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Engseld, Peter

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PO Box 117
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+46 46-222 00 00

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Conditioned Actions in Strategic Coordination Games

Peter Engseld*

May 30, 2005

Abstract

A simple symmetric 2×2 strategic coordination game is analyzed in an evolutionary environment under the assumption that agents are able to condition their actions on observations made of the opponent. Agents are assumed to be associated with a profile of characteristics, of which all agents can make a noisy observation. Actions can be conditioned on how the observed characteristics relates to that of their own. It is shown that there exist feasible states under which evolutionary pressure will transform any population conditioning its actions through a genetically induced continuous characteristic, such as body length, into a population conditioning its actions through Status, or how well agents have done in previous games. It is also shown that there does not exist feasible states by which a population conditioning its actions through Status could be invaded by any other strategies.

JEL classification: C70; C72

Keywords: Coordination; Hawk-Dove Games; Status; Positional Concerns; Conditioned Strategies; Evolutionary Equilibrium

1 Background

Two major approaches can be identified in the literature of theoretical models where agents are assumed to have concerns for exogenous variables; one assumes that agents have *direct* preferences for the variable, and the other argues that the concerns for the variables are *instrumental.* The former approach has been heavily criticized for being ambiguous in character, since these models can explain every imaginable action *ex post* just by letting the agents have "fitting" preferences, and thus not being able to explain anything *ex ante.*[1]

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*Dept of Economics, Lund University, P.O. Box 7082, 220 07 Lund, Sweden. Email: peter.engseld@nek.lu.se, Tel. +46-46-222 79 13, Fax: +46-46-222 46 13.

[1] For a more in-depth discussion, see Postlewaite (1998).

In the latter approach, it is often assumed that an exogenous variable is associated with some desirable ability of the individual, and that signaling this variable comes cheaper for individuals with this specific ability. Spence (1974) seminal work on separable equilibrium was the first to highlight this approach. He found that individuals with a higher productivity were willing to "burn money" by overspending on education in order to signal their abilities. Later, Frank (1985) showed that people overspend relatively in positional goods compared with non-positional goods in order to "Keep up with the Joneses". In the same manner, Cole et al. (1992) showed that agents tend to oversave in a "Rat race of the rich". Rege (2002) shows that positional goods such as Rolex watches and Armani suits can serve as signals of non-observable abilities in complementary interactions.

However, all the models above depend on that each individual's ability is observable through some proxy variable (signal) which is highly correlated with the individual's ability. In other words, it is assumed that relative position, e.g. high education or high status, is a good proxy for different desirable abilities. Consequently, given that there exists a separating equilibrium, preferences for relative position will emerge indirectly by this assumption.

The question we try to answer in this paper is whether there exists an inherent property, that in some context benefits high relative position, without assuming any correlation between a high relative position and ability. In this paper we introduce a third approach on how to model exogenous variables. Instead of using matching games, we analyze a generalized Hawk-Dove game under the assumption that agents can observe personal characteristics associated with each opponent, which in turn enables them to condition their actions on how their own personal characteristics relates to that of their opponent.

This class of models was partly analyzed by Maynard Smith (1982), where he shows that when agents condition their actions on the role of the opponent in a symmetric Hawk-Dove game with asymmetric agents, two evolutionarily stable strategies (*ESS*) emerges, which in theory are equally plausible. However, the original analysis of this game was made under the crucial assumption that the probability of an agent occupying a particular role is independent of its strategy, i.e. the case of status, or how well agents have done in previous games, was left unanalyzed. In the following paper we relax this assumption and analyze a generalized Hawk-Dove game where the agents are free to condition their actions on personal characteristics including strategies where the probability of an agent occupying a particular role is dependent of its strategy.

2 Outline of the Model

Consider a simple two-person symmetric 2×2 game $\Gamma(\alpha, \beta)$ as described in Figure 1. For $\alpha \in (0,1)$ and $\beta \in (0,1)$ we have a general class of games that includes both *Hawk-Dove games* ($\alpha > \beta$) and *Battle of the sexes games* ($\alpha < \beta$).

	H	D
H	0 0	β 1
D	1 β	α α

Figure 1: $\Gamma(\alpha, \beta)$

In a typical rational setting, the game $\Gamma(\alpha, \beta)$ reveals three Nash equilibria (*NE*). The pure *NE* implies that the row agent plays Hawk (*H*) and column agent plays Dove (*D*), or vice versa. This clearly suggests a coordination problem, since both agents would prefer the outcome where they play *H*, and the opponent play *D*. The third equilibrium, a mixed *NE*, implies that the agents randomize their actions such that they play H with probability $\frac{1-\alpha}{1+\beta-\alpha}$ and D with probability $\frac{\beta}{1+\beta-\alpha}$.

Let us now continue by viewing the game from an evolutionary perspective, where all the agents in a population are repeatedly matched to play $\Gamma(\alpha, \beta)$. It is easy to verify that we have an unique evolutionary stable equilibrium (*ESS*) if $\frac{1-\alpha}{1+\beta-\alpha}$ of the population plays H, and the remaining $\frac{\beta}{1+\beta-\alpha}$ of the population plays D. Note that the *ESS*, just as the mixed *NE*, results in a probability for coordination failure, i.e. not in a pure *NE*, that is $\left(\frac{\beta}{1+\beta-\alpha}\right)^2 + \left(\frac{1-\alpha}{1+\beta-\alpha}\right)^2 \geq \frac{1}{2}$. In other words, the probability for coordination failure equals or exceeds the probability for a successful coordination. This result raises the question whether there exist a feasible method ensuring that all matchings in the population results in a pure *NE*.

Assume that the agents have the ability to observe each other perfectly. The coordination problem could then, as Maynard Smith observed, be solved perfectly if all the agents condition their actions on e.g. how the body length of an opponent relates to that of the agent's own body length. One possible coordination rule where all matches results in pure *NE* can be found when the entire population plays D if their opponent is taller, and plays H if their opponent is shorter. This clearly implies that all matches in the population will become perfectly coordinated. However, the mirrored rule, where the agents play H if their opponent is taller and D if their opponent is shorter, would do just as well. The agents could of course instead use a different personal characteristics to condition their actions through, such as ability to run fast, intelligence, or body weight.

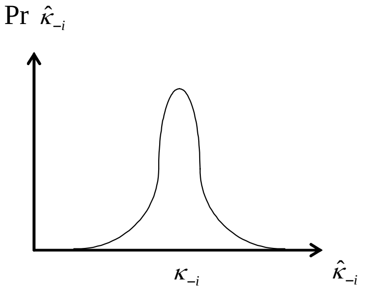
The obvious question is whether all conditioning rules are equally plausible in an evolutionary setting.

Regardless of which characteristic a population utilize to condition actions through, every feasible conditioning rule results in players who has been benefitted from the rule in previous periods, will continue to be benefitted in the coming periods as long as the population utilize an identical rule each period. In other words, all time consistent conditioning rules implies that agents who has done well in the past, will continue to do so. Consequently, a population conditioning through any time consistent rule will behave *as if* the agents were conditioning their actions on how well an agent has done in pervious games.

The main question we address in this paper is whether it is possible for a strategy conditioning actions on how well an agent has done in previous games to make a complete invasion of a population that is conditioning its actions by another time-consistent strategy.

3 Model

Imagine a continuum of agents in a population I, where each agent $i \in I$ is associated with a profile of characteristics $\Omega \equiv \left(\kappa^1, \kappa^2, \kappa^3, ...\right)$, such that none of the characteristics in the profile are perfectly correlated with another characteristics. Let κ denote an arbitrary characteristic in the characteristic profile Ω. Every characteristics $\kappa \in \Omega$ is normalized such that the range κ is $[\kappa^-, \kappa^+] \equiv \boldsymbol{\kappa}$. Let f_κ denote how the values of a characteristic κ are distributed over $\boldsymbol{\kappa}$. Each agent $i \in I$ is able to observe the value of each characteristics κ_i in their profile of characteristics Ω_i perfectly, but is restricted to make an imperfect observation of an opponents characteristics $\kappa_{-i} \in \Omega_{-i}$. Let the observed value of the characteristic $\widehat{\kappa}_{-i}$ be identically, symmetrically and unimodally distributed around the actual value $\kappa_{-i}, \forall i \in I$. Note that we hereby assume the prerequisites to observe characteristic accurately being equal for all characteristics.



