are Coase (1937) and Williamson (1985). The seminal contributions within the property-rights approach are Grossman and Hart (1986) and Hart and Moore (1990); see Hart (1995) and Holmström (1999) for clear and simple accounts of the basic logic.

[2] A party is subject to hold-up if another party threatens to withdraw from trade – in which case the specific asset would be inefficiently utilized – in order to appropriate all, or a large portion, of the surplus.

[3] Interestingly and importantly, there is a recent development of the property-rights approach stressing "con-

incentive schemes, moreover, depends on the measurement and contractibility characteristics of the activity subject to the make-or-buy decision. Baker (1992, 2002), Holmström and Milgrom (1991 and 1994), and Holmström (1999) have devised models where performance measures are manipulable and/or multi-dimensional, and brought these observations to bear in theoretical analyses of the make-or-buy decision.[4] There is, moreover, empirical work indicating that measurement aspects are important for explaining the make-or-buy decision: In their work on in-house versus independent sales forces, Anderson and Schmittlein (1984) and Anderson (1985) found measurement-related explanatory variables to stand out most strongly.

There is some literature dealing directly with the manipulability of performance measures. The focus of this literature is the limits that manipulability imposes on incentive provision, and it is generally concluded that manipulability does, indeed, limit the feasibility and the desirability of strong explicit incentives; this is true e.g. in Crocker and Slemrod (2007) and Goldman and Slezak (2006) and this point is also made in Baker (1992).[5] As far as we know, however, no work in this tradition explores the implications of the object of manipulation shifting with make vs. buy.

The related work by Acemoglu, Kremer and Mian (2008) builds on the core idea that market incentives sometimes induce too much "signaling effort," i.e. effort to inflate others' assessment of performance without promoting performance *per se*; they mention schooling and delegated asset management as examples where this may be a significant problem. Their analysis is devoted to exploring why incentives are, in general, weaker in firms and, even more so, in governments, than in markets.[6] Notably, they work in a "contract-free" environment, and

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tracts as reference points" (Hart and Moore, 2008; Hart, 2008). The focus of this literature, however, is on behavioral implications of the contract in place for renegotiation. If, e.g. a party is unhappy with the outcome of renegotiation (relative to a reference point influenced by the contract), that party may engage in "shading" performance.

[4]In Holmström and Milgrom (1991) it is argued that strong monetary incentives and rules governing how a task is accomplished are substitutes in structuring appropriate over-all incentives. In Holmström and Milgrom (1994), the *complementarities* among a set of instruments for affecting performance in a given task, and the implications of such complementarities for empirical work, are analyzed. Holmström (1999) explores how the power to structure incentives – a key trait of the firm – may or may not be determined by asset ownership.

[5]There are also instances of work exploring more indirect implications of manipulability; for example, Friebel and Guriev (2005) explore the implications or manipulation in a hierarchy where this gives lower-level managers leverage against their superiors in a way that turns out to undermine incentives.

[6]They consider a career-concerns model with a "good" and a "bad" component of effort; after showing that market incentives may be excessively strong, they argue that firms can remedy this by creating, by design, a moral-hazard-in-teams problem; they also argue that competition between firms allows remnants of market

hence do not address questions about the properties of actual incentive contracts.

Levin and Tadelis (2005) also study the make-or-buy decision by devising a simple theoretical model – driven by contract-administration costs – for generating and testing predictions from contracting by US cities.[7]

2 Basic framework

We will consider a simple linear principal-agent model where the principal has an exogenously given task that she cannot solve by herself. At the other end is an agent, who in the end solves the task; the agent may be thought of as a worker or a subdivision of a firm (make), or as a subcontractor (buy). The agent may exert effort on the task itself on the one hand, and on influencing – manipulating – the performance measure on the other.

The principal, P, is a risk neutral profit maximizer, whose benefit from completion of the task has a fixed component, $B > 0$, and a variable component that can be thought of as the net monetary outcome.

The agent, A, cares about income, y, and effort. He exerts effort a on the task and he exerts effort d on manipulating the performance measure. He is risk averse, and his utility from income y and effort (a, d) is

$$u_A(y; a) = -\exp\left\{-r_A\left[y - \left(a^2/2 + d^2/2\right)\right]\right\},$$

with $r_A > 0$ the agent's level of absolute risk aversion; the specific utility function is assumed for tractability. The agent has reservation payoff $\overline{u}_A$.[8] Without loss of generality, we will assume that $\overline{u}_A = -1$.

The agent exerts effort, a, on a task whose outcome – e.g. a measure of realized profits or cost savings – is x given by

$$x = a + \varepsilon,$$

where ε is a random variable reflecting the fact that the outcome is affected but not determined by the agent's effort; ε is normally distributed with mean zero and variance v. The contract

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incentives to trickle down to employees, and that this effect can be avoided by governments.

[7]Another instance of related work is Tadelis (2002), who draws on work by Bajari and Tadelis (2001) on fixed-price versus cost-plus contracting in procurement to argue that the *complexity* of an activity makes "make" a more likely outcome of the make-or-buy decision.

[8]As we will note below, one way to introduce interactions into the model would be to consider an interdependence between a and d in the agent's preferences; this was done in a previous version, but here we consider interactions through the technology.

governing the agent's reward, however, can be based only on a performance measure, z, that is subject to manipulation,

$$z = x + \gamma d = a + \varepsilon + \gamma d, \qquad (1)$$

where $\gamma \geq 0$ is a constant; for $\gamma = 0$, the problem is completely standard as we will note below. One may note that the agent may want to either inflate ($d > 0$) or deflate ($d < 0$) performance, which we allow; given z's dependence on d and the quadratic cost, either is equally costly.

Contracts are assumed to be linear in the relevant performance measures.[9] That is,

$$y = F + mz$$

for constants F and m.

Residual income and regimes   It is often presumed that incentives "originating in" an organization are weaker than incentives generated in contractual relations between organizations.[10] A core purpose of this paper is to provide some justification for this in a framwork where incentive contracts matter. We will make a precise distinction between an *employed agent* and an *independent agent* below, and we will refer to the two cases as two *regimes*, make and buy. The distinction between the regimes arises from output being subject to incomplete contracting, but in contrast to the property-rights approach there is a fully contractible performance measure that approximates output in our framework.

Output accrues through an *asset* that is, at least for practical purposes, indivisible; true performance, x, accrues as the proceeds of the asset and cannot be subject to a sharing contract. The assumptions about the asset are thus in line with the property-rights approach although it plays the sole role of carrying the proceeds of output.[11] An employed agent is defined by the principal owning the asset; the value produced by the efforts thus accrues to the principal, whose payoff is

$$B + x - R^{\text{emp}}(z), \qquad (2)$$

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[9] The most convincing rationale for linear contracts is provided by Holmström and Milgrom (1987); the essence of the argument is that when the agent can adjust effort when observing intermediate result – as modeled sophisticatedly in the paper – non-linearities can be exploited in a way that makes them unattractive.

[10] As we have noted, this is a widely shared presumption, articulated e.g. by Williamson (1998).

[11] The property-rights approach is presented by e.g. Hart (1995); while Hart dismisses unreflected reliance on "residual income" in modeling, he also stresses the point that residual income in most cases and for good reasons goes together with the *residual control rights* that come with ownership. Importantly, the dynamics that generate implications for investement incentives within the property-rights approach are absent from the model considered here.

where z is the available performance measure and where $R^{\mathrm{emp}}(z)$ is the remuneration to the agent. An independent agent, on the other hand, owns the asset and the value produced by the effort accrues directly to him; the principal's payoff is then

$$B - R^{\mathrm{ind}}(z) \tag{3}$$

where $R^{\mathrm{ind}}(z)$ is the contracted remuneration to the agent; the agent's income in this case is $R^{\mathrm{ind}}(z)+x$. Note that it may well be the case that $\partial R^{\mathrm{ind}}/\partial z$ is negative reflecting the principal's risk-sharing with an independent agent; this will in fact always be the case in the specification considered below. Note finally that z is the same across regimes; this purifies the analysis but it is clearly not a statement of fact and we will comment on it below.

2.1 Optimal contracts

An employed agent Consider first the case of an employed agent. The principal solves (where expectations are w.r.t. the distribution of ε)[12]

$$\max_{m,F} E\left(x - (F + mz)\right) = (a - (F + m(a + \gamma d)))$$

$$\text{s.t. } -\exp\left\{-r_A\left(F + m(a + \gamma d) - a^2/2 - d^2/2 - r_A m^2 v/2\right)\right\} \geq \overline{u}_A,$$

$$\text{and } (a,d) \in \arg\max\left(-\exp\left\{-r_A\left(F + m(a + \gamma d) - a^2/2 - d^2/2 - r_A m^2 v/2\right)\right\}\right).$$

Maximization by the agent yields

$$a = m, \quad \text{and} \quad d = \gamma m; \tag{4}$$

inserting this and taking logarithms we get

$$\max \; -F + (1 - m)m - m^2\gamma^2 \tag{5}$$

$$\text{s.t. } F + m^2 + m^2\gamma^2 - m^2/2 - m^2\gamma^2/2 - r_A m^2 v/2 \geq -\ln(-\overline{u}_A)/r_A.$$

Note that the right-hand side of the participation constraint is the "reservation certainty equivalent," and that this object is zero since we have assumed $\overline{u}_A = -1$. Solving the constraint –

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[12]Using the fact that

$$E\exp\left\{-r_A\left(F + m(a + \gamma d + \varepsilon) - (a^2/2 + d^2/2)\right)\right\}$$
$$= \exp\left\{-r_A\left(F + m(a + \gamma d) - (a^2/2 + d^2/2) - r_A m^2 v/2\right)\right\}.$$

which obviously must bind – for F, we get an unconstrained problem. Letting ϕ generically denote objective functions, P maximizes

$$\phi(m) = m - m^2/2 - m^2\gamma^2/2 - r_A m^2 v/2$$

with respect to m; the first-order condition is

$$\phi'(m) = 1 - m - \gamma^2 m - r_A m v = 0,$$

from which, with obvious notation,

$$m^{\text{emp}} = \frac{1}{1 + \gamma^2 + r_A v} \tag{6}$$

follows directly. The problem generates a value function for P from employing the agent,

$$\phi^{*\text{emp}} = m^{\text{emp}} - \left(1 + \gamma^2 + r_A v\right) (m^{\text{emp}})^2 /2, \tag{7}$$

giving

$$\phi^{*\text{emp}} = \frac{1}{2} \frac{1}{1 + \gamma^2 + r_A v}. \tag{8}$$

We see that the presence of manipulation, as parameterized by γ, weakens direct effort incentives; it is also costly from a welfare point of view.