The probability distribution of a noisy observation of κ_{-i}.

As a measure of how accurate an agent i can observe an opponent's characteristics, henceforth called the observational skill and denoted O_i, we use the inverse standard deviation.

Let the entire population play the game $\Gamma(\alpha, \beta)$ depicted in Figure 1. The action set is given by $\mathbf{A} = \{H, D\}$, where the actions are taken in discrete time, $t \in T = \{0, 1, ...\}$.

At the beginning of each period t are α and β randomly drawn from an interval $(0, 1)$ by two independent symmetric distributions. Each agent $i \in I$ is then randomly matched up with an opponent $-i$. Agent i then makes a perfect observation of her own characteristics κ_i and a noisy observation of the opponents characteristics $\widehat{\kappa}_{-i}$, where after an action $a_{i,-i} \in \mathbf{A}$ is taken. Let π_i^t denote agent i's expected payoff at each period t.[2]

If the agents are able to observe the characteristics, then they are also able to form strategies where they condition actions on how an opponents characteristics relates to their own. In this paper we restrict the analysis to the case when actions are conditioned on whether the opponents value of characteristic is higher or lower than that of the agent.

Consequently, an agent i are able to condition its actions through a characteristic κ in the following manner:

$$\text{"More is better"}: \begin{cases} a_{i,-i} = D & \mathit{if} \quad \kappa_i < \widehat{\kappa}_{-i} \\ a_{i,-i} = H & \mathit{if} \quad \kappa_i > \widehat{\kappa}_{-i} \end{cases} \tag{1}$$

The agent is also free to use the mirrored strategy, and condition its actions through κ as follows.

$$\text{"Less is better"}: \begin{cases} a_{i,-i} = H & \mathit{if} \quad \kappa_i^n < \widehat{\kappa}_{-i}^n \\ a_{i,-i} = D & \mathit{if} \quad \kappa_i^n > \widehat{\kappa}_{-i}^n \end{cases} \tag{2}$$

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[2] An infinite number of random matches within period t is one example of a matching procedure that generates π_i^t (see Kandori et al. (1993)).

3.1 Two Classes of Characteristics

Let us simplify the analysis by focusing on two characteristics, b and w, where b denotes a representative of an arbitrary *genetically endowed characteristic* and w denotes a representative of a *role-strategy dependent characteristic.*[3]

The genetically endowed characteristic have a distribution f_b where the characteristics b are *continuously*, *symmetrically* and *unimodally* distributed in a bell-shaped manner around a mean $\overline{b}$. We can consequently, without any loss of generality, focus on one strategy when agents condition their actions through b according to expression 1. Let this strategy be denoted $s_i^{\mathcal{B}}\left(b_i, \widehat{b}_{-i}\right)$, where

$$s_i^{\mathcal{B}}\left(b_i, \widehat{b}_{-i} \mid b_i < \widehat{b}_{-i}\right) = D, \text{ and } s_i^{\mathcal{B}}\left(b_i, \widehat{b}_{-i} \mid b_i > \widehat{b}_{-i}\right) = H.$$

The class of role-strategy dependent characteristics are modeled through *status.*

Definition 1 (*Status*) *The status of agent i at t is given by $w_i^t \equiv \rho\pi_i^{t-1} + (1-\rho)\, w_i^{t-1}$ where π_i^{t-1} is the payoff in the previous period and $0 < \rho < 1$.*

The distribution of status is of course solely dependent on the context. Intuitively, if the agents in a population were to utilize the strategy in expression 1 to condition actions by, this would result in an *expansion* in the distribution of status since this strategy favors agents that have been successful in previous games. If the agents instead where to use the mirrored strategy in expression 2, this would imply a *contraction* of the status distribution, because a less successful past is favored by this strategy. Since a contraction of the distribution clearly would lead to a higher degree of coordination failures, let us focus on the *Villain strategy* or $s^{\mathcal{V}}$ when agents condition actions through w as in expression 1.[4] Consequently, the *Villain strategy* takes actions such that

$$s_i^{\mathcal{V}}\left(w_i, \widehat{w}_{-i} \mid w_i < \widehat{w}_{-i}\right) = D \text{ and } s_i^{\mathcal{V}}\left(w_i, \widehat{w}_{-i} \mid w_i > \widehat{w}_{-i}\right) = H.$$

---

[3] Note that almost all personal characteristics can be considered as genetically endowed.

[4] For a more in-depth analysis, see Engseld (2003).

Let the mix of strategies present in a population I be given by Q_I and let $Q_{I:\mathcal{V}}, Q_{I:\mathcal{B}} \in [0,1]$ denote the fraction of individuals in the population I with strategy $s^{\mathcal{V}}$ respectively $s^{\mathcal{B}}$. We can characterize any mix of strategies as:

$$\Delta_Q \equiv \left\{ Q_I \in \mathbb{R}_+^2 \mid Q_{I:\mathcal{V}} + Q_{I:\mathcal{B}} = 1 \right\}.$$

3.2 Evolutionary Dynamic

As customary in evolutionary models, let each agent in the population in every period have equal probability to mutate. In this model both strategies and observational skills are subjected to mutations. Consequently, an agent has a very small probability in each period to change from $s^{\mathcal{B}}$ to $s^{\mathcal{V}}$, and vice versa. We assume that mutations in observational skill are just as likely to result in an improvment as a deterioration of the observational skill. Moreover:

- *Strategies*, *characteristics*, and *observational skill* associated with a higher payoff have more offsprings than those with a lower payoff. Let $g^t(s_i, b_i, \cdot)$ denote the growth at t for agent i endowed with b and playing s.

$$\text{sign}\left(\pi^t(s_i, b_i, O_i) - \pi^t(s_j, b_j, O_j)\right) = \text{sign}\left(g^t(s_i, b_i, O_i) - g^t(s_j, b_j, O_j)\right) \quad (3)$$

  The evolutionary pressure, i.e. the relationship between payoff and growth, is assumed to fluctuate over time, but always according to expression **??**, such that growth is weakly increasing in payoff. More specifically, every evolutionary pressure in accordance with expression **??** is assumed to have a positive probability to occur.

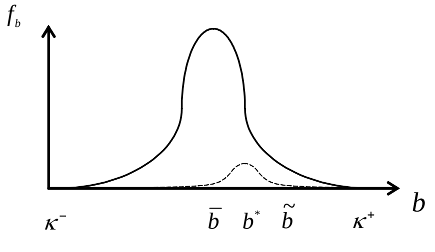
- *Strategies* are *identically* replicated into the next generation. This can be seen as if the offspring inherits the older generation's strategy by replicating the behavior of the parent or that it is genetically hardwired.

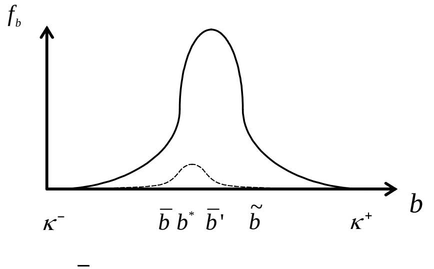
- *Role-strategy dependent characteristic* or *Status* is *identically* inherited by the next generation, which can be interpreted as if the parent in a transition stage acts on the behalf of the offspring, or the offspring by association inherits the status.

- *Genetically endowed characteristics* are distributed to the next generation through *mean regression.* The characteristics of newborns from each agent are continuously distributed with a full support on $\boldsymbol{\kappa}$, such that each agent will have offsprings with every feasible value of the characteristics. More specifically, the distribution of characteristics is unimodally dispersed around some b^* located between the mean $\overline{b}$, and the value of the parents characteristic $\widetilde{b}$, where $b^* \in \left[\overline{b}, \widetilde{b}\right)$ if $\widetilde{b} > \overline{b}$ and $b^* \in \left(\widetilde{b}, \overline{b}\right]$ if $\widetilde{b} < \overline{b}$. (see left hand figure below)
- *Observational skills* are distributed to the next generation through *mean regression.*

If a genetically endowed characteristic is utilized in a population for conditioning actions, we should expect the distribution to be transformed or at least translated such that the mean $\overline{b}$ increases from $\overline{b}$ to $\overline{b'}$. As an example, if it is favorable to be tall, then we should expect the average height in the population become taller as time goes by. Similar, if nature imposes a harsh evolutionary pressure, we should also expect an increase in $\overline{b}$.

However, any increase in $\overline{b}$ caused by evolutionary forces should be bounded, if nothing else due to physical limitations. That is, the mean of the distribution might change from $\overline{b}$ to $\overline{b'}$, whereas the charateristics are still distributed according to the old mean $\overline{b}$. (see right hand figure below)





In order to avoid confussion, let henceforth $\overline{b}$ denote the mean of the distribution f_b. We assume the evolutionary pressure to be constant over longer periods, such that the characteristics are *continuously*, *symmetrically* and *unimodally* distributed and f_b has a constant mean $\overline{b}$.

Property 1 *Every population conditioning its actions through some* $\kappa \in \mathbf{\Omega}$*, also have a corresponding distribution in status, denoted* $w(b)$*, which satisfies* $\sup w(b) - \inf w(b) = \kappa^{+} - \kappa^{-}$*.*

Property 1 is a direct result of definition 1 and the assumption that the prerequisites to observe characteristic are being equal for all characteristics. Let the distribution of Status through b be denoted $f_{w(b)}$.

Finally, assume that the adjustment process of the distribution is much faster than the growth process, which in turn is much faster than the process of mutations.

3.3 Evolutionary Stability

The rationale behind evolutionary stability, such as *ESS* and *NSS*, is based on the assumption that agents with a strategies yielding higher expected payoff are more frequent in the population next period. The implementation of this idea is not as straightforward in models with conditioned strategies as in standard evolutionary models with unconditioned strategies, since identical strategies can yield different payoffs dependent on the values of the characteristics. For example, one can envision an agent with a mutant strategy s' earning a higher payoff than some agents with strategy s and at the same time earn a lower payoff than other agents also using s, all due to the value of characteristic. Hence, in order to evaluate the model we need a modified criteria for evolutionary stability. Let us henceforth simplify the notation and exclude subscript and superscript when there is no risk for confusion.

Definition 2 $\overrightarrow{QQ'}$ *denotes a path connected curve in* Δ_Q *between* Q *and* Q'*, where* Q *can evolve to* Q'

- *through growth, or*
- *through mutations such that* $\exists s_i' \neq s_i$ *where* $\pi_i\left(s_i; Q_{I\setminus i}, \cdot\right) \leq \pi_i\left(s_i'; Q_{I\setminus i}, \cdot\right)$ *for some* $i \in I$*.*

Let us now define a *Mutation Proof Attraction Set* (*MAS*), which basically is a modified *Absorbing set* (see e.g. Samuelson (1998)), where the set is closed under *both* the growth mechanism and mutations.

Definition 3 (*MAS*) $\mathbf{Q}^{MAS}(\Gamma)$ *is a set of strategy mixes* $Q \in \mathbf{Q}^{MAS}(\Gamma)$ *such that*

- $\exists\ \overrightarrow{QQ'},\ \forall Q' \in \mathbf{Q}^{MAS}(\Gamma)$,
- $\exists\ \overrightarrow{Q'Q},\ \forall Q' \in \mathbf{Q}^{MAS}(\Gamma)$, *and*
- $\nexists\ \overrightarrow{QQ''}$ *for any* $Q'' \notin \mathbf{Q}^{MAS}(\Gamma)$*.*

Let $\Delta^{MAS}(\Gamma) \equiv \bigcup \mathbf{Q}^{MAS}(\Gamma)$*.*

Property 2 $\Delta^{MAS}(\Gamma) \neq \emptyset, \forall \Gamma$*.*

A population I belongs to a *MAS*, if the corresponding strategy mix Q belongs to an attraction set $\mathbf{Q}^{MAS}(\Gamma)$ where $\exists\ \overrightarrow{QQ'}$, and $\exists\ \overrightarrow{Q'Q},\ \forall Q' \in \mathbf{Q}^{MAS}(\Gamma)$. That is, each combination of strategies in the population that belongs to the attraction set $\mathbf{Q}^{MAS}(\Gamma)$ must be able to evolve to any other point in the attraction set through growth and/or mutations yielding at least as high payoff, i.e. neutral invasion. Moreover, there must not exist any path such that the population could evolve to a point $Q'' \notin \mathbf{Q}^{MAS}(\Gamma)$. Note that *MAS* yields identical equilibria on unconditioned strategies as *NSS* (see Maynard Smith (1982)).

4 Evaluating the Model

First consider $Q^{\mathcal{V}} \equiv \{Q \mid Q_{I:\mathcal{V}} = 1\}$, i.e. a homogenous strategy mix that solely consists of agents utilizing the *Villain strategy.*

Proposition 1 $Q^{\mathcal{V}} \in \Delta^{MAS}(\Gamma)$.

A population that condition actions through Status is a *MAS* and can consequently never be invaded.

Let us now instead consider $Q^{\mathcal{B}} \equiv \{Q \mid Q_{I:\mathcal{B}} = 1\}$ and focus on the agents observational skills. The assumption about agents ability to make imperfect observations of opponents characteristics is crucial for the model. So far we have not yet stated anything about how this ability might emerge or how well the agents can observe each other.

Lemma 1 *If* f_b *is symmetrically and continuously distributed over* $[\kappa^-, \kappa^+]$, *then* $\exists \delta > 0$ *such that*

$$\frac{\partial \pi_i\left(s^{\mathcal{B}}, b\right)}{\partial O_i} > 0, \forall b \in \left[\kappa^-, \overline{b} + \delta\right] : 1 - \alpha < \beta,$$
$$\frac{\partial \pi_i\left(s^{\mathcal{B}}, b\right)}{\partial O_i} > 0, \forall b \in \left[\overline{b} - \delta, \kappa^+\right] : 1 - \alpha > \beta.$$

Consequently, more than half of the population will always, regardless of what game is realized, receive a higher payoff if the observational skill is improved given the assumptions on f_b.

Corollary 1 $Q^{\mathcal{B}} \Rightarrow \frac{\partial \pi_i\left(s^{\mathcal{B}}, b\right)}{\partial O_i} > 0, \forall b \in \left[\overline{b} - \delta, \overline{b} + \delta\right]$, *and* $\forall \alpha, \beta \in (0, 1)$.

That is, there exist a interval around $\overline{b}$ where the payoff, regardless of what game is realized, is strictly increasing in observational skill.

Proposition 2 $Q^{\mathcal{B}} \Rightarrow \lim_{t \to \infty} O_i = \infty, \forall i \in I$.

Evolutionary pressure will consequently favor better observational skills and render agents will poor observational skills extinct. As a consequence whereof, we analyze the model when $O_i = O, \forall i \in I$ and when the characteristics are close to perfectly observable, i.e. $O \approx \infty$.

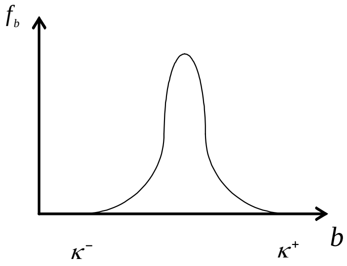
Let us now consider the case when an agent in $Q^{\mathcal{B}}$ change strategy from $s^{\mathcal{B}}$ to $s^{\mathcal{V}}$. From property 1 we know that agents in $Q^{\mathcal{B}}$ will behave as if they were conditioning their actions through status.