An independent agent Next, we will consider how the problem is modified if the principal opts for an independent agent. The management solves, having re-formulated the constraint as above and noting that the "reservation certainty equivalent" is zero by construction,

$$\max_{m,F} E\left(B - (F + mz)\right) = B - (F + m\left(a + \gamma d\right)),$$

$$\text{s.t. } F + m\left(a + \gamma d\right) + a - a^2/2 - d^2/2 - r_A\left(1 + m\right)^2 v/2 \geq 0;$$

the difference is that actual output enters the agent's rather than the principal's payoff and the agent's direct incentives to exert a are therefore strengthened by the direct effect while his incentives to exert d are unaffected. Maximization by the agent now yields

$$a = 1 + m; \quad \text{and} \quad d = \gamma m. \tag{9}$$

Note that the key difference between (4) and (9) is the difference in relative price. The direct performance incentive (incentive for a) is relatively stronger with an independent agent; note, however, that this may well call for $m < 0$ in the independence case.

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Inserting equilibrium effort and taking logarithms we get

$$\max \; -F - m\left(1+m\right) - m^2\gamma^2 \tag{10}$$

$$\text{s.t. } F + m\left(1+m\right) + m^2\gamma^2 + 1 + m - \left(1+m\right)^2/2 - m^2\gamma^2/2 - r_A\left(1+m\right)^2 v/2 \geq 0.$$

Solving the constraint as above, the principal maximizes the following objective function:

$$\phi(m) = 1 + m - \left(1+m\right)^2/2 - m^2\gamma^2/2 - r_A\left(1+m\right)^2 v/2$$

with respect to m, with first-order condition

$$\phi'(m) = 1 - \left(1+m\right) - \gamma^2 m - r_A\left(1+m\right)v = 0,$$

from which

$$m^{\text{ind}} = \frac{-r_A v}{1 + \gamma^2 + r_A v} \tag{11}$$

and from which the effective power of incentives facing the agent is

$$1 + m^{\text{ind}} = \frac{1 + \gamma^2}{1 + \gamma^2 + r_A v}. \tag{12}$$

Note that $m^{\text{ind}} < 0$, and that $1 + m^{\text{ind}} > m^{\text{emp}}$; we will come back to this.

The problem generates a value function for P from contracting with an independent agent,

$$\phi^{*\text{ind}} = 1 + m^{\text{ind}} - \left(1 + r_A v\right)\left(1 + m^{\text{ind}}\right)^2/2 - \gamma^2\left(m^{\text{ind}}\right)^2/2 \tag{13}$$

$$\phi^{*\text{ind}} = \frac{1 + \gamma^2}{1 + \gamma^2 + r_A v} - \left(1 + r_A v\right)\left(\frac{1 + \gamma^2}{1 + \gamma^2 + r_A v}\right)^2/2 - \gamma^2\left(\frac{-r_A v}{1 + \gamma^2 + r_A v}\right)^2/2 \tag{14}$$

$$\phi^{*\text{ind}} = \frac{2\left(1 + \gamma^2\right)\left(1 + \gamma^2 + r_A v\right) - \left(1 + r_A v\right)\left(1 + \gamma^2\right)^2 - \gamma^2\left(r_A v\right)^2}{2\left(1 + \gamma^2 + r_A v\right)^2} \tag{15}$$

which after some manipluation of the numerator (extracting $1 + \gamma^2 + r_A v$ as a common factor accordingt to the "simplification" below), gives

$$\phi^{*\text{ind}} = \frac{1 + \gamma^2\left(1 - r_A v\right)}{2\left(1 + \gamma^2 + r_A v\right)} \tag{16}$$

to be compared with $\phi^{*\text{emp}}$. We may note that effective effort incentives, $1 + m^{\text{ind}}$, are strengthened with increasing importance of manipulation (γ) while the value function is decreasing.[13] Thus, the power of the incentives coming directly from the contract (m^{ind}) are reduced in the independence case as well, and manipulation is costly here too.

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[13] The derivaitve of $1 + m^{\text{ind}}$ w.r.t. γ^2 is $r_A v/\left(1 + \gamma^2 + r_A v\right)^2$ and the derivative of the value function is $-r_A^2 v^2/2\left(1 + \gamma^2 + r_A v\right)^2$.

Simplification The numerator in (15) can be written:

$$2\left(1+\gamma^2\right)\left(1+\gamma^2+r_Av\right)-\left(1+\gamma^2+r_Av\right)-\gamma^2\left(1+\gamma^2\right)-\gamma^2r_Av\left(2+\gamma^2+r_Av\right)=$$

$$2\left(1+\gamma^2\right)\left(1+\gamma^2+r_Av\right)-\left(1+\gamma^2+r_Av\right)-\gamma^2\left(1+\gamma^2+r_Av\right)-\gamma^2r_Av\left(1+\gamma^2+r_Av\right)=$$

$$\left(1+\gamma^2+r_Av\right)\left(1+\gamma^2\left(1-r_Av\right)\right).$$

2.2 Comparisons

First, one may note the obvious fact that when $\gamma=0$, manipulation has no bite and the solutions in the two cases co-incide completely. Next, we start by comparing equilibrium productive effort, a, across regimes; the difference in terms of incentive intensity – which is equal to the difference in actual effort in this case – is clear cut (with obvious notation):

$$\Delta a = a^{*\mathrm{ind}} - a^{*\mathrm{emp}} = \left(1+m^{\mathrm{ind}}\right)-m^{\mathrm{emp}} = \frac{\gamma^2}{1+\gamma^2+r_Av} \geq 0. \tag{17}$$

In this setting, thus, an independent agent has unambiguously stronger performance incentives. When it comes to regime choice, we have:

PROPOSITION 1. *The principal prefers to contract with an independent agent rather than employing an agent precisely if $r_Av \leq 1$.*

Proof. The condition for an independent management to be optimal is that the following expression be non-negative:

$$\Delta^{\mathrm{profit}} = \phi^{*\mathrm{ind}} - \phi^{*\mathrm{emp}} = \frac{1+\gamma^2\left(1-r_Av\right)-1}{2\left(1+\gamma^2+r_Av\right)}.$$

The condition for an independent agent to be optimal is thus $r_Av \leq 1$.#

The result of the proposition is quite intuitive – risk exposure is unambiguously larger in the independence case. In fact, the result can be seen as a discontinuous version of the standard tradeoff between incentives and risk-sharing. The neutrality of regime choice with respect to γ, the impact of manipulation, is perhaps less intuitive, but it is consistent with the fact that manipulation is *costly* – the value functions being decreasing – in both cases.[14]

3 Externalities of manipulation

The above result is too simple in a way; while it conveys a basic trade-off, it is derived in a framework with little interaction between manipulation, which is purely redistributive, and

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[14]One may also note that whenever $r_Av \neq 1$, the difference between the value functions is increasing in γ.

productive effort which determines the outcome. The obvious way to proceed is to allow for interaction between manipulation and productive effort. Such interaction can operate through the agent's preferences and/or through the technology. In the following, we will develop a formulation where the interaction operates through the technology.[15]

In modeling terms, we will proceed by adding a term reflecting an externality to the linear formulation for output.[16] In this formulation, the externality affects output; as we will note below, one could also consider the case where the externality affects the principal independently of the regime; we will report that case briefly towards the end of this section, finding that the results are broadly similar.

We continue to denote "true output" by x, and we let

$$x = a + \varepsilon + \sigma\lambda(d), \tag{18}$$

where the externality is given by the function, $\lambda(d)$, weighted by σ. We will consider two functional relationships:

$$\lambda(d) = d \text{ or } \lambda(d) = |d|; \tag{19}$$

the significance of each of these specifications will be discussed shortly. The weight, σ, can be positive or negative.

Before going to interpretations, we must extend the definiton of the performance measure. We will assume that the performance measure is, similarly to the previous section, given by

$$z = a + \varepsilon + \gamma d.$$

This means that the externality does not affect the performance measure, but this reflects only an accounting convention.[17]

Interpretations The introduction of an externality enriches the model in two dimensions, reflected by σ and λ in the above formulation. The externality of manipulation may be either positive or negative (given by the sign of σ), and when performance is inflated and deflated, respectively, the externality may either work similarly or in opposite directions (the form of λ).

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[15]In a previous version we pursued the other route; we will comment on this below.

[16]The choice of the term "externality" is deliberate, reflecting the notion that manipulation as a primarily redistributive activity affects output in a way that is not directly dealt with by the contract.

[17]I.e., one could include the externality in the basis for z, with no implication for the result. More precisely, re-defining $\widehat{\gamma} = \gamma - \sigma$, and assuming $z = a + \varepsilon + \sigma d + \widehat{\gamma}d$ gives an equivalent formulation for the case $\lambda(d) = d$ (expressions would change but the substantive results would be the same); similarly with $\widehat{\gamma} = \gamma + \sigma$ for the other case.

While the significance of the sign of the externality has an obvious interpretation, the latter distinction may call for elaboration; it is a distinction between:

- the case where the activity of manipulation is costly (or beneficial) for output, in a way that leads to a similar externality if the agent tries to inflate performance ($d > 0$) or deflate performance ($d < 0$); in formal terms, this is manifest in the cost of manipulation depending on $|d|$; and,

- the case where the activity of manipulation has an impact on output that depends on whether the agent tries to inflate or deflate performance; the simplest case is that where the cost of manipulation depends on d.

With these distinctions introduced, there are four qualitative combinations illustrated in Table 1 below where we also sketch some examples.

	Positive ($\sigma > 0$)	**Negative ($\sigma < 0$)**
Externality depending on $|d|$	(iv) Perquisites	(i) Manipulating accounting data
Externality depending on d	(iii) Over-treatment	(ii) Excessive cost control

Table 1. Four cases and some example-interpretations of d.

To build a bit of intuition, we elaborate the four cases.

(i) This case applies e.g. when manipulation is a separate activity that is costly in terms of attention stolen from productive effort; this, obviously, may spill over negatively on the value of the asset/revenue stream itself independently of whether the agent aims at inflating or deflating measured performance.

(ii) Here, the agent can inflate the performance measure by e.g. keeping costs at bay by reducing the scope or quality of treatment, thus keeping down a verifiable set of activities in which the agent must share costs.[18] Another example would be the possibility of avoiding (unverifiable) costs by cutting corners (maintaining equipment only occasionally); if performance is e.g. measured profit or measured punctuality, this has likely a negative externality on the asset.

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[18] We will come back to this context of the agent being able to affect the set of activities. While an employed agent may want to the set of activities in order not to be punished on the margin, an independent agent may face an incentive, *ceteris paribus*, to expand the set of activities in order to have more costs shared by the principal.

(iii) The agent may e.g. inflate the performance measure by insisting on a higher-quality treatment (again with a verifiable set of activities) in cases where the agent has a positive stake in marginal performance. The difference between this case and case (ii) is that inflation of the performance measure generates a positive externality.

(iv) The final case is probably the least common in practice; manipulation is costly but has a positive externality on output independently of sign. An example that might exhibit a pattern like this (for small γ) may be perquisites, such as the possibility to mind certain private business at work which may increase the agent's motivation and spill over positively on the productive activity and thus the principal whether the direct effect on the performance measure is positive or negative.

3.1 Analysis

We now proceed by presenting the analysis for the two cases – the case where effort cost depends on d and where it depends on $|d|$ – in turn. The full analysis is available in the Appendix. We will, recalling that $x = a + \varepsilon + \sigma \lambda(d)$, and that measured performance is $z = a + \varepsilon + \gamma d$ proceed by presenting the key considerations in each case. In order to focus on interesting cases we will assume two bounds on the externality; we assume that:

1. $|\sigma| \leq \gamma$, which rules out the externality being the dominant force relative to performance manipulation in the trade-offs concerning d; this is to keep in line with the spirit of the model; and

2. $|\sigma| \gamma \leq 1$, which gurantees that the marginal returns to manipulation (through the externality) are no larger than the marginal returns to productive effort.

Externality depends on d We start by considering the analytically simpler case of the externality depending on d. The conclusion is quite simple in this case:

PROPOSITION 2. *The principal prefers to contract with an independent agent rather than employing an agent precisely if*
$$r_A v \leq \frac{1 - \sigma/\gamma}{1 + \sigma/\gamma}.$$

The full proof is provided in the Appendix, and the essence of the formal argument is outlined below; it is illustrated in Figure 1; as the externality grows in importance (i.e. grows relative to γ), the employment regime grows more attractive in the case of a positive externality,
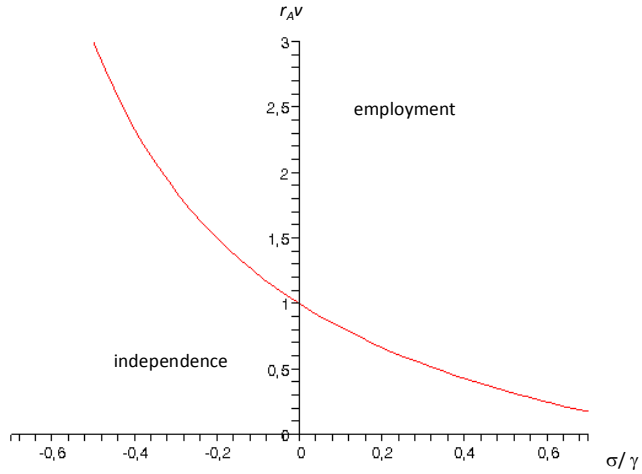
Figure 1: Basic result

and independence grows more attractive in the case of a negative externality. The condition has a simple interpretation: within the bounds assumed, a positive externality of manipulation on output makes employing the agent more attractive; a negative externality makes contracting with an independent agent more attractive.[19]

The basic intuition for the proposition stems from two forces. On the one hand, an independent agent internalizes the externality; this, obviously, favors independence. On the other hand, however, the basic incentive-vs-risk-sharing problem creates second-best distortions that are reduced in the employment regime when the externality is positive. More precisely, the constraint on the relative price between productive effort and manipulation makes incentives "positively aligned" in the employment case, and "divergent" in the independence case; with a positive externality, positively aligned incentives are desirable.[20]

Formal argument *Employed agent*: In this case, the agent faces the same situation as in the previous section, and he chooses

$$a = m, \quad \text{and} \quad d = \gamma m; \tag{20}$$

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[19] As is noted in the Appendix, the comparison is uncomplicated given the bound $|\sigma| \leq \gamma$; outside this bound, independence is, in fact, unambiguously preferable.

[20] In fact, when σ approaches 1, the incentives of an independent agent become more similar to those of a an employed agent, and the size of the advantage of employment decreases.

this choice feeds back to the principal's problem so as to give the following optimal incentive intensity:

$$m^{\text{emp}} = \frac{1 + \sigma\gamma}{1 + \gamma^2 + r_A v}. \tag{21}$$

The impact of the externality is not surprising; the principal will strengthen incentives in response to a positive externality and weaken incentives in response to a negative externality. Note that the assumption under 1. above ensures that $m^{\text{emp}} \geq 0$. This solution generates the following value function,

$$\phi^{*\text{emp}} = \frac{1}{2} \frac{(1 + \sigma\gamma)^2}{1 + \gamma^2 + r_A v}, \tag{22}$$

which reflects a strong dependence of the principal's payoff on the sign of the externality.

Independent agent: In this case, the agent faces the externality directly, and he chooses

$$a = m, \quad \text{and} \quad d = \gamma m + \sigma; \tag{23}$$

this choice feeds back to the principal's problem so as to give the same optimal incentive intensity as in the absence of the externality:

$$m^{\text{ind}} = \frac{-r_A v}{1 + \gamma^2 + r_A v}, \quad \text{and } m^{\text{ind}} + 1 = \frac{1 + \gamma^2}{1 + \gamma^2 + r_A v}. \tag{24}$$

The incentive scheme is thus independent of the externality, whereas the agent's manipulation depends on it. The joint effect on the value function is

$$\phi^{*\text{ind}} = \frac{1 + \gamma^2 (1 - r_A v)}{2 (1 + \gamma^2 + r_A v)} + \sigma^2/2; \tag{25}$$

here, the externality is beneficial independently of its sign, the intuition being that the agent can adjust to it and take advantage of it, i.e. that the agent internalizes it.

Comparisons: The ultimate goal of the analysis is to compare the principal's payoffs and thereby understand the choice of regime, but we start by comparing equilibrium effort, a. The difference in effort is equal to the difference in terms of the relevant incentive intensity:

$$\Delta a = a^{*\text{ind}} - a^{*\text{emp}} = 1 + m^{\text{ind}} - m^{\text{emp}} = \frac{\gamma^2 (1 - \sigma/\gamma)}{1 + \gamma^2 + r_A v}, \tag{26}$$

which is positive as long as $|\sigma| \leq \gamma$, as we have assumed. In this setting, thus, an independent agent has unambiguously stronger performance incentives. One may also note that manipulation is unambiguously positive in the employment case, whereas it is negative unless σ is too large in the independence case.

As to the choice of regime, the comparison of the value functions in (22) and (25) is straightforward, and the result in the proposition follows.

Externality depends on $|d|$ The case where the externality depends on $|d|$ turns out to produce a condition that is similar to that of Proposition 2. The analysis raises, however, a set of complications that need to be dealt with. The complications are due to the absolute value; as we noted above, manipulation is often negative in the independence case, and this means that the absolute value matters in a way that is not straightforward since d is endogenous. The way we will deal with this is to repeat the above analysis under the assumption that (with obvious notation) $d^{\text{ind}} < 0$, and then go back to see under what parameter configurations this is valid; finally we go through the remaining cases. The conclusion is:

PROPOSITION 3 *As long as*

$$\sigma \geq \frac{-\gamma r_A v}{1 + \gamma^2 + r_A v}$$

the principal prefers to contract with an independent agent rather than employing an agent precisely if

$$r_A v \leq \frac{1 - \sigma/\gamma}{1 + \sigma/\gamma}.$$

When

$$\sigma < \frac{-\gamma r_A v}{1 + \gamma^2 + r_A v}$$

and $\gamma \leq 1$ the principal always prefers to contract with an independent agent.

Again, we sketch the basic formal argument in the text below, while providing a full argument in the Appendix. The last case involves a bit more legwork in terms of computation and this is relegated to the Appendix in its entirety, but the result is simple under the condition that $\gamma \leq 1$ (which is a minor strengthening of $|\sigma|\gamma \leq 1$ since this is the case where σ is large negative); the condition is sufficient in the sense that even if γ is larger, independence is optimal under a large critical level of $r_A v$.

Formal argument   Since we always have $d > 0$ for an employed agent, the solution and value function remain the same when we turn to considering $\lambda(d) = |d|$; we thus proceed directly to the independence case.

We start by solving the problem given $d < 0$. In this case, the agent faces the externality directly, and the fact that $|d| = -d$ when $d < 0$ means that the the analysis is similar except we need to replace σ by $-\sigma$; the agent thus chooses

$$a = m, \quad \text{and} \quad d = \gamma m - \sigma; \tag{27}$$

and the property that the incentive scheme is unaffected by σ remains true. The expressions in (24) are therefore still valid; less obviously, the value function, too, is unaffected:

$$\phi^{*\text{ind}} = \frac{1 + \gamma^2 (1 - r_A v)}{2 (1 + \gamma^2 + r_A v)} + \sigma^2/2. \tag{28}$$

We can thus re-state the observation that the externality is beneficial independently of its sign. This, in turn, immediately implies that the comparisons in terms of productive effort, a, and regime choice remain unaffected.

When is $d < 0$? Next, we explore the conditions under which the preceeding analysis is valid. We saw that

$$d = \gamma m - \sigma = \frac{-\gamma r_A v}{1 + \gamma^2 + r_A v} - \sigma,$$

and the condition for $d \leq 0$ is thus

$$\sigma \geq \frac{-\gamma r_A v}{1 + \gamma^2 + r_A v};$$

i.e. that σ not be too negative.

In order to verify that this is indeed a solution, one must finally check that there does not exist superior solution to the agent's problem with $d > 0$ for some parameter values; this is done in the Appendix.

One may also note that effective performance incentive as measured by the equilibrium level of productive effort, a, are always stronger in the indedendence case for the parameter values covered by Propositon 3 (see the Appendix).

3.2   Results and interpretations

The idea of this paper is to find a simple and parsimonious framework for addressing the issue of how characteristics of activities map into make-or-buy choices. The intended contribution is both in terms of foundation – having set out a set of assumptions that generate two distinct regimes – and in terms of application. In terms of foundation, the framework synthesizes elements from incomplete-contracting and comprehensive-contracting frameworks in a way that is, arguably, conceptually appealing. On a general note, the ultimate incentives for exerting productive effort – as measured by actual effort – are always stronger in the independence case; this is in line with systematic as well as casual empirical observation.

In terms of application, manipulation possibilities clearly constitute a relevant element in many principal-agent relationships. Before we discuss empirical implications, we will elaborate a little bit about the results in terms of regime choice, i.e., the choice whether an activity is

undertaken within an organization or bought in the marketplace. The relative attractiveness of the two regimes depend on three properties within the model:

1. the joint measure of the risk exposure of the agent, measured by $r_A v$;

2. the effect that manipulation has on the underlying performance, termed the *externality* its sign and strength being measured by σ; and,

3. the extent, γ, to which manipulation affects the performance measure.

In addition to this, we have considered two specifications of the structure of the externality – its depending on d or $|d|$ – but the basic results have been similar. The principal's choice is between employing an agent or contracting with an independent agent, and the results are quite clear-cut:

- A higher risk exposure makes the choice of an employed agent more attractive. This is in line with expectation.

- The externality has a clear implication: a negative externality favors contracting with an independent agent, while a positive externality favors employing an agent.

- The sensitivity of the performance measure to manipulation has the effect of moderating the externality; as γ grows, the critical level of risk exposure tends towards one, the value that applies in the absence of an externality.