Proposition 3 *$\exists \boldsymbol{\kappa}_* \subseteq \boldsymbol{\kappa}$ in $Q^{\mathcal{B}}$ where $\pi\left(s^{\mathcal{V}}, w\left(b\right)\right) > \pi\left(s^{\mathcal{B}}, b\right), \forall b \in \boldsymbol{\kappa}_*$.*

Let $Q^c \equiv \{Q \mid Q_{I:\mathcal{V}} > 0, Q_{I:\mathcal{B}} > 0\}$.

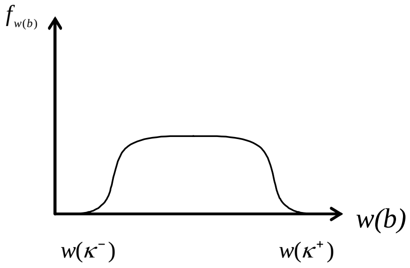
Corollary 2 *$\exists\ \overrightarrow{Q^{\mathcal{B}}Q'}$, where $Q' \in Q^c$.*

The intuition behind Proposition 3 is fairly straightforward; somewhere in f_b is the distribution of b dense enough, such that that a small change in b corresponds to a larger change in payoff, and thus also status $w\left(b\right)$, which in turn implies that conditioning actions through status implies less coordination failures than through the characteristic itself. Consequently, agents endowed with $b \in \boldsymbol{\kappa}_*$ playing $s^{\mathcal{V}}$ yields a higher payoff than playing $s^{\mathcal{B}}$.



The distribution of b

in the population



The corresponding

distribution of $w\left(b\right)$.

However, Proposition 3 and Corollary 2 is not enough to rule out that invasions of $Q^{\mathcal{B}}$ by $s^{\mathcal{V}}$ could be limited, such that there exist strategy mixes $Q^c \in \Delta^{MAS}\left(\Gamma\right)$. The question whether $s^{\mathcal{V}}$ is capable of making a complete invasion of $Q^{\mathcal{B}}$, i.e. $\exists\ \overrightarrow{Q^{\mathcal{B}}Q^{\mathcal{V}}}$, depends ultimately on the evolutionary pressure. Corollary 2 only states that there always exists an interval $\boldsymbol{\kappa}_* \subseteq \boldsymbol{\kappa}$ where $\pi\left(s^{\mathcal{V}}, w\left(b\right)\right) \geq \pi\left(s^{\mathcal{B}}, b\right), \forall b \in \boldsymbol{\kappa}_*$, this does not imply that the growth *in* $b \in \boldsymbol{\kappa}_*$ is higher by $s^{\mathcal{V}}$ than $s^{\mathcal{B}}$, but just that the growth *at* $b \in \boldsymbol{\kappa}_*$ is higher by $s^{\mathcal{V}}$ than $s^{\mathcal{B}}$. To see this, consider Q^c under an extreme harsh evolutionary pressure resulting in that the entire growth in the population stems from the agents endowed with b close to κ^+. For a typically bell-shaped distribution of characteristics, as depicted above, this implies that the entire growth stems from $\kappa \notin \boldsymbol{\kappa}_*$, where $\pi\left(s^{\mathcal{V}}, w\left(b\right)\right) < \pi\left(s^{\mathcal{B}}, b\right)$.

Definition 4 $\boldsymbol{\kappa}_{**} \subseteq \boldsymbol{\kappa}_{*}$ *such that* $g\left(s^{\mathcal{V}}, w\left(b\right)\right) \geq g\left(s^{\mathcal{B}}, b\right), \forall b \in \boldsymbol{\kappa}_{**}$.

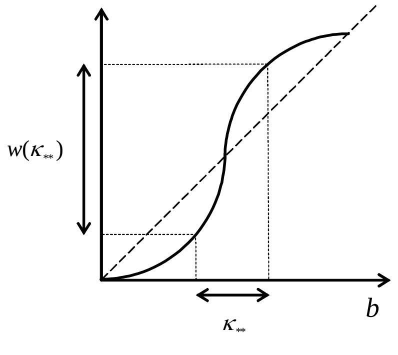
That is, $\boldsymbol{\kappa}_{**}$ is an interval where a higher payoff also implies a higher growth for $s^{\mathcal{V}}$. Let $w\left(\boldsymbol{\kappa}_{**}\right) \equiv \left\{w\left(b\right) \mid b \in \boldsymbol{\kappa}_{**}\right\}$.

Now consider an interval $\boldsymbol{\kappa}_{***} \subset \boldsymbol{\kappa}_{**}$. Analogous to above, let $w\left(\boldsymbol{\kappa}_{***}\right) \equiv \left\{w\left(b\right) \mid b \in \boldsymbol{\kappa}_{***}\right\}$. Let $Q_{w(\boldsymbol{\kappa}_{***})} \subset Q_{I}$ be the subset of the population that contain agent with status $w\left(b\right) \in w\left(\boldsymbol{\kappa}_{***}\right)$. Then let $Q^{\mathcal{V}}_{w(\boldsymbol{\kappa}_{***})}$ denotes the case when every agent in $Q_{w(\boldsymbol{\kappa}_{***})}$ utilize $s^{\mathcal{V}}$, i.e. $Q^{\mathcal{V}}_{w(\boldsymbol{\kappa}_{***})} \equiv \left\{Q_{w(\boldsymbol{\kappa}_{***})} \mid Q_{w(\boldsymbol{\kappa}_{***}):\mathcal{V}} = 1\right\}$.

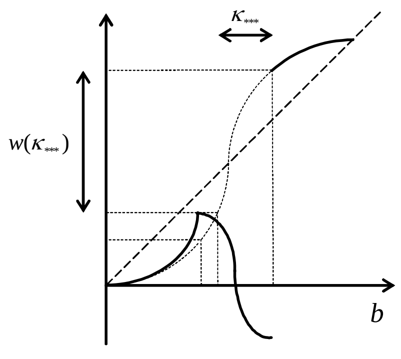
Lemma 2 $\boldsymbol{\kappa}_{**} \neq \emptyset \Rightarrow \exists\ \overrightarrow{Q^{\mathcal{B}}Q^{\tilde{c}}}$ *where* $Q^{\mathcal{V}}_{w(\boldsymbol{\kappa}_{***})} \subset Q^{\tilde{c}} \in Q^{c}$.

The driving force behind Lemma 2 is that $s^{\mathcal{V}}$ with $b \in \boldsymbol{\kappa}_{**}$ will both have a higher payoff and grow faster than $s^{\mathcal{B}}$. As $Q_{I:\mathcal{V}}$ grows, the difference in status between $s^{\mathcal{V}}$ and $s^{\mathcal{B}}$ with $b \in \boldsymbol{\kappa}_{**}$ will grow larger. An increased difference in status will also lead to an increase in coordination failures between $s^{\mathcal{B}}$ and $s^{\mathcal{V}}$ in $\boldsymbol{\kappa}_{**}$, which eventually results in that $s^{\mathcal{B}}$ with $b \in \boldsymbol{\kappa}_{***}$ will be pushed down and out of $w\left(\boldsymbol{\kappa}_{***}\right)$ as depicted in the right hand figure below.

Note that even though $g\left(s^{\mathcal{V}}, w\left(b\right)\right) \geq g\left(s^{\mathcal{B}}, b\right), \forall b \in \boldsymbol{\kappa}_{**}$, the status for $s^{\mathcal{B}}$ with b around $\inf \boldsymbol{\kappa}_{**}$ will increase, which is due to a higher success rate in coordinating with $s^{\mathcal{B}}$ with $w\left(b\right) < w\left(\boldsymbol{\kappa}_{***}\right)$ for $s^{\mathcal{B}}$ than $s^{\mathcal{V}}$.



Status distribution for $Q^{\mathcal{B}}$.



The change in status distribution in $Q_{I:\mathcal{B}}$ resulted by the invasion of $s^{\mathcal{V}}$ in $w\left(\boldsymbol{\kappa}_{***}\right)$.

Note that the existence of $\boldsymbol{\kappa}_{**} \neq \emptyset$ is guaranteed by the assumption of fluctuating evolutionary pressure. Consider for example $g\left(b\right) << g\left(\boldsymbol{\kappa}_{**}\right), \forall b <$

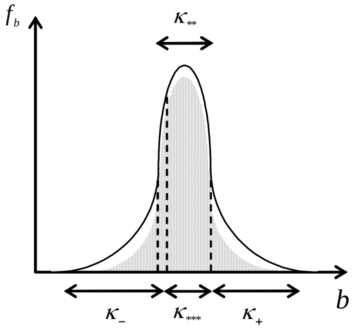
inf $\boldsymbol{\kappa}_{**}$ and $g(\boldsymbol{\kappa}_{**}) \lessapprox g(b), \forall b > \sup \boldsymbol{\kappa}_{**}$, where $\pi(b,\cdot) < \pi(b',\cdot) \Rightarrow g(b) < g(b'), \forall b, b' \in \boldsymbol{\kappa}_{**}$ and $\pi(b,\cdot) < \pi(b',\cdot) \Rightarrow g(b) = g(b'), \forall b, b' > \sup \boldsymbol{\kappa}_{**}$. Moreover, if $\alpha \gtrsim \beta$ and $1-\alpha$ is sufficiently large where growth rate is low for $b < \inf \boldsymbol{\kappa}_{***}$, then will the increase in status for $s^{\mathcal{B}}$ with b around inf $\boldsymbol{\kappa}_{**}$ be limited and thereby guarantee the existence of $\boldsymbol{\kappa}_{***} \neq \emptyset$.

Consequently, it is possible for $s^{\mathcal{V}}$ to make a complete invasion of $w(\boldsymbol{\kappa}_{***})$ in finite time frame. Hence, it follows directly that:

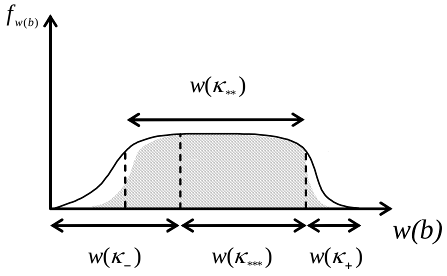
Corollary 3 $\exists$ $\overrightarrow{Q^{\mathcal{B}}Q^{\tilde{c}}}$ *where* $Q^{\mathcal{V}}_{w(\boldsymbol{\kappa}_{***})} \subset Q^{\tilde{c}} \in Q^{c}$.

Now consider the resulting distribution of $Q^{\tilde{c}}$ by b and $w(b)$, as qualitatively depicted down below, where the shaded areas represents $Q_{I:\mathcal{V}}$.

Let $\boldsymbol{\kappa}_{-} \equiv \{b < \inf \boldsymbol{\kappa}_{***}\}$ and $\boldsymbol{\kappa}_{+} \equiv \{b > \sup \boldsymbol{\kappa}_{***}\}$. Analogously, let $w(\boldsymbol{\kappa}_{-}) \equiv \{w(b) < \inf w(\boldsymbol{\kappa}_{***})\}$ and $w(\boldsymbol{\kappa}_{+}) \equiv \{w(b) > \sup w(\boldsymbol{\kappa}_{***})\}$.



Distribution of characteristics by $Q^{\tilde{c}}$.



The corresponding distribution of status by $Q^{\tilde{c}}$.

Let each agent be represented by a triplet indicating: strategy, endowed characteristics, and status level where the agent was born. As an example, $\left(s_i^{\mathcal{V}}, \boldsymbol{\kappa}_{-} : w(\boldsymbol{\kappa}_{+})\right)$ denotes an agent playing strategy $s_i^{\mathcal{V}}$ endowed with $b_i \in \boldsymbol{\kappa}_{-}$ and born in $w(\boldsymbol{\kappa}_{+})$.

Property 3 $Q^{\tilde{c}}$ *and* $\alpha > \beta \Rightarrow \left(s_i^{\mathcal{V}}, \mathbf{x} : w(\boldsymbol{\kappa})\right) \rightsquigarrow w_i(\mathbf{x}), \forall \mathbf{x} \in \{\boldsymbol{\kappa}_{-}, \boldsymbol{\kappa}_{***}, \boldsymbol{\kappa}_{+}\}$.

That is, every agent i playing $s_i^{\mathcal{V}}$ and endowed with e.g. a characteristic $b_i \in \boldsymbol{\kappa}_-$ will converge to the status interval $w_i(\boldsymbol{\kappa}_-)$, regardless of the initial status level.

Property 4 *$Q^{\widetilde{c}}$ and $\alpha > \beta \Rightarrow \left(s_i^{\mathcal{B}}, \mathbf{x} : w(\boldsymbol{\kappa})\right) \rightsquigarrow w_i(\boldsymbol{\kappa}_-), \forall \mathbf{x} \in \{\boldsymbol{\kappa}_-, \boldsymbol{\kappa}_{***}\}$, and $\left(s_i^{\mathcal{B}}, \boldsymbol{\kappa}_+ : w(\boldsymbol{\kappa}_+)\right) \rightsquigarrow w_i(\boldsymbol{\kappa}_+)$.*

Proposition 4 *$\boldsymbol{\kappa}_{**} \neq \emptyset \Rightarrow \exists\ \overrightarrow{Q^{\widetilde{c}} Q^{\mathcal{V}}}$.*

4.1 Summary

There exist feasible states of the world which entail that evolutionary pressure in a finite time frame could transform any population utlizing a time-consistent strategy that condition actions through a genetically endowed characteristic that is continuously distributed into a population conditioning its actions by the *Villain strategy.* Moreover, from Proposition 1 we know that there does not exist any feasible circumstances that could facilitate a successful invasion of $Q^{\mathcal{V}}$.

The question whether there might exist heterogenous strategy mixes with many different strategies that could be a *MAS* will for now be left unanswered. However, unless the strategies are arbitrarily closely distributed in f_w, a sufficiently harsh evolutionary pressure should be able to transform any heterogenous strategy mix to a homogenous strategy mix. Also note that two interesting genetically endowed characteristics, gender and age, remain unanalyzed. Whereas the former most likely could be incorporated in an extended model, the latter is much more problematic. How could we for example argue that old age would become more frequent in a population favouring old age?

5 Concluding Remarks

> “Beggars cannot be choosers about the source of their signal, or about its attractiveness compared with others that they can only wish were as

conspicuous." Schelling (1960)

The main purpose of this paper has been to argue the importance of status as a mean of coordination. Above it is demonstrated that the *Villain strategy*, under our assumptions, will prevail over any strategy conditioning actions through a continuously distributed time-independent genetically endowed characteristic.

While these finding by themselves might be interesting, the consequences hereof should be regarded as far more intriguing.

First notice that in a setting where status is partially private information, it should be in the agents interest to signal high status, through for example consumption, to ensure that their status is visible for every feasible opponent in the population. This is consistent with the ideas of Veblen (1899) who argued that the primary purpose for consumption was to signal one's rank in the society. Since Veblen never provided any profound behavioral reason for why agents should show rank, this model could, along with the different variations of Spence's model, serve as a rationale for such behavior.

Secondly, and perhaps even of greater significance, the results suggests an interdenpendency between economic situations. Assuming that agents utilize the *Villain strategy* in strategic coordination games as described in Figure 1 and that they observe Status through the cumulative payoff, this will induce concerns for status in *other* economic activities besides $\Gamma(\alpha, \beta)$. Consider for example the ultimatum game, the agents will here maximize their own payoff relative the opponent and the rest of the population. If an opponent receives a relatively high payoff will this affect the opponents status in a for the agent unfavorable way, since this results that the agent will earn a relatively lower payoff if they are matched up in a future coordination game. Consequently, it can never be evolutionary stable for agents as responder to accept offers close to zero.