The driving assumption is that the underlying outcome is owned by the principal in the case of an employed agent, but is owned by an independent agent. This implies that the incentives coming from the contract come on top of the direct incentives for an independent agent; since manipulation incentives come exclusively from the contract, the relative price between productive effort and manipulations differs across the two regimes.

The basic intuition for the results is that while an independent agent generally internalizes the externality in a way that an employed agent does not, this is more valuable in the case of a negative externality. Moreover, in the case of a positive externality, the restrictions placed on incentives by the two available sets of relative prices (resulting from the respective regimes) favor emplying the agent, in which case incentives for productive effort and manipulation are positively aligned.

We have mentioned the possibility that the externality would always hit the principal, rather than the owner of the asset. We have performed the analysis for this case as well with mostly

similar conclusions; the analysis is available from the author.[21] More precisely, the analysis is essentially unaffected for the case where the impact of the externality depends on d, whereas some results change in the other case. When the impact of the externality depends on $|d|$, the critical risk exposure stays constant (at $r_A v = 1$) within a range ($\sigma \in [-\gamma/2, \gamma/2]$); for more strongly negative externalities independence is always optimal, for more strongly positive externalities employment is always optimal.

We have made reference above to a previous version where there was no technological interaction, but where there was a dependence between the two efforts, a and d, in the agent's utility function. In more precise terms, there was a parameter, ρ, measuring the strength of competition between the efforts.[22] The results were similar in so far that the role that the externality plays in this framework was played by ρ in the other formulation (albeit inversely, a positive ρ having the the effect of a negative externality). Intuitively, ρ is a measure of the severity of the manipulation problem in a way that is akin to that of a negative externality.

Empirical implications The empirical implications of the framework and the analysis are closely tied to the interpretation of manipulation. In general terms, the conclusion that negative effects of manipulation on productive effort makes independence more likely seems quite reasonable; a simple example would be the intractability of employing an agent for an only-cash street sale in the absence of elaborate measures to prevent the agent from keeping a significant extra-contractual bonus.

On a completely different note, Anderson (1985) makes an interesting observation in the context of explaining the choice between in-house sales people and external representatives. Anderson notes that "But, contrary to expectations, the greater the possibility of the customer developing loyalty to the salesperson, the greater the likelyhood of using a rep." (p. 248). Loyalty between the salesperson and the customer expands the feasible set of manipulation that is not threatened by whistle-blowing by the customer, and thus plausibly re-inforces the potential for negative repercussions on the undertaking and on the principal.

One set of circumstances in which our framework seems relevant is that with an endogenous choice of the scope or the quality of an activity. Suppose that the agent can persuade the principal to affect the set of activities and that the agent's stake in the economic outcome is m. If the variation in economic outcome comes from costs and $m > 0$, the agent will want limit the number of activities. When $m < 0$, there may (this is beyond the letter of the model but

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[21] It is the second Appendix in this version.

[22] Even more precisely, the cost-of-effort function was $c(a, d) = a^2/2 + \rho a d + d^2/2$.

could straightforwardly be introduced) be scope for inflating reported costs in such a way that the contract-based cost-sharing is weighted more heavily than the true costs (in the extreme, true costs are borne according to $1 + m$; contracted costs are borne according to m); in such a case there would be an incentive for an independent agent to expand the set of activities. While a bit speculative, these respective patterns may be relevant in insurance-oriented and budgetary-based health insurance systems; in so far that there is a positive externality of excess activities – i.e. a negative externality of d – this is an argument in favor of insurance-oriented systems.

As a final example, one may note that the model by construction, but nevertheless importantly, illustrates that activities with similar external conditions may choose different regimes; a small change in a parameter starting close to the critical value can induce a jump. This is, for example, consistent with the dichotomy observed in the context of franchising. The dichotomy stems from the observation in e.g. retailing and fast-food restaurants that outlets run by an in-house management co-exist with franchised outlets, and that the incentives structures differ fundamentally; it is thus clearly a distinction between regimes with features similar to the *make* and *buy* features in our framework. The key consideration in these contexts is, arguably, a fear of quality shading; the prediction of our model is that the dichotomy is natural given variation in e.g. the severity of the measurement problem (γ) and the agent's risk aversion (r_A).

As a final remark we comment on the assumption that parameters – i.e. γ and the specification of the externality – are the same accross regimes. Beyond the fact that it keeps the model as simple as possible, it is important to note that the comparative-statics conclusions would not be sensitive to changing the assumption in this regard. For real-world outsourcing decisions, on the other hand, it is clearly possible that parameters differ across regimes. For instance, there may be less room for manipulation under employment ($\gamma^{\text{emp}} < \gamma^{\text{ind}}$) which obviously would favor employment *ceteris paribus*; such a difference would have a natural interpretation in terms of monitoring in the spirit of Alchian and Demsetz (1972).

4   Conclusions

This paper is an attempt to build a simple framework within which the distinction between make and buy – between performing an activity within a firm or organization (employment) and having it performed by an outside (independent) party – arises endogenously. Put somewhat pretentiously, the framework is a synthesis between an incomplete-contracting approach recognizing indivisibilities and limits of contracting, and a comprehensive-contracting approach

recognizing that there exist (imperfect) performance measures that can be subject to comprehensive contracting. The source of the imperfection of contractible performance measures is the fact that they can be manipulated by the agent in a simple principal-agent framework. The analysis shows that the way in which manipulation affects the agent's cost of effort and measured performance have implications for the choice between make and buy. The main conclusion is that a more severe manipulation problem – in the sense of manipulation exerting a stronger negative externality on the productive activity – makes it more attractive to buy, i.e. work with an independent agent.

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6 Appendix

In the Appendix we will provide the details of the analysis taking account of the externality, i.e., when the technology is given by:

$$x = a + \varepsilon + \sigma\lambda(d); \ z = a + \varepsilon + \gamma d$$

where either $\lambda(d) = d$ or $\lambda(d) = |d|$. We start with the former case.

6.1 Optimal contracts, $\lambda(d) = d$

An employed agent The principal's problem is

$$\max_{m,F} E\left(x - (F + mz)\right) = (a + \sigma d - (F + m(a + \gamma d)))$$

$$\text{s.t. } -\exp\left\{-r_A\left(F + m(a + \gamma d) - a^2/2 - d^2/2 - r_A m^2 v/2\right)\right\} \geq \bar{u}_A,$$

$$\text{and } (a,d) \in \arg\max\left(-\exp\left\{-r_A\left(F + m(a + \gamma d) - a^2/2 - d^2/2 - r_A m^2 v/2\right)\right\}\right).$$

Maximization by the agent yields

$$a = m, \text{ and } d = \gamma m; \tag{29}$$

inserting this and taking logarithms we get, using the assumption that $\bar{u}_A = -1$,

$$\max_{m,F} -F + (1 - m)m + \sigma\gamma m - m^2\gamma^2 \tag{30}$$

$$\text{s.t. } F + m^2 + m^2\gamma^2 - m^2/2 - m^2\gamma^2/2 - r_A m^2 v/2 \geq 0.$$

Note that the right-hand side of the participation constraint is the "reservation certainty equivalent."

Solving the constraint – which obviously must bind – for F, we get an unconstrained problem. Letting ϕ generically denote objective functions, P maximizes

$$\phi(m) = (1 + \sigma\gamma)m - m^2/2 - m^2\gamma^2/2 - r_A m^2 v/2$$

with respect to m; the first-order condition is

$$\phi'(m) = (1 + \sigma\gamma) - m - \gamma^2 m - r_A m v = 0,$$

from which

$$m^{\text{emp}} = \frac{1 + \sigma\gamma}{1 + \gamma^2 + r_A v} \tag{31}$$

follows directly. The problem generates a value function for P,

$$\phi^{*\text{emp}} = (1 + \sigma\gamma)m^{\text{emp}} - \left(1 + \gamma^2 + r_A v\right)(m^{\text{emp}})^2/2, \tag{32}$$

giving

$$\phi^{*\text{emp}} = \frac{1}{2}\frac{(1 + \sigma\gamma)^2}{1 + \gamma^2 + r_A v}. \tag{33}$$

An independent agent Next, we will consider how the problem is modified if the principal opts for an independent agent. The principal solves (again, recall that $\overline{u}_A = -1$)

$$\max_{m,F} E\left(B - (F + mz)\right) = B - (F + m\left(a + \gamma d\right)),$$

$$\text{s.t. } F + m\left(a + \gamma d\right) + a + \sigma d - a^2/2 - d^2/2 - r_A\left(1+m\right)^2 v/2 \geq 0;$$

the difference is that actual output enters the agent's rather than the principal's payoff and the agent's incentives to exert a are therefore strengthened by the direct effect while his incentives to exert d affected by the externality. Maximization by the agent now yields

$$a = 1 + m; \quad \text{and} \quad d = \gamma m + \sigma. \tag{34}$$

Inserting equilibrium effort, simplifying a bit, and taking logarithms we get

$$\max \; -F - m\left(1+m\right) - m^2\gamma^2 - \gamma m \sigma \tag{35}$$

$$\text{s.t. } F + m^2\gamma^2 + \gamma m \sigma + \sigma\left(\gamma m + \sigma\right) + \left(1+m\right)^2/2 - \left(\gamma m + \sigma\right)^2/2 - r_A\left(1+m\right)^2 v/2 \geq 0.$$