The concern for high status will also enter other economic situations, seemingly without any strategic components, such as lottery and insurance. In a lottery, it would in the long run be favorable to consider the status effect from the different outcomes of the lottery, since it would have an effect on the agents' outcome in a possible coordination game later.

In almost every economic analysis of the agents decision-making, there is often made an implicit assumption about independence between the economic situations. This study suggest the contrary; under our assumptions, we show how concern for high status evolves endogenously as a mean of coordination and thereby bring about interdependence between economic events.

A Appendix

Proof of Proposition 1. Since $Q^{\mathcal{V}}$ is a point and not a set in Δ_Q, it is sufficient to show $\nexists\ \overrightarrow{Q^{\mathcal{V}}Q'}$ for any $Q' \neq Q^{\mathcal{V}}$. From the assumption that no $\kappa \in \Omega$ are perfectly correlated, it follows directly that f_w will not be perfectly correlated to any f_κ. Thus any mutant strategy conditioning actions through a $\kappa \neq w$ will have a higher degree of coordination failures and thus also yield a lower payoff than the Villain strategy. ■

Proof of Lemma 1. Consider $Q^{\mathcal{B}}$ and an agent playing strategy $s^{\mathcal{B}}$.

$b_i > b_{-i}:$ Let p_i denote that agent i observes accurately. Note that $p_i > \frac{1}{2}, \forall i \in I$.

Let the payoff from meeting an opponent such that $b_i > b_{-i}$ be denoted $\pi_i\left(s^{\mathcal{B}}, b_i \mid b_i > b_{-i}\right) \equiv \pi^H_{i,-i}$.

Hence: $\pi^H_{i,-i} = p_i p_{-i} + \left(1 - p_i\right) p_{-i}\alpha + \left(1 - p_i\right)\left(1 - p_{-i}\right)\beta$.

We have that $\frac{\partial \pi^H_{i,-i}}{\partial p_i} = \left(1 - \alpha + \beta\right) p_{-i} - \beta$ where

$\frac{\partial \pi^H_{i,-i}}{\partial p_i} > 0 \Leftrightarrow p_{-i} > \frac{\beta}{1-\alpha+\beta}$. Since $p_{-i} > \frac{1}{2}$ it follows $1-\alpha > \beta \Rightarrow \frac{\partial \pi^H_{i,-i}}{\partial p_i} > 0$.

$b_i < b_{-i}:$ Analogously, let $\pi_i\left(s^{\mathcal{B}}, b_i \mid b_i < b_{-i}\right) \equiv \pi^L_{i,-i}$.

Hence: $\pi^L_{i,-i} = p_i p_{-i}\beta + p_i\left(1-p_{-i}\right)\alpha + \left(1-p_i\right)\left(1-p_{-i}\right)$.

$\frac{\partial \pi^L_{i,-i}}{\partial p_i} = \left(1-\alpha+\beta\right)p_{-i} + \alpha - 1$ where

$\frac{\partial \pi^L_{i,-i}}{\partial p_i} > 0 \Leftrightarrow p_{-i} > \frac{1-\alpha}{1-\alpha+\beta}$. Since $p_{-i} > \frac{1}{2}$ we have $1-\alpha < \beta \Rightarrow \frac{\partial \pi^L_{i,-i}}{\partial p_i} > 0$.

Hence $\frac{\partial \pi^H_{i,-i}}{\partial p_i} < 0 \Leftrightarrow p_{-i} \in \left[\frac{1}{2}, \frac{\beta}{1-\alpha+\beta}\right)$ and $\frac{\partial \pi^L_{i,-i}}{\partial p_i} < 0 \Leftrightarrow p_{-i} \in \left[\frac{1}{2}, \frac{1-\alpha}{1-\alpha+\beta}\right)$. To analyze $\frac{\partial \pi_i\left(s^{\mathcal{B}},\overline{b}_i\right)}{\partial p_i}$, where $\pi_i\left(s^{\mathcal{B}},\overline{b}_i\right) = \int_{-i\in I\setminus i}\pi_{i,-i}\left(s^{\mathcal{B}},\overline{b}_i \mid \overline{b}_i \lesseqgtr b_{-i}\right)$ it sufficient to consider pairs b_{-i}, b^*_{-i} at equal distance from $\overline{b}$, since b is *symmetrically* distributed around $\overline{b}$. Adding the pairs yields: $\frac{\partial \pi_{i,-i}\left(s^{\mathcal{B}},\overline{b}_i|\overline{b}_i>b_{-i}\right)}{\partial p_i} + \frac{\partial \pi_{i,-i}\left(s^{\mathcal{B}},\overline{b}_i|\overline{b}_i<b^*_{-i}\right)}{\partial p_i} = \left(1-\alpha+\beta\right)\left(2p_{-i}-1\right) > 0$, thus $\frac{\partial \pi_i\left(s^{\mathcal{B}},\overline{b}_i\right)}{\partial O_i} > 0$.

Note that sign $\frac{\partial \pi_i\left(s^{\mathcal{B}},\overline{b}_i\right)}{\partial p_i} =$ sign $\frac{\partial \pi_i\left(s^{\mathcal{B}},\overline{b}_i\right)}{\partial O_i}$. Since b are continuously distributed through f_b, we know that $\exists\delta > 0$, such that:

$1-\alpha < \beta \Rightarrow \frac{\partial \pi_i\left(s^{\mathcal{B}},b_i\right)}{\partial O_i} > 0, \forall b \in \left[\kappa^-, \overline{b}+\delta\right]$, and

$1-\alpha > \beta \Rightarrow \frac{\partial \pi_i\left(s^{\mathcal{B}},b_i\right)}{\partial O_i} > 0, \forall b \in \left[\overline{b}-\delta, \kappa^+\right]$. ■

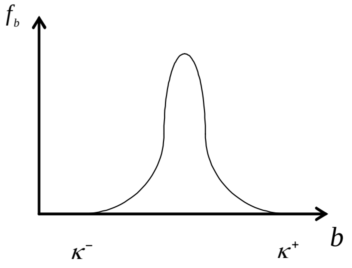
Proof of Proposition 2. From Lemma 1 we know:

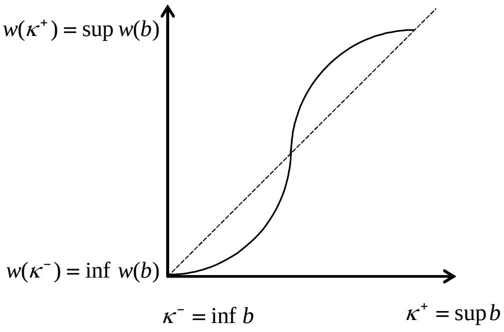
$\frac{\partial \pi_i\left(s^{\mathcal{B}},b\right)}{\partial O_i} > 0, \forall b \in \left[\kappa^-, \overline{b}+\delta\right] : 1-\alpha < \beta$,

$\frac{\partial \pi_i\left(s^{\mathcal{B}},b\right)}{\partial O_i} > 0, \forall b \in \left[\overline{b}-\delta, \kappa^+\right] : 1-\alpha > \beta$.

Hence, for any given α and β, more than half of the population will receive a higher payoff from an increase in observational skills. This implies that the probability for a successful mutation when the observational skills improve is higher than when it deteriorate. Since α and β are drawn from two independent symmetric distributions, it follows that $\Pr\left(1-\alpha < \beta\right) = \Pr\left(1-\alpha > \beta\right)$. Consequently, from symmetry it follows that the observational skills in the population will improve through mutations and growth. Moreover, improvements of observational skills in the population will result in an increase of δ, which in turn will increase the probability of a successful observational skill enhancing mutation and thereby speed up the improvement process of observational skills. ■

Proof of Proposition 3. Consider $Q^{\mathcal{B}}$ where the characteristics b are continuously, symmetrically and unimodally distributed by f_b as qualitatively depicted in the left-hand figure below. From Property 1 we know that each distribution of b has a corresponding distribution of $w(b)$, since $\pi\left(s^{\mathcal{B}}, b\right) = w(b)$. In the right-hand figure below we depict a qualitative graph over how b relates to $w(b)$ when observational skills are close to perfect.