Solving the constraint as above, the principal maximizes the following objective function:

$$\phi(m) = 1 + m + \sigma\left(\gamma m + \sigma\right) - \left(1+m\right)^2/2 - \frac{1}{2}\left(\gamma m + \sigma\right)^2 - r_A\left(1+m\right)^2 v/2$$

with respect to m, with first-order condition

$$\phi'(m) = 1 + \sigma\gamma - \left(1+m\right) - \gamma\left(\gamma m + \sigma\right) - r_A\left(1+m\right)v = 0,$$

from which

$$m^{\text{ind}} = \frac{-r_A v}{1 + \gamma^2 + r_A v} \tag{36}$$

and from which the effective power of incentives facing the agent is

$$m^{\text{ind}} + 1 = \frac{1 + \gamma^2}{1 + \gamma^2 + r_A v}. \tag{37}$$

The problem generates a value function for P,

$$\phi^{*\text{ind}} = 1 + m^{\text{ind}} + \sigma\left(\gamma m^{\text{ind}} + \sigma\right) - \left(1 + r_A v\right)\left(1 + m^{\text{ind}}\right)^2/2 - \left(\gamma m^{\text{ind}} + \sigma\right)^2/2 \tag{38}$$

$$\phi^{*\text{ind}} = 1 + m^{\text{ind}} + \sigma\gamma m^{\text{ind}} + \sigma^2 - \left(1 + r_A v\right)\left(1 + m^{\text{ind}}\right)^2/2 - \gamma^2\left(m^{\text{ind}}\right)^2/2 - \sigma\gamma m^{\text{ind}} - \sigma^2/2 \tag{39}$$

$$\phi^{*\text{ind}} = 1 + m^{\text{ind}} + \sigma^2/2 - \left(1 + r_A v\right)\left(1 + m^{\text{ind}}\right)^2/2 - \gamma^2\left(m^{\text{ind}}\right)^2/2 \tag{40}$$

$$\phi^{*\text{ind}} = \frac{1 + \gamma^2}{1 + \gamma^2 + r_A v} + \sigma^2/2 - \left(1 + r_A v\right)\left(\frac{1 + \gamma^2}{1 + \gamma^2 + r_A v}\right)^2/2 - \gamma^2\left(\frac{-r_A v}{1 + \gamma^2 + r_A v}\right)^2/2 \tag{41}$$

$$\phi^{*\text{ind}} = \frac{2\left(1+\gamma^2\right)\left(1+\gamma^2+r_A v\right) - \left(1+r_A v\right)\left(1+\gamma^2\right)^2 - \gamma^2\left(r_A v\right)^2}{2\left(1+\gamma^2+r_A v\right)^2} + \sigma^2/2, \qquad (42)$$

using the simplification below,

$$\phi^{*\text{ind}} = \frac{\left(1+\gamma^2+r_A v\right)\left(1+\gamma^2\left(1-r_A v\right)\right)}{2\left(1+\gamma^2+r_A v\right)^2} + \sigma^2/2, \qquad (43)$$

$$\phi^{*\text{ind}} = \frac{\left(1+\gamma^2\left(1-r_A v\right)\right)}{2\left(1+\gamma^2+r_A v\right)} + \sigma^2/2, \qquad (44)$$

to be compared with $\phi^{*\text{emp}}$ above.

Simplification

$$2\left(1+\gamma^2\right)\left(1+\gamma^2+r_A v\right) - \left(1+\gamma^2+r_A v+\gamma^2 r_A v\right)\left(1+\gamma^2\right) - \gamma^2\left(r_A v\right)^2 =$$

$$\left(1+\gamma^2\right)\left(1+\gamma^2+r_A v\right) - \gamma^2 r_A v\left(1+\gamma^2+r_A v\right) =$$

$$\left(1+\gamma^2+r_A v\right)\left(1+\gamma^2\left(1-r_A v\right)\right)$$

#

6.2   Comparisons $\lambda(d) = d$

We start by comparing equilibrium effort, a; this difference is equal to the difference in terms of incentive intensity in this case:

$$\Delta a = 1 + m^{\text{ind}} - m^{\text{emp}} = \frac{\gamma^2 - \sigma\gamma}{1+\gamma^2+r_A v} = \frac{\gamma^2\left(1-\sigma/\gamma\right)}{1+\gamma^2+r_A v}, \qquad (45)$$

which is positive as long as $|\sigma| \leq \gamma$ as assumed. In this setting, thus, an independent agent has unambiguously stronger performance incentives.

The condition for an independent management to be optimal is that the following expression be non-negative:

$$\Delta^{\text{profit}} = \phi^{*\text{ind}} - \phi^{*\text{emp}},$$

$$\phi^{*\text{emp}} = \frac{1}{2}\frac{\left(1+\sigma\gamma\right)^2}{1+\gamma^2+r_A v}. \qquad (46)$$

$$\phi^{*\text{ind}} = \frac{\left(1+\gamma^2\left(1-r_A v\right)\right)}{2\left(1+\gamma^2+r_A v\right)} + \sigma^2/2, \qquad (47)$$

$$\Delta^{\text{profit}} = \frac{\left(1+\gamma^2\left(1-r_A v\right)\right) - \left(1+\sigma\gamma\right)^2}{2\left(1+\gamma^2+r_A v\right)} + \frac{\sigma^2}{2}.$$

$$\Delta^{\text{profit}} = \frac{\left(1+\gamma^2\left(1-r_A v\right)\right) - \left(1+\sigma\gamma\right)^2 + \sigma^2\left(1+\gamma^2+r_A v\right)}{2\left(1+\gamma^2+r_A v\right)}.$$

The condition for an independent agent to be optimal is thus,

$$1 + \gamma^2 \left(1 - r_A v\right) - 1 - 2\sigma\gamma - \sigma^2\gamma^2 + \sigma^2 \left(1 + \gamma^2 + r_A v\right) \geq 0.$$

$$\gamma^2 \left(1 - r_A v\right) - 2\sigma\gamma + \sigma^2 \left(1 + r_A v\right) \geq 0.$$

$$r_A v \left(\gamma^2 - \sigma^2\right) \leq \gamma^2 - 2\sigma\gamma + \sigma^2 = (\gamma - \sigma)^2$$

$$r_A v \left(\gamma + \sigma\right)\left(\gamma - \sigma\right) \leq (\gamma - \sigma)^2$$

Re-writing, we get,[23]

$$r_A v \leq \frac{\gamma - \sigma}{\gamma + \sigma} = \frac{1 - \sigma/\gamma}{1 + \sigma/\gamma}.$$

REMARK: this expression can be re-written,

$$\frac{\sigma}{\gamma} \leq \frac{1 - r_A v}{1 + r_A v}.$$

6.3 Optimal contracts, $\lambda(d) = |d|$

As we noted in the text, the employment case is unaffected by the change of λ; we thus proceed directly to the independence case.

An independent agent We are going to use the fact that since $|d| = -d$ when $d < 0$, we can simply use $(-d)$ in the analysis as long as we can be sure that d is negative. We will then check that this is valid and explore the remaining cases. The management solves

$$\max_{m,F} E\left(B - (F + mz)\right) = B - \left(F + m\left(a + \gamma d\right)\right),$$

$$\text{s.t. } F + m\left(a + \gamma d\right) + a - \sigma d - a^2/2 - d^2/2 - r_A\left(1 + m\right)^2 v/2 \geq 0.$$

Maximization by the agent now yields

$$a = 1 + m; \quad \text{and} \quad d = \gamma m - \sigma. \tag{48}$$

Inserting equilibrium effort, simplifying a bit, and taking logarithms we get

$$\max \; -F - m\left(1 + m\right) - m^2\gamma^2 + \gamma m\sigma \tag{49}$$

$$\text{s.t. } F + m^2\gamma^2 - \gamma m\sigma - \sigma\left(\gamma m - \sigma\right) + \left(1 + m\right)^2/2 - \left(\gamma m - \sigma\right)^2/2 - r_A\left(1 + m\right)^2 v/2 \geq 0.$$

Solving the constraint as above, the principal maximizes the following objective function:

$$\phi(m) = 1 + m - \sigma\left(\gamma m - \sigma\right) - \left(1 + m\right)^2/2 - \frac{1}{2}\left(\gamma m - \sigma\right)^2 - r_A\left(1 + m\right)^2 v/2 \tag{50}$$

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[23]One may note that whenever $|\sigma| > \gamma$, independence is optimal since the RHS is postive, while the LHS is negative; for $\sigma = \gamma$, there is indifference. Note that this seemingly entails a discontinuity at $\sigma = \gamma$, but the actual profit difference is zero at $\sigma = \gamma$; the intuition is that the internalization effect then becomes dominant.

with respect to m, with first-order condition

$$\phi'(m) = 1 - \sigma\gamma - (1 + m) - \gamma(\gamma m - \sigma) - r_A(1 + m)v = 0,$$

from which

$$m^{\text{ind}} = \frac{-r_A v}{1 + \gamma^2 + r_A v} \tag{51}$$

and from which the effective power of incentives facing the agent is

$$m^{\text{ind}} + 1 = \frac{1 + \gamma^2}{1 + \gamma^2 + r_A v}. \tag{52}$$

The value function for P is

$$\phi^{*\text{ind}} = 1 + m^{\text{ind}} - \sigma\left(\gamma m^{\text{ind}} - \sigma\right) - (1 + r_A v)\left(1 + m^{\text{ind}}\right)^2/2 - \left(\gamma m^{\text{ind}} - \sigma\right)^2/2 \tag{53}$$

$$\phi^{*\text{ind}} = 1 + m^{\text{ind}} - \sigma\gamma m^{\text{ind}} + \sigma^2 - (1 + r_A v)\left(1 + m^{\text{ind}}\right)^2/2 - \gamma^2\left(m^{\text{ind}}\right)^2/2 + \sigma\gamma m^{\text{ind}} - \sigma^2/2 \tag{54}$$

$$\phi^{*\text{ind}} = 1 + m^{\text{ind}} + \sigma^2/2 - (1 + r_A v)\left(1 + m^{\text{ind}}\right)^2/2 - \gamma^2\left(m^{\text{ind}}\right)^2/2 \tag{55}$$

$$\phi^{*\text{ind}} = \frac{1 + \gamma^2}{1 + \gamma^2 + r_A v} + \sigma^2/2 - (1 + r_A v)\left(\frac{1 + \gamma^2}{1 + \gamma^2 + r_A v}\right)^2/2 - \gamma^2\left(\frac{-r_A v}{1 + \gamma^2 + r_A v}\right)^2/2 \tag{56}$$

$$\phi^{*\text{ind}} = \frac{2\left(1 + \gamma^2\right)\left(1 + \gamma^2 + r_A v\right) - (1 + r_A v)\left(1 + \gamma^2\right)^2 - \gamma^2\left(r_A v\right)^2}{2\left(1 + \gamma^2 + r_A v\right)^2} + \sigma^2/2 \tag{57}$$

to be compared with $\phi^{*\text{emp}}$ above. Note that this value function is identical to (42), and the same anlysis thus applies.

Next, we need to look at actual d:

$$d^{\text{ind}} = \gamma m^{\text{ind}} - \sigma = \frac{-\gamma r_A v}{1 + \gamma^2 + r_A v} - \sigma;$$

d is non-positive if

$$-\sigma \leq \frac{\gamma r_A v}{1 + \gamma^2 + r_A v}.$$

In order to verify that this is indeed a solution, one must check that there does not exist another solution (superior for the agent) with $d > 0$ for some parameter values. The analysis of this case was done above and we saw that with $\lambda(d) = d$ (which coincides with $\lambda(d) = |d|$ for $d > 0$) we have, recalling that m is the same in the two cases

$$d = \gamma m + \sigma = \frac{-\gamma r_A v}{1 + \gamma^2 + r_A v} + \sigma;$$

to rule out $d > 0$ *as a solution*, this must be non-positive,

$$\sigma \leq \frac{\gamma r_A v}{1 + \gamma^2 + r_A v},$$

providing an upper bound on the externality (we explore the case where this is not satisfied below).

In sum, if

$$|\sigma| \leq \frac{\gamma r_A v}{1 + \gamma^2 + r_A v},$$

d is unambiguously negative under both specifications. Note that this interval expands with $r_A v$; it shrinks to zero when risk vanishes.