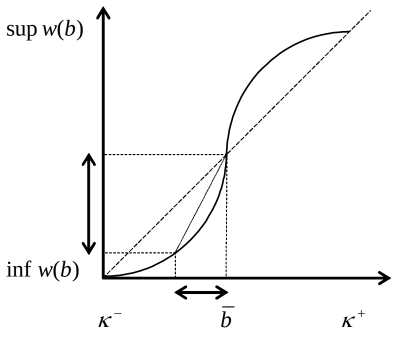


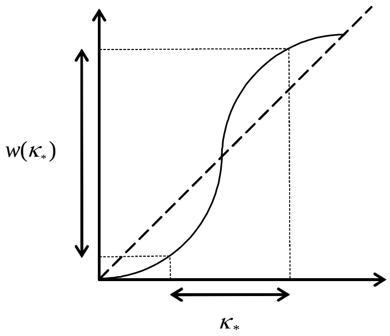


Now consider an agent with characteristic $\bar{b}$.

From the left-hand figure below it is easy to be convinced that $\left|w\left(\bar{b}\right) - w(b)\right| > \left|\bar{b} - b\right|, \forall b \in (\kappa^-, \kappa^+)$, and $\left|w\left(\bar{b}\right) - w(b)\right| = \left|\bar{b} - b\right|, b = \kappa^-$ or κ^+.

Since the payoff is strictly decreasing in the degree of coordination failures, we have that $\pi_{i,-i}\left(s_i^{\mathcal{V}}, w_i\left(\bar{b}\right), w_{-i}(b)\right) > \pi_{i,-i}\left(s_i^{\mathcal{B}}, \bar{b}_i, b_{-i}\right), \forall b_{-i} \in (\kappa^-, \kappa^+)$ and $\pi_{i,-i}\left(s_i^{\mathcal{V}}, w_i\left(\bar{b}\right), w_{-i}(b)\right) = \pi_{i,-i}\left(s_i^{\mathcal{B}}, \bar{b}_i, b_{-i}\right), b_{-i} = \kappa^-$ or κ^+. Hence it follows that $\pi\left(s^{\mathcal{V}}, w\left(\bar{b}\right)\right) > \pi\left(s^{\mathcal{B}}, \bar{b}\right)$.





Since f_b is continuous, we know that $\exists \boldsymbol{\kappa}_* \subseteq \boldsymbol{\kappa}$, such that $\pi\left(s^{\mathcal{V}}, w(b)\right) > \pi\left(s^{\mathcal{B}}, b\right), \forall b \in \boldsymbol{\kappa}_*$. ■

Proof of Lemma 2. Let us consider $Q^{\mathcal{B}}$ and $\Gamma(\alpha,\beta)$ when $\alpha > \beta$.

Imagine an agent mutate from $s^{\mathcal{B}}$ and $s^{\mathcal{V}}$. Regardless of where in $\boldsymbol{\kappa}$ the mutation takes place, the mutating agent will eventually have offsprings dispersed over the entire $\boldsymbol{\kappa}$.

Focus on $b \in \boldsymbol{\kappa}_{**}$. From Proposition 3 we know $\pi\left(s^{\mathcal{V}}, w(b)\right) > \pi\left(s^{\mathcal{B}}, b\right)$, and from Definition 4 it follows that $g\left(s^{\mathcal{V}}, w(b)\right) \geq g\left(s^{\mathcal{B}}, b\right)$.

If $\pi\left(s^{\mathcal{V}}, w(b)\right) > \pi\left(s^{\mathcal{B}}, b\right)$ then status of $s^{\mathcal{V}}$ will become higher than $s^{\mathcal{B}}$, denoted $w\left(b : s^{\mathcal{V}}\right) > w\left(b : s^{\mathcal{B}}\right)$. This in turn will lead to increased coordination failures between $s^{\mathcal{V}}$ and $s^{\mathcal{B}}$ in cases where $b \geq b'$ but $w\left(b' : s^{\mathcal{V}}\right) > w\left(b : s^{\mathcal{B}}\right)$, since both $s^{\mathcal{V}}$ and $s^{\mathcal{B}}$ will have a high probability to choose action *H*. Initially when $Q_{w(\boldsymbol{\kappa}_{**}):\mathcal{V}} << Q_{w(\boldsymbol{\kappa}_{**}):\mathcal{B}}$, these coordination failures will strike harder on $s^{\mathcal{V}}$ than $s^{\mathcal{B}}$ such that the decrease in payoff/status of $s^{\mathcal{V}}$ becomes larger than of $s^{\mathcal{B}}$. However, $w\left(b : s^{\mathcal{V}}\right) = w\left(b : s^{\mathcal{B}}\right) \Rightarrow \pi\left(s^{\mathcal{V}}, w(b)\right) > \pi\left(s^{\mathcal{B}}, b\right), \forall b \in \boldsymbol{\kappa}_{**}$ and since f_b is continuous, we have $w\left(b : s^{\mathcal{V}}\right) > w\left(b : s^{\mathcal{B}}\right)$ and $\pi\left(s^{\mathcal{V}}, w(b)\right) > \pi\left(s^{\mathcal{B}}, b\right), \forall b \in \boldsymbol{\kappa}_{**}$.

As $Q_{w(\boldsymbol{\kappa}_{**}):\mathcal{V}}$ grow, coordination failures will strike $s^{\mathcal{B}}$ in $\boldsymbol{\kappa}_{**}$ harder. When the fractions of $s^{\mathcal{V}}$ after a finite periods becomes larger than $s^{\mathcal{B}}$ at some $b \in \boldsymbol{\kappa}_{**}$, the coordination failures will result in a larger decrease in payoff/status for $s^{\mathcal{B}}$ than for $s^{\mathcal{V}}$. The decrease in status for $s^{\mathcal{B}}$ will subsequently lead to more coordination failures, which in turn entails an even lower status. This self-enforcing process will result in that almost all $s^{\mathcal{B}}$ with $b \in \boldsymbol{\kappa}_{**}$ will be pushed down and out of $w(\boldsymbol{\kappa}_{**})$.

Note that agents playing $s^{\mathcal{B}}$ and endowed with $b' \approx \inf \boldsymbol{\kappa}_{**}$ will have an increase in status. As agents playing $s^{\mathcal{B}}$ with $b \in \boldsymbol{\kappa}_{**}$ due to coordination failures with $s^{\mathcal{V}}$ will be pushed down, agents playing $s^{\mathcal{V}}$ with $b'' < b$, but with $w(b'') > w(b)$ will fail to coordinate with these agents and thus have a decrease in status. In comparison, agents playing $s^{\mathcal{B}}$ with b' will clearly have a much higher success rate in coordinating with both $s^{\mathcal{B}}$ with b'' and thus yield a higher status.

It is easy to see that the magnitude of the increase in status for agents playing $s^{\mathcal{B}}$ with b' is dependent on how α and β relates to 1. Consequently, if $\alpha \gtrsim \beta$ and $1-\alpha$ is sufficiently large, then the increase in payoff/status for agents playing $s^{\mathcal{B}}$ with b' will be limited such that $\exists \boldsymbol{\kappa}_{***} \neq \emptyset$ where $\boldsymbol{\kappa}_{***} \subset \boldsymbol{\kappa}_{**}$ which result in that $Q_{w(\boldsymbol{\kappa}_{***}):\mathcal{V}} = 1$. $\blacksquare$

Proof of Proposition 4. Consider the case when the evolutionary pressure imposes a sufficiently low growth in $\boldsymbol{\kappa}_{-}$ and $\boldsymbol{\kappa}_{+}$ relatively $\boldsymbol{\kappa}_{**}$, e.g. $g(\boldsymbol{\kappa}_{-}) << g(\boldsymbol{\kappa}_{**}) \lessapprox g(\boldsymbol{\kappa}_{+})$.
Let $\alpha > \beta$.