Remaining cases

Strong positive externality   When the externality is strong enough, there are two consistent solutions, with d positive and negative, to the agent's proplem, which is

$$F + m(a + \gamma d) + a + \sigma |d| - a^2/2 - d^2/2 - r_A (1 + m)^2 v/2.$$

- If $d > 0$, the objective is

$$F + m(a + \gamma d) + a + \sigma d - a^2/2 - d^2/2 - r_A (1 + m)^2 v/2,$$

with solution and value:

$$d = \gamma m + \sigma; \ (\gamma m + \sigma)^2/2.$$

- If $d < 0$, the objective is

$$F + m(a + \gamma d) + a - \sigma d - a^2/2 - d^2/2 - r_A (1 + m)^2 v/2,$$

with solution and value:

$$d = \gamma m - \sigma; \ (\gamma m - \sigma)^2/2.$$

- Since $m < 0$, the latter possibility is obviously superior, and the solution with $d < 0$ is the relevant one in this case too.

Strong negative externality   We now turn to the final case of a strong negative externality; while the result in this case is clear-cut, the argument is a bit technical.

Agent's best reply: The agent's objective is still:

$$F + m(a + \gamma d) + a + \sigma |d| - a^2/2 - d^2/2 - r_A (1 + m)^2 v/2.$$

Now, for fixed m, there are three possibilities:

- $d > 0$: then d must solve

$$\max\ F + m\left(a + \gamma d\right) + a + \sigma d - a^2/2 - d^2/2 - r_A\left(1 + m\right)^2 v/2$$

which is a well-behaved problem with solution $d = \gamma m + \sigma$; it can thus only be relevant if $\gamma m + \sigma > 0$, which is ruled out by $m < 0$ and $\sigma < 0$.

- $d < 0$: then d must solve

$$\max\ F + m\left(a + \gamma d\right) + a - \sigma d - a^2/2 - d^2/2 - r_A\left(1 + m\right)^2 v/2$$

which is a well-behaved problem with solution $d = \gamma m - \sigma$; it can thus only be relevant if $\gamma m - \sigma < 0$, which was the case considered above.

- $d = 0$: If neither of these hold – i.e. if $\gamma m - \sigma > 0$ – the remaining possibility is $d = 0$.

Remark: In sum, this gives us a complete map of the agent's best reply given m:

$$d = \min\left\{\gamma m - \sigma, 0\right\}.$$

Principal's response to $d=0$: The principal's problem assuming $d = 0$ is

$$\max_{m,F}\ E\left(B - \left(F + mz\right)\right) = B - \left(F + ma\right),$$

$$\text{s.t. } F + ma + a - a^2/2 - r_A\left(1 + m\right)^2 v/2 \geq 0;$$

obviously, $a = 1 + m$, and one gets straightforwardly,

$$\phi^0(m) = (m + 1) - (1 + r_A v)\left(m + 1\right)^2/2 \tag{58}$$

with (well known) solution

$$m^{d=0} + 1 = \frac{1}{1 + r_A v}.$$

The value function for this case – where m is not subject to any constraint – is thus

$$\phi^{*0} = \frac{1}{1 + r_A v} - \frac{1 + r_A v}{2\left(1 + r_A v\right)^2} = \frac{1}{2\left(1 + r_A v\right)}. \tag{59}$$

Now, since $m^{d=0} < m^{\text{ind}}$ (i.e. $\left|m^{d=0}\right| > \left|m^{\text{ind}}\right|$),

$$\gamma m^{d=0} - \sigma < \gamma m^{\text{ind}} - \sigma.$$

This means that there are cases where the agent's best reply to m^{ind} is $d = 0$ ($\gamma m^{\text{ind}} - \sigma \geq 0$) whereas the principal's myopically (i.e., taking d as given) optimal response to $d = 0$ creates an incentive for $d < 0$ ($\gamma m^{d=0} - \sigma < 0$). Denote by $\widehat{m}$ the principal's ultimate choice (i.e. incorporating the agent's response); it is clear that:

- it cannot be the case that $\gamma\widehat{m} - \sigma < 0$ since the principal's relevant objective function then is (50), which is well-behaved and which produces the solution m^{ind} unless there are constraints;

- consider next $\gamma\widehat{m} - \sigma \geq 0$; the relevant objective function is then (58) and the unconstrained solution to the maximization of it is $m^{d=0}$ which, however, violates the constraint; the solution is obviously choosing $\widehat{m}$ such that

$$\gamma\widehat{m} - \sigma = 0.$$

In conclusion, whenever $\gamma m^{\text{ind}} - \sigma > 0$, i.e. when

$$\sigma < \frac{-\gamma r_A v}{1 + \gamma^2 + r_A v},$$

the solution for the independence case entails the agent choosing $d = 0$, and the incentive scheme is given by

$$m = \max\left\{\frac{-r_A v}{1 + r_A v}, \frac{\sigma}{\gamma}\right\}.$$

Comparisons: The principal's value function given $d = 0$ in is to be compared with

$$\phi^{*\text{emp}} = \frac{1}{2}\frac{(1 + \sigma\gamma)^2}{1 + \gamma^2 + r_A v}. \tag{60}$$

We have two cases:

- In the simpler case, there is no constraint on d and (59) is the relevant value; the difference is then

$$\phi^{*0} - \phi^{*\text{emp}} = \frac{1}{2(1 + r_A v)} - \frac{1}{2}\frac{(1 + \sigma\gamma)^2}{1 + \gamma^2 + r_A v} = \frac{(1 + \gamma^2 + r_A v) - (1 + r_A v)(1 + \sigma\gamma)^2}{2(1 + r_A v)(1 + \gamma^2 + r_A v)},$$

or, one step further,

$$\frac{(1 + \gamma^2 + r_A v) - (1 + r_A v)(1 + 2\sigma\gamma + \sigma^2\gamma^2)}{2(1 + r_A v)(1 + \gamma^2 + r_A v)} = \frac{\gamma^2 - \sigma\gamma(1 + r_A v)(2 + \sigma\gamma)}{2(1 + r_A v)(1 + \gamma^2 + r_A v)}.$$

When $\widehat{m}$ is not limited by the constraint that $\gamma\widehat{m} - \sigma \geq 0$ (i.e. when $\gamma m^{d=0} - \sigma \geq 0$), and under the assumed parameter restrictions ($|\sigma\gamma| \leq 1$) independence is thus unambiguously preferable.

- The more cumbersome case is that where $d = 0$ but where the principal is bound by the constraint that $\gamma\widehat{m} - \sigma \geq 0$ so that $\widehat{m} \in [-r_A v/(1 + r_A v), -r_A v/(1 + \gamma^2 + r_A v)]$. In this case, it is harder to obtain a tractable tight bound, and we will provide a sufficient condition for independence to be optimal in this case too. The principal's objective is still

defined by $d = 0$ and the objective function – a function of m – in (58) applies; a notable feature of this objective (following naturally from $d = 0$) is that it does not depend on σ. Consider the difference $\phi^{**}(m) = \phi^0(m) - \phi^{*\mathrm{emp}}$:

$$\phi^{**}(m) = (m+1) - \frac{1 + r_A v}{2}(m+1)^2 - \frac{1}{2}\frac{(1+\sigma\gamma)^2}{1+\gamma^2 + r_A v}$$

at boundary where σ is just negative enough to give $\gamma m^{\mathrm{ind}} - \sigma \geq 0$ (i.e. where $\gamma m^{\mathrm{ind}} - \sigma = 0$). When σ turns more negative from that point, two things happen: (i) σ has a direct effect through $\phi^{*\mathrm{emp}}$; and (ii) the constaint on m is relaxed. Both these effects make independence more attractive. The condition at the boundary is

$$r_A v \leq \frac{1 - \sigma/\gamma}{1 + \sigma/\gamma};$$

with σ/γ defined by $\gamma m^{\mathrm{ind}} - \sigma = 0$ so that

$$\sigma/\gamma = \frac{-r_A v}{1 + \gamma^2 + r_A v};$$

this can be expressed

$$r_A v \leq \frac{1 + r_A v / \left(1 + \gamma^2 + r_A v\right)}{1 - r_A v / \left(1 + \gamma^2 + r_A v\right)} = \frac{1 + \gamma^2 + 2 r_A v}{1 + \gamma^2} = 1 + \frac{2}{1+\gamma^2} r_A v,$$

or

$$r_A v \left(1 - \frac{2}{1+\gamma^2}\right) \leq 1; \quad r_A v \left(\frac{\gamma^2 - 1}{\gamma^2 + 1}\right) \leq 1.$$

We see that if $\gamma \leq 1$ independence is unambiguously preferable. Note that given that this case concerns σ strongly negative, the condition that $\gamma \leq 1$ is quite weak given $|\sigma|\gamma \leq 1$.

Comparing effort: Let first the principal's solution be unconstrained:

$$m^{d=0} + 1 = \frac{1}{1 + r_A v} \tag{61}$$

and note that $\sigma \leq -\gamma r_A v / \left(1 + r_A v\right)$ in

$$m^{\mathrm{emp}} = \frac{1 + \sigma\gamma}{1 + \gamma^2 + r_A v} \tag{62}$$

$$\Delta a = \frac{1}{1 + r_A v} - \frac{1 + \sigma\gamma}{1 + \gamma^2 + r_A v}$$

$$\Delta a \geq \frac{1}{1 + r_A v} - \frac{1 + \gamma^2 \left(-r_A v / \left(1 + r_A v\right)\right)}{1 + \gamma^2 + r_A v} = \frac{1 + \gamma^2 + r_A v - \left(1 - \gamma^2 r_A v\right)}{\left(1 + \gamma^2 + r_A v\right)\left(1 + r_A v\right)}$$

$$\Delta a \geq \frac{\gamma^2 + r_A v + \gamma^2 r_A v}{\left(1 + \gamma^2 + r_A v\right)\left(1 + r_A v\right)} > 0.$$

Next,

$$m^{d=0} + 1 \geq \frac{1}{1 + r_A v} \tag{63}$$

and note that $\sigma \leq -\gamma r_A v / \left(1 + \gamma^2 + r_A v\right)$ in

$$m^{\text{emp}} = \frac{1 + \sigma\gamma}{1 + \gamma^2 + r_A v} \tag{64}$$

$$\Delta a = \frac{1}{1 + r_A v} - \frac{1 + \sigma\gamma}{1 + \gamma^2 + r_A v}$$

$$\Delta a \geq \frac{1}{1 + r_A v} - \frac{1 + \gamma^2 \left(-r_A v / \left(1 + \gamma^2 + r_A v\right)\right)}{1 + \gamma^2 + r_A v}$$

$$\Delta a \geq \frac{\left(1 + \gamma^2 + r_A v\right)^2 - (1 + r_A v)\left(1 + \gamma^2 + r_A v\right) + \gamma^2 r_A v\left(1 + r_A v\right)}{(1 + r_A v)\left(1 + \gamma^2 + r_A v\right)^2}$$

$$\Delta a \geq \frac{\left(1 + \gamma^2 + r_A v\right) - (1 + r_A v)}{(1 + r_A v)\left(1 + \gamma^2 + r_A v\right)} + \frac{\gamma^2 r_A v\left(1 + r_A v\right)}{(1 + r_A v)\left(1 + \gamma^2 + r_A v\right)^2} > 0.$$

Summary of cases

- d with sign – no issues;

- $|d|$:

  - $\sigma > \gamma r_A v / \left(1 + \gamma^2 + r_A v\right)$, $d < 0$ and the analysis of that case applies;
  - $|\sigma| \leq \gamma r_A v / \left(1 + \gamma^2 + r_A v\right)$, $d < 0$ and this goes through consistently;
  - $\sigma \leq -\gamma r_A v / \left(1 + r_A v\right)$, the agent chooses $d = 0$ and this works through consistently;
  - $-\gamma r_A v / \left(1 + r_A v\right) \leq \sigma \leq -\gamma r_A v / \left(1 + \gamma^2 + r_A v\right)$, still $d = 0$, but a bit more complicated (the principal re-optimizes m but this does not change d).

7 Externality on principal

First, we note that with an employed agent this analysis is similar to the main case; the externality affects the principal.

An independent agent Next, we will consider how the problem is modified if the principal opts for an independent agent (the externality depends on d). The principal solves

$$\max_{m,F} \; E\left(B - (F + mz)\right) + \sigma d = B - (F + m\left(a + \gamma d\right)) + \sigma d,$$

$$\text{s.t. } F + m\left(a + \gamma d\right) + a - a^2/2 - d^2/2 - r_A\left(1 + m\right)^2 v/2 \geq 0;$$

maximization by the agent yields

$$a = 1 + m; \quad \text{and} \quad d = \gamma m,$$

and inserting equilibrium effort and taking logarithms we get

$$\max \; -F - m\left(1 + m\right) - m^2\gamma^2 + \sigma\gamma m \tag{65}$$

s.t. $F + m\left(1 + m\right) + m^2\gamma^2 + 1 + m - \left(1 + m\right)^2/2 - \left(\gamma m\right)^2/2 - r_A\left(1 + m\right)^2 v/2 \geq 0.$

Solving the constraint, the principal maximizes the following objective function:

$$\phi(m) = 1 + m + \sigma\left(\gamma m\right) - \left(1 + m\right)^2/2 - \frac{1}{2}\left(\gamma m\right)^2 - r_A\left(1 + m\right)^2 v/2$$

with respect to m, with first-order condition

$$\phi'(m) = 1 + \sigma\gamma - \left(1 + m\right) - \gamma^2 m - r_A\left(1 + m\right)v = 0,$$

from which

$$m^{\text{ind}} = \frac{\sigma\gamma - r_A v}{1 + \gamma^2 + r_A v} \tag{66}$$

and from which the effective power of incentives facing the agent is

$$m^{\text{ind}} + 1 = \frac{1 + \gamma^2 + \sigma\gamma}{1 + \gamma^2 + r_A v}. \tag{67}$$

The problem generates a value function for P,

$$\phi^{*\text{ind}} = 1 + m^{\text{ind}} + \sigma\left(\gamma m^{\text{ind}}\right) - \left(1 + r_A v\right)\left(1 + m^{\text{ind}}\right)^2/2 - \left(\gamma m^{\text{ind}}\right)^2/2 \tag{68}$$

$$\phi^{*\text{ind}} = 1 + \left(1 + \sigma\gamma\right)m^{\text{ind}} - \left(1 + r_A v\right)\left(1 + m^{\text{ind}}\right)^2/2 - \gamma^2\left(m^{\text{ind}}\right)^2/2 \tag{69}$$

$$\phi^{*\text{ind}} = 1 + \left(1 + \sigma\gamma\right)\frac{\sigma\gamma - r_A v}{1 + \gamma^2 + r_A v} - \left(1 + r_A v\right)\left(\frac{1 + \gamma^2 + \sigma\gamma}{1 + \gamma^2 + r_A v}\right)^2/2 - \gamma^2\left(\frac{\sigma\gamma - r_A v}{1 + \gamma^2 + r_A v}\right)^2/2 \tag{70}$$

$$\phi^{*\text{ind}} = \frac{1 + \gamma^2 + \sigma\gamma + \sigma\gamma\left(\sigma\gamma - r_A v\right)}{1 + \gamma^2 + r_A v} - \left(1 + r_A v\right)\left(\frac{1 + \gamma^2 + \sigma\gamma}{1 + \gamma^2 + r_A v}\right)^2/2 - \gamma^2\left(\frac{\sigma\gamma - r_A v}{1 + \gamma^2 + r_A v}\right)^2/2 \tag{71}$$

$$\phi^{*\text{ind}} = \frac{2\left(1 + \gamma^2 + \sigma\gamma + \sigma\gamma\left(\sigma\gamma - r_A v\right)\right)\left(1 + \gamma^2 + r_A v\right) - \left(1 + r_A v\right)\left(1 + \gamma^2 + \sigma\gamma\right)^2 - \gamma^2\left(\sigma\gamma - r_A v\right)^2}{2\left(1 + \gamma^2 + r_A v\right)^2} \tag{72}$$

according to the simplification below:

$$\phi^{*\text{ind}} = \frac{2\left(1 + \gamma^2 + \sigma\gamma + \sigma^2\gamma^2 - \sigma\gamma r_A v\right) - \left[\sigma^2\gamma^2 + 1 + \gamma^2 + 2\sigma\gamma + \gamma^2 r_A v\right]}{2\left(1 + \gamma^2 + r_A v\right)} \tag{73}$$

$$\phi^{*\text{ind}} = \frac{1 + \gamma^2 + \sigma^2\gamma^2 - 2\sigma\gamma r_A v - \gamma^2 r_A v}{2\left(1 + \gamma^2 + r_A v\right)} \tag{74}$$

$$\phi^{*\text{ind}} = \frac{1 + \gamma^2 + \sigma^2\gamma^2 - \left(2\sigma\gamma + \gamma^2\right)r_A v}{2\left(1 + \gamma^2 + r_A v\right)} \tag{75}$$

to be compared with $\phi^{*\text{emp}}$ from the text.

Simplification:

$$(1 + r_A v)\left(1 + \gamma^2 + \sigma\gamma\right)^2 + \gamma^2\left(\sigma\gamma - r_A v\right)^2$$

$$(1 + r_A v)\left(\left(1 + \gamma^2\right)^2 + 2\left(1 + \gamma^2\right)\sigma\gamma + \sigma^2\gamma^2\right) + \gamma^2\left(\sigma^2\gamma^2 - 2r_A v\sigma\gamma + (r_A v)^2\right)$$

$$\sigma^2\gamma^2\left(1 + \gamma^2 + r_A v\right) + (1 + r_A v)\left(1 + \gamma^2\right)\left(\left(1 + \gamma^2\right) + 2\sigma\gamma\right) + \gamma^2 r_A v\left(-2\sigma\gamma + r_A v\right)$$

$$\sigma^2\gamma^2\left(1 + \gamma^2 + r_A v\right) + \left(1 + \gamma^2 + r_A v\right)\left(\left(1 + \gamma^2\right) + 2\sigma\gamma\right) + \gamma^2 r_A v\left(\left(1 + \gamma^2 + r_A v\right)\right)$$

$$\left(1 + \gamma^2 + r_A v\right)\left[\sigma^2\gamma^2 + \left(1 + \gamma^2 + 2\sigma\gamma\right) + \gamma^2 r_A v\right]$$

#

Comparisons We start by comparing equilibrium effort, a:

$$\Delta a = 1 + m^{\mathrm{ind}} - m^{\mathrm{emp}} = \frac{\gamma^2}{1 + \gamma^2 + r_A v}. \tag{76}$$

In this setting, thus, an independent agent has unambiguously stronger performance incentives.

The condition for an independent management to be optimal is that the following expression be non-negative:

$$\Delta^{\mathrm{profit}} = \phi^{*\mathrm{ind}} - \phi^{*\mathrm{emp}},$$

$$\phi^{*\mathrm{emp}} = \frac{1}{2}\frac{(1 + \sigma\gamma)^2}{1 + \gamma^2 + r_A v}. \tag{77}$$

$$\phi^{*\mathrm{ind}} = \frac{1 + \gamma^2 + \sigma^2\gamma^2 - \left(2\sigma\gamma + \gamma^2\right)r_A v}{2\left(1 + \gamma^2 + r_A v\right)} = \frac{(1 + \sigma\gamma)^2 + \gamma^2 - 2\sigma\gamma - \left(2\sigma\gamma + \gamma^2\right)r_A v}{2\left(1 + \gamma^2 + r_A v\right)} \tag{78}$$

$$\Delta^{\mathrm{profit}} = \frac{(1 + \sigma\gamma)^2 + \gamma^2 - 2\sigma\gamma - \left(2\sigma\gamma + \gamma^2\right)r_A v - (1 + \sigma\gamma)^2}{2\left(1 + \gamma^2 + r_A v\right)}$$

The condition for an independent agent to be optimal is thus,

$$\gamma^2 - 2\sigma\gamma \geq \left(\gamma^2 + 2\sigma\gamma\right)r_A v \tag{79}$$

$$r_A v \leq \frac{1 - 2\sigma/\gamma}{1 + 2\sigma/\gamma}.$$

For $|\sigma| \leq \gamma/2$ we thus have a result that is similar to the case in the text, re-scaled by a factor two; for σ outside these bounds, it is easily seen from (79) that for $\sigma < -\gamma/2$, independence is always optimal, and $\sigma > \gamma/2$ for employment is always optimal.