> Consider $Q_{w(\boldsymbol{\kappa}_{+})}$. We know that $\pi\left(s^{\mathcal{B}}, b\right) > \pi\left(s^{\mathcal{V}}, w(b)\right), \forall b \in \boldsymbol{\kappa}_{+}$. But since the agents are dispersed through mean regression, we know that the grow in $\boldsymbol{\kappa}_{+}$ stems from the entire $\boldsymbol{\kappa}$ and not just from $\boldsymbol{\kappa}_{+}$. From Property 3 we know that and all agents endowed with $b \in \boldsymbol{\kappa}_{+}$ and playing $s^{\mathcal{V}}$ will converge to $w(\boldsymbol{\kappa}_{+})$. From Property 4 we know that the growth of $s^{\mathcal{B}}$ outside $\boldsymbol{\kappa}_{+}$ will decrease, since agents endowed with $b \in \boldsymbol{\kappa}_{***}$ will converge to $w(\boldsymbol{\kappa}_{-})$. We also know that not all $\left(s^{\mathcal{B}}, \boldsymbol{\kappa}_{+} : w(\boldsymbol{\kappa}_{-})\right)$ will converge to $w(\boldsymbol{\kappa}_{+})$. That is, the growth in $\boldsymbol{\kappa}_{+}$ stemming from $\boldsymbol{\kappa} \setminus \boldsymbol{\kappa}_{+}$ will decrease for $s^{\mathcal{B}}$ and remain unchanged for $s^{\mathcal{V}}$.
>
> Note that $O \to \infty \Rightarrow \pi\left(s^{\mathcal{V}}, w(b)\right) \to \pi\left(s^{\mathcal{B}}, b\right), \forall b \in \boldsymbol{\kappa}$.
>
> Consequently, $\exists O < \infty$ where $s^{\mathcal{V}}$ grow faster than $s^{\mathcal{B}}$ in $\boldsymbol{\kappa}_{+}$, such that $Q_{w(\boldsymbol{\kappa}_{+}):\mathcal{V}}$ will increase (and $Q_{w(\boldsymbol{\kappa}_{+}):\mathcal{B}}$ decrease), which in a finite sequence of games will result in that the fraction playing $s^{\mathcal{V}}$ in $w(\boldsymbol{\kappa}_{+})$ becomes larger then fraction playing $s^{\mathcal{B}}$ in $w(\boldsymbol{\kappa}_{+})$. Note that we still have that $\pi\left(s^{\mathcal{B}}, b\right) > \pi\left(s^{\mathcal{V}}, w(b)\right)$, but $g\left(s^{\mathcal{B}}, b\right) < g\left(s^{\mathcal{V}}, w(b)\right), , \forall b \in \boldsymbol{\kappa}_{+}$.

Let $\alpha < \beta$.

> Let $b < b' < b''$, and $w(b) < w(b')$. First consider agents $\left(s^{\mathcal{B}}, b', w(b')\right)$ and $\left(s^{\mathcal{V}}, b', w(b')\right)$, i.e. agents endowed with b' at status level $w(b')$ and

playing $s^{\mathcal{B}}$ and $s^{\mathcal{V}}$. Now consider agents $\left(s^{\mathcal{V}}, b, w\left(b\right)\right)$ and $\left(s^{\mathcal{V}}, b'', w\left(b\right)\right)$, where $\left(s^{\mathcal{V}}, b'', w\left(b\right)\right)$ can be considered as a new born endowed with b'' but born at $w\left(b\right)$ and playing $s^{\mathcal{V}}$.

$\left(s^{\mathcal{B}}, b', w\left(b'\right)\right)$ will play H against $\left(s^{\mathcal{V}}, b, w\left(b\right)\right)$ and D against $\left(s^{\mathcal{V}}, b'', w\left(b\right)\right)$.

$\left(s^{\mathcal{V}}, b', w\left(b'\right)\right)$ will play H against both $\left(s^{\mathcal{V}}, b, w\left(b\right)\right)$ and $\left(s^{\mathcal{V}}, b'', w\left(b\right)\right)$.

Since both $\left(s^{\mathcal{V}}, b, w\left(b\right)\right)$ and $\left(s^{\mathcal{V}}, b'', w\left(b\right)\right)$ will play D against $\left(s^{\mathcal{B}}, b', w\left(b'\right)\right)$ and $\left(s^{\mathcal{V}}, b', w\left(b'\right)\right)$, this results in that $\pi\left(s^{\mathcal{V}}, b'', w\left(b\right)\right) < \pi\left(s^{\mathcal{V}}, b, w\left(b\right)\right)$ and that $\left(s^{\mathcal{V}}, b', w\left(b'\right)\right)$ receive a higher payoff against $\left(s^{\mathcal{V}}, b'', w\left(b\right)\right)$.

Consequently, if $\alpha < \beta$ then $\left(s^{\mathcal{V}}, \boldsymbol{\kappa}_{+} : w\left(\mathbf{x}\right)\right) \rightsquigarrow w\left(\boldsymbol{\kappa}_{-}\right), \forall \mathbf{x} \in \left\{\boldsymbol{\kappa}_{-}, \boldsymbol{\kappa}_{***}\right\}$ and thereby fail to coordinate with $\left(s^{\mathcal{B}}, \boldsymbol{\kappa}_{+} : w\left(\boldsymbol{\kappa}_{+}\right)\right)$ which results in a payoff/status decrease for both $\left(s^{\mathcal{B}}, \boldsymbol{\kappa}_{+} : w\left(\boldsymbol{\kappa}_{+}\right)\right)$ and $\left(s^{\mathcal{V}}, \boldsymbol{\kappa}_{+} : w\left(\mathbf{x}\right)\right), \forall \mathbf{x} \in \left\{\boldsymbol{\kappa}_{-}, \boldsymbol{\kappa}_{***}\right\}$. $\left(s^{\mathcal{V}}, \boldsymbol{\kappa}_{+} : w\left(\boldsymbol{\kappa}_{+}\right)\right)$ will just as successful in coordination with $\left(s^{\mathcal{V}}, \boldsymbol{\kappa}_{+} : w\left(\mathbf{x}\right)\right), \forall \mathbf{x} \in \left\{\boldsymbol{\kappa}_{-}, \boldsymbol{\kappa}_{***}\right\}$ as before.

We know that $O \to \infty \Rightarrow \pi\left(s^{\mathcal{V}}, w\left(b\right)\right) \to \pi\left(s^{\mathcal{B}}, b\right), \forall b \in \boldsymbol{\kappa}$. Hence $\exists O < \infty$ such that $\pi\left(s^{\mathcal{B}}, b\right) < \pi\left(s^{\mathcal{V}}, w\left(b\right)\right), \forall b \in \boldsymbol{\kappa}_{+}$. Just as in Lemma 2 will this result in coordination failures between $s^{\mathcal{B}}$ and $s^{\mathcal{V}}$ with same b. Given the fractions of $s^{\mathcal{V}}$ is larger than $s^{\mathcal{B}}$ at every $b \in \boldsymbol{\kappa}_{+}$, the coordination failures will result in a larger decrease in payoff/status for $s^{\mathcal{B}}$ than for $s^{\mathcal{V}}$. This will decrease the status for $s^{\mathcal{B}}$ and subsequently lead to more coordination failures, which will result in that until all $s^{\mathcal{B}}$ with $b \in \boldsymbol{\kappa}_{+}$ first will be pushed down and out of $w\left(\boldsymbol{\kappa}_{+}\right)$ and then $w\left(\boldsymbol{\kappa}_{***}\right)$ into $w\left(\boldsymbol{\kappa}_{-}\right)$.

When both $w\left(\boldsymbol{\kappa}_{***}\right)$ and $w\left(\boldsymbol{\kappa}_{+}\right)$ exclusively consist of agents coordinating their actions through $s^{\mathcal{V}}$, the lower growth rate in $w\left(\boldsymbol{\kappa}_{-}\right)$ will eventually make $s^{\mathcal{B}}$ extinct. ■

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