Independence – externality depending on $|d|$ and actual d negative   We assume that $d < 0$ and then verify when this is valid and collect the remaining cases. The management solves

$$\max_{m,F} E\left(B - (F + mz)\right) + \sigma d = B - (F + m(a + \gamma d)) - \sigma d,$$

$$\text{s.t.} \quad F + m(a + \gamma d) + a - a^2/2 - d^2/2 - r_A(1 + m)^2 v/2 \geq 0;$$

the agent chooses

$$a = 1 + m; \quad \text{and} \quad d = \gamma m,$$

and we get

$$\phi(m) = 1 + m - \sigma(\gamma m) - (1 + m)^2/2 - \frac{1}{2}(\gamma m)^2 - r_A(1 + m)^2 v/2$$

with first-order condition with respect to m,

$$\phi'(m) = 1 - \sigma\gamma - (1 + m) - \gamma^2 m - r_A(1 + m)v = 0,$$

from which

$$m^{\text{ind}} = \frac{-\sigma\gamma - r_A v}{1 + \gamma^2 + r_A v} \tag{80}$$

and from which the effective power of incentives facing the agent is

$$m^{\text{ind}} + 1 = \frac{1 + \gamma^2 - \sigma\gamma}{1 + \gamma^2 + r_A v}. \tag{81}$$

The value function for P is

$$\phi^{*\text{ind}} = 1 + m^{\text{ind}} - \sigma\left(\gamma m^{\text{ind}}\right) - (1 + r_A v)\left(1 + m^{\text{ind}}\right)^2/2 - \left(\gamma m^{\text{ind}}\right)^2/2 \tag{82}$$

$$\phi^{*\text{ind}} = 1 + (1 - \sigma\gamma)m^{\text{ind}} - (1 + r_A v)\left(1 + m^{\text{ind}}\right)^2/2 - \gamma^2\left(m^{\text{ind}}\right)^2/2 \tag{83}$$

$$\phi^{*\text{ind}} = 1 + (1 - \sigma\gamma)\frac{-\sigma\gamma - r_A v}{1 + \gamma^2 + r_A v} - (1 + r_A v)\left(\frac{1 + \gamma^2 - \sigma\gamma}{1 + \gamma^2 + r_A v}\right)^2/2 - \gamma^2\left(\frac{-\sigma\gamma - r_A v}{1 + \gamma^2 + r_A v}\right)^2/2 \tag{84}$$

$$\phi^{*\text{ind}} = \frac{1 + \gamma^2 - \sigma\gamma - \sigma\gamma(-\sigma\gamma - r_A v)}{1 + \gamma^2 + r_A v} - (1 + r_A v)\left(\frac{1 + \gamma^2 - \sigma\gamma}{1 + \gamma^2 + r_A v}\right)^2/2 - \gamma^2\left(\frac{-\sigma\gamma - r_A v}{1 + \gamma^2 + r_A v}\right)^2/2 \tag{85}$$

$$\phi^{*\text{ind}} = \frac{2\left(1 + \gamma^2 - \sigma\gamma + \sigma\gamma(\sigma\gamma + r_A v)\right)\left(1 + \gamma^2 + r_A v\right) - (1 + r_A v)\left(1 + \gamma^2 - \sigma\gamma\right)^2 - \gamma^2(\sigma\gamma + r_A v)^2}{2\left(1 + \gamma^2 + r_A v\right)^2} \tag{86}$$

using the simplification below,

$$\phi^{*\text{ind}} = \frac{2\left(1 + \gamma^2 - \sigma\gamma + \sigma^2\gamma^2 + \sigma\gamma r_A v\right) - \left(\sigma^2\gamma^2 + 1 + \gamma^2 - 2\sigma\gamma + \gamma^2 r_A v\right)}{2\left(1 + \gamma^2 + r_A v\right)} \tag{87}$$

$$\phi^{*\text{ind}} = \frac{1 + \gamma^2 + \sigma^2\gamma^2 + \left(2\sigma\gamma - \gamma^2\right)r_A v}{2\left(1 + \gamma^2 + r_A v\right)} \tag{88}$$

to be compared with $\phi^{*\text{emp}}$ in the text.

Simplification:

$$(1 + r_A v) \left(1 + \gamma^2 - \sigma\gamma\right)^2 + \gamma^2 \left(\sigma\gamma + r_A v\right)^2$$

$$(1 + r_A v) \left(\left(1 + \gamma^2\right)^2 - 2\left(1 + \gamma^2\right)\sigma\gamma + \sigma^2\gamma^2\right) + \gamma^2 \left(\sigma^2\gamma^2 + 2\sigma\gamma r_A v + (r_A v)^2\right)$$

$$\sigma^2\gamma^2 \left(1 + \gamma^2 + r_A v\right) + (1 + r_A v)\left(1 + \gamma^2\right)\left(\left(1 + \gamma^2\right) - 2\sigma\gamma\right) + \gamma^2 r_A v \left(2\sigma\gamma + r_A v\right)$$

$$\sigma^2\gamma^2 \left(1 + \gamma^2 + r_A v\right) + \left(1 + \gamma^2 + r_A v\right)\left(1 + \gamma^2 - 2\sigma\gamma\right) + \gamma^2 r_A v \left(1 + \gamma^2 - 2\sigma\gamma\right) + \gamma^2 r_A v \left(2\sigma\gamma + r_A v\right)$$

$$\sigma^2\gamma^2 \left(1 + \gamma^2 + r_A v\right) + \left(1 + \gamma^2 + r_A v\right)\left(1 + \gamma^2 - 2\sigma\gamma\right) + \gamma^2 r_A v \left(1 + \gamma^2 + r_A v\right)$$

$$\left(1 + \gamma^2 + r_A v\right)\left(\sigma^2\gamma^2 + \left(1 + \gamma^2 - 2\sigma\gamma\right) + \gamma^2 r_A v\right)$$

Comparisons   We start by comparing equilibrium effort, a:

$$\Delta a = 1 + m^{\text{ind}} - m^{\text{emp}} = \frac{\gamma^2 - 2\sigma\gamma}{1 + \gamma^2 + r_A v}. \tag{89}$$

In this setting, thus, an independent agent has unambiguously stronger performance incentives within the bounds $|\sigma| \leq \gamma/2$.

The condition for an independent management to be optimal is that the following expression be non-negative:

$$\Delta^{\text{profit}} = \phi^{*\text{ind}} - \phi^{*\text{emp}},$$

$$\phi^{*\text{emp}} = \frac{1}{2}\frac{(1 + \sigma\gamma)^2}{1 + \gamma^2 + r_A v} \tag{90}$$

$$\phi^{*\text{ind}} = \frac{1 + \gamma^2 + \sigma^2\gamma^2 + \left(2\sigma\gamma - \gamma^2\right)r_A v}{2\left(1 + \gamma^2 + r_A v\right)} = \frac{(1 + \sigma\gamma)^2 + \gamma^2 - 2\sigma\gamma + \left(2\sigma\gamma - \gamma^2\right)r_A v}{2\left(1 + \gamma^2 + r_A v\right)} \tag{91}$$

$$\Delta^{\text{profit}} = \frac{(1 + \sigma\gamma)^2 + \gamma^2 - 2\sigma\gamma + \left(2\sigma\gamma - \gamma^2\right)r_A v - (1 + \sigma\gamma)^2}{2\left(1 + \gamma^2 + r_A v\right)}$$

The condition for an independent agent to be optimal is thus,

$$\gamma^2 - 2\sigma\gamma \geq \left(\gamma^2 - 2\sigma\gamma\right)r_A v,$$

i.e.

$$r_A v \leq 1.$$

The sign issue − when is $d < 0$ We have:

- loss function σd:

$$m^{\text{ind+}} = \frac{\sigma\gamma - r_A v}{1 + \gamma^2 + r_A v}; \tag{92}$$

- loss function $-\sigma d$:

$$m^{\text{ind-}} = \frac{-\sigma\gamma - r_A v}{1 + \gamma^2 + r_A v}; \tag{93}$$

- for the true loss function, $\sigma |d|$: we have three cases

1.

$$|\sigma\gamma| \leq r_A v \implies m^{\text{ind}} \leq 0 \implies \text{loss function } -\sigma d \text{ gives correct solution}$$

2.

$$\sigma\gamma > r_A v \implies \text{potential multiplicity due to benefits from manipulation of any sign}$$

The two canidate solutions generated by $m^{\text{ind}+}$ and $m^{\text{ind}-}$:

$$\phi^{*\text{ind}+} = \frac{1 + \gamma^2 + \sigma^2\gamma^2 - (2\sigma\gamma + \gamma^2)\, r_A v}{2\,(1 + \gamma^2 + r_A v)}; \tag{94}$$

$$\phi^{*\text{ind}-} = \frac{1 + \gamma^2 + \sigma^2\gamma^2 + (2\sigma\gamma - \gamma^2)\, r_A v}{2\,(1 + \gamma^2 + r_A v)}; \tag{95}$$

now,

$$\phi^{*\text{ind}-} - \phi^{*\text{ind}+} = \frac{1 + \gamma^2 + \sigma^2\gamma^2 + (2\sigma\gamma - \gamma^2)\, r_A v}{2\,(1 + \gamma^2 + r_A v)} - \frac{1 + \gamma^2 + \sigma^2\gamma^2 - (2\sigma\gamma + \gamma^2)\, r_A v}{2\,(1 + \gamma^2 + r_A v)}$$

$$\phi^{*\text{ind}-} - \phi^{*\text{ind}+} = \frac{4\sigma\gamma}{2\,(1 + \gamma^2 + r_A v)} > 0.$$

In conclusion, the loss function $-\sigma d$ applies here too.

3.

$$-\sigma\gamma > r_A v$$

Inserint the agent's response into the principal's problem gives a value function,

$$\phi(m) = \sigma\gamma\,|m| + 1 + m - \frac{1 + r_A v}{2}\,(1 + m)^2 - \frac{1}{2}\gamma^2 m^2$$

There are three possibilities for m:

- - * $m > 0$: since the objective function is well-behaved this implies that $|m| = m$ in ϕ and that
    $$m = \frac{\sigma\gamma - r_A v}{1 + \gamma^2 + r_A v};$$
    this is possible only if $\sigma\gamma > r_A v$, and thus inconsistent;
  - * $m < 0$: since the objective function is well-behaved this implies that $|m| = -m$ in ϕ and that
    $$m = \frac{-\sigma\gamma - r_A v}{1 + \gamma^2 + r_A v};$$
    this is possible only if $-\sigma\gamma < r_A v$, and thus consistent;
  - * $m = 0$: this is the only possibility.

We finally need to test $m = 0$:

$$\phi^{*\text{ind}0} = 1 - (1 + r_A v)/2 \tag{96}$$

and compare with

$$\phi^{*\text{emp}} = \frac{1}{2} \frac{(1 + \sigma\gamma)^2}{1 + \gamma^2 + r_A v}; \tag{97}$$

independence is optimal if

$$\phi^{*\text{ind}0} - \phi^{*\text{emp}} = \frac{(1 - r_A v)(1 + \gamma^2 + r_A v) - (1 + \sigma\gamma)^2}{2(1 + \gamma^2 + r_A v)} \geq 0.$$

Now, confining interest to the case where $|\sigma\gamma| \leq 1$ – which implies that $r_A v < 1$ since $-\sigma\gamma > r_A v$ – this is always satisfied:

$$(1 - r_A v)(1 + \gamma^2 + r_A v) - (1 + \sigma\gamma)^2 \geq (1 + \sigma\gamma)(1 + \gamma^2 + r_A v) - (1 + \sigma\gamma)^2 \geq$$

$$\geq (1 + \sigma\gamma)\left[(1 + \gamma^2 + r_A v) - (1 + \sigma\gamma)\right] > 0.$$

Summary of cases

- d with sign – no issues;

- $|d|$

    - $\sigma\gamma > r_A v$ a bit special but ultimately covered by analysis of the next case;

    - $|\sigma\gamma| < r_A v$ implies $m < 0$ and $r_A v = 1$ is uniformly critical;

    - $\sigma\gamma < -r_A v$, as long as $|\sigma\gamma| \leq 1$, independence is always optimal.